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The Time-Dependent Reduction of Sliding Cohesion due to Rock Bridges Along Discontinuities: A Fracture Mechanics Approach

By

J. Kemeny

Department of Mining and Geological Engineering, University of Arizona, Tucson, U.S.A.

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Summary

In this paper, a fracture mechanics model is developed to illustrate the importance of time-dependence for brittle fractured rock. In particular a model is developed for the time-dependent degradation of rock joint cohesion. Degradation of joint cohesion is modeled as the time-dependent breaking of intact patches or rock bridges along the joint surface. A fracture mechanics model is developed utilizing subcritical crack growth, which results in a closed-form solution for joint cohesion as a function of time. As an example, a rock block containing rock bridges subjected to plane sliding is analyzed. The cohesion is found to continually decrease, at first slowly and then more rapidly. At a particular value of time the cohesion reduces to value that results in slope instability. A second example is given where variations in some of the material parameters are assumed. A probabilistic slope analysis is conducted, and the probability of failure as a function of time is predicted. The probability of failure is found to increase with time, from an initial value of 5% to a value at 100 years of over 40%. These examples show the importance of being able to predict the time-dependent behavior of a rock mass containing discontinuities, even for relatively short-term rock structures.

Keywords: Rock fracture mechanics, time dependent, subcritical crack growth, slope stability, rock joint, rock joint, rock fracture, joint cohesion, joint friction angle, rock bridge.

1. Introduction

Many of the most important issues in rock mechanics are concerned with the presence of fractures in rocks, and the opening, closing, growth or sliding of these fractures when subjected to forces and displacements. At first glance, time dependence may not appear to fit in this category. For instance, it is well known that steady-state creep in salt rocks is controlled by ductile rather than fracturing mechanisms. However, in brittle rocks, studies have shown a strong connection between time-dependent behavior and fracture phenomena. For instance, creep tests conducted on brittle rocks consistently show an association with both acoustic emissions and decreasing P-wave velocities, both strong indicators of crack growth and/or crack sliding e.g. (Sano et al., 1982). Fracture-mechanics models utilizing time-dependent microcrack growth also support the importance of fracture phenomena in the creep of brittle rocks. For instance, utilizing subcritical crack growth, Kemeny (1991) derived a closed-form solution for transient creep strain that is proportional to the logarithm of time, precisely the form predicted by many empirically-based models (Jaeger and Cook, 1979). Tertiary creep or creep rupture is also predicted in the models presented in Kemeny (1991), due to the interaction and coalescence of cracks.

The time-dependent properties of brittle rocks are often ignored in the design of rock structures, due in part to the small additional strains that occur in the life of these structures. However, it is necessary to consider the time-dependent properties of brittle rocks when designing excavations that must remain open for long periods of time, such as for the long-term storage of high-level nuclear waste (Kemeny and Cook, 1991; Kicker et al., 2000). Also, by understanding the origin of time-dependent behavior in brittle rocks, models can be developed to determine those situations where time-dependence must be considered even for the short-term life of rock structures. It is shown in this paper, for instance, that when time-dependence is considered for relatively short-term structures, the probability of failure over the life of the structure can increase dramatically compared with a time-independent design.

It is reasonable to assume that the time-dependent behavior of rock masses will be controlled by the time-dependent opening or sliding that occurs along discontinuities. Large-scale discontinuities include faults, joints, bedding planes, and other types of fractures. As an example, imagine a wedge of rock in the roof or wall of an underground opening formed by the intersection of joints. In many instances, the kinetic and kinematic requirements for unstable movement of the block are met, but the block does not fall immediately after the excavation is made. The block may fall after some period of time, or the block may remain stable for the life of the underground opening. What are the physical mechanisms responsible for the time-dependent stability of this rock wedge? In many cases, the time-dependent stability of the block is controlled by intact or unbroken segments along the joint surface, referred to as rock bridges and illustrated in Figure 1. These intact rock segments provide a joint cohesion (in shear and tension). Depending on the applied stresses, the joint cohesion provided by the intact segments may deteriorate with time, resulting in time-dependent discontinuity behavior. By understanding the mechanics of this process, the time-dependence of rock fall or rock sliding problems can be predicted, which can improve the safety of underground or surface excavations. At the present time, only empirical approaches are available to predict the time-dependence of rock fall and rock sliding events (Kaiser et al., 1992; Hedley, 1992; Mojtabai and Beattie, 1996).

