Open-System Magma Chamber Evolution: an Energy-constrained Geochemical Model Incorporating the Effects of Concurrent Eruption, Recharge, Variable Assimilation and Fractional Crystallization (EC-E′RAχFC)

FRANK J. SPERA[1]* AND WENDY A. BOHRSON[2]

[1]INSTITUTE FOR CRUSTAL STUDIES AND DEPARTMENT OF EARTH SCIENCES, UNIVERSITY OF CALIFORNIA, SANTA BARBARA, CA 93106, USA

[2]DEPARTMENT OF GEOLOGICAL SCIENCES, CENTRAL WASHINGTON UNIVERSITY, ELLENSBURG, WA 98926, USA

Significant petrogenetic processes governing the geochemical evolution of magma bodies include magma **R**echarge (including formation of 'quenched inclusions' or enclaves), heating and concomitant partial melting of country rock with possible 'contamination' of the evolving magma body (**A**ssimilation), and formation and separation of cumulates by **F**ractional **C**rystallization (RAFC). Although the importance of modeling such open-system magma chambers subject to energy conservation has been demonstrated, the effects of concurrent removal of magma by eruption and/or variable assimilation (involving imperfect extraction of anatectic melt from wall rock) have not been considered. In this study, we extend the EC-RAFC model to include the effects of **E**ruption and variable amounts of assimilation, **A**χ. This model, called EC-E′RAχFC, tracks the compositions (trace elements and isotopes), temperatures, and masses of magma body liquid (melt), eruptive magma, cumulates and enclaves within a composite magmatic system undergoing simultaneous eruption, recharge, assimilation and fractional crystallization. The model is formulated as a set of $4 + t + i + s$ coupled nonlinear differential equations, where the number of trace elements, radiogenic and stable isotope ratios modeled are t, i and s, respectively. Solution of the EC-E′RAχFC equations provides values for the average temperature of wall rock (T_a), mass of melt within the magma body (M_m), masses of cumulates (M_{ct}), enclaves (M_{en}) and wall rock (M_a^o) and the masses of anatectic melt generated (M_a^*) and assimilated (χM_a^*). In addition, t trace element concentrations and i + s isotopic ratios in melt and eruptive magma $(C_m, \varepsilon_m, \delta_m)$, cumulates $(C_{ct}, \varepsilon_m, \delta_m)$,

enclaves $(C_{en}, \varepsilon_r^o, \delta_r^o)$ and anatectic melt $(C_a, \varepsilon_a^o, \delta_a^o)$ as a function of magma temperature (T_m) are also computed. Input parameters include the (user-defined) equilibration temperature (T_{eq}), a factor describing the efficiency of addition of anatectic melt (χ) from country rock to host magma, the initial temperature and composition of pristine host melt $(T_m^o, C_m^o, \varepsilon_m^o, \delta_m^o)$, recharge melt $(T_r^o, C_r^o, \varepsilon_r^o, \delta_r^o)$ and wall rock $(T_a^o, C_a^o, \varepsilon_a^o, \delta_a^o)$, distribution coefficients (D_m, D_r, D_a) and their temperature dependences $(\Delta H_m, \Delta H_r, \Delta H_a)$, latent heats of transition (melting or crystallization) for wall rock (Δh_a), pristine magma (Δh_m) and recharge magma (Δh_r) as well as the isobaric specific heat capacity of assimilant $(C_{p,a})$, pristine $(C_{p,m})$ and recharge $(C_{p,r})$ melts. The magma recharge mass and eruptive magma mass functions, $M_r(T_m)$ and $M_e(T_m)$, respectively, are specified a priori. $M_r(T_m)$ and $M_e(T_m)$ are modeled as either continuous or episodic (step-like) processes. Melt productivity functions, which prescribe the relationship between melt mass fraction and temperature, are defined for end-member bulk compositions characterizing the local geologic site. EC-E′RAχFC has potential for addressing fundamental questions in igneous petrology such as: What are intrusive to extrusive ratios (I/E) for particular magmatic systems, and how does this factor relate to rates of crustal growth? How does I/E vary temporally at single, long-lived magmatic centers? What system characteristics are most profoundly influenced by eruption? What is the quantitative relationship between recharge and assimilation? In cases where the extraction efficiency can be shown to be less than unity, what geologic criteria are important and

*Corresponding author: Department of Geological Sciences, University of California, Santa Barbara, CA 93106-9630, USA. Telephone: 805-893-4880. Fax: 805-893-2314. E-mail: spera@geol.ucsb.edu

can these criteria be linked to field observations? A critical aspect of the energy-constrained approach is that it requires integration of field, geochronological, petrologic, and geochemical data, and, thus, the EC-ERAFC 'systems' approach provides a means for answering broad questions while unifying observations from a number of disciplines relevant to the study of igneous rocks.

INTRODUCTION

It is generally understood that magma bodies evolve as open systems exchanging material and heat with their surroundings under far-from-equilibrium conditions. This interaction gives rise to the formation of what nowadays are recognized as self-organized dissipative structures (Nicolis & Prigogine, 1977) at a variety of spatiotemporal scales. In magmatic systems, these structures may include, for example, rhythmically layered cumulates, compositionally zoned melt bodies, epithermal ore deposits formed by reaction between magmatic–hydrothermal fluids and country rock, as well as complex compositional zoning profiles within single crystals (for detailed arguments spanning a range of spatial and temporal scales see, e.g. Korzhinskii, 1970; Taylor, 1974; McBirney & Noyes, 1979; Rose & Burt, 1979; Smith, 1979; McBirney, 1980; Brimhall & Crerar, 1987; Feldstein *et al.*, 1994; Halden, 1996; L'Heureux & Fowler, 1996; Tepley *et al.*, 2000). In magmatic environments, the largest source of entropy production is the transport of heat between magma and its surroundings; entropy generation by magma mixing and irreversibility associated with growth and nucleation kinetics are additional, but generally smaller, sources. The mechanisms of heat transport include heat conduction into wall rock, heat transport by hydrothermal convection and the advection of heat associated with both recharge of fresh magma into an existing magma body and eruption of magma from that body. The mechanisms of mixing include advective mixing driven by thermal and compositional buoyancy of melt, multi-phase convection (Bergantz & Ni, 1999), and, to a lesser degree, simple molecular diffusion (Trial & Spera, 1990).

Although the study of heat transfer in magmatic systems has a venerable history (e.g. Ingersoll *et al.*, 1954; Shaw, 1965; Jaeger, 1968; Spera, 1979; Carrigan, 1988; Huppert & Sparks, 1988; Bergantz, 1989; Marsh, 1989), the explicit connection between heat transfer and the trace element and isotopic evolution of magma in open systems has been investigated less thoroughly despite the obvious coupling. As noted by Taylor (1980; Taylor & Sheppard, 1986; see also DePaolo, 1981), it was

N. L. Bowen (Bowen, 1928) who pointed out that assimilation is not a simple two-component mixing process. Instead, at a minimum, it represents a three end-member problem involving the coupled energetics (both sensible and latent) among magma, country rock assimilant and cumulates. If one allows for magma replenishment (addition) and magma removal (eruption), it is evident that there are many degrees of freedom available to magma undergoing open-system evolution. The geochemical path followed by an open system is dictated by the efficacy and coupling among the processes of eruption, recharge, mixing, assimilation and crystallization, and is therefore highly contingent on inherent vagaries particular to a given magmatic system, no two of which are alike in most respects.

Although geochemists and petrologists have long appreciated, in principle, the concept of open-system behavior, the translation of the concept into a practical geochemical model applicable to natural systems has proved challenging. A prominent contribution to this subject is the collective works of M. J. O'Hara and co-workers over the past quarter-century (e.g. O'Hara, 1977, 1980; O'Hara & Mathews, 1981; O'Hara & Fry, 1996; O'Hara & Herzberg, 2002). O'Hara and collaborators have consistently argued that many magmas on Earth exhibit characteristics suggesting they are not primary (e.g. O'Hara, 1980, 2000); among the most important of these characteristics is evidence of low-pressure crystallization, implying residence in shallow-level magma reservoirs where processes such as magma mixing, contamination and periodic eruption can potentially modify mantle 'signatures'. O'Hara has also examined the question of how basalts (and other magmas) acquire their chemical signatures by investigating the consequences of magma migration through the crust. At the heart of this is the 'space problem' (e.g. O'Hara 1998). O'Hara (1998) noted that prior to 1950, the 'space problem' was connected to the study of assimilation, contamination and hybridization of evolving calc-alkaline plutons, the large volumes of which made the 'space problem' a visible issue. Field evidence, primarily from the margins of plutons, provided support for the importance of assimilation and hybridization in the petrogenesis of magmas. As basalts became increasingly important in understanding magma petrogenesis and deciphering sub-solidus convective mixing in the mantle, the 'space problem' and the relevance of assimilation became issues of secondary importance. However, as O'Hara particularly noted in his 1998 contribution on the thermal and geochemical consequences of large-scale assimilation in ocean island development, magmas emplaced at crustal levels must make space for themselves, and one possible mechanism is assimilation of crustal material as the magma body migrates. Thermal considerations also provide support for crustal assimilation

(O'Hara, 1998). Through numerous quantitative models and associated analysis, O'Hara and colleagues have provided a framework in which the impact of Eruption, Recharge, Assimilation and Fractional Crystallization (ERAFC) processes on major and trace element and isotopic characteristics of magmas can be evaluated.

A critical step in further quantifying the types of models that O'Hara and colleagues have generated is development of a model for open-system magmatic behavior using coupled mass, momentum, species and energy conservation. Although, in principle, this seems straightforward, complicating matters quickly arise when conservation principles are applied to natural systems. For example, rarely is anything known regarding the three-dimensional form of a magma body. Without such knowledge, rates of heat transport are impossible to quantify, and this uncertainty makes it impossible to estimate rates of assimilation, magma mixing and fractional crystallization for specific magmatic systems. Similarly, phase equilibria models including minor and trace element partitioning and activity–composition relations for many important phases are still lacking. Incomplete information on the transport properties of magmatic materials such as thermal conductivity and the kinetics of transport phenomena introduces additional uncertainty into the modeling of magmatic systems. Unknown thermophysical property variations in host rock environments, such as the three-dimensional structure of the permeability, which can influence the rate of heat transfer between magma and country rock, serve to further obfuscate attempts to model particular natural systems.

In light of these difficulties, is there any hope for developing a model for the trace element and isotopic evolution of natural (open) magmatic systems? We believe the answer is yes. The phenomena of eruption, recharge and the mixing of magmas, assimilation, and fractional crystallization (i.e. ERAFC processes) are intimately linked through energy conservation, independent of the complicating details of momentum transport. This linkage can be exploited to develop a thermodynamic model for the open-system geochemical evolution of magma undergoing ERAFC processes. This model can then serve as a preliminary 'reference state' from which to evaluate the full complexity of geochemical evolution. Although this approach cannot provide absolute temporal information and is not a model for the dynamical evolution of magma, it does provide a relative chronology and material inventory for the succession of melt and solid compositions produced during ERAFC evolution and can be applied to natural systems. Significantly, model predictions can be compared with observed compositions and masses or relative masses of melts and crystalline products from particular natural systems. Indeed, when combined with trace and isotope material balance expressions, energy conservation leads to a self-consistent algorithm for determination of the geochemical path followed by magma along the path to thermal equilibrium from its initial far-from-equilibrium state. In the EC-ERAFC model, the path refers to the progression of wall-rock temperatures (T_a), trace element concentrations and isotope ratios in anatectic melt, host melt, eruptive magma and crystallized solids as a function of the melt temperature, T_m. The solid products of ERAFC evolution include cumulate rocks produced by fractional crystallization as well as solids produced by 'instantaneous' chilling of a portion of recharge melt. The latter solids, which are non-equilibrium with respect to the magma body, are identified with the class of inclusions ubiquitous in plutons and lavas denoted by the term 'enclave' in the classical petrologic literature (Best & Christiansen, 2001).

The purpose of this study is to present a description of the open-system model. Details of earlier EC-AFC and EC-RAFC models forming the backbone of EC-E'RAχFC have been presented elsewhere (Bohrson & Spera, 2001, 2003; Spera & Bohrson, 2001, 2002). In the current work, attention is focused on the novel elements introduced by allowance for eruption (E') and for variable addition of anatectic melt into the host magma body (Aχ). In the EC-AFC model, all melt generated in country rock is added to the evolving magma body. In the present model, that condition is relaxed. EC-E'RAχFC is valid for arbitrary eruptive mass, $\overline{M}_e^o$ (the non-dimensional eruptive mass), in the limit of no addition of anatectic melt into the magma body, $\chi = 0$ (i.e. no contamination) and $\overline{M}_r^o = 0$ (no recharge). It also is valid in the limit of $\overline{M}_e^o = 0$ (no eruption) with arbitrary recharge mass, $\overline{M}_r^o$, for all χ such that $0 \leq \chi \leq 1$. It should be noted that χ is the mass fraction of anatectic liquid generated in the country rock that is added to and mixes with pre-existing host magma. For arbitrary M_r^o and χ, the EC-E'RAχFC presented in this study is limited to $\overline{M}_e^o < 0.4$, approximately. The superscript on the E' is included to remind the reader that the current model is not valid for scenarios in which $\overline{M}_e^o$ exceeds approximately 0.4. A summary of all the variables in the model is provided in Table 1.

