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A Methodology for Reliability-Based Design of Rock Slopes

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Summary

A reliability-based methodology for the design of rock slopes, that can easily be implemented by the practicing engineers is proposed. The advanced first-order second-moment (AFOSM) method is adopted as the reliability assessment model and its application is illustrated for the case of plane failure. A model is developed within the framework of first-order second-moment approach to analyze the uncertainties underlying the in situ shear strength properties of rock discontinuities. Here, particular emphasis is given on the assessment of uncertainties related to the shear characteristics of clean, unfilled rock discontinuities under low normal stress levels. An extensive literature survey on the shear characteristics of discontinuities is carried out in order to collect data for the quantification of uncertainties. The data extracted from this literature survey are classified and reprocessed so that they can be utilized in the uncertainty analysis model. A user friendly software called ROCKREL is developed to carry out the numerical computations and to make the proposed design format more practical.

Keywords: Rock slope, reliability-based design, discontinuity shear strength, uncertainty analysis, AFOSM method.

1. Introduction

The design of a safe slope or the assessment of the safety of an existing one is among the most complicated and popular problems in rock engineering. The main complication comes from the uncertainties involved in the design parameters. In the conventional design procedures safety is achieved based on factor of safety and the influence of a parameter on the calculated safety factor is investigated through a sensitivity analysis. In fact such sensitivity analyses are necessitated from the recognition of the fact that there are many uncertainties associated with the design parameters, particularly those related to the strength and geometry of the discontinuities.

The utilization of probabilistic methods in rock engineering, however, permits a rational treatment of various sources of uncertainties that significantly influence the safety of a rock slope. Moreover, probabilistic approaches offer a systematic way of treating uncertainties and of quantifying the reliability of a design (Kirsten, 1983). The applications of probabilistic techniques in the evaluation of rock slope stability (i.e. Shuk, 1970; McMahon, 1971; Piteau and Martin, 1977; Kim et al., 1978; Major et al., 1978; Baecher and Einstein, 1978; Marek and Savely, 1978; Glynn and Einstein, 1979; McPhail and Fourie, 1980; Priest and Brown, 1983; Kirsten, 1983; Einstein et al., 1983; Morriss and Stoter, 1983; McCracken, 1983; Rosenbaum and Jarvis, 1985; Bolle et al., 1987; Kulatilake, 1988; Esterhuizen, 1990; Genske and Walz, 1991; Muralha, 1991; Kimmance and Howe, 1991; Nathanail and Rosenbaum, 1991; Sandroni, 1993; Muralha and Trunk, 1993; Trunk, 1993; Düzgün, 1994; Quek and Leung, 1995; Whittlestone et al., 1995; Düzgün et al., 1995) were limited usually to hypothetical case studies and such methods did not gain widespread application for the design of rock slopes in practice. The main reason for this is the lack of sufficient information for the quantification of uncertainties and the lack of acceptable design criteria, like acceptable deterministic safety factors of 1.3 and 1.5. If these two problems can be solved, than the utilization of such methods will find widespread implementation in practice, allowing a systematic analysis of uncertainties and hence resulting into more realistic rock slope designs. Moreover, the uncertainty analysis gives the design engineers an opportunity for estimating the design parameters in a systematic way rather than by heuristic or judgmental decisions.

Regarding the rock slope stability problem, it is quite clear that the shear strength of discontinuities, which is one of the key parameters influencing the safety of a slope, involves a high degree of uncertainty. In the deterministic design formats, the shear strength of discontinuities is assigned a single value based on the laboratory or in situ shear strength test results, supplemented with engineering judgment, generally exercised in a conservative manner to account for uncertainties. In the probabilistic approach, on the other hand, this estimation is performed in a more systematic, rational and consistent way by utilizing all sources of information, including laboratory and in situ measurements, data from similar sites, expert-opinion and using quantitative measures of uncertainty.

In this study, a practical reliability-based design procedure is developed with the aim that the practicing engineers can easily make use of this approach. For this purpose, advanced first-order second-moment (AFOSM) reliability method is adopted as the principal design format and its implementation is carried out for the case of plane failure. Particular emphasis is given to the assessment of uncertainties related to the shear strength characteristics of rock discontinuities. An uncertainty model is developed within the framework of first-order second-moment approach to analyze the uncertainties associated with this parameter. A design criterion for choosing the acceptable probability of failure and reliability index is proposed and the application of the proposed model is illustrated through a real life case study. A software called ROCKREL is developed for carrying out the numerical computations and for the easy usage of the design methodology introduced in this study.

2. Analysis of Uncertainties

In the reliability-based design and safety checking, the first step is the assessment of uncertainties involved in the key parameters influencing safety. In the case of rock slopes, the key parameter is the shear strength of rock discontinuities. This parameter has two components, namely friction angle and cohesion. In this study because of several ambiguities on the existence of cohesion for rough rock discontinuities (Barton, 1976; Hoek and Bray, 1981), the friction angle is accepted as the dominant component contributing to the shear strength. Consequently, throughout the study cohesion is almost always ignored. It is to be noted that the discontinuities considered herein are clean unfilled discontinuities. Moreover, since rock slope failures usually occur at low normal stress levels, the usage of peak friction angle seems to be more realistic in design. Hence during data extraction from literature, only peak friction angle of unfilled rock discontinuities at low normal stress levels is taken into account and the uncertainty analysis is concentrated on the assessment of the variability and errors in peak friction angle.

If the true but unknown peak friction angle of rock discontinuity is denoted by ϕ, and the average peak friction angle as measured in the laboratory without being corrected for discrepancies as $\hat{\phi}$, then the following expression forms the basis for uncertainty analysis and correction for biases:

$$\phi = N_o(N_1, \ldots, N_k)\hat{\phi}. \tag{1}$$

In Eq. 1, errors resulting from the inherent variability of peak friction angle and those stemming from the discrepancies between the laboratory and in situ conditions are modeled by the random correction factors, N_i's. Here, N_o is the correction factor with mean 1.0 and c.o.v., Δ_o, accounting for the error in the mean peak friction angle due to insufficient sampling (limited number of samples). The errors due to discrepancies between the laboratory and in situ conditions are corrected by N_i's, with respective mean, $\overline{N}_i$ and c.o.v., Δ_i. Among the various factors contributing to these discrepancies, only the effects of scale, anisotropy and water saturation are taken into consideration, since sufficient data were only available for these factors. According to the first-order second-moment format and based on Eq. 1, the mean in situ peak friction angle is expressed as follows:

$$\mu_\phi \cong \overline{N}_1\overline{N}_2 \ldots \overline{N}_k\bar{\phi} = \overline{N}_\phi\bar{\phi}. \tag{2}$$

Here, $\bar{\phi}$ is the mean value of $\hat{\phi}$. Assuming complete statistical independence among all variables involved in Eq. 1, the overall c.o.v. of $\hat{\phi}$, Ω_ϕ, is:

$$\Omega_\phi \cong \sqrt{\Delta_o^2 + \sum_{i=1}^{k} \Delta_i^2 + \delta_\phi^2}, \tag{3}$$

where, δ_ϕ is the c.o.v. of $\hat{\phi}$, reflecting the degree of inherent variability. Δ_o represents the uncertainty due to insufficient sampling with sample size of n.