This paper focuses on the time-dependent behavior of discontinuities, and in particular time-dependent discontinuity cohesion. The assumption is made in this paper that a rock bridge along a discontinuity can be modeled as the "patch" between two coplanar fractures, and that the time-dependent shearing of the patch under load is responsible for time-dependent deformation and sliding along discontinuities. It is assumed that the time-dependent shearing of the rock bridges is due to subcritical crack growth (Atkinson, 1984; Atkinson, 1987). Only shear failure of the rock bridges is considered at this time, even though tensile failure could be added in a straightforward manner. As an example, a rock block containing rock bridges subjected to plane sliding is analyzed. Due to the progressive shearing through the rock bridges, the cohesion is found to continually decrease, at first slowly and then more rapidly. Eventually the cohesion reduces to a value that results in slope instability. A second example is given where a variation in some of the material parameters is assumed. A probabilistic slope analysis is conducted, and the probability of failure as a function of time is predicted. The probability of failure is found to increase with time, from an initial value of 5% to a value over 40% at 100 years. These examples show the importance of being able to predict the time-dependent behavior of a rock mass containing discontinuities, even for relatively short-term rock structures.

The outline of this paper is as follows. The fracture-mechanics model for time-dependent discontinuity cohesion is presented in Section 2. Simple examples using this model are presented in Section 3. A discussion of the results, limitations of the model, and some other issues are presented in Section 4. Finally, conclusions are presented in Section 5.

2. Fracture Mechanics Model

A simple fracture mechanics model for the rock bridge is shown in Fig. 1. The rock bridge itself is modeled as a patch of intact material between two coplanar cracks. The boundary conditions consist of a rock bridge of width $2a$ contained in a body of width $2w$ under a far-field shear stress τ and normal stress σ_n. The Mode II stress intensity factor for this problem is given by (Rooke and Cartwright, 1976,

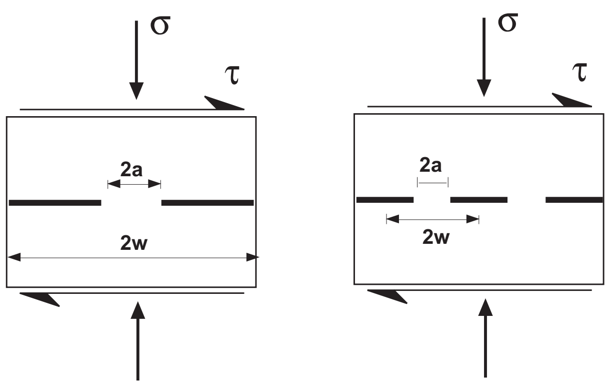

Fig. 1. Fracture mechanics models, **a)** single rock bridge under far field normal and shear stresses, **b)** multiple rock bridges under far field normal and shear stresses

assumes $a \ll w$):

$$K_{\mathrm{II}} = \frac{\tau 2w}{\sqrt{\pi a}}. \tag{1}$$

This solution assumes no friction along the fracture surfaces. If there is friction along the fracture surfaces, then utilizing the principle of superposition, the shear stress τ in Eq. (1) can be substituted by the effective shear stress above friction, $\tau - \sigma_n \tan\phi$, to give:

$$K_{\mathrm{II}} = \frac{(\tau - \sigma_n \tan\phi)2w}{\sqrt{\pi a}}, \tag{2}$$

where ϕ is the friction angle for the joint surfaces on either side of the rock bridge. Also, since this is an asymptotic solution for small a/w, it is equally valid for a discontinuity that contains not one but a number of rock bridges of width $2a$, where the bridge to bridge spacing is equal to $2w$, as shown in Fig. 1b.