QUALITATIVE DESCRIPTION OF EC-E'RAχFC MODEL: A SYSTEMS APPROACH

Figure 1 illustrates a crustal-scale view of magma transport, thereby providing a context for the ERAFC magma evolution model. Not included in the illustration are the regions where magma is generated, perhaps by isentropic pressure-release melting (Verhoogen, 1954; Asimow, 2000), by volatile-induced solidus depression (Bailey, 1970), or by segregation from residual source material

Table 1: Nomenclature

Symbol	Definition	Units
T_m^o	Initial host melt temperature	K
T_r^o	Initial recharge melt temperature	K
$T_{l,m}$	Pristine host melt liquidus temperature	K
T_{eq}	Equilibration temperature	K
T_m	Host melt temperature	K
T_a	Temperature of country rock	K
$T_{l,a}$	Wall-rock liquidus temperature	K
T_a^o	Initial country rock temperature	K
$T_{l,r}$	Recharge melt liquidus temperature	K
$T_{r,i}$	Temperature mid-point of ith recharge episode	K
$T_{e,i}$	Temperature mid-point of ith eruptive episode	K
Δh_a	Enthalpy of fusion of assimilant	J/kg
Δh_m	Enthalpy of crystallization of host magma	J/kg
Δh_r	Enthalpy of crystallization of recharge magma	J/kg
$C_{p,m}$	Magma isobaric specific heat capacity	J/kg K
$C_{p,a}$	Assimilant isobaric specific heat capacity	J/kg K
$C_{p,r}$	Recharge melt specific heat capacity	J/kg K
$M_r(T_m)$	Mass of recharge magma	kg
$M_e(T_m)$	Mass of eruptive magma	kg
M_r^o	Total mass of recharge magma added during ERAFC event	kg
M_e^o	Total mass of eruptive magma added during ERAFC event	kg
M_o	Mass of initial magma	kg
M_{ct}	Mass of cumulates	kg
M_{en}	Mass of enclaves	kg
M_s	Total mass of solids (cumulates plus enclaves)	kg
$\Delta M_{r,i}$	ith increment of recharge mass addition	kg
N_r	Number of episodes of recharge addition	
m_i	Parameter in recharge addition mass function	K^{-1}
d_i	Parameter in recharge addition mass function	
h_{lib}	Total heat liberated by magma	J
h_{abs}	total heat absorbed by country rock	J
Δh_a	Specific fusion enthalpy of wall rock	J/kg

Symbol	Definition	Units
Δh_m	Specific crystallization enthalpy of host magma	J/kg
Δh_r	specific crystallization enthalpy of recharge melt	J/kg
M_m	Mass of melt in magma body	kg
M_a^*	Mass of anatectic melt	kg
M_a^o	Mass of country rock involved in RAFC event	kg
ΔH_a	Enthalpy of anatectic melt/ anatectic melt partition coefficient	J/mol
ΔH_m	Enthalpy of cumulate/host melt partition coefficient	J/mol
ΔH_r	Enthalpy of enclave/recharge melt partition coefficient	J/mol
χ	Melt extraction efficiency	
C_a	concentration of trace element in country rock*	ppm
C_r	Concentration of trace element in recharge melt	ppm
C_m	Concentration of trace element in host melt	ppm
C_{en}	Concentration of trace element in enclave	ppm
$\hat{C}_a$	Average concentration of anatectic melt along path $T_a^o \rightarrow T_m$	ppm
C_{ct}	Instantaneous concentration of trace element in cumulate	ppm
$\hat{C}_{ct}$	average concentration of cumulates along path $T_m^o \rightarrow T_m$	ppm
$\hat{C}_{en}$	Average concentration of enclaves along path $T_m^o \rightarrow T_m$	ppm
D_m	Bulk distribution coefficient between cumulate and melt	
D_a	Bulk distribution coefficient between wall rock and anatectic melt	
D_r	Bulk distribution coefficient between enclave and recharge melt	
$s \equiv \dfrac{C_a^o}{C_m^o}$	Ratio of initial concentration of trace element in country rock and pristine magma	
$t \equiv \dfrac{C_r^o}{C_m^o}$	Ratio of initial concentration of trace element in recharge melt and pristine magma	
ε_m	Isotopic ratio in host melt	
ε_r	Isotopic ratio in recharge melt	
ε	Isotopic ratio in assimilant	
δ_a	Oxygen isotopic composition of assimilant	

Table 1: continued

Symbol	Definition	Units
δ	Oxygen isotopic composition of host magma	
δ_r	Oxygen isotopic composition of recharge melt	
$f_m(T)$	Melt productivity of pristine initial host melt composition	
$f_a(T)$	Melt productivity of wall-rock composition	
$f_r(T)$	Melt productivity of recharge melt composition	
$\overline{M}_{en} \equiv \dfrac{M_{en}}{M_o}$	Non-dimensional mass of enclaves	
$\overline{M}_{ct} \equiv \dfrac{M_{ct}}{M_o}$	Non-dimensional mass of cumulates	
$\overline{M}_s \equiv \dfrac{M_s}{M_o}$	Non-dimensional mass of all solids (cumulates plus enclaves)	
$\overline{M}_a^o \equiv \dfrac{M_a^o}{M_o}$	Non-dimensional mass of assimilant	
$\overline{T}_{eq} \equiv \dfrac{T_{eq}}{T_{l,m}}$	Non-dimensional equilibration temperature	
$\overline{T}_a \equiv \dfrac{T_a}{T_{l,m}}$	Non-dimensional assimilant temperature	
$\overline{T}_a \equiv \dfrac{T_a}{T_{l,m}}$	Non-dimensional magma temperature	
$\overline{T}_m \equiv \dfrac{T_m}{T_{l,m}}$	Non-dimensional melt fraction in magma body	
$\overline{M}_m \equiv \dfrac{M_m}{M_o}$	Non-dimensional concentration of trace element in anatectic melt	
$\overline{C}_a \equiv \dfrac{C_a}{C_a^o}$	Non-dimensional concentration of trace element in recharge magma	
$\overline{C}_r \equiv \dfrac{C_r}{C_r^o}$	Non-dimensional concentration of trace element in melt	

*In text and equations, initial values of trace element concentrations and isotopic ratios are denoted by superscript ^{o} (e.g. C_a^o is the initial concentration of trace element in wall rock).

Fig. 1. Schematic depiction of magmatic transport phenomena after Hill *et al.* (2002). Early stages of melt generation and segregation are not portrayed. Magma ascends from source region via crack network or as discrete pod-shaped bodies. Magma bodies develop in regions where upward ascent of magma is impeded, such as at the Moho or the brittle–ductile transition, as a result of density contrast or rheological gradients, respectively. Rates of heat transfer between magma and country rock are modulated by the thermophysical properties of country rock, the presence or absence of hydrothermal systems and the character of magma body–country rock contact area. Deeper bodies emplaced in low-porosity crust may lose heat mainly by conduction whereas in shallower environments hydrothermal convection may be more significant. A magma storage body is one component of the composite E′RAχFC system portrayed in detail in Fig. 2.

by percolation. Once segregated from residuum, melt rises as a result of the collective action of favorable pressure and buoyancy forces. The form (e.g. isolated pods or plexus of propagating cracks) and transport rate of magma through the lithosphere depends on a complex interplay of factors including the distribution of magma pressure and buoyancy forces, the magnitude, orientation and spatial variation of the principal stresses, and the thermophysical properties of both magma and country rock (e.g. see Petford & Koenders, 1988; Hart, 1993; Rubin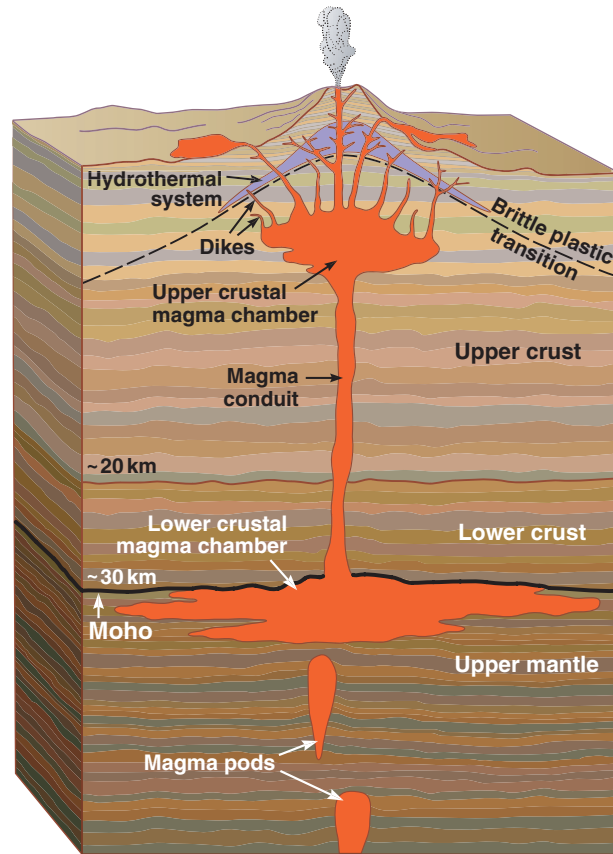, 1995). The loss of heat from the magma body depends upon the local temperature and thermophysical properties of the country rock as well as the relative importance of porous medium thermal–salinity convection compared with heat conduction (e.g. Norton & Cathles, 1979; Norton & Taylor, 1979; Taylor, 1986; Carrigan, 1988; Cathles *et al.*, 1997; Schoofs & Spera, 2003). At shallow depths where porosity–permeability relations are favorable, large-scale hydrothermal circulation systems may develop and allow relatively efficient transfer of magmatic heat into country rock. In this case, although a large volume of country rock is heated, the fraction of wall rock undergoing partial melting may be restricted because of the relatively rapid transport of heat

away from the magma body. In contrast, at greater depths where fluids may be absent or the permeability very low, heat conduction dominates heat loss. Although a smaller mass of country rock is heated, the local rise in temperature may lead to significant country rock anatexis. Germane to ERAFC evolution is that magma may sometimes be 'stored' in finite-volume bodies at depth. Two illustrative storage depths, drawn from a large number of possibilities, are illustrated in Fig. 1. One is the Moho, where a sharp contrast in density between mantle and lower crust exists, and another is the brittle–ductile rheological transition, where country rock thermophysical properties and the state of stress might impede upward magma transport. Once a substantial volume of magma is stored within the crust, the EC-E'RAχFC model may be applied.

In EC-E'RAχFC, a composite system is envisioned that is isolated adiabatically from its environment (Fig. 2). The composite system comprises four sub-systems with boundaries that may be open, closed or semi-permeable with respect to mass, and adiabatic or diathermal with respect to energy. In general, a stipulation regarding both heat and matter exchange between each sub-system is required to define an EC-E'RAχFC evolution. The four sub-systems in the ERAFC model include country rock, magma body, recharge reservoir and an effusive (eruptive) volume. Wall rock (synonymous with country rock) is separated from the magma body by diathermal, semipermeable boundaries. That is, heat can freely pass across the boundary that is permeable to fraction χ of anatectic melt generated in the country rock by partial fusion. The mass of country rock involved in E'RAχFC evolution is governed by an integral energy balance that enforces energy conservation among all four sub-systems along the path towards thermal equilibration. The magma body consists of host melt, cumulates and enclaves. Two additional sub-systems include a reservoir of recharge melt of arbitrary mass, specific enthalpy and composition, and an eruptive or extrusive reservoir formed by partial eruption of the magma body. During eruptive episodes, the boundary between the magma body and the eruptive reservoir is perfectly open with respect to both heat and material transport. During episodes of recharge, the boundary between the recharge reservoir and the magma body is also open with respect to matter and heat. Boundaries between the recharge and eruptive reservoirs and the country rock are closed and adiabatic.