Data to quantify the statistical parameters of the correction factors are usually unavailable or insufficient. However, based on the results reported in literature, the range of correction factors can be determined. By prescribing simple distributions, such as triangular or uniform, over the respective ranges, estimates of N_i and Δ_i can be obtained. The choice of distribution type depends on the engineer's opinion. If it is believed that the expected value of N_i lies closer to the lower limit of the range, lower triangular distribution, LTD, can be selected. On the contrary, if it is closer to the upper limit, upper triangular distribution, UTD, will be the suitable one. If the expected value of N_i lies around the mid-range, symmetric triangular distribution, STD, can be chosen. On the other hand, if any value of N_i is equally likely over the selected range, a uniform (rectangular) distribution, UD, will be more appropriate (Yucemen et al., 1973).

2.1 Inherent Variability

The natural or inherent variability in a rock medium can conveniently be quantified by the coefficient of variation (c.o.v.), δ_ϕ. Consequently, the uncertainty associated with the correction factor N_o, measured by the c.o.v. Δ_o, is to be calculated from the following expression:

$$\Delta_o = \frac{\delta_\phi}{\sqrt{n}} \tag{4}$$

where,

δ_ϕ = c.o.v. of peak friction angle as obtained from laboratory testing
n = Number of tests conducted
Δ_o = c.o.v. of N_o, representing the uncertainty due to limited number of test samples

It is to be noted that Eq. 4 expresses the standard error in the mean in terms of the coefficient of variation and implicitly it assumes that all of the n observations are statistically independent.

The inherent variability of peak friction angle can be determined in two ways depending on the type of samples obtained from the field. If it is possible to extract samples from the discontinuity on which the slope is to be constructed, then these samples reflect the roughness of discontinuities. Accordingly both components of the peak friction angle, namely: resistance due to the material strength reflected by the basic friction angle and resistance due to the roughness of the discontinuity planes, will be encountered in the laboratory. Then the inherent variability δ_ϕ will reflect the effect of both components and Δ_o can easily be calculated from Eq. 4. On the other hand, $\bar{\phi}$ is simply the average of the peak friction angles measured in the laboratory.

In the second case, it may not be possible to acquire samples containing roughness profiles existing in the field or the number of such samples may not be sufficient to run a meaningful statistical analysis. In such a situation, usually tests are performed on planar saw cut surfaces of rock that give only the basic friction angle of the considered discontinuity. In order to estimate the average peak friction angle, $\bar{\phi}$, from the mean value of the basic friction angle, $\bar{\phi}_b$, that is obtained from saw cut surfaces, the correction factor N^* is introduced. The proposed model for $\bar{\phi}$ in this case is as follows:

$$\bar{\phi} = \bar{N}^* \bar{\phi}_b. \tag{5}$$

For the quantification of N^* a comprehensive literature survey is carried out on the effect of roughness on the discontinuity shear behavior. Research results on peak friction angle values (Richard, 1975; Barton and Choubey, 1977; Robertson, 1977; Barla et al., 1985; Nilsen, 1985; Reeves, 1985; Zongqi and Ming, 1990; Sfondrini and Sterlacchini, 1996; Kabeya and Legge, 1996) corresponding to various discontinuity profiles are "filtered" for low normal stress levels. Then, by evaluating the ratio of the filtered ϕ to the derived ϕ_b, the values of N^* are computed from Eq. 5 for various discontinuity surfaces associated with different rock types.

It is necessary to classify the computed N^* values according to a certain roughness measure so that the practicing engineers can select the proper value of N^*. Roughness of the discontinuity surfaces is expressed in terms of many variables. The most widely used parameter is Barton's (1973) joint (discontinuity) roughness coefficient (JRC). The ranges of N^* for different discontinuity profiles are estimated after a comprehensive literature survey and by processing the reported results according to Eq. 5. In Table 1, the proposed ranges of N^* and the corresponding JRC ranges for various discontinuity profiles are listed, together with the statistical parameters of N^* based on different distributional assumptions.

Now Δ_o, which reflects the effect of inherent variability in proportion to the sample size, n, can be calculated from the following relationship, where the uncertainty in the two components are aggregated according to the first-order second-moment model:

$$\Delta_o \cong \sqrt{\Delta^{*2} + \frac{\delta_{\phi_b}^2}{n}}. \tag{6}$$

Here, Δ^* is the c.o.v. of N^* accounting for the errors resulting from the estimation of the peak friction angle based on the basic friction angle, ϕ_b, which is obtained from the measurements conducted on saw cut surfaces. δ_{ϕ_b} is the c.o.v. of the basic friction angle.

2.2 Discrepancies Between Laboratory Measured and In Situ Discontinuity Shear Strength Values

The major factors creating discrepancies between the laboratory measured and the in situ shear strength values, which are considered in this study, are scale, anisotropy and water saturation. For each effect a correction factor, N_i, is introduced. For the assessment of the statistical parameters of a correction factor, a comprehensive literature survey is conducted related to the corresponding effect. Then a range for the correction factor is estimated based on the data compiled in this way. It is to be noted that the correction factors (N^*, N_1 and N_2) are classified according to JRC value. In this respect JRC value of a discontinuity becomes the most

Table 1. Ranges and statistical parameters of the correction factor, N^*, accounting for the roughness of discontinuity surfaces

JRC range	Estimated range of N^*	Assumed distribution	$\bar{N}^*$	Δ^*
0–2	1.03–1.24	UD	1.12	0.053
		STD	1.12	0.038
		UTD	1.17	0.042
		LTD	1.10	0.035
2–4	1.10–1.55	UD	1.33	0.098
		STD	1.33	0.069
		UTD	1.40	0.076
		LTD	1.25	0.085
4–6	1.24–1.77	UD	1.51	0.101
		STD	1.51	0.071
		UTD	1.59	0.078
		LTD	1.42	0.088
6–8	1.16–1.76	UD	1.46	0.119
		STD	1.46	0.084
		UTD	1.56	0.091
		LTD	1.36	0.104
8–10	1.30–1.84	UD	1.57	0.099
		STD	1.57	0.070
		UTD	1.66	0.077
		LTD	1.48	0.086
10–12	1.43–2.13	UD	1.78	0.114
		STD	1.78	0.080
		UTD	1.90	0.087
		LTD	1.66	0.099
12–14	1.53–2.10	UD	1.82	0.091
		STD	1.82	0.064
		UTD	1.91	0.070
		LTD	1.72	0.078
14–16	1.66–2.06	UD	1.86	0.062
		STD	1.86	0.044
		UTD	1.93	0.049
		LTD	1.79	0.053
16–18	1.67–2.08	UD	1.88	0.063
		STD	1.88	0.045
		UTD	1.94	0.050
		LTD	1.81	0.053
18–20	1.66–2.83	UD	2.25	0.150
		STD	2.25	0.106
		UTD	2.44	0.113
		LTD	2.05	0.135

critical parameter in selecting the appropriate correction factor. In the following the literature survey and the results are summarized with respect to these three factors.

