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Precipitation calculation method based on parameterization of distribution function evolution and its performance in global spectral atmospheric model

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Abstract

A new method of precipitation calculation, based on cloud microphysics parameterization is presented. The main idea of the method is to parameterize the evolution of distribution function during precipitation formation process. Gravitational and turbulent coalescence of cloud particles are assumed to be the main processes leading to the rain formation. The method considers the precipitation formation in liquid and mixed phase clouds. The phase state of a cloudy layer is calculated as a function of temperature. The effects of precipitation forcing in underlying cloudy layers, melting and evaporation are taken into account. The proposed method was introduced to a large-scale condensation scheme of the global spectral atmospheric model of the Hydrometcentre of Russia. Numerical experiments with the new version of model were conducted. The numerical results were verified using precipitation observations. The experiments show that the new method improves the skill of precipitation forecast for lead times 24–72 h in summer and 24–36 h in spring.

Keywords: Weather forecast; Precipitation; Clouds; Microphysics; Numerical modeling; Verification

1. Introduction

The accuracy of calculation of precipitation spatial distribution and amount is one of important and actual problems of meteorology, which has a particular significance in forecasting such phenomena connected with precipitation as floods, high waters on the rivers, snow drifts, etc. In this connection, a method of precipitation calculation, which reproduces the physics of precipitation formation most reliably, should be applied when slowing the problems of large-scale modeling of atmospheric processes and weather forecasting. On the other hand, the application of such method should not make a set of model equations too complicated. On a hemispheric scale, the optimal approach to this problem is a parametric description of microphysical processes. In recent years, many methods of precipitation calculation based on parameterization of microphysics were proposed. But by certain reasons, application of these methods in a global model could not improve the prognostic precipitation considerably.

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The first reason is related to the basic idea of precipitation beginning, namely, the formulation of autoconversion process. The autoconversion is a starting process, which leads to precipitation formation in clouds and the accuracy of its definition is a main factor in the rain formation. The autoconversion is described (Kessler, 1969; Rutledge and Hobbs, 1983) as the process generating precipitation after the cloud water content reaches some value, named an autoconversion threshold. Another formula for autoconversion was proposed in Sundqvist (1978). However, this formula is also based on the autoconversion threshold. It is worth to note that the autoconversion thresholds used in Kessler (1969), Sundqvist (1978), Heise and Roeckner (1990), Smith (1990) and Zhao and Carr, 1997 are quite different. The usage of such values will lead to a large spread of calculated precipitation intensities. In works of Rotstayn (1997) and Wilson and Ballard (1999), the autoconversion rate was defined through the cloud water content and the concentration of droplets by the formula proposed in Tripoli and Cotton (1980). But this formula also uses some threshold value as a starting point for precipitation. In Rotstayn (1997), the autoconversion threshold was calculated from the concentration and the mean radius of cloud droplets. But the mean radius value was assumed to be a constant. This contradicts with the fact, that cloud particles are growing during the precipitation formation process.

Why is there such a great variety of autoconversion formulations and autoconversion threshold values? The possible answer is because this value could not be measured. Therefore, it is impossible to evaluate the value of this threshold from measurements. Thus, the question arises if this value exists in nature. A lot of observation data (Mason, 1971; Matveev, 2000) shows that clouds begin to precipitate when the cloud water content reaches quite different values. Moreover, some observations indicate that precipitation was formed in clouds when the cloud water content decreased or remained constant (Cloud Physics, 1961; Litvinov, 1980). The solution of this problem is a revision of the conception of precipitation beginning. It is more correct to define precipitation beginning as a formation of part of cloud spectra consisting of large drops, which fall out as precipitation. Such an approach requires a description of the evolution of distribution function during the precipitation formation. This evolution can be described by solving the kinetic coalescence equation proposed in Smoluchowsky (1916). But now, this approach can be applied only in local modeling tasks (Voloshyk and Sedynov, 1975). In global atmospheric modeling, the possible solution of this problem is a parameterization of the distribution function evolution in a cloudy layer. This approach is presented in this paper.