In the most general formulation of EC-E'RAχFC, each of the four sub-systems can be viewed as individual composite systems. In principle, this allows one to model effects such as compositionally zoned magma bodies, recharge magma of varying specific enthalpy and composition, the imperfect extraction of anatectic melts and eruption of magma with a different solid to melt ratio compared with the magma body at the time of eruption. The EC-E'RAχFC versions presented here and in previous work (EC-AFC, EC-RAFC) apply to homogeneous (non-zoned) magma bodies replenished by recharge magma of fixed composition and temperature (specific enthalpy). Eruptive magma is at the same temperature as host magma, and its composition reflects that of host melt and average cumulates at T_m. Crystallinity is approximated from that of pristine magma at T_m. In EC-E'RAχFC, the anatectic melt extraction factor, χ, is set by the investigator in the range $0 \leq \chi \leq 1$. For $\chi = 1$, all anatectic melt generated by partial melting of wall rock is added to and homogenized within the magma body. Alternatively, for $\chi = 0$, none of the wall-rock anatectic melt is allowed to enter the magma body, although the energy needed for its generation is accounted for (Petford & Gallagher, 2001; Petford, 2003). The earlier EC-AFC and EC-RAFC models (Bohrson & Spera, 2001, 2003; Spera & Bohrson, 2001, 2002) implicitly set $\chi = 1$, and are, therefore, 'maximal contamination' RAFC scenarios. In EC-E'RAχFC, magma removed during eruption is a mixture of crystals and melt present in the same ratio as that in the magma body during the eruption, ignoring the effects of recharge and assimilation. That is, no fractionation between solids and melt is permitted during eruption and the contribution to the ratio of crystals and melts owing to recharge and assimilation is ignored. The latter constraint is not strictly correct but is a good approximation provided $\overline{M}_e$ is less than 0·4. This limit was empirically determined by comparing the solid to melt ratio within the magma body between cases with $\chi = 1$ and finite $\overline{M}_r^o$ and with zero $\overline{M}_e^o$ and $\chi = 0$.

To uniquely define an E'RAχFC event, an integral energy balance is invoked to provide a connection between the mass and thermal properties of all subsystems, given a set of initial conditions and a chosen equilibration temperature (T_{eq}). The integral equation allows determination of the mass of wall rock, M_a^o, which thermally equilibrates with the magma body sub-system during the ERAFC event. Once M_a^o has been determined, path-dependent parameters, such as trace element and isotope characteristics of melt and solids (cumulates and enclaves) are determined by solution of a set of differential equations. These differential equations express conservation of energy, mass, species and isotope balance as a function of melt temperature for $T_m \subset [T_m^o, T_{eq}]$. There can be a positive or negative correlation between the extent of anatexis and the addition of recharge magma depending upon the initial temperature and thermodynamic properties of recharge melt relative to T_m at the time of replenishment. The removal of heat as a result of eruption always decreases the heat available for partial melting of country rock.
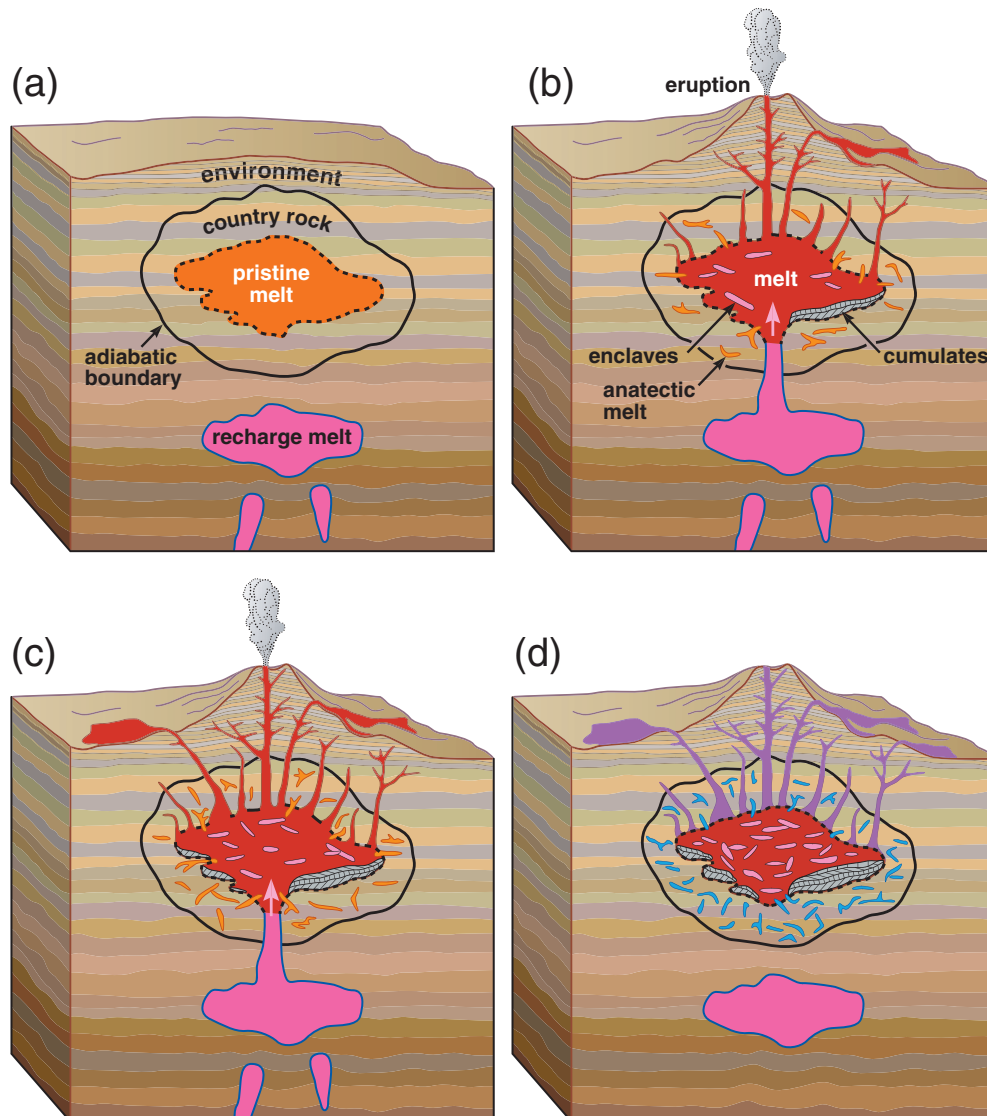
Fig. 2. Schematic illustration of E′RAχFC evolution. (a) The initial state for E′RAχFC evolution. A batch of magma of initial mass M_o and temperature T_m^o is emplaced into country rock of temperature T_a^o. (b) Early stage of E′RAχFC evolution. Recharge melt is being added to magma body at prescribed rate, $M_r(T_m)$ with formation of enclaves and mixing of recharge and pristine melts. Removal by eruption of homogenized magma takes place according to the prescription, $M_e(T_m)$. Heating of country rock generates anatectic melt, a fraction (χ) of which contaminates evolving magma. Cumulate rocks form by fractional crystallization. (c) Further E′RAχFC evolution takes place as the temperature of melt (T_m) progresses along the trajectory $T_m^o \rightarrow T_{eq}$. (d) Final state with thermal condition $T_m = T_a = T_{eq}$. Magma body consists of cumulates, enclaves and homogeneous melt of mass M_{ct}, M_{en} and M_m, respectively. The total mass of recharge is $\overline{M}_r^o$ and the total mass of eruptive magma is $\overline{M}_e^o$. The trace element and isotopic compositions of all materials are defined along the path to equilibration.

DETAILED DESCRIPTION OF EC-E′RAχFC MODEL

The thermodynamic description of the composite system during E′RAχFC evolution (Fig. 2) illustrates the progression of states along the geochemical EC-E′RAχFC path. The initial state (Fig. 2a) is a composite system composed of four sub-systems: country rock of mass M_a^o at mean initial temperature T_a^o, a pristine batch of chemically homogeneous isothermal melt of mass M_o and temperature T_m^o, a compositionally distinct (but homogeneous) recharge melt reservoir of mass M_r^o and temperature T_r^o and a 'virtual' eruptive reservoir (initially empty). The trace element and isotopic compositions of country rock, pristine magma, and recharge magma are specified. The country rock–magma body boundary is diathermal and semi-permeable, allowing exchange of enthalpy and mass fraction χ of anatectic melt generated by partial melting in the country rock. Recharge melt is added during the E′RAχFC event according to an *ab initio*

(user-defined) prescription $M_r = M_r(T_m)$, where T_m is the temperature of host melt. Eruptive magma is similarly removed during E′RAχFC according to an *ab initio* (user-defined) prescription $M_e = M_e(T_m)$. Figure 2b, c and d shows successive steps along the evolutionary path as T_m varies from T_m^o, the initial temperature, to the final or equilibration temperature, T_{eq}, specified by the user (Fig. 2d). Although time is not considered in the thermodynamic model (indeed, that is precisely why the model is of general utility), T_m serves as the progress variable in the differential equations defining the geochemical path (see Edwards & Russell, 1998; Hawkesworth *et al.*, 2000). T_m monotonically falls from the initial value, T_m^o, to the final or equilibration temperature, T_{eq}, and points in the same direction as the 'arrow of time'. At T_{eq} (Fig. 2d) the composite system has reached thermal equilibrium and entropy production associated with heat transfer vanishes within the composite system. The profoundness of the thermal interaction between wall rock and the magma body is measured by the value chosen for T_{eq}.

The thermal consequences of addition of recharge melt to the magma body depend on the initial temperature (T_r^o) and the melt fraction–temperature relationship [herein termed the melt productivity, $f_r(T_m)$] of recharge melt as well as the host melt temperature T_m at the time of recharge addition. The initial temperature of recharge melt, T_r^o, is set equal to its liquidus temperature, $T_{r,l}$. If T_r^o is less than T_m, thermal energy required to warm recharge melt comes from sensible heat stored in the host melt and latent heat associated with *in situ* cumulate formation; once local thermal equilibrium is reached, the remaining recharge melt homogenizes with host magma. In contrast, if T_r^o exceeds T_m, the fraction $1 - f_r(T_m)$ of recharge melt 'instantaneously' solidifies upon injection into cooler host magma. The solid 'quench' products of this thermal interaction are termed enclaves in the E′RAχFC formulation. Such enclaves are commonly found in granitic (*sensu lato*) plutons and their volcanic equivalents and are generally interpreted as the chilled remnants of mafic magma injected into cooler magma (see, e.g. Furman & Spera, 1985; Didier & Barbarin, 1991; Wiebe & Snyder, 1993; Wiebe, 1994; Wiebe & Adams, 1997; Snyder & Tait, 1998; Waight *et al.*, 2001, and references therein).

During the course of E′RAχFC evolution (Fig. 2b and c), magma is removed by eruption from the magma body sub-system according to the *a priori* prescription $M_e(T_m)$. Magma removed by eruption is in thermal equilibrium with host melt and consists of a mixture of melt and solids in the same proportion as would be present in the pristine magma body at T_m; this is an approximation of the melt–crystal state of a body that has undergone assimilation and recharge. EC-E′RAχFC evolution is complete (Fig. 2d) when the temperature of the wall-rock restite (that part of country rock that remains solid) is equal to

the melt temperature, T_m, which, in turn, is equal to T_{eq}. That is, at the completion of the E′RAχFC event, $T_a = T_m = T_{eq}$. In cases where all the country rock involved in the ERAFC interaction melts, the equilibration condition is simply $T_m = T_{eq}$. When T_{eq} is reached, the sub-systems include: (1) mass M_m of host melt of homogeneous composition at temperature T_{eq}; (2) mass M_{ct} of cumulates of variable (but known) composition formed by fractional crystallization; (3) mass M_{en} of enclaves also of variable (but known) composition formed by closed-system fractional crystallization of recharge melt; (4) mass $M_a^o - \chi M_a^*$ of residual wall rock (restite) plus anatectic melt trapped in the country rock reservoir; (5) mass M_e^o of eruptive magma of known composition and temperature.

In the EC-E′RAχFC model, host melt and wall rock have unique trace element (C_m^o, C_a^o) and isotopic (ε_m^o, δ_m^o, ε_a^o, δ_a^o, where ε_m is radiogenic and δ_m is oxygen) initial compositions. Recharge melt, with distinct trace element (C_r^o) and isotopic (ε_r^o, δ_r^o) compositions at initial temperature (T_r^o), is added to host magma according to user-defined recharge mass function $M_r(T_m)$. The recharge mass added to the magma body during E′RAχFC evolution is M_r^o, where $M_r^o = M_r(T_{eq})$. The extrusive mass is likewise specified *a priori* by definition of the eruptive mass function, $M_e(T_m)$, and its composition is that of the host magma (melt plus average crystals) at the time of eruption.

Results from both experimental phase equilibria and thermodynamic modeling (e.g. MELTS, Ghiorso, 1997) are used to constrain the thermodynamics of melting. These constraints include melt productivity functions for wall rock, pristine magma and recharge magma and the phase assemblages as a function of temperature for estimation of bulk partition coefficients. The solution of the conservation equations provides the mass of heated wall rock (M_a^o), the amount of anatectic melt (M_a^*) generated in wall rock and added to host magma (χM_a^*), the mass of melt in the chamber (M_m), and the mass of cumulates (M_{ct}) and enclaves (M_{en}) as a function of T_m along the path $T_m^o \rightarrow T_{eq}$. In addition to masses, trace element concentrations and radiogenic and oxygen isotope ratios in melt, cumulates and enclaves at each temperature along the path $T_m^o \rightarrow T_{eq}$ are determined.