It is assumed that shear crack growth will occur when the mode II stress intensity factor, K_{II} reaches the critical fracture toughness under mode II loading, $K_{\mathrm{II}C}$. The effect of shear crack growth is to decrease the size of the rock bridge. Also, it is assumed that crack growth will occur in the plane of the cracks, which is the direction of maximum shear stress around the crack tips. Experiments have shown that rock bridges in shear will ultimately fail in the maximum shear direction, even though initial crack growth may be in the direction of maximum hoop tension (Bobet and Einstein, 1998; Rao et al., 1999). Experimentally determined values of $K_{\mathrm{II}C}$ take into account the energy for both the initial extensile microcrack growth and subsequent shear failure. Rearranging Eq. (2) gives:

$$\tau = \frac{K_{\mathrm{II}C}\sqrt{\pi a}}{2w} + \sigma_n \tan\phi. \tag{3}$$

Equation (3) is the failure criterion for the discontinuity, and is made up of two terms, a cohesion term and a frictional term. The first term on the right hand side of the equation is the joint cohesion due to the rock bridge:

$$C_0 = \text{cohesion} = \frac{K_{\mathrm{II}C}\sqrt{\pi a}}{2w}. \tag{4}$$

Equation (4) shows that the cohesion depends on the mode II fracture toughness, the size of the rock bridge, and the size of the two cracks on both sides of the rock bridge. As an example, using the parameters $K_{\mathrm{II}C} = 0.5$ MPa $\sqrt{\text{m}}$, $w = 0.5$ m, and $a = 0.0127$ m, Eq. (4) gives a cohesion of 0.1 MPa. This is the initial cohesion immediately after a load is applied. With time and due to the applied loads, the size of the rock bridge will decrease due to subcritical crack growth, causing the cohesion to decrease with time as given by Eq. (4).

A formulation for the change in the rock bridge size with time and applied loads is developed using subcritical crack growth. Tests on rock fractures subjected to quasi-static loading or creep indicate a power-law dependence of crack velocity on the stress intensity factor. One popular formulation is the Charles power law (Charles, 1958). The rock bridge problem in terms of Charles power law is given by:

$$a'(t) = A\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{IC}}}\right)^n, \tag{5}$$

where $a'(t)$ is the time-dependent reduction in the rock bridge width and A and n are material constants. Even though the Charles formulation is mainly used in mode I problems, its use for important mode II problems (including earthquake rupture) is discussed in Atkinson (1984) and Kemeny (1993). Using Eq. (2), Eq. (5) can be rewritten as:

$$a'(t) = A\left[\frac{2w(\tau - \sigma_n \tan\phi)}{K_{\mathrm{IIC}}\sqrt{\pi a(t)}}\right]^n. \tag{6}$$

Solving Eq. (6) with the initial condition $a = a_0$ when $t = 0$ gives:

$$a = \left[a_o^{1+n/2} - \left(1 + \frac{n}{2}\right)At\left[\frac{2w(\tau - \sigma_n \tan\phi)}{K_{\mathrm{IIC}}\sqrt{\pi}}\right]^n\right]^{1/(1+n/2)}, \tag{7}$$

where t is time, in seconds. Plugging this result into Eq. (4) gives for the time-dependent cohesion:

$$C_0 = \frac{\sqrt{\pi}\left[a_o^{1+n/2} - \left(1 + \frac{n}{2}\right)At\left[\frac{2w(\tau - \sigma_n \tan\phi)}{K_{\mathrm{IIC}}\sqrt{\pi}}\right]^n\right]^{1/(2+n)}}{2w}. \tag{8}$$

The usefulness of Eq. (8) will be demonstrated in the examples shown below.