The studies encountered in the literature generally indicate a considerable effect of size on the strength of rock discontinuities (Krsmanovic and Popovic, 1966; Locher and Rieder, 1970; Barton, 1973; Pratt el al., 1974; Warcham and Sherwood, 1974; Barton, 1976; Barton and Choubey, 1977; Brown et al., 1977; Leichnitz and Natau, 1979; Krahn and Morgenstern, 1979; Barton and Bandis, 1980; Bandis et al., 1981; Barton and Bandis, 1982; McMahon, 1985; Peres-Rodrigues and Charrua-Graca, 1985; Swan, 1985a,b; Swan and Zongqi, 1985;

Table 2. Ranges and the statistical parameters of the correction factor, N_1, accounting for the scale effect

Discontinuity description	JRC range	Estimated range of N_1	Assumed distribution	$\bar{N}_1$	Δ_1
Slightly rough to almost smooth, slightly undulating	0–4	0.90–0.95	UD	0.93	0.016
			STD	0.93	0.011
			UTD	0.93	0.013
			LTD	0.92	0.013
Moderately undulating, rough	4–8	0.80–0.88	UD	0.84	0.027
			STD	0.84	0.019
			UTD	0.85	0.022
			LTD	0.83	0.023
Undulating, very rough	8–14	0.70–0.79	UD	0.75	0.035
			STD	0.75	0.025
			UTD	0.76	0.028
			LTD	0.73	0.029
Strongly undulating, very rough	14–20	0.60–0.70	UD	0.65	0.044
			STD	0.65	0.031
			UTD	0.67	0.035
			LTD	0.63	0.037

Bandis, 1990; Barton, 1990; Cunha, 1990; Muralha and Cunha, 1990a,b; Pistone, 1990; Sage et al., 1990; Cunha, 1991; Yoshinaka et al., 1991; Al-Harthi and Hencher, 1993; Cunha, 1993; Hencher et al., 1993; Ohnishi et al., 1993; Yoshinaka et al., 1993; Bakhtar and Barton, 1994; Cunha and Muralha, 1995; Giani et al., 1995; Ohnishi and Yoshinaka, 1995; Lumsden and Hencher, 1996; Seidel et al., 1996). Accordingly, in the design of rock slopes the peak friction angle of discontinuities should be corrected for the scale effect. This is achieved by the introduction of a correction factor, denoted by N_1 and defined as the ratio of peak friction angle found from large scale tests (assumed to represent the in situ values) to that obtained from small scale tests (assumed to represent laboratory measured values). The studies listed above have considered discontinuity samples with similar roughness characteristics, but of different sizes and have consistently reported lower shear strength values for in situ or large-scale discontinuities. In computing the values of N_1, firstly the investigated discontinuity types are classified based on the JRC ranges as shown in Table 2. Then the data assessed from the studies mentioned above are grouped according to this classification and the corresponding N_1 values are computed for each case based on the definition of N_1 given above. In Table 2, N_1 values corresponding to various discontinuity surface types and JRC ranges are listed. On the same table the mean and c.o.v. of N_1 are also given. It is observed in Table 2 that for the smoother and planar discontinuities the scale effect is rather insignificant compared to the rougher and undulating discontinuity surfaces. Therefore, the lower values of N_1 are recommended for rougher and more undulating surfaces, while values close to 1.0 are suggested for the smoother and planar discontinuities.

It is to be noted that N_1 is a "macro" correction factor accounting for the scale effect without taking into consideration, explicitly, the contributions from the specific characteristics of joints (such as, nature, matching etc). Here, the classification for the scale effect is done by considering only the surface type and the JRC ranges. However, in future studies, such micro effects can be incorporated by expressing N_1 in terms of component correction factors, accounting for these effects. The proposed model permits such refinements, provided that sufficient information and data become available for the assessment of the required statistical parameters.

Generally, the shear strength of rock discontinuities is estimated by using direct shear, push and pull or tilt tests, without taking into consideration the shearing direction. However, studies on the effect of anisotropy on the shear strength of rock discontinuities (Jaeger, 1960; Donath, 1964; Deklotz and Brown, 1967; Herget, 1970; Donath, 1972; Humston, 1972; Jackson and Dunn, 1974; Lafountain and Dunn, 1974; Vutukuri et al., 1974; Huang and Doong, 1990; Kimura et al., 1993; Kulatilake, 1995; Aydan et al., 1996) have concluded that friction angle varies with the orientation of shearing. Hence, due to the effect of anisotropy any difference between the in situ and laboratory shearing directions will lead to an error. This error will be accounted for by the correction factor N_2. This correction factor is quite important in design situations, where the slope to be constructed has a probable sliding direction that is different from the laboratory shearing direction. In such a situation, the rock discontinuity on which the sliding is expected usually does not exhibit uniform roughness characteristics in different shearing directions. Consequently, the peak friction angle obtained from the laboratory tests that are performed on samples taken from the same discontinuity surface, may not be a good estimate, if the laboratory and in situ shearing directions are different. This difference is due to the fact that, even on the same discontinuity surface the degree of roughness changes in different directions. The best way to estimate the peak friction angle for rock slope designs is to obtain samples from the discontinuity surface in the expected shearing direction and to conduct shear tests along this direction. Nevertheless, obtaining such samples is generally quite difficult and sometimes impossible. As a result, the correction of the friction angle for the directional differences is an essential part of rock slope design.

As mentioned above, the correction factor N_2 is introduced to account for this anisotropy effect and it is attempted to quantify N_2 by extracting the available data from literature. The following procedure is utilized for the determination of N_2 values. Firstly, the discontinuity types investigated in literature (Jaeger, 1960; Donath, 1964; Deklotz and Brown, 1967; Herget, 1970; Donath, 1972; Humston, 1972; Jackson and Dunn, 1974; Lafountain and Dunn, 1974; Vutukuri et al., 1974; Huang and Doong, 1990; Kimura et al., 1993; Kulatilake, 1995; Aydan et al., 1996) are classified according to their roughness degree, selecting the JRC value as the classification measure. Then a reference testing direction is established and depending on the value of peak friction angle obtained from each testing direction, they are grouped as the ones corresponding to direction resisting sliding and direction favoring sliding. Finally, the N_2 values are computed by dividing the peak friction angle measured along the reference testing direction by the ones obtained from the resisting or favoring sliding directions.