In the following sections, mathematical details of the EC-E′RAχFC algorithm are presented. The first section details the parameterization of the mass eruptive function, $M_e(T_m)$. The non-linear melt productivity functions $f_a(T)$, $f_m(T)$ and $f_r(T)$ and the mass recharge function $M_r(T_m)$ have been presented elsewhere (Spera & Bohrson, 2002) and are used but not derived here. The derivation of the EC-E′RAχFC algorithm is given in two parts. The first is an integral energy conservation statement that provides a set of ordered pairs (T_{eq}, M_a^o) for particular thermodynamic properties, initial conditions,

mass eruption history, $M_e(T_m)$, and total recharge mass, M_r^o. A particular choice of T_{eq} is made based upon geologic knowledge of the magmatic system under study. Once T_{eq} has been chosen, the second part of the calculation gives the solution to the path-dependent differential equations defining the chemical and thermal evolution of the melt, wall rock, cumulates and enclaves along the E'RAχFC temperature trajectory $T_m^o \rightarrow T_{eq}$.

In (2), the factors $m_{e,i}$ and g_i control the width and slope of the eruption mass function, $T_{e,i}$ is the mid-point of the temperature interval during which the ith eruptive pulse occurs, and $\Delta M_{e,i}$ is the mass of the ith pulse. By use of (2), specification of eruptive episodes of arbitrary mass at specific temperatures in the interval $T_m^o \rightarrow T_{eq}$ is accomplished. An illustrative example is given in Fig. 3 for an E'RAχFC event made up of three eruptive pulses.

Differentiation of (2) gives a differential equation for the variation of the eruptive mass with melt temperature T_m:

$$\frac{d\overline{M}_e}{d\overline{T}_m} = -\sum_{i=1}^{N_e} \frac{\exp[m_i(T_{l,m}\overline{T}_m - T_{e,i})]T_{l,m}g_i m_{e,i}\Delta\overline{M}_{e,i}\{1 + \exp[m_{e,i}(T_{l,m}\overline{T}_m - T_{e,i})]\}^{(g_i-1)}}{\{1 + \exp[m_{e,i}(T_{l,m}\overline{T}_m - T_{e,i})]\}^{2g_i}}. \tag{3}$$

Mathematical details of the E'RAχFC algorithm

Magma eruptive function

In EC-E'RAχFC, magma is removed by eruption from the magma body during the approach to thermal equilibrium. In all cases, erupted material includes melt and crystals in the proportion they would exist within the pristine magma reservoir at temperature T_m. Two forms are useful as 'end-member' models for the eruption of magma during RAχFC evolution. The simplest eruptive function is the linear one,

$$\overline{M}_e(\overline{T}_m) = \overline{M}_e^o (T_{l,m}\overline{T}_m - T_m^o)/(T_{eq} - T_m^o) \tag{1}$$

for which magma is removed as a linear function of T_m. A more realistic mass eruption model is the multiple pulse or episodic model, which is modeled as follows. At some set of predetermined small temperature intervals (the ith temperature interval having midpoint $T_{e,i}$), the ith pulse of eruptive magma of mass $\Delta M_{e,i}$ is removed from the host magma body. There may be an arbitrary number (N_e) of eruptive episodes; the current code allows for up to 20 such episodes in any single E'RAχFC simulation. The logistic form gives the cumulative mass of eruptive magma for multiple eruption pulses:

$$\overline{M}_e(\overline{T}_m) = \sum_{i=1}^{N_e} \frac{\Delta\overline{M}_{e,i}}{\{1 + \exp[m_{e,i}(T_{l,m}\overline{T}_m - T_{e,i})]\}^{g_i}}. \tag{2}$$

Equation (3) is needed for computation of the E'RAχFC path.

Integral enthalpy balance

The integral enthalpy balance provides a fundamental constraint on the geochemical path followed by sub-systems during E'RAχFC evolution. The balance defines a relation among four quantities characterizing E'RAχFC evolution: the mass of country rock, M_a^o, the total mass of recharge added, M_r^o, the eruptive mass function, $M_e(T_m)$, and the equilibration temperature, T_{eq}. The integral enthalpy balance incorporates heating and partial melting of country rock, addition of recharge melt of arbitrary composition and temperature, eruption of magma, magma cooling, and heat exchange and solidification associated with enclaves and cumulates. The required thermodynamic parameters include the melt productivity functions for country rock, pristine host magma and recharge magma (f_a, f_m and f_r, respectively), enthalpy of crystallization of host and recharge magmas (Δh_m and Δh_r), the fusion enthalpy of country rock (Δh_a), and the average isobaric specific heat capacity of all compositions ($C_{p,a}$, $C_{p,m}$ and $C_{p,r}$). The expression derived in the Appendix is

$$\overline{M}_a^o = \{C_{p,m}(T_m^o - T_{l,m}\overline{T}_{eq}) + \overline{M}_r^o C_{p,r}(T_r^o - T_{l,m}\overline{T}_{eq}) + \Delta h_m[1 - f_m(\overline{T}_{eq})] + \overline{M}_r^o \Delta h_r[1 - f_r(\overline{T}_{eq})]$$
$$+ [A] + [B]\}/[C_{p,a}(T_{l,m}\overline{T}_{eq} - T_a^o) + \Delta h_a f_a(\overline{T}_{eq})] \tag{4}$$

where [A] and [B], non-zero only if $M_e(T_m)$ is non-zero, are defined as

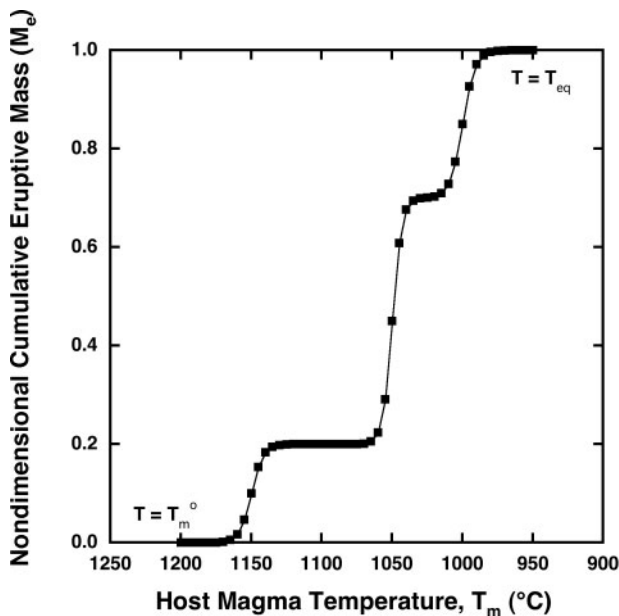$$[A] = -C_{p,m}\int_{T_m^o}^{T_{eq}} \overline{M}_e(T_m)dT_m \tag{5a}$$

Fig. 3. Example of episodic eruption from a magma body. $M_e(T_m)$ is the non-dimensional cumulative mass of eruptive magma as a function of host melt temperature, T_m. T_m falls from the initial value, T_r^o, to the equilibration value, T_{eq}. The function $M_e(T_m)$ is defined by specifying the number of eruptive episodes (N_e). This example is for a three-pulse E′RAχFC event ($N_e = 3$). Four parameters are needed to define each eruptive pulse [see equation (2) in text]. For the example shown, they are [$\Delta M_{e,i}$, $T_{e,i}$, $m_{e,i}$, g_i]: (0·2, 1150°C, 0·12, 2), (0·5, 1050°C, 0·15, 2), (0·3, 1000°C, 0·15, 1·5) with $T_m^o = 1200$°C and $T_{eq} = 950$°C. The sum of the three pulses is $M_e^o = 1·0$.

and

$$[B] = -h_m \int_{T_m^o}^{T_{eq}} \frac{df_m}{dT_m} \overline{M}_e(T_m) dT_m. \qquad (5b)$$

If both $f_m(T_m)$ and $M_e(T_m)$ are linear in T_m, then integral [A] and [B] are evaluated analytically. In this case, equation (4) becomes

$$\overline{M}_a^o = \{C_{p,m}(T_m^o - T_{l,m}\overline{T}_{eq}) + \overline{M}_r^o C_{p,r}(T_r^o - T_{l,m}\overline{T}_{eq}) + \Delta h_m[1 - f_m(\overline{T}_{eq})] + \overline{M}_r^o \Delta h_r[1 - f_r(\overline{T}_{eq})]$$
$$+ \tfrac{1}{2}\overline{M}_e^o(T_{eq} - T_m^o)[C_{p,m} + h_m/(T_{l,m} - T_s)]\}/[C_{p,a}(T_{l,m}\overline{T}_{eq} - T_a^o) + \Delta h_a f_a(\overline{T}_{eq})]. \qquad (6)$$

In the more general case when f_m, M_e or both are non-linear (logistic) functions (Spera & Bohrson, 2002), the integrals [A] and [B] are integrated numerically. Once $M_e(T_m)$, $\overline{M}_r^o$, melt productivity functions and thermodynamic properties of all materials are specified, a unique relationship exists between T_{eq} and M_a^o.

Calculation of thermal and geochemical paths

To compute the thermal, trace element and isotopic paths of anatectic melt, cumulates, enclaves and homogenized host melt within all sub-systems, a set of coupled ordinary nonlinear differential equations expressing conservation of enthalpy, mass, species and isotopic

composition is solved. The output of this computation includes the temperature trajectory of the country rock restite (T_a), the mass of partially molten assimilant produced (M_a^*), the mass of partially molten assimilant incorporated in host melt (χM_a^*), the mass of homogenized melt (M_m) within the magma body, the mass of cumulates formed by fractional crystallization (M_{ct}), the mass of enclaves formed by 'quenching' a portion of recharge magma (M_{en}), the mass of erupted magma M_e as well as the concentration of trace elements in both host melt and erupted melt (C_m), and solids [cumulates (C_{ct}) and possible enclaves (C_{en})]. In addition, the isotopic compositions (radiogenic, ε_m and oxygen, δ_m) in melt and crystalline solids are also determined. The independent variable is the melt temperature T_m, and the calculation ends when T_{eq}, set *a priori*, is reached. We emphasize that T_{eq} must be specified in order to compute the path in temperature–composition space because M_a^o, the mass of country rock involved in E′RAχFC, is a function of T_{eq} given in equation (4).

The model consists of $4 + t + i + s$ differential equations where t is the number of trace elements, i the number of radiogenic isotopic ratios and s the number of stable isotopes considered in the calculation. There are no formal limitations on t, s or i except the patience of the geochemist in dealing with the tyranny of numbers. Pressure is accommodated in EC-E′RAχFC approximately by adjustment of liquidi and the solidus temperatures ($T_{l,a}$, $T_{l,m}$, $T_{l,r}$ and T_s) and melt properties, when such variations can be deduced and are critical to the analysis.

The first two differential equations are known *a priori* and are the differential forms for the mass recharge function and the mass eruptive function. The form of the recharge function has been given by Spera & Bohrson (2002) and is not repeated here. The eruptive mass function is either linear [equation (1)] or logistic [equation (2)]. The third differential equation expresses conservation of energy along the path as country rock heats up, partially melts and thermally equilibrates with host melt that simultaneously undergoes recharge and eruption. Energy conservation leads to a differential expression for the country rock temperature, T_a, as a function of melt temperature, T_m. Anatectic melt of mass χM_a^* generated by country rock partial fusion is assumed to rapidly mix with host melt. The mass of anatectic melt remaining trapped in country rock is $(1 - \chi)M_a^*$. Along the equilibration path, restite {the sum of crystalline country rock of mass $[1 - f_a(T_a)]M_a^o$, and trapped anatectic melt of mass $(1 - \chi)M_a^*$} is not immediately brought into thermal equilibrium with

magma. In contrast, assimilated anatectic melt of mass χM_a^* attains local thermal equilibrium with host melt at each point along the equilibration path $T_m^o \to T_{eq}$. Incomplete extraction of anatectic melt from country

conservation of energy that incorporates all of the above features gives the derivative of the country rock temperature T_a with respect to T_m along the EC-E′RAχFC path to thermal equilibrium at T_{eq}:

$$
\frac{d\overline{T}_a}{d\overline{T}_m} = \left(\frac{-1}{\overline{M}_a^o}\right)\left(\frac{T_{l,m}C_{p,m} + \Delta h_m f'_m(\overline{T}_m) + \chi\overline{M}_a^o C_{p,a}T_{l,m}f_a(\overline{T}_a) + [T_{l,m}C_{p,r} + \Delta h_r f'_r(T_m)]\overline{M}_r(\overline{T}_m)}{T_{l,m}C_{p,a}[1 - \chi f_a(\overline{T}_a)] + [\Delta h_a + \chi C_{p,a}T_{l,m}(\overline{T}_m - \overline{T}_a)]f'_a(\overline{T}_a)}\right.
$$

$$
\left. + \frac{\{T_{l,m}C_{p,r}(\overline{T}_r^o - \overline{T}_m) + [1 - f_r(\overline{T}_m)]\Delta h_r\}\dfrac{d\overline{M}_r}{d\overline{T}_m} - [T_{l,m}C_{p,m} + \Delta h_m f'_m(\overline{T}_m)]\overline{M}_e(T_m)}{T_{l,m}C_{p,a}[1 - \chi f_a(\overline{T}_a)] + [\Delta h_a + \chi C_{p,a}T_{l,m}(\overline{T}_m - \overline{T}_a)]f'_a(\overline{T}_a)}\right) \tag{7}
$$

rock is allowed for by specification of the extraction efficiency factor, χ. In EC-E′RAχFC, χ is a parameter of the simulation ($0 \leq \chi \leq 1$) set *a priori* by the investigator based on available geological evidence. In cases where T_{eq} is less than $T_{l,a}$, the thermal equilibrium condition is $T_a = T_m = T_{eq}$. When $T_{eq} > T_{l,a}$, the equilibrium condition is $T_m = T_{eq}$, as no crystalline restite remains (i.e. $M_a^o = M_a^*$).