The second reason is related to the number of additional equations, which are included into the model to describe cloud variables and parameterize cloud processes. It was proposed in Sundqvist (1978) to use an additional equation for the cloud water content along with equations for temperature and humidity. Two additional equations for cloud water and ice are used in Rutledge and Hobbs (1983) and Rotstayn (1997). In Wilson and Ballard (1999), this system is supplemented by equations for rainfall and snow. The partitioning of solid precipitation into different parts—ice, snow and grauppel—is presented in Wacker (1995). Additional equation for droplet concentration is described in Lohmann et al. (1999). In my opinion, the application of additional equations is more suitable for mesoscale modeling. Too much computer time is needed to solve a separate equation for each cloud variable in global model. Also, a lack of initial data on cloud water and ice content on a global scale will lead to large spin up times for these variables. This has a negative impact on precipitation, especially for short-range forecasts. Therefore, approach similar to Smith (1990), in which the cloud ice content, rain and snow are diagnostic variables, is most appropriate for large-scale modeling.

Such a conception is adopted for definition of cloud variables described in this paper. The cloud water content, a relation between drops and ice crystals in cloudy layer, precipitation intensity and its phase are diagnostic values whose descriptions are based on a vertical distribution of temperature and humidity resulting from the integration of the model. This conception makes this method applicable to each model that has heat influx and humidity transfer equations. Also, this method can be applied to diagnose precipitation from the observed humidity and temperature values.

2. Method description

A first step in parametrizing the processes of the precipitation formation is to define the boundaries of the cloudy layer, based on the analysis of temperature and humidity vertical distribution in the atmosphere. This information can be obtained from Smagorinsky's relations. After that, it is necessary to define cloud water content δ, and mean radius of cloud particles r_1 for this layer.

For calculation of cloud water content, different types of statistical and empirical relations can be proposed (Cloud Physics, 1961; Sasamori, 1975; Matveev, 1981; Hence and Heise, 1984; Cloud and Cloudy Atmosphere, 1989). In this work, an empirical relation obtained from a generalization of aircraft measurements in clouds (Matveev, 1981) was chosen

$$\delta = \begin{cases} 0.27\dfrac{T}{p}\exp\left(17.86\left(1 - \dfrac{258.0}{T}\right)\right), & \text{at } s \geq s_{CR} \\ 0, & \text{at } s < s_{CR} \end{cases} \tag{1}$$

where δ is the cloud water content (kg/m^3), T is the temperature (K), p is the pressure (hPa), s is the humidity (kg kg^{-1}), s_{CR} is the humidity level above which clouds exist.

A mean radius r_1 can be defined as a function of cloud water content by the following empirical relation (Cloud and Cloudy Atmosphere, 1989)

$$r_1 = a\delta + b \tag{2}$$

where r_1 is the mean radius (m km), $a = 11.0$, and $b = 4.0$.

Let us suppose that this values of δ and r_1 characterize the initial time moment t_0 of precipitation formation processes. The main task is to define the intensity of precipitation, which will be formed in a cloudy layer after time Δt. The precipitation intensity I_P (mm/h) can be calculated from the difference between δ and δ_{CR} with the next formula (Akimov, 2001a,b)

$$I_P = \frac{3.6 \times 10^6}{\rho_W} \frac{(\delta(t_0) - \delta_{CR}(t_0 + \Delta t))}{\Delta t} \Delta H \tag{3}$$

where ΔH is the thickness of a cloudy layer (m), ρ_W is the density of water (kg/m^3), δ_{CR} is the critical value of cloud water content, which characterize a fraction of water remaining in the cloudy layer after time Δt.

The critical cloud water content can be defined as a part of cloudy drop spectra, formed by the particles smaller than the critical radius. This definition of δ_{CR} value can be expressed by the following formula (Dmitrieva-Arrago and Akimov, 1996):

$$\delta_{CR} = \frac{4}{3}\pi\rho_W n \int_0^{r_{CR}} r^3 f(r)\mathrm{d}r, \tag{4}$$

where r_{CR} is the critical radius defined as a minimum value of the drop radius, at which it can overcome the vertical velocity in the cloudy layer and precipitate, $f(r)$ is the drop size distribution function, n is the concentration of drops (M^{-3}).