In the implementation of this correction factor, the first step is to determine whether there is a difference between the laboratory and in situ shearing direc-

Table 3. Ranges and the statistical parameters of the correction factor, N_2, accounting for the effect of anisotropy for Cases I and II

JRC range	Estimated range of N_2	Assumed distribution	Case I		Case II	
			$\bar{N}_2$	Δ_2	$\bar{N}_2$	Δ_2
0–2	0.89–0.91	UD	0.90	0.006	1.11	0.005
		STD	0.90	0.005	1.11	0.004
		UTD	0.90	0.005	1.11	0.004
		LTD	0.90	0.005	1.11	0.004
2–4	0.88–0.93	UD	0.91	0.016	1.11	0.028
		STD	0.91	0.011	1.11	0.020
		UTD	0.91	0.013	1.12	0.013
		LTD	0.91	0.013	1.11	0.013
4–6	0.85–0.90	UD	0.88	0.016	1.15	0.031
		STD	0.88	0.012	1.15	0.022
		UTD	0.88	0.013	1.16	0.014
		LTD	0.88	0.014	1.13	0.015
6–8	0.83–0.87	UD	0.85	0.014	1.18	0.021
		STD	0.85	0.010	1.18	0.015
		UTD	0.86	0.011	1.18	0.010
		LTD	0.84	0.011	1.17	0.010
8–12	0.80–0.97	UD	0.89	0.055	1.14	0.056
		STD	0.89	0.039	1.14	0.039
		UTD	0.91	0.043	1.18	0.044
		LTD	0.86	0.047	1.10	0.047
12–18	0.72–0.92	UD	0.82	0.070	1.24	0.070
		STD	0.82	0.050	1.24	0.049
		UTD	0.85	0.055	1.29	0.055
		LTD	0.79	0.060	1.19	0.059

tions. If such a difference is noted, then the uniformity or the regularity of the considered roughness surface should be stated. If the discontinuity has a uniform or a regular roughness profile, implying an insignificant anisotropy effect then, the mean value of N_2 can be taken as 1.0 in the uncertainty model. Otherwise, the type of the directional difference between the laboratory and the in situ conditions, i.e., whether direction favoring or resisting sliding, should be ascertained. If the in situ probable shearing direction is favoring sliding and the laboratory shearing direction is resisting sliding the values of N_2 given in Table 3, under Case I column should be used. However, if the in situ probable shearing direction is resisting sliding and the laboratory shearing direction is favoring sliding, then the N_2 values given in Table 3, under Case II column will be utilized. Note that the N_2 values listed for Cases I and II are reciprocals of each other. The statistical parameters of N_2 under different distributional assumptions are obtained and displayed in Table 3 for these two cases.

In the design and analysis of rock slopes, the presence of water on the discontinuity surface or in a tension crack intersecting the slope, is handled by adding an uplift force, which favors sliding. Additionally, the influence of a rise in the water table bisecting a rock slope is accommodated by using saturated rock density in the design procedure. However, the presence of water in a rock discontinuity leads to several mechanical and some chemical effects causing a change in the disconti-

Table 4. Ranges and the statistical parameters of the correction factor, N_3, accounting for the water saturation effect

Estimated range of N_3	Assumed distribution	$\bar{N}_3$	Δ_3
0.70–0.95	UD	0.83	0.087
	STD	0.83	0.062
	UTD	0.87	0.068
	LTD	0.78	0.075

nuity shear strength. Generally the discontinuity shear strength decreases in case of saturation, due to adverse effect of water on compressive and tensile strengths of rock (Barton, 1973 and 1976). Hence, if the test specimens are dry samples and there is a possibility of saturation of discontinuity surface by rainfalls or by the rise of water table, the probable change in the shear strength of rock discontinuity should be incorporated into the uncertainty analysis. For this purpose, the correction factor N_3 is introduced and the available data for the quantification of N_3 are assessed from a detailed literature survey (Jaeger, 1959; Patton, 1966; Jaeger and Rosengren, 1968; Rosengren, 1968; Duncan, 1969; Coulson, 1970; Barton, 1973; Barton, 1976). In the case of rougher discontinuities, water saturation has greater influence on shear strength. Hence the values closer to the lower bound of the range for N_3 should be chosen for very rough, undulating discontinuities, if there is a possibility of water saturation. On the contrary, the values closer to the upper limit of the selected range is recommended for smooth planar discontinuities, since the reducing effect of water on the shear strength is quite small in this case. Table 4 illustrates the recommended range for N_3 and the corresponding values of the mean and c.o.v. according to different distributional assumptions.

3. Reliability-Based Design Model for Plane Failure Mechanism

In the reliability-based design, safety of a slope is measured by the reliability index or by the probability of survival (or equivalently by probability of failure) rather than the classical safety factor. The reliability index, is similar to the safety factor of deterministic approaches, but it includes also the effects of uncertainties and errors in the input parameters in an explicit way. In the simplest technical terms the reliability index can be defined as the minimum distance from the origin of normalized basic variables to the failure surface. A normalized variable has mean zero and standard deviation one. Engineering reliability problems can generally be reduced to the comparison of demand and supply in meeting a specified performance requirement. For example, the safety of a structure depends on the strength of the structure (supply) and the applied load (demand), which are treated as random variables or random functions. In the advanced first-order second-moment (AFOSM) method, random variables are described only by their first and second statistical moments (i.e. mean, variance and correlation characteristics). Although this method has been implemented in various fields of engineering for more than two decades, its application to rock slope stability analysis is quite recent. Most of the recent probabilistic slope stability studies prefer this method (Düzgün et al., 1994; Düzgün et al., 1995; Quek and Leung, 1995; Chen at al., 1998) on the grounds that it is simple and avoids the shortcomings of the classical reliability methods.

In this study the plane failure mechanism is assumed to form the basic failure model for the reliability assessment of a rock slope. The implementation of the AFOSM method to the plane failure mode requires firstly, the formulation of the resisting and driving forces, identification of the probabilistic and deterministic parameters and characterization of the geometry. The basic mechanism of plane failure is best described by a sliding mass on an inclined plane. The mechanical principles state that sliding occurs when the total driving forces exceed the total resisting forces. Here, it should be noted that in the design of rock slopes it is assumed that the kinematic feasibility is assessed on the basis of a given block's potential to move.

The analysis of this failure mechanism has two stages; the first is essentially geometrical and the second deals with the analysis of forces. Geometrical analysis is usually the most complicated aspect. The method given by Priest (1993) is adapted here, since it is more amenable to computer programming and reliability-based analysis. Figure 1 illustrates the geometry of a typical plane failure case.

In the analysis of the system of forces, the stability of a unit slice of rock, measured normal to the plane of the cross-section in Fig. 1, is considered.