All recharge melt initially enters the magma body at temperature $T_{l,r} = T_r^o$, but is thermally equilibrated at the local T_m before mixing with host magma. The process of thermal equilibration depends on the melt temperature, T_m, at the time of replenishment. If $T_r^o > T_m$, then because recharge magma is assumed to be initially intruded at its liquidus temperature ($T_{l,r}$), a fraction of the recharge melt crystallizes to form enclaves. The fraction of recharge melt that solidifies is $1 - f_r(T_m)$ and the resulting increment of mass of enclaves is $dM_{en} = [1 - f_r(T_m)]dM_r$. Enclaves are not in isotopic or trace element

where f'_a and f'_m represent derivatives of the melt productivity functions with respect to T_a and T_m, respectively, and $\overline{M}_a^o$ is computed from equation (4). A derivation of (7) is presented in the Appendix.

The fourth constraint on the geochemical path is conservation of mass and explicitly provides an expression for the variation of the mass of liquid (melt) within the magma body as a function of T_m. The derivative of the mass of melt (M_m) in the host magma body with respect to magma temperature T_m is expressed in terms of the melt productivity functions and their derivatives and the mass recharge function and its derivative with respect to T_m. The mass of melt in the magma body along the path is related to the amount of melt initially present (M_o), the amount added by assimilation of anatectic melt (χM_a^*) and by recharge (M_r, but allowing for enclave formation), the amount removed by cumulate formation (M_{ct}) and, finally, the melt removed by eruption (M_e). The expression has the differential form

$$
\frac{d\overline{M}_m}{d\overline{T}_m} = \overline{M}_a^o \chi f'_a(\overline{T}_a)\frac{d\overline{T}_a}{d\overline{T}_m} + f'_m(\overline{T}_m) + f'_r(\overline{T}_m)\overline{M}_r(\overline{T}_m) + f_r(\overline{T}_m)\frac{d\overline{M}_r}{d\overline{T}_m}
$$

$$
- f'_m(\overline{T}_m)\overline{M}_e(\overline{T}_m) - f_m(\overline{T}_m)\frac{d\overline{M}_e}{d\overline{T}_m} \tag{8}
$$

equilibrium with host melt; they are distinct from cumulates formed by fractional crystallization of host melt. That fraction of recharge melt that does not 'quench' is assumed to mix with host magma (at T_m) and deliver its excess heat, which then becomes available for anatexis of country rock. In this way, there is an intimate connection between recharge and host rock heating and possible contamination, depending on the value of χ. If, on the other hand, $T_r^o < T_m$ at the time of intrusion, heat is extracted from host magma and added to the recharge melt. This can give rise to a 'wave' of cumulate formation and puts limits on the extent of anatexis. Finally, the heat removed associated with the eruption of magma must be accounted for. The differential equation expressing

where primes on f_a, f_m and f_r denote temperature derivatives.

Conservation of species provides the basis for determining trace element abundance in melt as a function of T_m. Initial trace element concentrations in country rock, host melt and recharge melt are C_a^o, C_m^o and C_r^o, respectively. We assume that partial melting of country rock is described by fractional melting so that the concentration of a trace element in the anatectic melt is given by

$$
C_a = \frac{C_a^o}{D_a}[1 - f_a(\overline{T}_a)]^{[(1 - D_a)/D_a]} \tag{9}
$$

where D_a is a function of temperature [see Spera & Bohrson (2001, appendix I)]. Additionally, a distinct bulk melt–solid partition coefficient D_m, also dependent upon temperature, is defined to account for fractionation of the trace element between cumulate and melt. In the enclaves, the distribution of trace element takes place by closed-system fractional crystallization of the recharge melt. Chilling of the recharge melt by the cooler host magma precludes significant chemical mixing. The trace element bulk distribution coefficient that describes the fractionation of a trace element between recharge melt and its associated solid enclave is D_r, which may also be temperature dependent. With these expressions, the species balance expression for the variations of the concentration of a trace element in the host melt within the well-mixed magma body as a function of T_m is

Finally, the differential equation expressing the oxygen isotope balance in host melt is

$$
\frac{d\delta_m}{d\overline{T}_m} = \frac{1}{\overline{M}_m}\left\{ (\delta_a^o - \delta_m)k_{oxy}^a \overline{M}_a^o \chi f'_a(\overline{T}_a)\frac{d\overline{T}_a}{d\overline{T}_m} \right.
$$
$$
\left. + (\delta_r^o - \delta_m)k_{oxy}^r \ f_r(\overline{T}_m)]\frac{d\overline{M}_r}{d\overline{T}_m} \right\}. \tag{12}
$$

Temperature-dependent oxygen fractionation is neglected in (12). This effect is small in magmatic systems (1 or 2‰). In cases where magma and country rock have nearly the same oxygen isotopic ratio, temperature effects may be important, and (12) should be modified to include temperature-dependent oxygen isotope fractionation. The ratios k_{oxy}^a and k_{oxy}^r represent respectively, the mass fraction ratios of oxygen in assimilant to melt and

$$
\frac{d\overline{C}_m}{d\overline{T}_m} = \frac{1}{\overline{M}_m}\left\{ \overline{M}_a^o\chi(s\overline{C}_a - \overline{C}_m)f'_a(\overline{T}_a)\frac{d\overline{T}_a}{d\overline{T}_m} + \overline{C}_m(D_m - 1)[f'_m(\overline{T}_m) + \overline{M}_r(\overline{T}_m)f'_r(\overline{T}_m)] \right.
$$
$$
\left. + t[f_r(\overline{T}_m)^{D_r - 1} - \overline{C}_m]f_r(\overline{T}_m)\frac{d\overline{M}_r}{d\overline{T}_m} - [(D_m - 1)\overline{C}_mf_m(T_m)]\overline{M}_e(T_m) \right\}. \tag{10}
$$

The species balance equation accounts for the formation of enclaves and cumulates, the introduction of anatectic melt derived by fractional fusion of country rock (assimilant), and both recharge and eruption of magma. The composition of the recharge magma is an initial condition whereas the composition of melt removed during eruption is identical to melt within the host magma body at the moment of eruption.

For an isotopic ratio in the host melt ε_m, the differential equation is

$$
\frac{d\bar{\varepsilon}_m}{d\overline{T}} = \frac{1}{\overline{M}_m}\left\{ s\frac{\overline{C}_a}{\overline{C}_m}(\varepsilon_a^o - \varepsilon_m)\overline{M}_a^o\chi f'_a(\overline{T}_a)\frac{d\overline{T}_a}{d\overline{T}_m} \right.
$$
$$
\left. + \frac{t}{\overline{C}_m}[(f_r(\overline{T}_m)]^{D_r - 1}(\varepsilon_r^o - \varepsilon_m)f_r(\overline{T}_m)\frac{d\overline{M}_r}{d\overline{T}_m} \right\} \tag{11}
$$

where ε_a^o, ε_r^o and ε_m represent the isotopic ratio [e.g. $\varepsilon_m = (^{87}Sr/^{86}Sr)_{melt}$] of anatectic melt (identical to country rock because we assume there is no fractionation of isotopes during partial melting), recharge melt and host melt at T_m, respectively. It should be noted that the epsilons refer to isotope ratios rather than the conventional definitions. Radiogenic in-growth and temperature-dependent isotopic fractionation are neglected in (11). Isotopic equilibrium is assumed to prevail between cumulates and melt; enclaves, on the other hand, are assumed to be in isotopic equilibrium with recharge melt of initial isotopic composition ε_r^o.

recharge melt to host melt before ERAFC processes. The mass fraction of oxygen in most natural compositions is about 47% and varies relatively little.

In addition to the primary variables computed by solution of the differential equations, other quantities may be calculated. For example, in a natural system, compositional data are sometimes available for the crystalline products (cumulates and enclaves), anatectic melt and eruptive products. The EC-E′RAχFC solution links all parts of the composite system to one another. When modeling natural systems, it is important to consider the composition of cumulates, enclaves, and anatectic melt as well as the composition of the evolving host melt so as to obtain as robust a solution as possible. The mass, trace element and isotopic compositions (path average and instantaneous) of all solids (cumulates and enclaves) and of anatectic melt along the thermal equilibration path $T_m^o \to T_{eq}$ are part of the E′RAχFC solution. Expressions for these quantities are given in the Appendix.

The set of $4 + t + i + s$ coupled ordinary differential equations representing E′RAχFC evolution is posed as an initial value problem with T_m as the independent variable. This set of differential equations is subject to the following initial conditions: at $T = T_m^o$, $T_a = T_a^o$, $T_r = T_r^o$, $M_m = M_o$, $C_m = C_m^o$, $C_a = C_a^o$, and $C_r = C_r^o$ for t trace element species, $\varepsilon_m = \varepsilon_m^o$, $\varepsilon_a = \varepsilon_a^o$, and $\varepsilon_r = \varepsilon_r^o$ for i isotope species and $\delta_m = \delta_m^o$, $\delta_a = \delta_a^o$, and $\delta_r = \delta_r^o$ for oxygen. Once cast into dimensionless form, the system of equations are numerically solved by a fourth-order Runge–Kutta method. The input and

output are presented in a code programmed in Visual Basic. A copy of the EC-E'RAχFC code is available at http://magma.geol.ucsb.edu/ and on the *Journal of Petrology* web site at http://www.petrology.oupjournals.org.

APPLICATIONS OF EC-E'RAχFC

The utility, generality and robustness of the EC-E'RAχFC model is best discovered by systematic application to a wide range of magmatic systems. This work has just begun [e.g. see Fowler *et al.* (2004) for application to a portion of the British Tertiary Igneous Province]. Here we provide a few examples to illustrate application of EC-E'RAχFC to situations where extraction of wall-rock partial melt may be variably efficient and where eruption and recharge interact non-linearly to alter the geochemical path of the sequence of melts along the approach to thermal equilibrium. These examples are not intended to be exhaustive, only illustrative.

Geologic evidence suggests that in some cases not all of the wall-rock melt generated during an ERAFC interaction is assimilated into the host magma body (e.g. Grove & Kinzler, 1986; see also James, 1981). For example, migmatites probably represent crustal sections that have undergone partial melting, but where melt has not been fully extracted (Johannes & Gupta, 1982). It is therefore important to examine the consequences of imperfect extraction (and hence incomplete addition) of anatectic melt to an evolving magma body. There is also abundant evidence that suggests recharge and eruption may be linked in open systems (Sparks & Sigurdsson, 1977; Blake, 1981). The ability to accommodate distinct $M_e(T_m)$ vs $M_r(T_m)$ paths in EC-E'RAχFC allows us to investigate the chemical consequences of the relationships between these two processes. Below, we begin by illustrating selected geochemical traits of a lower-crustal magma body undergoing E'RAχFC; we follow this with specific comparisons of systems that are variably affected by assimilation, recharge and eruption.

EC-E'RAχFC models of intrusion of mafic magma into lower crust of mafic–intermediate composition

Figure 4a–d illustrates selected geochemical characteristics of a basaltic magma intruded into lower crust of mafic–intermediate composition initially at 600°C. The geochemical paths revealed in the figure simulate evolution from 1320°C ($T_m^o = T_{l,m}$) to an equilibration temperature of 1090°C (T_{eq}). Four cases are shown: AFC where $\chi = 1$; AχFC where $\chi = 0.5$; E'RAFC where M_e and M_r each are 0·3, are continuous, and $\chi = 1$; E'RAχFC where M_e and M_r each are 0·3 and are continuous, and $\chi = 0.5$. Modeling parameters are summarized in Table 2.

