Geophys. J. Int. (2004) **158,** 443–445 doi: 10.1111/j.1365-246X.2004.02321.x

FAST TRACK PAPER

Is 600 Myr long enough for the random palaeogeographic test of the geomagnetic axial dipole assumption?

P. L. McFadden

Geoscience Australia, GPO Box 378, *Canberra,* ACT 2601, *Australia*

Accepted 2004 March 8. Received 2004 March 8; in original form 2003 October 23

SUMMARY

A simple analysis suggests that a time interval of more than about 5000 Myr is needed for the random palaeogeographic test to be effective in assessing the geomagnetic axial dipole assumption. This time is so long as to render the test ineffective. Consequently, intervals of 600 Myr should be recognized as providing only a small sample that is likely to be biased towards low-inclination values.

Key words: geocentric axial dipole hypothesis, palaeomagnetism.

INTRODUCTION

The assumption of a time-average geocentric axial dipole (GAD) field is fundamental to most palaeomagnetic studies. It is relatively easy to test this assumption if the positions of the continents (and therefore any sampling sites) are known by independent means. Thus it has been relatively easy to test this hypothesis for the past 5 Myr (McElhinny & McFadden 2000) because the continents have been in essentially the same position as they are today. However, if the latitude and orientation of a continent have been determined palaeomagnetically, then that determination has been based on the GAD assumption and it is not possible to test that assumption directly.

Evans (1976) suggested an indirect test, the random palaeogeographic test (RPT), that made use of the frequency distribution of inclination data over an extended interval of time. He argued that, given enough time, the random motion of the continents on the surface of the globe meant that they would occupy the different latitudes in accordance with the surface area of the globe at each latitude. He then argued that 600 Myr would be sufficient time for this to occur so that a uniform random sampling in time over such an interval would be the equivalent of a spatially random sample of latitude with a frequency proportional to $\sin\theta$, where θ is the co-latitude. The test would then be a comparison of the observed distribution of the absolute values of magnetic inclinations with that to be expected from a uniform random sampling over the globe of a geocentric axial dipole field.

While the GAD assumption seems to hold reasonably well for the past 5 Myr (McElhinny *et al.* 1996; Hatakeyama & Kono 2002), on the basis of the RPT it has been challenged for much earlier times (Piper & Grant 1989; Kent & Smethurst 1998; Van der Voo & Torsvik 2001; Torsvik & Van der Voo 2002). Authors have suggested persistent quadrupolar and octupolar fields to account for the difference between observed and expected distributions, particularly for the observed excess of low-inclination values. McElhinny & McFadden (2000) suggested that an alternative, and perhaps more likely, explanation is that the underlying assumption of random sampling of the surface of the globe through continental drift during the Palaeozoic and Precambrian is invalid. In examining this matter it is important to recognize that there are two distinct questions. First, is an interval of 600 Myr actually long enough for the continental motions to satisfy the assumption that each co-latitude band has been occupied for a time that is proportional to the area at that co-latitude? For example, has the co-latitude band 40°–50° been occupied for eight times as long as the polar co-latitude band 0°–10°? Only if this is so have the continental motions provided a sampling of co-latitude that is equivalent to a uniform random sample of the surface of the globe. Second, has our sampling of the magnetic inclinations provided us with an acceptably uniform random sample in time and has this, combined with the motion of the continents, given us the equivalent of a uniform random sample in space? Only the first question is addressed here, for failure of the underlying assumption is fatal to the test. Should 600 Myr be inadequate then the fundamental assumption of the RPT is invalid, and, depending on the time and particular continental motions, it would be possible to conclude at a given level of confidence that the field was dipolar even if it were not and at other times to conclude that it was not dipolar even if it were.

Meert *et al.* (2003) have undertaken an excellent random walk analysis to assess the validity of the RPT. Their analysis suggests that an interval of 600 Myr is unlikely to be sufficient. However, their approach does not separate the two questions identified above and does not facilitate an understanding of some of the finer points at issue.

MINIMUM TIME REQUIRED

First, it is necessary to determine the minimum time required for a single point moving on the globe to occupy each co-latitude band for a time that is proportional to the area at that co-latitude band. Interestingly, and importantly, should the point occupy the pole for a finite time then, because there is no area at the pole, it will, formally, require an infinite amount of time to occupy the other latitudes for times that are in the presumed proportion. A consequence is that the minimum time required depends on the width of the co-latitude band chosen for observation and analysis.