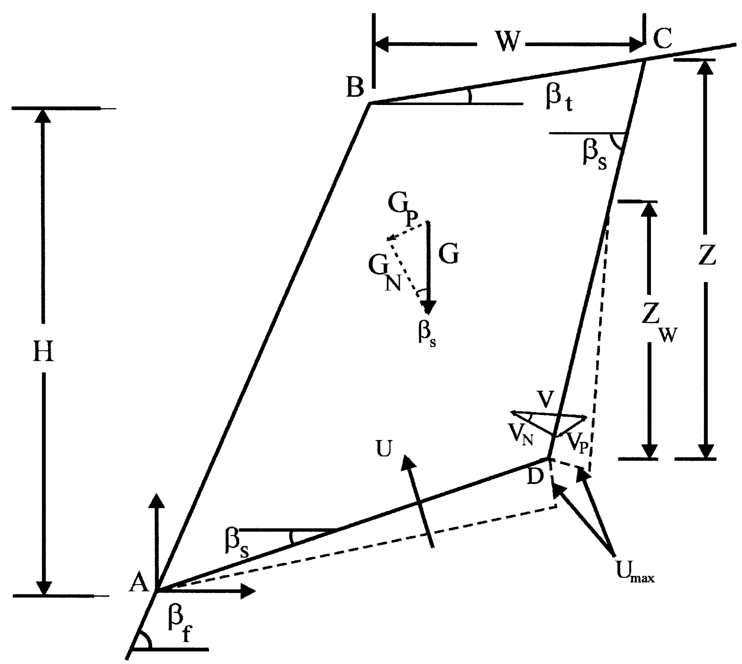

Fig. 1. Geometry of plane failure (after Priest, 1993)

Table 5. Forces acting on the sliding block shown in Fig. 1

Force	Parallel component	Normal component
G	$G_P = G\sin\beta_s$	$G_N = G\cos\beta_s$
U	$U_P = 0$	$U_N = -U$
V	$V_P = V\sin(\beta_c - \beta_s)$	$V_N = -V\cos(\beta_c - \beta_s)$

G: Weight of the sliding block
U: Water pressure on the sliding plane
V: Water pressure in the tension crack

Although the block may be extensive along the crest of the slope, it is assumed that vertical discontinuities or some other features help to the release of the block and allow it to slide along the plane AD, without significant lateral constraints. Hence, it is convenient to analyze the forces G, U and V in terms of their components that lie parallel to the sliding plane, which form the driving forces, and that are normal to the sliding plane, which contribute to the resisting frictional strength. The parallel and normal force components are listed in Table 5. Forces that tend to activate sliding or compress the sliding plane are taken positive. The details of this formulation is given by Priest (1993).

The formulation of a performance function (failure function) or a limit state equation is the second step in the AFOSM method. In the rock slope stability problem, the performance function, $g(\underset{\sim}{x})$, is defined as the difference of the resisting forces, R_f and the driving forces, D_f, as given in Eq. 7 below.

$$g(\underset{\sim}{x}) = R_f - D_f; \tag{7}$$

where:

$$R_f = cL_{AD} + (G_N + U_N + V_N)\tan\phi, \tag{8}$$

$$D_f = G_P + U_P + V_P. \tag{9}$$

The parameters introduced in Eqs. 8 and 9 are defined as follows:

$\underset{\sim}{x}$: Vector of basic variables
c: Cohesion
ϕ: Discontinuity friction angle
L_{AD}: Length of sliding plane
G_N: Vertical component of the weight of the block
U_N: Vertical component of the water force on the sliding plane
V_N: Vertical component of the water force in the tension crack
Gp: Horizontal component of the weight of the block
Up: Horizontal component of the water force on the sliding plane
Vp: Horizontal component of the water force in the tension crack

The limit state condition is achieved when $g(\underset{\sim}{x}) = 0$ and in this case the limit state equation becomes:

$$cL_{AD} + (G_N + U_N + V_N)\tan\phi - (G_P + U_P + V_P) = 0. \tag{10}$$

The identification of the random basic variables in the limit state equation is the third step in AFOSM analysis. In this study the basic variables to be considered are the shear strength parameters, namely: cohesion (x_1) and friction angle (x_2). Therefore the expressions L_{AD}, $(G_N + U_N + V_N)$ and $(G_P + U_P + V_P)$ can be treated as deterministic quantities, denoted by a_1, a_2, a_3, respectively. Accordingly, the performance function takes the following simplified form:

$$g(x_1, x_2) = a_1 x_1 + a_2 \tan(x_2) - a_3. \tag{11}$$

The resulting performance function is non-linear due to the presence of the $\tan\phi$ term. Since sufficient data for the justification of site-specific probability distributions for cohesion and friction angle are generally not available, the recommendations given in literature are taken into consideration in selecting the probability distributions for these two basic variables. In the literature, usually the lognormal distribution is assigned to the strength parameters, since lognormal distribution does not permit negative values and strength values are always positive (Muralha and Trunk, 1993; Ang and Tang, 1984). Truncated normal distribution with no negative values and type I asymptotic distribution are also listed as suitable distributions for strength parameters (Ang and Tang, 1984). Here, both cohesion and friction angle are assumed to be lognormally distributed. The normal distribution is also considered in order to check the sensitivity of AFOSM model to the distribution type and the results obtained from both distributions are compared.

Finally, the reliability index, β, is computed by following an iterative algorithm proposed by Ang and Tang (1984). A software package, called ROCKREL (Duzgun, 2000), is prepared to carry out the numerical computations according to this algorithm. ROCKREL (Rock Reliability) is coded in Visual Basic 6.0 in order to create a simple tool for the engineers to implement the proposed uncertainty analysis model and the reliability-based design approach. For easy access and use ROCKREL is prepared to run on Windows'95 or 98 environments. It is a menu driven software consisting of four main menus called: File, Design Parameters, Analysis and About. Each main menu has its related sub menus. In Fig. 2 the menu structure of this software is shown.

File menu is composed of six sub menus in which, file operations such as creating a new file (New), retrieving a previous file (Retrieve), saving updates (Save), saving the changes under a new file name (Save As), printing inputs and outputs (Print) and quitting the program (Exit) are performed. Design Parameters menu has sub menus of Slope Parameters and Uncertainty Analysis. Slope Parameters sub menu serves for inputting the slope parameters, such as geometrical, strength and mechanical parameters for reliability and safety factor calculations. Uncertainty Analysis sub menu allows the computation of estimated mean value of peak friction angle and total uncertainty. After entering the input parameters by using Slope Parameters sub menu and performing the uncertainty analysis, reliability of the given slope can be evaluated from the Analysis menu. This menu has three sub menus named as Run, Design Angle and Graph. Run sub menu evaluates the reliability index either based on AFOSM or Monte Carlo Simulation algorithms. The later algorithm is not utilized in this study. Design Angle sub menu serves for