3. Plane Sliding Examples

As a demonstration of the usefulness of Eq. (8), consider the simple plane-sliding example shown in Fig. 2. A block of weight W rests on a discontinuity with slope angle θ and surface area A_s. For this simple slope stability problem, the shear strength of the discontinuity is given by:

$$\tau = C_0 + \sigma_n \tan\phi, \tag{9}$$

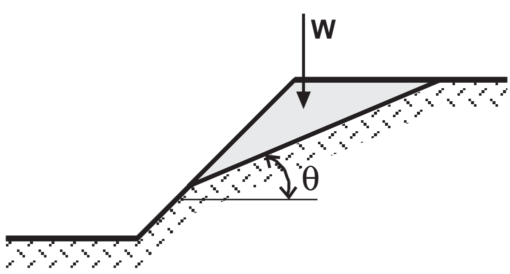

Fig. 2. Rock block of weight W resting on a slope with angle θ, subjected only to gravitational forces

Table 1. Material and slope parameters for the first example

Discontinuity slope angle, θ	35
Discontinuity friction angle, ϕ	25
Block weight, W	25 MN
Discontinuity surface area	100 m^2
Subcritical parameter A	10^{-5} m/s
Subcritical parameter n	25
Shear fracture toughness, K_{IIC}	0.5 MPa $\sqrt{m}$
Initial rock bridge half-width, a_0	0.0127 m
Rock bridge spacing, $2w$	1 m

and the factor of safety is given by:

$$FS = \frac{C_0 + \dfrac{W}{A_s}\cos\theta\tan\phi}{\dfrac{W}{A_s}\sin\theta}. \tag{10}$$

In both of these equations, by replacing the static cohesion C_0 by the time-dependent cohesion given in Eq. (8), time-dependent slope stability can be analyzed. Notice that if the slope angle θ is greater than the discontinuity friction angle ϕ, stability depends on the cohesion and unstable sliding will occur once the cohesion decreases to a critical value (when the factor of safety equals unity).

Consider first an example using the properties shown in Table 1. In this case the friction angle is less than the slope angle, and slope failure is expected to occur as the cohesion decreases with time due to subcritical crack growth. The subcritical crack growth parameters are taken from a thermo-mechanical study of underground excavations in tuff at Yucca Mountain (EPRI, 1996). The rock bridge size and spacing were picked to give an initial discontinuity cohesion of 0.1 MPa, which is reasonable for joints in tuff at Yucca Mountain (Kicker et al., 2000).

Figure 3a shows the cohesion as a function of time. In the first 250 years, the cohesion slowly decreases from its initial value of 0.1 MPa to a value of about 0.095 MPa. After 250 years the decrease is much more rapid. In order for the factor of safety to reach 1, the cohesion must decrease to a value of 0.048 MPa. Figure 3b shows the safety factor as a function of time. The safety factor starts at a value of about 1.36 and reaches a value of 1 at a time of 264.81 years in this particular example.

As shown in Fig. 3, for a given set of parameters the subcritical crack growth model predicts slope failure at a very specific value of time. This failure time will of course vary with changes in the material or slope parameters. Since material and slope parameters are not known exactly and will vary spatially, a probabilistic slope analysis can be used to determine the variation in failure times for a particular problem. At any given time, a histogram of factors of safety can be determined, and the probability of failure for that time is the percent of safety factors less than one.