Assume that we wish to bin the observations into co-latitude cells of width ϕ. The minimum time for the point to move from the pole and occupy all the co-latitudes in the first cell is of course the time for the point to move along a meridional line. If the point is moving at a constant speed s on a globe of radius r, then this time τ_ϕ is given by

$$\tau_\phi = \frac{r\phi}{s}. \tag{1}$$

The area A_ϕ of this first cell is given by

$$A_\phi = 2\pi r^2(1 - \cos\phi). \tag{2}$$

The incremental area at co-latitude θ is given by

$$\delta A = 2\pi r \sin\theta r \delta\theta, \tag{3}$$

so the time δt that must be spent at co-latitude θ to get the time spent there to be proportional to the area of the globe at that co-latitude is given by

$$\delta t = \frac{\delta A}{A_\phi}\tau_\phi. \tag{4}$$

Letting $T_{\min}$ be the minimum time needed for the point to occupy each co-latitude band for a time proportional to the area of that co-latitude band, $T_{\min}$ is just the sum of τ_ϕ and the incremental times δt from ϕ to the equator for both hemispheres. It is necessary to sum it over both hemispheres because a point moving randomly near the equator is bound to change hemispheres, and we have no independent evidence to know this has happened and restrict observations to one hemisphere or the other. Hence, with ϕ in radians,

$$\begin{aligned} T_{\min} &= 2\left(\tau_\phi + \int_\phi^{\frac{\pi}{2}} \frac{2\pi r \sin\theta\, r\, d\theta}{A_\phi}\tau_\phi\right) \\ &= \frac{2r\phi}{s}\left(1 + \frac{\cos\phi}{1 - \cos\phi}\right). \end{aligned} \tag{5}$$

If the point is moving on the Earth ($r = 6378$ km) at a constant speed s of 60 mm yr^{-1}, then for a 5° cell $T_{\min}$ is 4857 Myr, for a 10° cell it is 2405 Myr, and for a 15° cell 1578 Myr. Typically it would be best to use a 5° cell and unwise to use a cell any coarser than 10°. Obviously these times scale inversely with the speed.

Fig. 1 shows one of many paths that would take this minimum time but which, for a 10° co-latitude cell, occupies each co-latitude band for a time that is proportional to the area on the globe in that co-latitude band. For example, the distance travelled (and therefore time spent) in the equatorial co-latitude belt 80°–90° is 11.4 times the distance travelled in the polar co-latitude belt 0°–10°. For a 5° cell the distance travelled in the 5°–10° co-latitude band would have to be three times that travelled in the 0°–5° co-latitude band (as compared with equal times being acceptable in the 10° cell situation) and similarly the distances travelled in each of the other co-latitude bands would have to increase to maintain the correct proportions. Eq. (5) shows that $T_{\min} \to \infty$ as $\phi \to 0$, confirming that, formally, an infinite time is required if there is finite time spent at the pole.

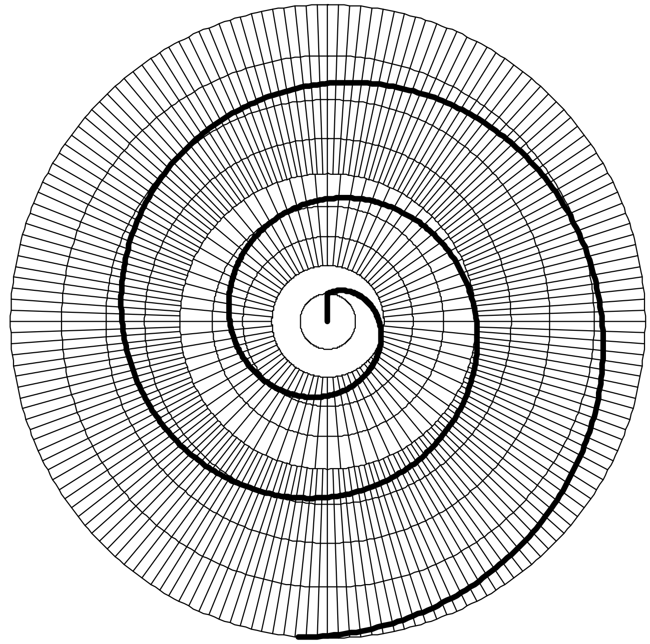

Figure 1. Polar stereographic projection of a (non-unique) path for a point moving with constant speed to occupy 10° co-latitude bands for times proportional to the area of those bands.

A point travelling directly from one pole to the other along a meridional line at 60 mm yr^{-1} would take 334 Myr for the transit and would pass through the equatorial regions far too quickly. This simple observation suggests that 600 Myr is a woefully inadequate time to average out over a random motion.

EFFECT OF LARGE AND MULTIPLE CONTINENTS

The times $T_{\min}$ quoted above are the absolute minimum time intervals required for a single point to occupy the different latitudes for times that are in the right ratio to satisfy the underlying assumption of the RPT. It would undoubtedly take several times these intervals to average out appropriately for the actual continental motion. However, the reality is that there are several continents rather than a single point moving on the surface of the globe.