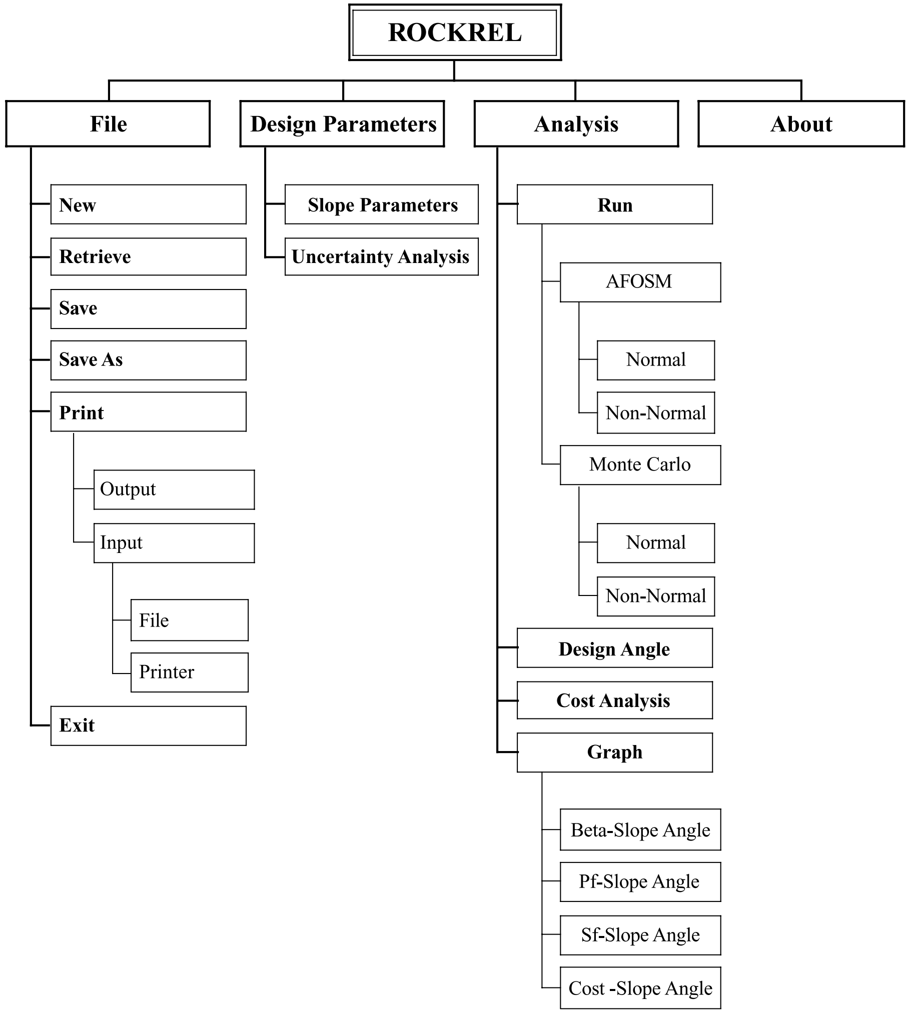

Fig. 2. Menu structure of ROCKREL

the purpose of selecting the appropriate design angle corresponding to a specified reliability index. Graph sub menu is for the graphical representation of results. About menu contains general information about ROCKREL.

4. A Case Study

In order to show the implementation of the proposed reliability-based design methodology a case study is considered. In this case study the reliability of the West Wall of Kanmantoo Mine in South Australia is examined. The basic source of data and information for this application is the study of McMahon (1981), in which he has redesigned the existing rock slope. He modeled the possible discontinuity orientations by Monte Carlo simulation and performed a stability analysis. However, in McMahon's study, strength of the discontinuities is considered to be a deterministic parameter. Since there was limited information available about the shear strength characteristics of the rock discontinuities, a detailed uncertainty analysis as explained in Sections 2 and 3, was not possible. Here we assumed that all correction factors have an expected value of 1.0, implying that the in situ shear strength parameters are estimated without any bias. However, we considered the uncertainties resulting from the discrepancies between the laboratory measured and in situ values of the shear strength parameters as explained in the following.

Initial studies by Jaeger (1970) led to a recommendation that the pit be excavated at a slope angle of 55° between haul road segments on the walls, subject to a review after the fresh rock was exposed (McMahon, 1982). In 1972 after the pit had been excavated to a maximum depth of 48 m, minor slope failures on the west wall caused sufficient concern. McMahon (1981) suggested 46° of slope angle for the west wall after revising the stability analysis. The rock types within the west wall consisted of garnet-andalusite-biotite-schist and garnet-chlorite schist. A large number of direct shear tests by both McMahon (1981) and triaxial tests by Jaeger (1970) were carried out on discontinuity surfaces. Thus, these measurements directly give the peak friction angle, which includes the effect of roughness. Also McMahon (1981) used 35° friction angle for his analysis and assumed zero cohesion. In our analysis, we implicitly assumed that this is an unbiased estimate of the in situ value of the peak friction angle and took the mean value of the peak friction angle as 35°. This is equivalent to setting the mean values of all the correction factors to 1.0. Since there was no direct information available about the inherent variability in the friction angle, a c.o.v. of 0.12 (corresponding to a standard deviation of 4.2°) is assumed based on the study of Ozgenoglu et al. (1982). Furthermore, we arbitrarily assumed that McMahon's estimate of 35° is based on 25 measurements. With respect to the uncertainties associated with the scale effect and anisotropy we selected the largest c.o.v. values associated with the symmetric triangular distribution from Tables 2 and 3, respectively, which are, $\Delta_1 = 0.031$ and $\Delta_2 = 0.049$. Using Eq. 3 with these values, the total uncertainty in the peak friction angle is computed as:

$$\Omega_\phi = \sqrt{0.12^2/25 + 0.031^2 + 0.049^2 + 0.12^2} = 0.14$$

Here, since it is assumed that the slope is in dry condition, the effect of water saturation is ignored, i.e. $\Delta_3 = 0$. For the sake of completeness, unlike McMahon's study, we assumed that a certain degree of cohesion exists with a mean value of 20 kPa (Vutukuri et al., 1974) and a total c.o.v. 0.20. The c.o.v. of 0.20, which corresponds to a standard deviation of 4 kPa, is selected based on the fact that variability in cohesion is usually more than that of friction angle (Muralha and Trunk, 1993; Chowhury, 1986). Since the quantification of the uncertainties in peak friction angle is done without any sound basis and does not depend on site-specific data, the effect of uncertainties in peak friction angle on safety is investigated through a set of sensitivity studies.

Table 6. "Best" estimate values of the geometrical and mechanical parameters used in the reliability-based design of the West Wall of Kanmantoo Mine

Geometrical parameters	Mechanical parameters
Height (H) = 110 m	Mean friction angle (μ_ϕ) = 35°
Width (W) = 130 m	Standard deviation of friction angle (σ_ϕ) = 4.2°
Tension crack depth (z) = 50 m	Mean cohesion (μ_c) = 20 kPa
Water depth in tension crack (z_w) = 0 m	Standard deviation of cohesion (σ_c) = 4 kPa
Slope angle (β_f) = 35°–90°	Unit weight of rock (γ_r) = 28 kN/m^3
Tension crack angle (β_c) = 90°	
Slope top angle (β_t) = 0°	
Discontinuity dip (β_s) = 27°	

McMahon (1981) reported that the most critical discontinuities endangering the stability of the slope have dip angles ranging from 27° to 33° and used 27°. In this study the discontinuity dip is also selected as 27°. No information was available on the unit weight of rock, but it is assumed to be 28 kN/m^3 as reported in Vutukuri et al. (1974).