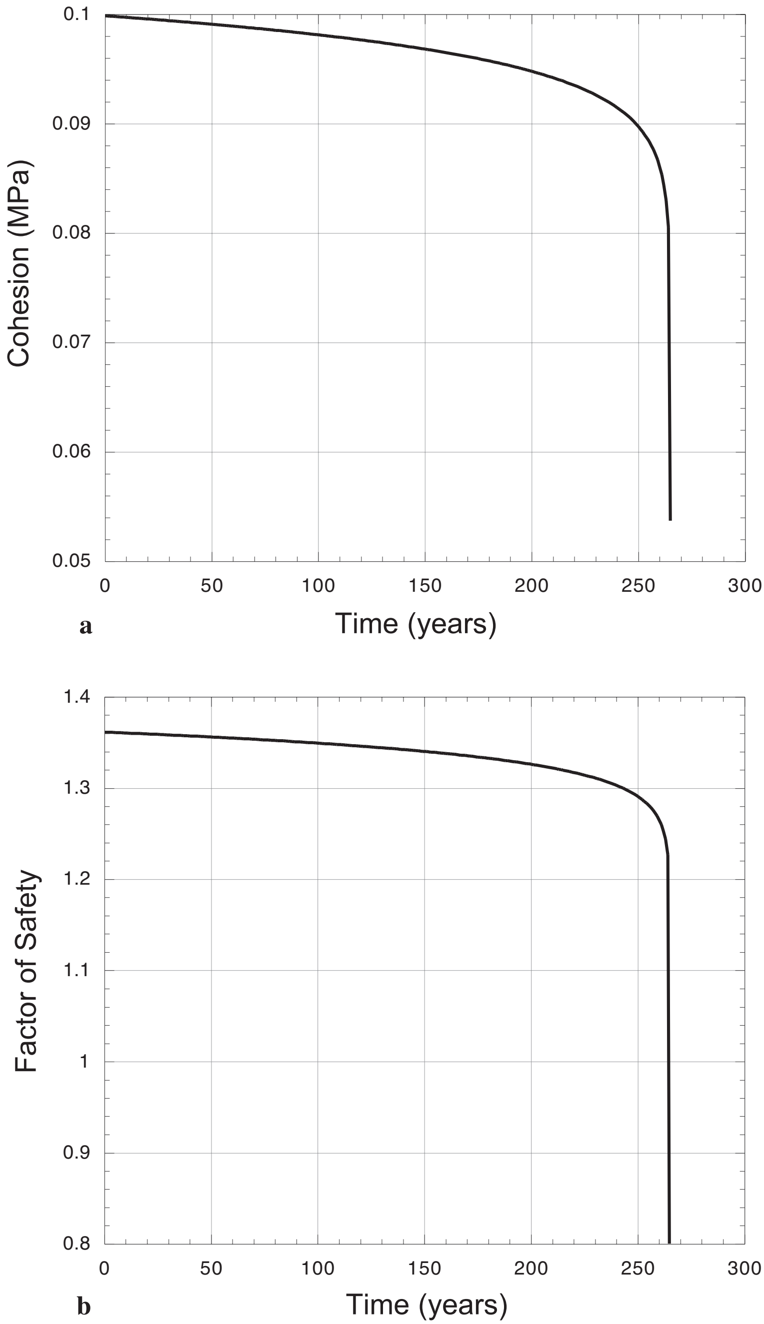

Fig. 3. Cohesion and safety factor as a function of time using the parameters given in Table 1

Table 2. Material and slope parameters for the second example

Discontinuity slope angle, θ	35
Mean discontinuity friction angle, ϕ	25
S.D. of friction angle	7
Block weight, W	25 MN
Discontinuity surface area	100 m^2
Subcritical parameter A	10^{-5} m/s
Subcritical parameter n	25
Shear fracture toughness, K_{IIC}	0.5 MPa $\sqrt{m}$
Mean initial rock bridge half-width, a_0	0.0127 m
S.D. of initial rock bridge half-width	0.0011 m
Rock bridge spacing, $2w$	1 m