Formula (4) expresses δ_{CR} as a function of two variables $f(r)$ and n. Using a connection between the cloud water content, cube-mean radius r_3^3, and the concentration—$\delta = 4\pi r_3^3 n/3$, formula (4) can be transformed to the following form:

$$\delta_{CR} = \frac{\delta}{r_3^3} \int_0^{r_{CR}} r^3 f(r)\mathrm{d}r. \tag{5}$$

Thus, to calculate δ_{CR}, it is necessary to know variation of the drop size distribution $f(r)$ within the time interval Δt. Let us suppose that the $f(r)$ function can be expressed as a gamma distribution with two parameters α and β (Cloud and Cloudy Atmosphere, 1989):

$$f(r) = \frac{1}{\Gamma(\alpha+1)\beta^{\alpha+1}} r^\alpha \exp\left(-\frac{r}{\beta}\right). \tag{6}$$

It was shown (Cloud and Cloudy Atmosphere, 1989) that many experimentally measured distributions of cloudy drops could be approximated by a constant parameter $\alpha = 2$ (it is well known as the Khrgian–Mazin distribution). In this case, the evolution of the drop size distribution function during the process of precipitation formation is defined by the evolution of parameter β. The parameter β characterizes the mean radius of cloud particles r_1 and is connected with it by the following relation (Cloud and Cloudy Atmosphere, 1989):

$$r_1 = (\alpha + 1)\beta. \tag{7}$$

On substituting the distribution function (6) into Eq. (5), we can obtain:

$$\delta_{CR} = \frac{\delta}{r_3^3 \Gamma(\alpha+1)\beta_1^{\alpha+1}} \int_0^{r_{CR}} r^{\alpha+3} \exp\left(-\frac{r}{\beta_1}\right) dr. \quad (8)$$

where β_1 is the distribution function parameter at time $t_0 + \Delta t$.

The integral in the right hand of formula (8) is an incomplete gamma function, which can be presented as a numerical sum. Thus, the following formula for δ_{CR} calculation can be obtained:

$$\delta_{CR}(t_0 + \Delta t) = \delta(t_0)\Gamma_C(\beta_1, r_{CR}), \quad (9)$$

where

$$\Gamma_C(\beta_1, r_{CR}) = 1 - \frac{1}{m!}\exp\left(-\frac{r_{CR}}{\beta_1}\right) \times \sum_{i=0}^{m} \frac{m!}{(m-i)!}\left(\frac{r_{CR}}{\beta_1}\right)^{m-i}, \quad m = 3 + \alpha.$$

In order to calculate δ_{CR} with the help of formula (5), it is necessary to know the evolution of the parameter β during the time interval Δt. For this purpose, let us define the variation of the mean radius of cloud particles within the time interval Δt, as follows

$$\Delta r_1 = \int_0^{\infty} \Delta r f(r) dr, \quad (10)$$

where Δr is the variation of cloud particle radii due to precipitation formation processes.

First, let us consider a growing of drops in liquid clouds. In this type of clouds, the main microphysical process, leading to precipitation formation, is gravitational coalescence. The variation of cloud particle radii due to gravitation coalescence Δr_G has the next form (Rogers, 1988)

$$\Delta r_G = \frac{\bar{E}_d \delta}{4\rho_W} U(r) \Delta t \quad (11)$$

where $\bar{E}_d$ is the mean coefficient of cloud drops collection, $U(r)$ is the fall velocity of cloud drops.

The fall velocity of cloud drops can be expressed using the Stoks formula, which approximate the fall velocities of the drops with a radius of 1–100 m km

$$U(r) = k_1 r^2 \quad (12)$$

where $k_1 = 1.19 \times 10^8\ \mathrm{m}^{-1}\ \mathrm{s}^{-1}$ is a constant.

Following the results described in Almeida (1979), Koziol and Leighton (1996), Pinsky and Khain (1997) and Shaw et al., 1998, the turbulent coalescence was chosen as an additional microphysical process, which leads to drops growing and rain formation. The variation of cloud particles radii due to influence of cloud turbulence Δr_T has the next form:

$$\Delta r_T = \frac{16}{3} \frac{\delta}{\rho_W} r \left|\frac{\partial V}{\partial s}\right| \Delta t \quad (13)$$

where $|\partial V/\partial s|$ is the value of turbulent gradient which is connected with turbulent energy ε and kinematic viscosity of air ν by the following expression (Cloud Physics, 1961):

$$\left|\frac{\partial V}{\partial s}\right| = \sqrt{\frac{2\varepsilon}{15\pi\nu}}. \quad (14)$$