Figure 4a shows the [Sr] vs ^{87}Sr/^{86}Sr trajectory for the four cases. The flat trends near $T = T_m^o$ for the two AFC cases reflect heating up of wall rock to its solidus. Over approximately the same temperature range, the decrease in ^{87}Sr/^{86}Sr for the two cases involving recharge and eruption reflects addition of recharge magma that has a less radiogenic Sr isotope value. As T_m continues to fall in all four cases, the degree of heterogeneity in Sr isotope values over a small range of [Sr] is a consequence of wall-rock fractional melting of an element that is incompatible (wall-rock $D_{Sr} = 0.05$); the initial, low-degree melting of the wall rock releases high concentrations of Sr into the host magma, yielding distinct changes in ^{87}Sr/^{86}Sr as well as an increase in [Sr] in the host magma (despite Sr being compatible in magma; $D_{Sr} = 1.5$). The flattening of all trends as T_m becomes close to T_{eq} is also an outcome of fractional melting of an incompatible element; most of the Sr has been stripped from the wall rock, so while mass exchange is still continuing (i.e. assimilation is continuing), little Sr is being added from the wall rock to the host magma. Thus, little change in the isotope ratio occurs, although for the cases involving recharge, there is a slight decrease in ^{87}Sr/^{86}Sr because of the addition of the less radiogenic recharge magma. An interesting observation regarding all of the [Sr] vs ^{87}Sr/^{86}Sr trends is the large degree of isotopic heterogeneity evident over a fairly restricted, relatively elevated range of Sr. ^{87}Sr/^{86}Sr varies from ~0·705 to ~0·7105 over a range of ~550 to 700 ppm Sr. In the absence of consideration of the influence that shallow-level processes might have on such trends, this type of compositional variation might incorrectly be attributed to mantle heterogeneity or the mixing of two isotopically distinct primary liquids.

The marked differences in Sr paths between cases where $\chi = 1.0$ and $\chi = 0.5$ illustrate the sensitivity of geochemical parameters to the efficiency of extraction of wall-rock partial melt. For these cases, full extraction yields ^{87}Sr/^{86}Sr of ~0·7105, whereas 50% extraction yields lower ratios near 0·7082.

More subtle distinctions in isotopic ratios are evident through comparison of the AFC and E'RAFC cases. In the cases illustrated here, compared with the AFC cases, continuous eruption and recharge ($M_r = M_e = 0.3$) yield slightly more radiogenic ^{87}Sr/^{86}Sr values. Compared with the AFC cases, the slightly more crustal signatures associated with the E'RAFC cases are at least partly a consequence of their higher M_a^* (at T_{eq}: AFC, $\chi = 1$, $M_a^* = 0.59$; E'RAFC, $\chi = 1$, $M_a^* = 0.67$; and AχFC, $\chi = 0.5$, $M_a^* = 0.295$; E'RAχFC, $\chi = 0.5$, $M_a^* = 0.335$), which reflect the additional energy provided by recharge magma. In these cases, the net effect of recharge and eruption is to increase the amount of energy available for heating and melting of wall rock. Although the total masses of material continuously added by recharge and subtracted by eruption are equal (0·3), the recharge magma has
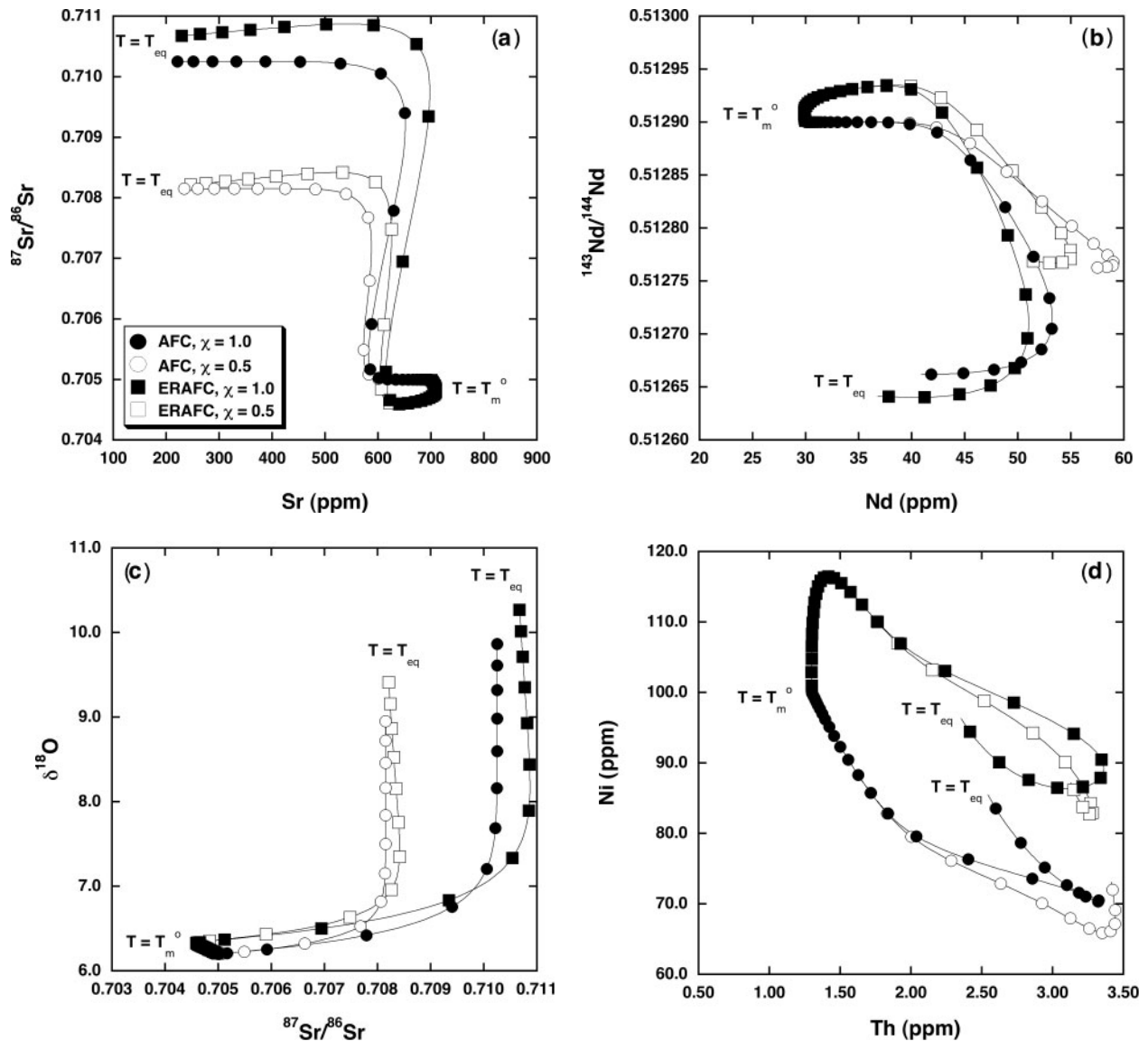
Fig. 4. (a) Sr (ppm) vs $^{87}Sr/^{86}Sr$, (b) Nd (ppm) vs $^{143}Nd/^{144}Nd$, (c) $^{87}Sr/^{86}Sr$ vs $\delta^{18}O$, and (d) Th (ppm) vs Ni (ppm) for case where mafic magma intrudes into lower crust of mafic–intermediate composition. Two EC-AχFC models are illustrated, one with $\chi = 1{\cdot}0$ and one with $\chi = 0{\cdot}5$. E'RAχFC models reflect continuous recharge and eruption ($M_e = M_r = 0{\cdot}3$), and results for both $\chi = 1{\cdot}0$ and $\chi = 0{\cdot}5$ are shown. Recharge magma is modeled as more mafic than pristine magma. Models run from T_m^o to T_{eq}, and each symbol represents a fall of approximately 7°C. Additional model parameters are listed in Table 2.

$T_r^o = T_{l,r} = 1320°C$, whereas magma is removed at each T_m as the simulation runs from $T_m^o \rightarrow T_{eq}$. Thus, for these cases, the amount of energy added by recharge exceeds the amount removed by eruption.

The trajectories for Nd are similar in form to those for Sr. The AFC trends show an initial flat trend, followed at lower T_m by marked changes in $^{143}Nd/^{144}Nd$ over a relatively restricted range of [Nd]. The trajectories terminate with relatively flat trends as Nd decreases. The E'RAχFC are similar, although recharge at relatively high T_m yields an increase in $^{143}Nd/^{144}Nd$ as a result of the radiogenic nature of

recharge magma. As T_m approaches T_{eq}, [Nd] decreases despite it being incompatible in all sub-systems ($D_{Nd} = 0{\cdot}25$, Table 2). These decreases are a consequence of the 'dilution effect' that occurs when most of an incompatible element has been stripped from the wall rock by fractional melting. At lower T_m, although assimilant is still being added, there is so little Nd being contributed to the host melt that its concentration in the host is actually diluted. By comparison of the $\chi = 1{\cdot}0$ vs $\chi = 0{\cdot}5$ cases, it is evident that the degree of dilution is less in the cases where incomplete extraction occurs.

Table 2: EC-E′RAχFC parameters for intrusion of mafic magma into lower crust of mafic–intermediate composition

Thermal parameters

Pristine magma liquidus temperature, $T_{l,m}$	1320°C	Crystallization enthalpy, Δh_m (J/kg)	396000
Pristine magma initial temperature, T_m^o	1320°C	Isobaric specific heat of magma, $C_{p,m}$ (J/kg K)	1484
Recharge magma liquidus temperature, $T_{l,m}$	1320°C	Crystallization enthalpy, Δh_r (J/kg)	396000
Recharge magma initial temperature, T_m^o	1320°C	Isobaric specific heat of magma, $C_{p,m}$ (J/kg K)	1484
Assimilant liquidus temperature, $T_{l,a}$	1100°C	Fusion enthalpy, Δh_a (J/kg)	354000
Assimilant initial temperature, T_a^o	600°C	Isobaric specific heat of assimilant, $C_{p,a}$ (J/kg K)	1388
Solidus temperature, T_s	950°C		
Equilibration temperature, T_{eq}	1090°C		

$T_m - M_r^o - M_e^o$ *for episodic recharge cases*

	Recharge before eruption	Eruption before recharge
Recharge ($M_r^o = 0.3$)	1225°C	1180°C
Eruption ($M_e^o = 0.3$)	1180°C	1225°C

Recharge and eruption occur in single pulse at temperatures indicated.

Compositional parameters

	Sr	Nd	Th	Ni	$\delta^{18}O$
Pristine magma initial concentration (ppm), C_m^o	700	30	1·3	100	
Pristine magma isotope ratio, ε_m	0·7050	0·5129			6·2
Pristine magma trace element distribution coefficient, D_m	1·5	0·25	0·1	2	
Recharge magma initial concentration (ppm), C_r^o	1000	20	1	300	
Recharge magma isotope ratio, ε_r	0·7028	0·5132			7·0
Recharge magma trace element distribution coefficient, D_r	1·5	0·25	0·1	2	
Assimilant initial concentration (ppm), C_a^o	230	12·7	1·06	135	
Assimilant isotope ratio, ε_a	0·7200	0·5120			12·0
Assimilant trace element distribution coefficient, D_a	0·05	0·25	0·1	2	

Non-linear parameterization of $f_a(T)$, $f_m(T)$, and $f_r(T)$ used.

Figure 4c illustrates Sr isotope–O isotope trajectories for the four cases. For the cases modeled, at higher T_m, Sr isotope signatures vary markedly whereas the variations in $\delta^{18}O$ are relatively restricted. At lower T_m, distinct changes in $\delta^{18}O$ are accompanied by modest variations in $^{87}Sr/^{86}Sr$. An instructive observation regarding these trends is the concavity of the curves. Based on previous AFC models, such concavity has been linked to 'source' contamination rather than 'crustal' contamination (e.g. James, 1981; Taylor & Sheppard, 1986). This figure therefore underscores the complex coupling among energy, mass, and species, and also emphasizes the importance of accommodating the compositional changes experienced by wall rock as partial melting occurs. EC-E′RAχFC results such as these further emphasize the necessity of assessing shallow-level processes prior to discussing mantle characteristics.