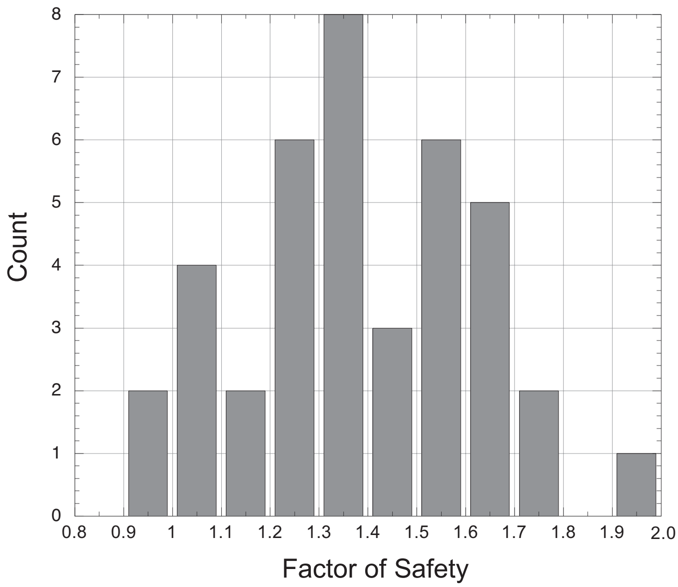

Fig. 4. Histogram of safety factors at $t = 0$ using the parameters given in Table 2. The percent of cases less than the 1 is the probability of failure

Consider a second example using the properties shown in Table 2. The mean values for all the parameters are the same as those used in example 1. Now, however, variations are made in two of the parameters, the initial rock bridge width and the discontinuity friction angle. Standard deviations for both of these parameters are listed in Table 2, and a normal distribution is assumed for both of these parameters. A Monte Carlo simulation with 40 trials is used to generate a histogram of factors of safety. The initial histogram of factors of safety at $t = 0$ for this example is shown in Fig. 4. A mean factor of safety of about 1.4 is predicted, which agrees with the results of the static example given above. Figure 4 shows that the factors of safety at $t = 0$ vary from 0.9 to 2, with a probability of failure of

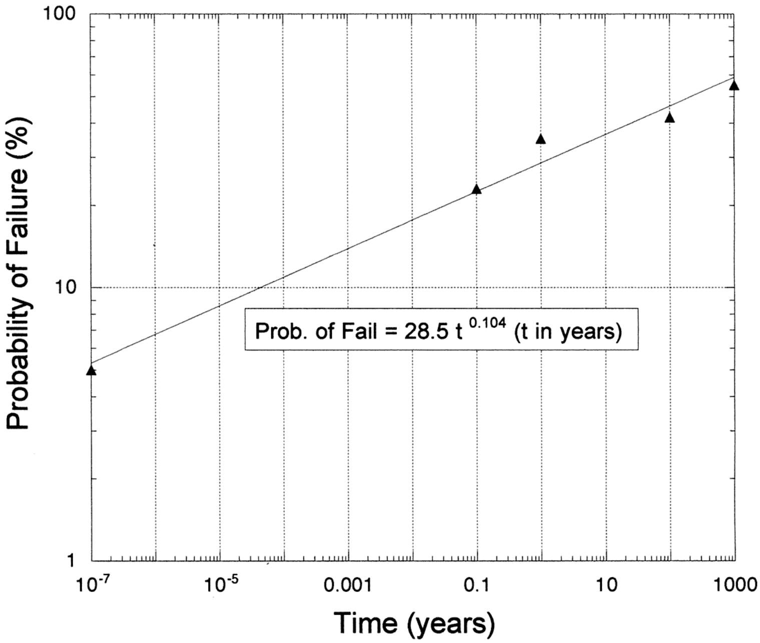

Fig. 5. Probability of failure vs. time using the parameters given in Table 2

5%. This represents a fairly stable slope, and the time-independent analysis would stop here.

With time the factors of safety decrease and, correspondingly, the probabilities of failure increase with time. At a number of time steps, histograms of factor of safety were calculated along with a new probability of failure for each time step. Figure 5 presents a plot of probability of failure vs. time. This figure shows several interesting things. First of all, it shows that the probability of failure increases from a static value (static value of t set to $t = 1$ sec) of 5% to a value over 42% at 100 years and to a value of 55% at 1000 years. This alone is an important result and shows how a seemingly stable slope may become unstable with time. The second interesting aspect of Fig. 5 is the apparent linear trend on the log probability of failure vs. log time plot. This indicates a relationship of the form:

$$\text{Probability of Failure} = Bt^{\alpha} \tag{8}$$

where B and α are constants. For the results shown in Fig. 5, $\alpha = 0.104$ and $B = 28.5$ (with t measured in years). Further analysis is required to see if this relationship holds for other cases.