The expression for variation of cloud particles radii is obtained as a sum of Eqs. (11) and (13) with substitution in it (Eqs. (12) and (14)):

$$\Delta r = \Delta r_G + \Delta r_T = \left[\frac{\bar{E}_d \delta}{4\rho_W} k_1 r^2 + \frac{16}{3}\frac{\delta}{\rho_W} r \sqrt{\frac{2\varepsilon}{15\pi\nu}}\right] \Delta t \quad (15)$$

Substituting expression (12) in Eq. (10) with $f(r)$ defined accordingly to Eq. (6) and express mean radius from parameter β using Eq. (7) after solving an integral, the following formula for distribution parameter variation was obtained

$$\beta_1 = \beta_0 + \left((\alpha+2)\frac{\bar{E}_d \delta}{4\rho_W} k_1 \beta_0^2 + \frac{16}{3}\frac{\delta}{\rho_W}\beta_0 \sqrt{\frac{2\varepsilon}{15\pi\nu}}\right)\Delta t \quad (16)$$

where β_0 is the distribution function parameter value at time t_0.

The calculation of precipitation intensity, using formulas (3), (9), (16) depends on values of parameters included in these formulas. There are critical radius r_{CR} in formula (9) and mean coefficient of collection $\bar{E}_d$, turbulence energy in cloud ε and β_0 in formula (16). The distribution parameter β_0 at time moment t_0 can be calculated from expression (7) using the mean radius values r_1 defined from relation (2). The estimations which have been carried out in Dmitrieva-Arrago and Akimov (1996) and Akimov

(2001a,b) have shown that optimum values of parameters r_{CR} and $\bar{E}_d$ are 50 m km and 0.15, respectively. It has been shown in Akimov (2001a, b) that parameter ε can be approximately assumed as a constant value, different for stratiform and convective clouds ($\varepsilon = 14.0 \times 10^{-4}$ m^2/s^3 for stratiform and $\varepsilon = 100.0 \times 10^{-4}$ m^2/s^3 for convective clouds).

In order to generalize the proposed method for mixed phase cloudy layer, it should be noted that in this case, cloud water content δ represent the quantity of water in two phases. Therefore, it is necessary to define the part of ice crystals in cloud layer P. This value can be calculated as a function of temperature using the following formula (Sundqvist, 1993):

$$P = 1 - A(1 - \exp(-x^2)), \tag{17}$$

where $A = 1.058$ is a constant, $x = (T - 232)/24.04$. Formula (17) approximates P as function of temperature in interval 232 ÷ 273 K. At $T > 273$ K, $P = 0$, and at $T < 232$ K, $P = 1$.

For parameterization of rain formation processes in mixed phase cloudy layer, let us suppose that ice crystals can be described as spherical particles with mass equivalent to mass of crystals. In this case, cloud particles can be characterized by their equivalent mean radius. Thus, the formula (16) may be adopted for mixed cloudy layer with substitution ρ_W for ρ_I, $\bar{E}_d$ for $\bar{E}_i$ and α for α_I. Here ρ_I is the density of ice, $\bar{E}_i = 0.45$ is the mean coefficient of collection between ice crystals and drops, $\alpha_I = 3$ is the distribution parameter value obtained from the measurements in mixed phase clouds (Cloud and Cloudy Atmosphere, 1989).

In mixed phase clouds, the Berjeron–Findaizen process, resulted in intensive sublimation of water vapour on ice crystals, leads to precipitation formation in addition to coalescence processes. In order to describe this process, let us consider the variation of particles radii due to condensation–sublimation process Δr_C, which is expressed by the following formula (Matveev, 2000)

$$\Delta r_C = \frac{\rho_A D}{\rho r} \Delta s \Delta t \tag{18}$$

where $D = 0.22 \times 10^{-4}$ m^2/s is the diffusion coefficient, ρ_A is the density of air, ρ is the density of water or ice in order of particle type, Δs is the oversaturation in surrounding air (kg kg^{-1}).