Figure 4d provides an example of E′RAχFC trends in element–element space. In this instance, the behavior of an incompatible element (Th) is compared with that of a compatible element (Ni). For the AFC cases, at higher T_m, [Th] initially increases, consistent with its incompatible behavior in all sub-systems. [Ni] decreases because of its compatible nature. However, at higher T_m, [Th] decreases whereas [Ni] increases. The reversals in the trends of these elemental concentrations are tied to melting in the wall rock. Th, being incompatible, is stripped from the wall rock, and at lower T_m, little is being added (i.e. dilution effect). In contrast, for fractional melting of a compatible element, higher degrees of fractional melting
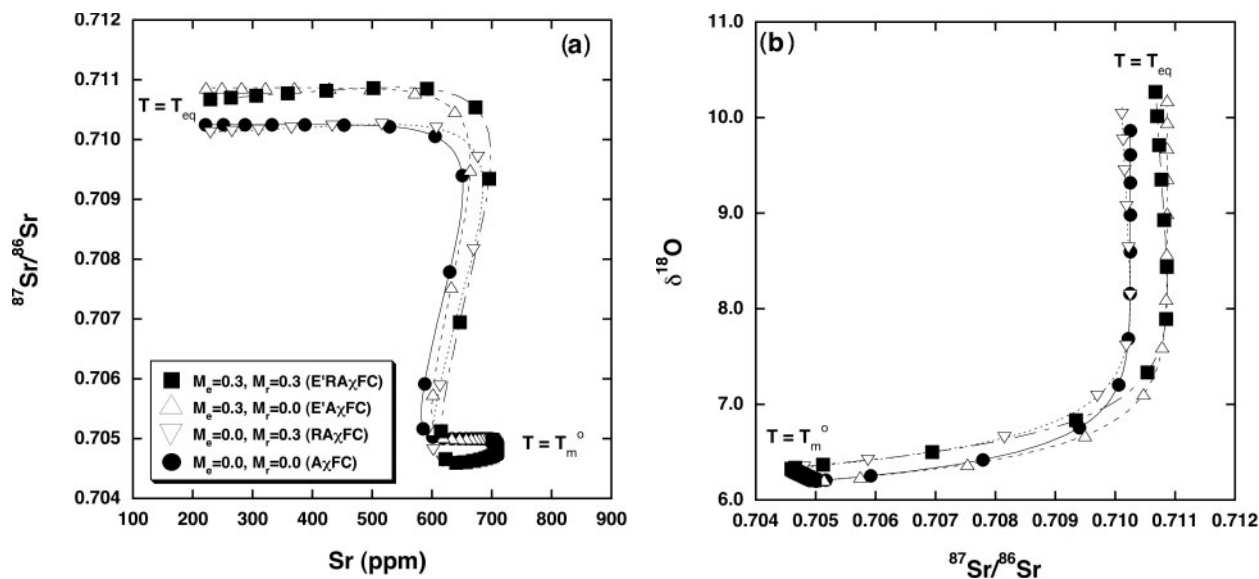
Fig. 5. (a) Sr (ppm) vs $^{87}Sr/^{86}Sr$ and (b) $^{87}Sr/^{86}Sr$ vs $\delta^{18}O$ model results for continuous recharge and eruption. Four cases are shown: E'RAχFC where $M_e = 0.3$ and $M_r = 0.3$; E'AχFC where $M_e = 0.3$ and $M_r = 0.0$; RAχFC where $M_e = 0.0$ and $M_r = 0.3$; AχFC where $M_e = 0.0$ and $M_r = 0.0$. Models run from T_m^o to T_{eq}, and each symbol represents a fall of approximately 7°C. Additional model parameters are listed in Table 2.

yield higher concentrations of the element. Thus, [Ni] increases. The effects of recharge are seen in the E'RAFC cases, where Ni initially increases because of its higher concentration in the recharge magma. A critical aspect of note for these trends is that EC-E'RAχFC does not necessarily yield monotonic trends in element–element space; thus, the magma parcel that has experienced the most E'RAFC is not necessarily the one with the highest Th or the lowest Ni. Such results emphasize the need to use a multifaceted approach when attempting to identify open-system magmas.

Chemical consequences of continuous eruption and recharge

There is abundant evidence to suggest that the dynamics of recharge and eruption may be linked (e.g. Sparks & Sigurdsson, 1977; Blake, 1981). To highlight the compositional consequences of this coupling, Fig. 5 summarizes cases in which combinations of recharge and eruptive masses are varied. Four cases are illustrated: E'RAχFC where $M_e = 0.3$ and $M_r = 0.3$; E'AχFC where $M_e = 0.3$ and $M_r = 0.0$; RAχFC where $M_e = 0.0$ (no eruption) and $M_r = 0.3$; AχFC where $M_e = 0.0$ and $M_r = 0.0$. For all cases, recharge and eruptive masses are linear functions of temperature (i.e. continuous recharge and/or eruption) and $\chi = 1.0$. All other parameters are the same as those listed in Table 2.

At T_{eq}, Sr isotopes (Fig. 5a) vary, although the four cases cluster into two groups—the two characterized by eruption ($^{87}Sr/^{86}Sr \approx 0.7108$), and the two lacking eruption ($^{87}Sr/^{86}Sr \approx 0.7102$). Sr concentrations at T_{eq}

are similar among the four cases. Examination of Fig. 5b, $^{87}Sr/^{86}Sr$ vs $\delta^{18}O$, shows the same groupings, with the cases involving eruption having slightly more crust-like O and Sr isotopes. These results are indicative of the complex consequences of the energetic and mass coupling of the sub-systems. A predictable consequence of an eruption is that less energy is available for transfer to country rock. Indeed, for the E'AχFC case, the normalized mass of wall rock melted and assimilated into the host chamber (χM_a^*) is the smallest, whereas that for the RAχFC case is the largest (χM_a^*: E'AχFC $= 0.49$, RAχFC $= 0.77$, E'RAχFC $= 0.67$, and AχFC $= 0.59$). The relatively strong signature of crustal contamination in the E'AχFC case is a function of the coupling between M_a^* and M_m, which is also predictably the smallest of the four cases (M_m: E'AχFC $= 0.58$, RAχFC $= 0.94$, E'RAχFC $= 0.80$, and AχFC $= 0.72$). Thus, at T_{eq}, although the mass of melt assimilated is the smallest, the total mass of melt is also the smallest. Together with the other parameters of the simulation, these system characteristics yield the most contaminated signatures. A key point to appreciate is that because of the coupling, predictions about how open systems will behave as they evolve are not straightforward. Rules of thumb that have guided assessment of the importance of open-system processes (such as a correlation between geochemical signature and amount of contamination) therefore need to be abandoned because of the inherent non-linearity implied when the energetics of melting is coupled to trace element conservation linked through the processes of partial melting, recharge, incomplete contamination ($\chi < 1$) and eruption.
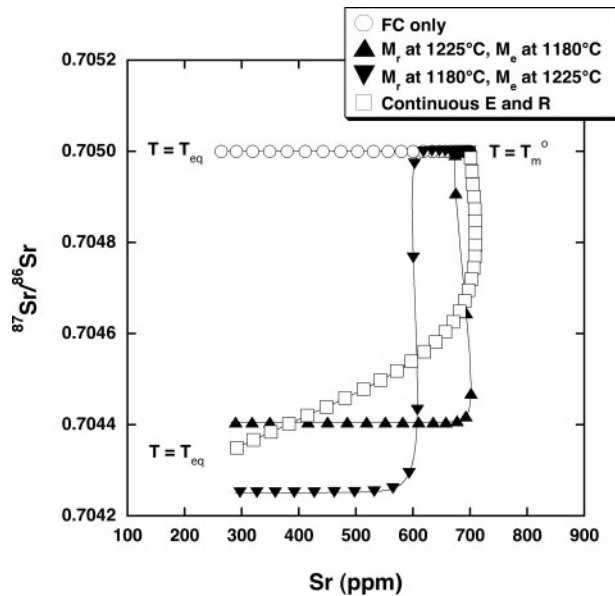
Fig. 6. Sr (ppm) vs ^{87}Sr/^{86}Sr model results for two cases of episodic recharge and eruption where the magma temperature at which the pulse occurs varies ($M_r = 0{\cdot}3$ at 1225°C and $M_e = 0{\cdot}3$ at 1180°C; $M_r = 0{\cdot}3$ at 1180°C and $M_e = 0{\cdot}3$ at 1225°C). Also shown for comparison are a case of continuous recharge and eruption ($M_e = M_r = 0{\cdot}3$) and a case where no recharge or eruption occurs ($M_e = M_r = 0{\cdot}0$). To emphasize the effects of episodic eruption and recharge, for all cases, χ was set at zero. Models run from T_m^o to T_{eq}, and each symbol represents a fall of approximately 7°C. Additional model parameters are listed in Table 2.

Chemical consequences of episodic eruption and recharge

How sensitive is magma composition to the timing of recharge and eruption? Figure 6 summarizes results of simulations involving episodic recharge and eruption ($M_r = 0{\cdot}3$ at 1225°C and $M_e = 0{\cdot}3$ at 1180°C; $M_r = 0{\cdot}3$ at 1180°C and $M_e = 0{\cdot}3$ at 1225°C). The first case, in which recharge occurs at a higher T_m than eruption, is probably geologically more likely, but both cases are shown for purposes of illustration. Two additional cases are also portrayed for comparison: continuous $M_e = M_r = 0{\cdot}3$ (continuous recharge and eruption), and $M_e = M_r = 0{\cdot}0$ (no recharge or eruption). To better discuss the effects of episodic recharge and eruption, for these cases, χ was set equal to zero. This means that the evolving magma body is not contaminated by introduction of anatectic melts. That is, partial melts necessarily generated in the country rock remain there throughout the E′RFC evolution. Input parameters for these models are listed in Table 2; where appropriate, parameters are the same as those for the cases shown in Figs 4 and 5.

The model in which eruption occurs at a higher T_m than recharge yields the most extreme ^{87}Sr/^{86}Sr (i.e. that most like recharge magma). This is because at relatively high T_m, 30% of the original normalized mass of the

magma body is erupted ($M_e = 0{\cdot}3$), and thus, the mass of Sr in the chamber is reduced. When recharge occurs at a lower T_m, its Sr, which is less radiogenic than that in the pristine magma, has a proportionally greater effect on the host magma, thereby yielding a less radiogenic ^{87}Sr/^{86}Sr as $T_m \rightarrow T_{eq}$.

For the case in which recharge occurs at a higher T_m than eruption, at the T_m of the recharge event, ^{87}Sr/^{86}Sr of the host magma decreases because recharge magma has a less radiogenic signature than pristine magma. Because the mass of the magma chamber at this temperature is larger than that for the previous case, the lever effect from recharge is not as large, and therefore the Sr isotope signature is higher. For the parameters of these cases, continuous recharge and eruption yield ^{87}Sr/^{86}Sr between those of the two episodic cases.

Systems that undergo episodic recharge and eruption can develop distinctly different geochemical characteristics; thus, the geochemical evolution (i.e. T_m–geochemical path) of these systems may be very different. Because T_m is the E′RAχFC progress variable, it can be considered as a proxy for time, and thus, in natural systems in which there is geologic evidence for episodic recharge and eruption, it may be possible to hypothesize about the relative timing and magnitude of such events.

Finally, the evolution of systems characterized by episodes of recharge and eruption ($\pm$ assimilation) may be distinct from those that undergo continuous recharge and eruption ($\pm$ assimilation). Distinguishing between these scenarios geochemically relies not only upon high-quality geochemical data, but also on the ability to place samples in an evolutionary framework. For these reasons, the collection of relative and absolute geochronological information on igneous systems is of paramount importance.

SUMMARY

EC-E′RAχFC is a thermodynamic model that enforces energy conservation and total mass, trace element and isotope material balance in the composite system composed of magma body, country rock, replenishment reservoir and extrusive (eruptive) reservoir. EC-E′RAχFC represents an extension of the earlier EC-AFC and EC-RAFC models and includes the effects of: (1) episodic or continuous magma removal by eruption; (2) variable addition of anatectic melt generated by wall-rock partial fusion. Results provide information on the trace element and isotopic composition of host melt, eruptive magma and crystalline products (cumulates and non-equilibrium enclaves) and the average wall-rock temperature along the equilibration path. Input parameters are based on knowledge of the magmatic system of interest as well as relevant thermodynamic properties such as the isobaric heat capacity, enthalpies of transition

and bulk partition coefficients for trace elements. In this study we have not focused attention on the predicted compositions of cumulates and enclaves, although the EC-E'RAχFC model makes predictions that can be tested, provided samples are available.

Several forward models provide insight into the theoretical behavior of E'RAχFC magmatic systems. The geochemical consequences of imperfect extraction of wall-rock melt into the host magma body can be profound. A fruitful area of research involves linking geochemical signatures of melt and associated solids with field evidence of imperfect extraction from wall rock. In addition, theoretical work that characterizes how melt migrates may reveal systematics regarding how transport processes work in different magmatic environments. Simulations that illustrate the effects of different recharge–eruption–temperature paths provide convincing evidence that isotope and trace element signatures can be dramatically affected by E'RAχFC processes. Hence, it is clear that the crustal-level geochemical evolution of a magma body must be evaluated from a mass, species and energy context before assignment of mantle characteristics can reliably be made. In addition, we emphasize that documenting magma chamber processes is inextricably linked to placing samples in evolutionary context. To develop meaningful models of how magmas evolve, collection of high-quality geochemical and geochronological data is required.