Oddly enough the presence of a large continent does not assist the process; instead it hinders. If the requirement were for the different latitudes all to be occupied for the same amount of time then the presence of a large continent would assist. However, in the current situation, if the point at the centre of a continent occupies a given latitude for the appropriate amount of time then the latitude at the low-latitude edge of the continent is occupied for too short a time and that at the high-latitude edge for too long a time. Therefore the existence of continents does not reduce the minimum times given above.

Obviously the presence of multiple continents assists the process, if the continents move independently. The very nature of plate tectonics forces some constraints and correlations on the continental movements and this may have some impact on how effectively the different continents will assist in the averaging process. Regardless of this, the number of continents is small and it seems unlikely that there are enough to reduce the time needed for effective averaging much below the absolute minimum times calculated above for a single point. The analysis of Meert *et al.* (2003) bears this out.

EFFECTS OF A SMALL TIME INTERVAL

The above analysis makes it clear that 600 Myr is nowhere near long enough an interval to undertake the RPT. Any sampling over an interval of less than about 5000 Myr has to be considered as a small and potentially biased sample. The bias comes about because the continent is not free to appear at random locations but has to move along a deterministic path from one location to another in a process that might seem to us somewhat random.

At a random point in time a continent is much more likely to be at a low latitude than at a high latitude, simply because of the greater area at low latitudes. It cannot then randomly translate to a high latitude but will continue to occupy low latitudes for a reasonable fraction of a short observing interval. Thus in sampling over too short an interval it is likely that low-latitude (and therefore low magnetic inclination) observations will be over-represented, as has been observed.

Conversely, should a continent actually sit over the pole for any significant amount of time (as with Antarctica now) it would then take forever (quite literally) to occupy other latitudes for long enough to average out the time spent at the pole.

CONCLUSION

The analysis suggests that, in order to perform the RPT by binning observations into 5° co-latitude cells, a time interval of more than about 4800 Myr is needed to satisfy the underlying assumption that the motion of the continents on the surface of the globe ensures that the different latitudes have been occupied in accordance with the surface area of the globe at each latitude. This is longer than the known age of the Earth and so does not permit the RPT to be used to determine any evolution in the structure of the geomagnetic field. Even a 10° cell size requires more than about 2400 Myr.

Evans (1976), Piper & Grant (1989) and Kent & Smethurst (1998) all assumed that 600 Myr is a sufficient time interval for the RPT. In accord with the conclusions of Meert *et al.* (2003), the analysis here shows that this is not so. Consequently, conclusions based on the RPT that the geomagnetic field was significantly non-geocentric axial dipolar are unreliable. Furthermore, the observation of an apparent excess of low-inclination values can be explained in terms of a short sample leading to a bias without calling upon persistent quadrupolar or octupolar fields.

ACKNOWLEDGMENTS

I thank Joe Meert for useful comments as a reviewer. This paper is published with the permission of the Chief Executive Officer, Geoscience Australia.

REFERENCES

Evans, M.E., 1976. Test of the dipolar nature of the geomagnetic field throughout Phanerozoic time, *Nature,* **262,** 676–677.

Hatakeyama, T. & Kono, M., 2002. Geomagnetic field model for the last 5 My: time-averaged field and secular variation, *Phys. Earth planet. Inter.,* **133,** 181–215.

Kent, D.V. & Smethurst, M.A., 1998. Shallow bias of paleomagnetic inclinations in the Paleozoic and Precambrian, *Earth planet. Sci. Lett.,* **160,** 391–402.

McElhinny, M.W. & McFadden, P.L., 2000. *Paleomagnetism: Continents and Oceans*, Academic Press, San Diego.

McElhinny, M.W., McFadden, P.L. & Merrill, R.T., 1996. The time-averaged paleomagnetic field 0–5 Ma, *J. geophys. Res.,* **101,** 25 007–25 027.

Meert, J.G., Tamrat, E. & Spearman, J., 2003. Non-dipole fields and inclination bias: Insights from a random walk analysis, *Earth planet. Sci. Lett.,* **214,** 395–408.

Piper, J.D.A. & Grant, S., 1989. A palaeomagnetic test of the axial dipole assumption and implications for continental distribution throughout geological time, *Phys. Earth. planet. Inter.,* **55,** 37–53.

Torsvik, T.H. & Van der Voo, R., 2002. Refining Gondwana and Pangea palaeogeography: estimates of Phanerozoic non-dipole (octupole) fields, *Geophys. J. Int.,* **151,** 771–794.

Van der Voo, R. & Torsvik, T.H., 2001. Evidence for Permian and Mesozoic non-dipole fields provides an explanation for Pangea reconstruction problems, *Earth planet. Sci. Lett.,* **187,** 71–81.