Substituting expression (18) in Eq. (10) and considering that cloudy layer consists from P part of crystals and $1 - P$ part of drops, the next expression for mean radius is resulted

$$\Delta r_1 = (1 - P) \int_0^\infty \frac{\rho_A D}{\rho_W r} \Delta s_W \Delta t f(r) \mathrm{d}r + P \int_0^\infty \frac{\rho_A D}{\rho_I r} \Delta s_I \Delta t f(r) \mathrm{d}r \tag{19}$$

where Δs_W is the oversaturation related to drops, Δs_I is the oversaturation related to crystals.

Using formula (6) for $f(r)$ and expressing mean radius from parameter β according to (7) after solving an integral, the following formula for distribution parameter variance can be obtained:

$$\Delta \beta_C = \left((1 - P) \frac{\rho_A D}{2\rho_W(\alpha + 1)\beta} \Delta s_W + P \frac{\rho_A D}{2\rho_I(\alpha_I + 1)\beta} \Delta s_I \right) \Delta t \tag{20}$$

Taking into account that Δs_I accordingly to measurements in clouds (Kachurin and Morachaevsky, 1965) is usually on three orders greater than Δs_W, the first term in Eq. (20) is negligible in comparison with the second.

In order to define oversaturation related to crystals Δs_I, let us note that this value depends on relation between drops and ice crystals in cloudy layer (Kachurin and Morachaevsky, 1965). Taking into account that saturation at moment when ice crystals begin to appear in cloudy layer is near to saturation above the water, the following condition for oversaturation value can be written:

$$\Delta s_I \to \Delta s_{WI}, \qquad \text{when } P \to 0 \tag{21}$$

where Δs_{WI} is the difference between saturation humidity values above water and above ice.

Also it should be noted that oversaturation in ice clouds falls down to very small values about 10^{-10} (Matveev, 2000). Accordingly, the next condition for oversaturation can be written:

$$\Delta s_I \to 0, \qquad \text{when } P \to 1 \tag{22}$$

Using linear approximation which satisfies the conditions (21) and (23), the following relation for oversaturation was obtained:

$$\Delta s_{\rm I} = (1 - P)\Delta s_{\rm WI} \tag{23}$$

Formula (20), with substitution (23) can be transformed to the next form:

$$\Delta\beta_{\rm C} = P(1 - P)\frac{\rho_{\rm A} D}{2\rho_{\rm I}(\alpha_{\rm I} + 1)\beta}\Delta s_{\rm WI}\Delta t \tag{24}$$

Taking into account additional term expressed according to Eq. (24), the next formula for distribution parameter variation in mixed phase cloud can be obtained:

$$\beta_1 = \beta_0 + \left((\alpha_{\rm I} + 2)\frac{\bar{E}_{\rm i}\delta}{4\rho_{\rm I}}k_1\beta_0^2 + \frac{16}{3}\frac{\delta}{\rho_{\rm I}}\beta_0\sqrt{\frac{2\varepsilon}{15\pi\nu}} + P(1 - P)\frac{\rho_{\rm A} D}{2\rho_{\rm I}(\alpha_{\rm I} + 1)\beta_0}\Delta s_{\rm WI}\right)\Delta t \tag{25}$$

Thus precipitation intensity from liquid cloud layer can be calculated using formulas (3), (9) and (16). Precipitation intensity from mixed cloudy layer is calculated with substitution (16) for Eq. (25).

Intensity $I_{\rm P}$ represent only quantity of precipitation which falls out from single cloudy layer. This value changes during falling out of precipitation particles to the surface. The main processes that influence precipitation quantity are $I_{\rm P}$ forcing due to coalescence of precipitation particles with drops in underlying cloudy layer and $I_{\rm P}$ decreasing due to evaporation in unsaturated layer. The phase of precipitation depends on melting rate of precipitation particles. Parameterization of these processes (description is presented in Appendix A) are based on assuming that distribution of precipitation particles is described as gamma function (Shlesinger et al., 1988) instead of widely used Marshall–Palmer distribution. The parameterization also takes into account variation of mean radius of precipitation particles together with precipitation intensity variation. Proposed parameterization does not include different equation for snow and rain intensity. On the basis of precipitation measurements (Litvinov, 1980), particles, which fall out from mixed cloudy layer are assumed as snow. Snow and rain coexist only below zero isotherm in melting layer and proportion between snow and rain quantity is defined by decreasing of snow intensity due to melting $I_{\rm S}$. This allows to predict such phenomenon as occurrence of mixed phase precipitation (wet snow) near the surface.