McMahon (1981) stated that the most probable failure mode is plane failure with or without a tension crack. Hence plane failure geometry is taken into account with 0° angle of slope top and 90° angle of tension crack.

The effects of the geometrical and mechanical parameters on safety are examined by running a set of sensitivity studies. In a sensitivity study, the value of a single parameter is changed, while the other parameters are fixed to their "best estimate" values and the variation of safety in terms of β and Pf is examined. The "best" estimate values of the geometrical and mechanical parameters used in the reliability-based design are listed in Table 6.

In order to select the design slope angle for the West Wall of Kanmantoo Mine, AFOSM method with normal and lognormal basic variables is utilized. The reliability index (β) and the probability of failure (Pf) for slope angles between 35° and 90° are computed for these two distributions, and the results are compared. For the sake of keeping parallelism with deterministic approaches, the corresponding mean safety factor (SF) values are also presented. Since the depth of the mine changes as the mine proceeds and deepens, analyses are conducted for various values of slope height. These analyses also reflect the effect of slope height on the safety level of the slope. Similarly, the influence of the variability in shear strength parameters are determined by repeating the analysis for different c.o.v. values.

The design slope angles are determined based on McMahon's observations. He reported that the mine was excavated to 110 m depth at 46° slope angle and the slope was stable. He also stated that it was not possible to say whether or not the slope would have failed if it had been excavated at 55°, which was recommended by Jaeger (1970) during the initial design. Hence, for 110 m slope height and for a slope angle of 46° by using the best estimate values listed in Table 6, β is computed as 1.71 and 1.88, assuming normal and lognormal variates, respectively. Since the slope was in stable condition, these β values are considered to correspond

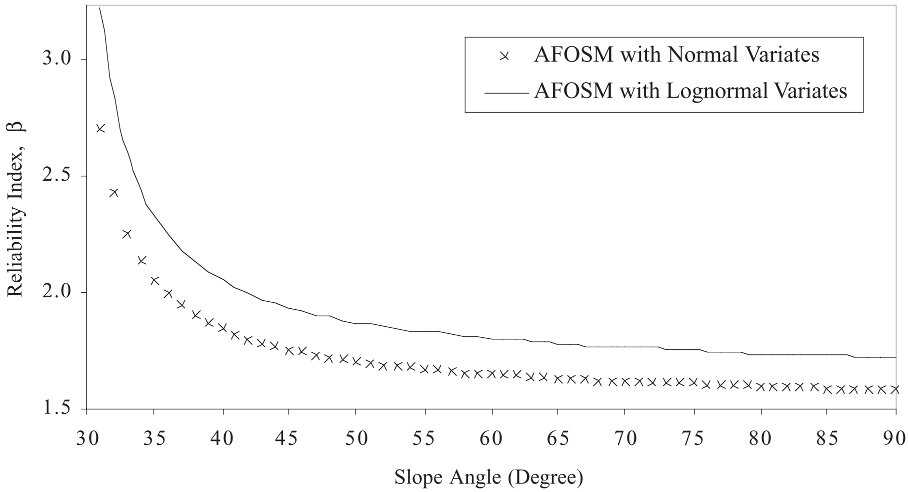

Fig. 3. Variation of reliability index with slope angle for normal and lognormal basic variables

to acceptable safety levels and could be recommended for design. It is clear from Fig. 3 that AFOSM model with normal basic variables results in slightly lower β values than the case with lognormal basic variables. This is due to the fact that lognormal distribution does not permit negative values, yielding to a thicker right tail corresponding to more likely higher strength values.

From this point on AFOSM model with lognormal variates is taken as the basic case and the sensitivity studies are carried out according to this model. The preference of the lognormal model is based on the fact that lognormal distribution is generally recommended for the description of the random characteristics of the strength parameters (Muralha and Trunk, 1993; Chowdhury, 1986).

The reliability index, probability of failure and mean safety factor values are computed for different slope height values ranging from 75 to 200 m, while keeping the other parameters constant and equal to the values listed in Table 6. All of the analyses are performed assuming the slope to be in dry condition. The variation of reliability index (β) with the slope angle for slope heights of 75, 100, 125, 150, 175 and 200 m is illustrated in Fig. 4.

As expected, reliability index decreases with increasing slope angle. It is clear from Fig. 4 that the difference in β values is more dramatic for lower slope angles (35°–40°), since these angles are only few degrees greater than the assumed discontinuity dip (27°), and hence the gravitational load on the slope is quite small compared to the higher slope angles. It is also to be noted that lower β values are obtained for higher slope heights. The curve for 75 m of slope height in Fig. 4, is more distant from the others, indicating significantly higher safety levels, especially for slope angles less than 45°. This is due to a higher width to height ratio (W/H) when $H = 75$ m as compared to other slope height values. For example, the gravitational load corresponding to $H = 100$ m ($W/H = 1.3$) is approximately seven times more than that obtained for $H = 75$ m ($W/H = 1.73$); whereas the

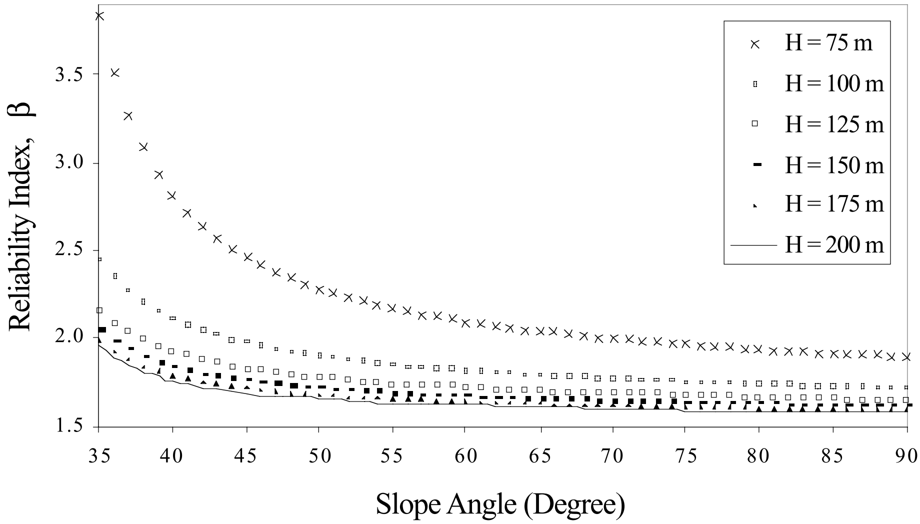

Fig. 4. Variation of reliability index with slope angle for various slope heights

Table 7. Recommended design slope angles for various slope heights

Slope height (m)	Recommended design angle (°)
75	90
100	52
125	42
150	37
175	37
200	37

gravitational load computed for $H = 125$ m ($W/H = 1.04$) is only 1.5 times more than that found for $H = 100$ m ($W/H = 1.3$).