4. Discussion

In this paper, a simple model was developed to illustrate the importance of time-dependence for brittle fractured rock. The results indicated that this is important even for the design of relatively short-term structures. This has important implications for auto and railroad tunnels, for rock slopes, and for a host of civil and mining underground excavations. Most of these structures are required to remain stable for at least 100 years.

At the present time the time-dependence of these structures is either ignored or analyzed using empirical methods. Even if sophisticated numerical models are used, the time-dependent properties are ambiguous and difficult to determine. The hope is to develop a more mechanistic basis for time-dependence in brittle rocks, and in particular for time-dependent joint behavior. This behavior could then easily be implemented in finite element or discrete element numerical models. The model developed in this paper, though simple, was able to predict changes in the probability of failure with time for a simple slope problem. In particular, for a seemingly stable slope with a static probability of failure of 5%, the model gave a probability of failure over 30% at 10 years and over 40% at 100 years. Additional case studies need to be conducted, including actual field case studies of importance to the civil and mining industries. Time-dependent laboratory tests on rock fractures should also be conducted to verify the relationships derived in this paper.

Applying the techniques in this paper to field applications involves the determination of certain material parameters. The subcritical crack growth parameters (A and n) can be determined from laboratory tests (Atkinson, 1984), even though this is not a standard rock mechanics laboratory test at the present time. Another approach is to determine the subcritical crack growth parameters from shear tests conducted at different loading rates, since the loading rate effect and the time-dependent properties should be governed by the same parameters (Kemeny, 1991). The other set of parameters that must be determined involve the distribution of unbroken rock bridges along the joint surface. These could be determined with existing field characterization techniques such as information on joint length and spacing (i.e., joint persistence), and possibly developing new techniques that focus on the bridges themselves. Rock bridge information could also be determined from laboratory shear tests (using Eq. 4), assuming samples could be obtained with intact segments.

The model presented in this paper is very simple and idealized. It does not take into account the roughness of discontinuities, and it assumes that the two fractures on both sides of the rock bridge are coplanar. In reality, the two fractures separating the rock bridge are often offset, and depending on the direction of the offset relative to the shear direction, the bridge can fail in tension or shear. The advantage of assuming coplanar fractures separating the bridge is that a closed-form solution is available. To model a more realistic discontinuity that has roughness and non-coplanar bridges, numerical modeling is required. This would be a natural extension to the work presented in this paper. Because of the simplicity of the model presented in this paper, it is not clear whether the model can be used for actual engineering design. In the near future, the model will be used as a basis for several underground and surface case studies, and the outcome of these case studies will hopefully give some insight into the usefulness of the model for engineering design.

5. Conclusions

In this paper a model was developed for the time-dependent degradation of rock joint cohesion. Degradation of joint cohesion was modeled as the time-dependent breaking of intact patches or rock bridges along the joint surface. A fracture mechanics model was developed utilizing subcritical crack growth, which results in a closed-form solution for joint cohesion as a function of time. As an example, a rock block subjected to plane sliding was analyzed. The cohesion was found to continually decrease, at first slowly and then more rapidly. At a particular value of time the cohesion reduced to value that resulted in slope instability. A second example was given where a variation in some of the material parameters was assumed. A probabilistic slope analysis was conducted, and the probability of failure as a function of time was predicted. The probability of failure was found to increase with time, from an initial value of 5% to a value at 100 years of over 40%. These examples show the importance of being able to predict the time-dependent behavior of a rock mass containing discontinuities, even for relatively short-term rock structures. In the future some actual field case studies using the model will be conducted, to evaluate the usefulness of the simple model for actual engineering design.

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Author's address: Dr. John Kemeny, Department of Mining and Geological Engineering, Building 12, Room 229, University of Arizona, Tucson, AZ 85721, U.S.A. e-mail: kemeny@u.arizona.edu