3. Performance of new method in the model and estimation criteria

For evaluating the impact in precipitation forecast, which is resulted from usage of new parameterization, the proposed method was included as a precipitation parameterization module in global spectral model T85L31 of the Hydrometcentre of Russia (Kurbatkin et al., 1994). The spectral model is based on a set of equations for horizontal wind components, heat influx equation and humidity transfer equation. The equations are solved in σ coordinate system. Horizontal spectral resolution of the model is T85 that corresponds to 128 points on longitude and 73 points on latitude. Vertical structure of model is 31σ levels, from $\sigma = 0.9952\ 0.01$. The initial data for model run are fields of heopotential, wind velocity, temperature and humidity analysis. The analysis fields are produced in the Hydrometcentre of Russia on $2.5 \times 2.5°$ regular latitude–longitude grid in p coordinate system and then transferred to fields in σ system. The set of physical parameterizations in model includes parameterization of turbulence based on work of Bourke et al. (1977), parameterization of radiation processes (Geleyn and Hollingsworth, 1979), parameterization of surface processes (Deardorff, 1978) and parameterization of moist convection (Kuo, 1974). Condensation parameterization is based on calculation of condensation heat influx when humidity reaches its saturation value. The difference between humidity and saturation humidity defines the condensate value. The precipitation is calculated using Kessler's approach (Kessler, 1969).

The performance of new method in model was resulted only in substitution precipitation calculation module based on Kessler parameterization by new one. On the one hand, this is not quite correct because inclusion of new precipitation calculation method demands more realistic representation of other physical processes, especially this is related to condensation and moist convection parameterizations. But on the other hand, this allows to compare precipitation forecasts resulted from application of two different methods, because all other parameterization modules remain unchanged.

With the new version of model, a series of numerical experiments were conducted. Precipitation fields were calculated as accumulating sums for 12 h interval, for lead times from 24 to 72 h. The results of numerical experiments with new version of model were compared with results obtained from the old version of model, which is currently used for precipitation fields forecast. The spatial distribution of precipitation, obtained as a result of an integration of model, was received in points of the regular latitude–longitude grid with spatial step $2.5° \times 2.5°$. The comparison of precipitation was made using observed 12 h precipitation sums. The precipitation forecast values were compared with the observed values represented as data at single stations on the European part of Russia, and as a spatial distribution on $2.5° \times 2.5°$ grid, obtained by observation data averaging on quadrates of grid.

For comparison of precipitation fields calculated using new method, with precipitation fields obtained from the old version of model, the set of statistical scores were used (description of each score is presented in Appendix B). This set of scores was recommended by Russian Hydrometeorological Service for verification of precipitation forecasts (Methodical Instructions, 1991). The Pearcy–Obychov criterion (also known as Hanssen and Kuipers discriminant) is assumed as the main score, which describes degree of coincidence between prognostic and observed spatial distributions of precipitation. The value of this score varies from $T = 1$ for ideal forecast to $T = -1$ for absolutely incorrect forecast. The precipitation and no precipitation cases are divided by threshold value 0.1 mm/12 h as recommended in Methodical Instructions (1991). The accuracy of precipitation quantity calculation is evaluated by root-mean-