E'RAχFC represents the culmination of a number of years of model development. Based on modeling results from several natural systems (Bohrson & Spera, 2001, 2003), the approach holds promise for enhancing our understanding of the behavior of open-system magmatic processes. In particular, the most detailed application to date, by Fowler *et al.* (2004) on rocks of the British Tertiary Igneous Complex, illustrates the potential the model has for fingerprinting mantle vs crustal processes. Further exploration of issues such as this, and application of E'RAχFC to other natural datasets, will continue to build on contributions that M. J. O'Hara has made to understanding open-system magma chambers over the last 25 years.

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SUPPLEMENTARY DATA

Supplementary data for this paper are available at *Journal of Petrology* online.

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APPENDIX

Derivation of integral energy balance

The starting point in the derivation is the differential form for the heat available from cooling and crystallization of magma (h_{avail}) and the heat absorbed by country rock for heating and partial fusion (h_{abs}). The expressions are from Spera & Bohrson (2002) with addition of a term associated with the removal of magma from the geochemically evolving magma body. The expressions for the available and absorbed enthalpy are

Similarly, equation (A2) is integrated between the limits T_a^o and T_{eq} to give the heat absorbed (by convention a positive quantity) by country rock of M_a^o mass initially at temperature T_a^o to raise its temperature to T_{eq} and partially melt it to the extent $f_a(T_{eq})$. The result is

$$h_{abs} = h_{abs}(T_{eq}) - h_{abs}(T_a^o)$$
$$= M_a^o C_{p,a}(T_{eq} - T_a^o) + f_a(T_{eq})M_a^o h_a. \qquad (A4)$$

$$\frac{dh_{avail}}{dT_m} = [M_o - M_e(T_m)][C_{p,m} + h_m f'_m(T_m)] + M_r(T_m)[C_{p,r} + h_r f'_r(T_m)]$$
$$+ \frac{dM_r}{dT_m}\{C_{p,r}(T_m - T_r^o) + h_r[f_r(T_m) - 1]\} \qquad (A1)$$

and

$$\frac{dh_{abs}}{dT_a} = M_a^o C_{p,a}[1 - \chi f_a(T_a)] + M_a^o \chi C_{p,a} f'_a(T_a)(T_m - T_a)$$
$$+ M_a^o \chi C_{p,a} f_a(T_a)\frac{dT_a}{dT_m} + M_a^o h_a f'_a(T_a). \qquad (A2)$$

Now, equation (A1) is integrated between the limits T_m^o and T_{eq} using integration by parts and allowing for nonlinear melting and eruptive mass functions, $f_m(T_m)$ and $M_e(T_m)$, respectively. This gives the amount of heat available for heating cool country rock (by convention a negative quantity) by the cooling and crystallization of magma allowing for heat exchange owing to the processes of recharge and eruption. The result after algebraic simplification is

Now, energy conservation demands that

$$\Delta h_{abs} + \Delta h_{avail} = 0. \qquad (A5)$$

A value M_a^o of as a function of T_{eq} may be found by combining (A3) and (A4) and invoking energy conservation. The result is

$$h_{avail} = h_{avail}(T_{eq}) - h_{avail}(T_m^o) = M_o\{C_{p,m}(T_{eq} - T_m^o) + \Delta h_m[f_m(T_{eq}) - 1]\} + M_r^o\{C_{p,r}(T_{eq} - T_r^o)$$
$$+ \Delta h_r[f_r(T_{eq}) - 1]\} - C_{p,m}\int_{T_m^o}^{T_{eq}} \overline{M}_e(T_m)dT_m - h_m\int_{T_m^o}^{T_{eq}} \frac{df_m}{dT_m}\overline{M}_e(T_m)dT_m. \qquad (A3)$$

$$\overline{M}_a^o = \{C_{p,m}(T_m^o - T_{l,m}\overline{T}_{eq}) + \overline{M}_r^o C_{p,r}(T_r^o - T_{l,m}\overline{T}_{eq}) + \Delta h_m[1 - f_m(\overline{T}_{eq})] + \overline{M}_r^o \Delta h_r[1 - f_r(\overline{T}_{eq})]$$
$$+ [A] + [B]\}/[C_{p,a}(T_{l,m}\overline{T}_{eq} - T_a^o) + \Delta h_a f_a(\overline{T}_{eq})] \tag{A6}$$

where

$$[A] = -C_{p,m}\int_{T_m^o}^{T_{eq}} \overline{M}_e(T_m)\mathrm{d}T_m \tag{A7}$$

and

$$[B] = -h_m\int_{T_m^o}^{T_{eq}} \frac{\mathrm{d}f_m}{\mathrm{d}T_m}\overline{M}_e(T_m)\mathrm{d}T_m \tag{A8}$$

which is identical to equations (4), (5a) and (5b) in the text. It should be noted that M_a^o is independent of χ, as the heat absorbed by country rock depends only on the initial and final temperature of country rock (T_a^o and T_{eq}, respectively) and the extent of partial melting.

Derivation of path energy conservation equation

The energy conservation principle is used to compute the mean temperature of country rock, of mass M_a^o, involved in E′RAχFC evolution as a function of the host magma temperature, T_m. The quantity M_a^o is determined, as outlined above, by the integral energy balance and is needed for solution of the differential equation relating T_a to T_m.

Differentiation of equation (A5) with respect to T_m and use of the chain rule gives

$$\frac{\mathrm{d}h_{abs}}{\mathrm{d}T_a}\frac{\mathrm{d}T_a}{\mathrm{d}T_m} + \frac{\mathrm{d}h_{avail}}{\mathrm{d}T_m} = 0. \tag{A9}$$

Average and instantaneous compositions of solids along ERAFC path

Once the concentration of trace element in host melt is known, trace element concentrations in solids (cumulates and enclaves) may be determined. E′RAχFC computed compositions provide a relative compositional chronology of cumulates and possible enclaves.

The instantaneous concentration of a trace element in enclave (at $T = T_m$) is

$$C_{en} = C_r^o D_r[f_r(\overline{T}_m)]^{D_r - 1} \tag{A11}$$

whereas the average trace element concentration in enclaves along the path $T_m^o \to T_m$ found by integration of (A11) along the ERAFC path is

$$\hat{C}_{en} = \frac{C_r^o [1 - f_r^{D_r}(\overline{T}_m)]}{[1 - f_r(\overline{T}_m)]}. \tag{A12}$$

The instantaneous concentration of trace element in cumulate at T_m is given by

$$C_{ct} = C_m^o \overline{C}_m D_m \tag{A13}$$

where $\overline{C}_m$ is calculated from equation (10) in the text. The average trace element composition in cumulates formed along the path $T_m^o \to T_{eq}$ is

$$C_{ct} = \frac{C_m^o D_m \left[\int_{T_m^o}^{T_{eq}} f_r(\overline{T}_m)M_r(\overline{T}_m)\overline{C}_m\mathrm{d}T_m + \int_{T_m^o}^{T_{eq}} f_m(\overline{T}_m)\overline{C}_m\mathrm{d}T_m \right]}{\int_{T_m^o}^{T_{eq}} f_r(\overline{T}_m)M_r(\overline{T}_m)\mathrm{d}T_m + \int_{T_m^o}^{T_{eq}} f_m(\overline{T}_m)\mathrm{d}T_m}. \tag{A14}$$

Now, equations (A1) and (A2) are substituted into (A9) and the result is simplified to give

$$\frac{\mathrm{d}\overline{T}_a}{\mathrm{d}\overline{T}_m} = \left(\frac{-1}{\overline{M}_a^o}\right)$$
$$\left(\frac{T_{l,m}C_{p,m} + \Delta h_m f_m'(\overline{T}_m) + \overline{M}_a^o \chi C_{p,a}T_{l,m}f_a(\overline{T}_a) + [T_{l,m}C_{p,r} + \Delta h_r f_r'(\overline{T}_m)]\overline{M}_r(\overline{T}_m)}{T_{l,m}C_{p,a}[1 - \chi f_a(\overline{T}_a)] + [f_a'(\overline{T}_a)\Delta h_a + C_{p,a}T_{l,m}f_a'(\overline{T}_a)\chi(\overline{T}_m - \overline{T}_a)]}\right.$$
$$\left. + \frac{\{T_{l,m}C_{p,r}(T_r^o - \overline{T}_m) + [1 - f_r(\overline{T}_m)]\Delta h_r\}\frac{\mathrm{d}\overline{M}_r}{\mathrm{d}\overline{T}_m} - [T_{l,m}C_{p,m} + \Delta h_m f_m'(\overline{T}_m)]\overline{M}_e(T_m)}{T_{l,m}C_{p,a}[1 - \chi f_a(\overline{M}_a)] + [f_a'(\overline{T}_a)\Delta h_a + C_{p,a}T_{l,m}f_a'(\overline{T}_a)\chi(\overline{T}_m - \overline{T}_a)]}\right) \tag{A10}$$

which is identical to equation (7) in the text.

Recharge melt can be either a source (if $T_r^o > T_m$) or sink (if $T_r^o < T_m$) of enthalpy (heat). Enclaves form in the former case as a result of chilling of recharge melt intruded into cooler host magma. When $T_r^o > T_m$, heat is also available from non-equilibrium crystallization of enclaves liberating specific latent heat Δh_r. The mass of enclaves is given by the integral

$$M_{en} = \int_{T_M^o}^{T_{eq}} M_r'(T_m)[1 - f_r(T_m)]dT_m. \qquad (A15)$$

The total mass of cumulates (M_{ct}) formed by fractional crystallization is

$$M_{ct} = \int_{T_M^o}^{T_{eq}} - f_r'(T_m)\overline{M}_r(\overline{T}_m)d\overline{T}_m$$
$$- \int_{T_M^o}^{T_{eq}} f_m'(\overline{T}_m)d\overline{T}_m. \qquad (A16)$$

The primes in the above expressions mean that derivatives with respect to T_m have been taken. The total mass of all solid products generated during an ERAFC event (M_s) is given by $M_s = M_{en} + M_{ct}$ using (A15) and (A16).

For partial melting in wall rock, we assume fractional melting occurs. The concentration of trace element in anatectic liquid at temperature T_a is

$$C_a = \frac{C_a^o}{D_a}[1 - f_a(\overline{T}_a)]^{[(1 - D_a)/D_a]} \qquad (A17)$$

where D_a is the distribution coefficient (the equilibrium constant of the trace element distribution reaction) between anatectic melt and residual (unassimilated) wall rock. The average concentration of trace element of anatectic melt generated by partial fusion of country rock in the temperature interval $T_a^o \to T_a$ is

$$\hat{C}_a = \frac{C_a^o}{f_a(T_a)}\left\{1 - [-f_a(T_a)]^{1/D_a(T_a)}\right\}. \qquad (A18)$$

When the initial temperature of recharge melt exceeds that of host magma (i.e. $T_r^o > T_m$) the fraction $[1 - f_r(T_m)]dM_r$ of recharge chills to form solid enclaves. The initial recharge melt temperature (T_r^o) is assumed

equal to its liquidus temperature, $T_{l,r}$. The concentration of trace element available for mixing into host magma differs from that in pristine recharge melt because of depletion or enrichment of trace element as a result of enclave crystallization. This process is modeled as closed-system fractional crystallization according to

$$C_r = C_r^o f_r(T_m)]^{D_r - 1}. \qquad (A19)$$

In (A19), C_r represents the trace element concentration in residual recharge melt after enclave formation, C_r^o is the concentration of trace element in pristine recharge melt and D_r is the partition coefficient between enclave solid and recharge melt. All trace element equilibrium constants are taken as functions of temperature:

$$D_a = D_a^o\left(\exp\frac{-\Delta H_a}{RT_{l,m}} \cdot \frac{1}{\overline{T}_a}\right) \qquad (A20a)$$

$$D_m = D_m^o\left(\exp\frac{-\Delta H_m}{RT_{l,m}} \cdot \frac{1}{\overline{T}_m}\right) \qquad (A20b)$$

$$D_r = D_r^o\left(\exp\frac{-\Delta H_r}{RT_{l,m}} \cdot \frac{1}{\overline{T}_m}\right). \qquad (A20c)$$

In (A20), $\Delta H_j\ j \subset \{a, m, r\}$ represent the effective enthalpies of the reactions governing bulk partitioning of trace element between anatectic melt and country rock restite, host melt and cumulates, and pristine recharge melt and enclaves, respectively. The ΔH_j values are 'effective' values in the sense that the dependence of D_j on phase assemblage is parameterized implicitly using the fictive temperature dependence of D_j. That is, 'effective' values ΔH_j are chosen by consideration of phase equilibria relevant to the bulk compositions and equilibrium phase assemblages of $\{a, m$ and $r\}$ along the temperature trajectory $T_m^o \to T_{eq}$. If this auxiliary information is not available or poorly known, constant bulk partition coefficients D_j may be used by setting ΔH_j equal to zero.