As expected the probability of failure (Pf) increases as the slope angle and/or slope height increases, whereas the mean safety factor exhibits a very similar trend to that of the reliability index.

Since the slope was reported to be in safe state, the β value of 1.88 (with log-normally distributed shear strength parameters) is taken as the target β value for the design of this slope. In Table 7, the recommended design angles for various slope heights, computed based on this target β value of 1.88 with lognormal variates, are presented. Note that this β value corresponds to a failure probability of 0.03. This level of safety is consistent with those reported in literature (e.g. Kirsten, 1983; Whitman, 1984; Sandroni, 1993).

The mean safety factor values corresponding to each recommended design angle are all found to be 1.38, which is an acceptable value for the conventional

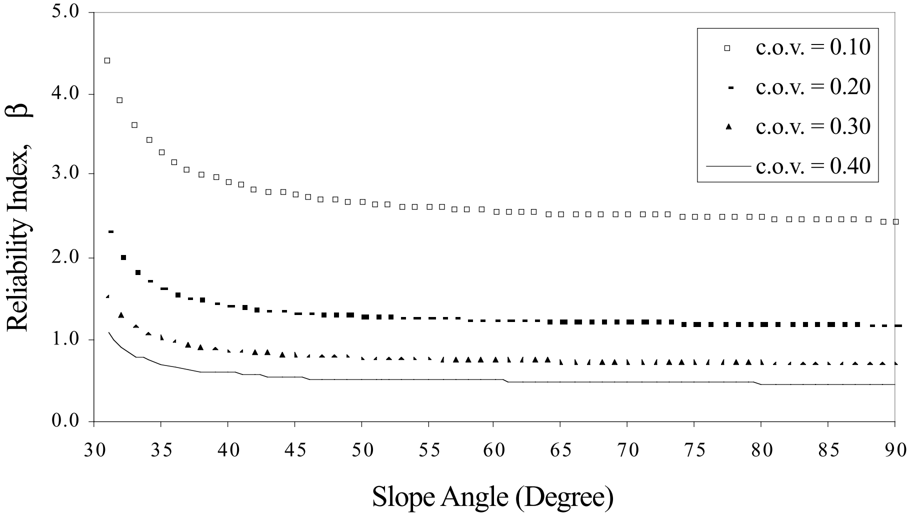

Fig. 5. Effect of variability in friction angle on reliability index

design practice, where the acceptable safety factor values range between 1.3 and 1.5. This demonstrates the existence of a correspondence between the mean safety factor and β.

The sensitivity of safety to the variability in peak friction angle is investigated by computing the β value corresponding to different c.o.v. values. All parameters are again fixed to the values given in Table 6. The influence of the variability in friction angle on safety is examined by computing the reliability index for total c.o.v. values of 0.10, 0.20, 0.30 and 0.40 and is shown in Fig. 5.

Figure 5 indicates that, for a given slope angle, when the uncertainty in friction angle increases the reliability index decreases. Hence efforts to decrease the uncertainty in friction angle by collecting additional information, such as obtaining more samples from the site, collecting more data on properties of existing discontinuities, etc., will contribute significantly to the design of more reliable slopes.

The sensitivity analysis clearly shows that any future data gathering effort should concentrate on the reduction of uncertainties associated with the friction angle, which will lead to a slope with a higher reliability index, i.e. higher safety for the same design, or more economical design for the specified β. Providing the necessary tools for optimal planning of additional data acquisition activities is one of the advantages of the probabilistic approach. On the other hand, since deterministic analysis cannot take the uncertainties directly into account, the safety factor remains the same for different levels of uncertainty.

As a final note, depending on the geometry of the slope, which changes as the pit proceeds, we recommend that the design slope angles should be revised by following the design procedure described above.

5. Conclusions

A reliability-based design methodology is developed through which many sources of uncertainties can be modeled and systematically analyzed in evaluating the safety of rock slopes. In the assessment of the stability of rock slopes, the major source of uncertainty is the incomplete knowledge of the in situ value of the average peak friction angle along a critical discontinuity.

The main sources of errors and uncertainties associated with the peak friction angle of rock discontinuities are modeled and directly incorporated into the design in terms of random correction factors. These correction factors were evaluated based on an extensive literature survey. The suggested ranges of variation of each correction factor, together with the statistical parameters obtained based on prescribed simple probability distributions, may serve as a guide to engineers in assessing the uncertainties for his specific case.

A case study, involving the analysis of the West Wall of Kanmantoo Mine is presented to show the implementation of the proposed model and to obtain an estimate of the target β value. Since the information available on the shear strength characteristics of the rock discontinuities was rather limited, a detailed uncertainty analysis, involving the full utilization of the correction factors as explained in Sections 2 and 3, was not possible. However, still the case study illustrated the general approach of the model and served for the purpose of calibration of the safety level in terms of β. On the basis of the uncertainty analysis conducted in study, for the West Wall of Kanmantoo Mine, the reliability index is computed as 1.88 assuming lognormally distributed shear strength parameters. Considering the fact that this slope was in stable condition, a reliability index value around 1.9 seems to correspond to the level of safety implicit in the current design practice of rock slopes. However, acceptable values of reliability index should be based on more comprehensive calibration analyses.

In specific cases, if extensive field exploration, testing, data collection are performed and/or if reliable expert opinion is available, then the level of uncertainties will be reduced. Accordingly, the design prior to this additional data accumulation will now imply a higher reliability index and a smaller failure probability. The quantification of failure probability associated with a design alternative enables an engineer to compare the relative reliability of alternative designs. This information, which can only be obtained in the reliability-based design procedure, is needed in the selection of the optimal design alternative based on a trade-off between risk of failure and cost of construction.

The main difficulty in the implementation of the proposed methodology is the assessment of uncertainties. In other words, within the context of AFOSM model, the statistical parameters of the correction factors should be quantified. The uncertainty analysis conducted in this study for the peak friction angle provides the necessary guidelines for the quantification of these uncertainties. Here, only the effects of roughness, scale, anisotropy and water saturation are considered and the corresponding correction factors are evaluated. However, further studies on the assessment of uncertainties due to other factors, such as shearing rate, weathering and characteristics of filling material are needed.

In the proposed uncertainty model, the factors causing discrepancies between laboratory measured and in situ values of peak friction angle are considered to be independent of each other. This assumption needs further justification. It is also necessary to investigate and quantify the spatial correlation exhibited by cohesion and peak friction angle.

For many years in rock engineering, the design and analysis have been based on deterministic methods. However, the reliability-based design model presented in this study, together with its application, clearly demonstrates the appropriateness of the probabilistic and statistical methods in dealing with problems that inherently involve uncertainties and variability. Besides, based on the system reliability concepts it is possible to consider the possibility of different failure modes (e.g. wedge, plane, toppling), failure along different failure surfaces and their overall effect on slope safety.

Finally, we would like to emphasize the fact that the probabilistic approaches face the same problems as the deterministic approaches, since they also utilize the same failure models. However, the probabilistic approaches broaden and open new horizons for the practicing engineers.

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