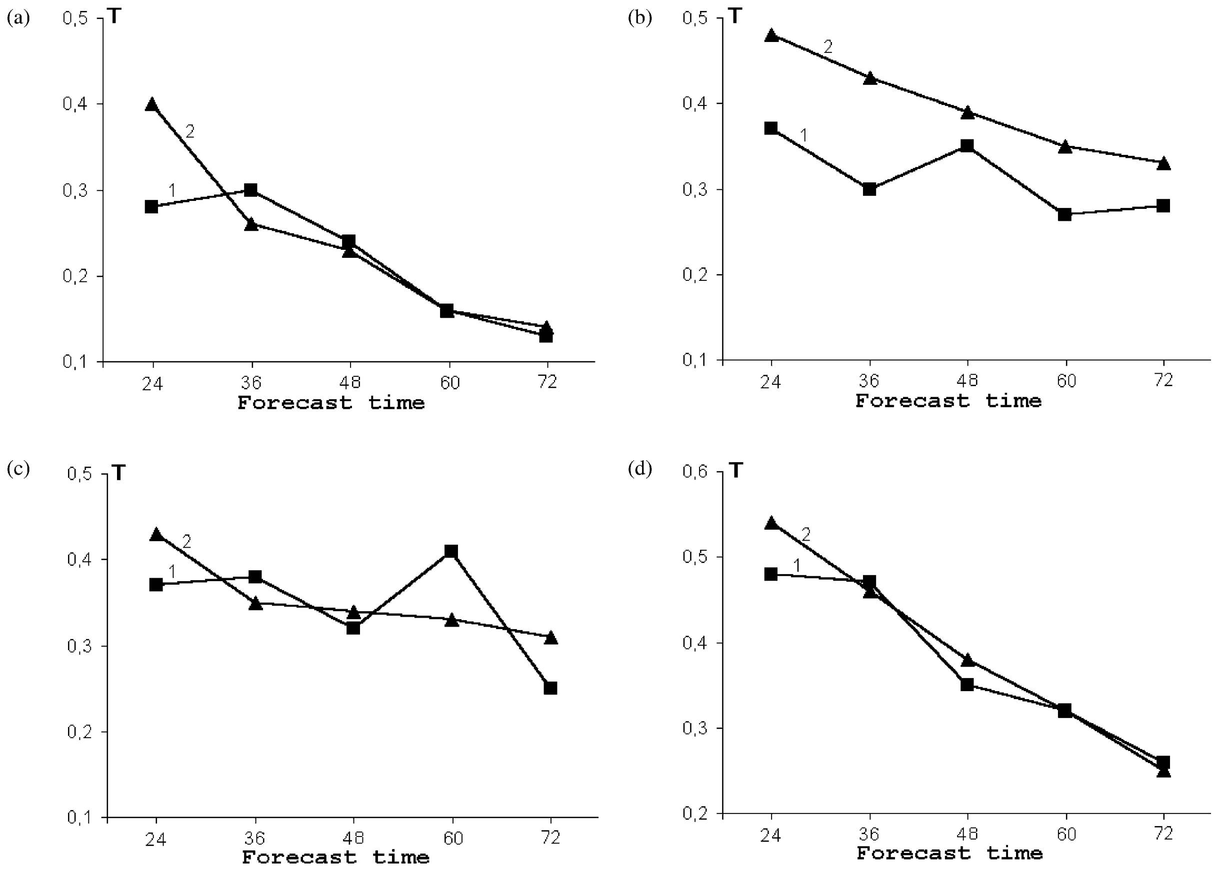

Fig. 1. Pearcy-Obychov criterion values (T) calculated for March 2002 in four regions using two different model versions: 1- old version, 2- version including new method of precipitation calculation. Regions of estimations: European part of Russia (a), Central Europe (b), West Europe (c), North America (d).

square σ_Q and systematic δQ errors of precipitation forecast.

4. Results and discussion

The first stage of comparison of precipitation fields calculated using new method with precipitation fields obtained from the old version of model was an estimation of spatial distribution of precipitation forecasts in some regions of northern hemisphere. For this problem, the four regions characterized by the densest mesh of synoptic observations were chosen: European part of Russia (20–55°E; 50–65°N), Central Europe (0–20°E; 40–60°N), West Europe (0–20°W; 35–60°N), North America (70–100°W; 30–50°N).

The Pearcy–Obychov criterion T values calculated for the four above named regions for March 2001 are presented in Fig. 1. Figure shows that the use of the new method considerably improves precipitation forecast of the model only for lead times 24–36 h. For forecast times 36–72 h, the results of both methods are close. In Fig. 2, the Pearcy–Obychov criterion values calculated for the same regions for July 2001 are presented. In this case, application of the new method considerably improves the precipitation forecast for all lead times 24–72 h. In general, from results presented in Figs. 1 and 2, it is seen that the new method improves precipitation forecast ($T = 0.3–0.4$), especially in cases, when the forecast resulted from old version of model gives rather moderate scores ($T = 0.1–0.2$).

The root-mean-square forecast error σ_Q calculated for July 2001 in four regions are presented in Fig. 3. In three regions, the precipitation forecast obtained from the new version of model is characterized by smaller error ($\sigma_Q = 1.5–2.5$) than the old version results ($\sigma_Q = 2.5–4.0$). But in North America, the forecast

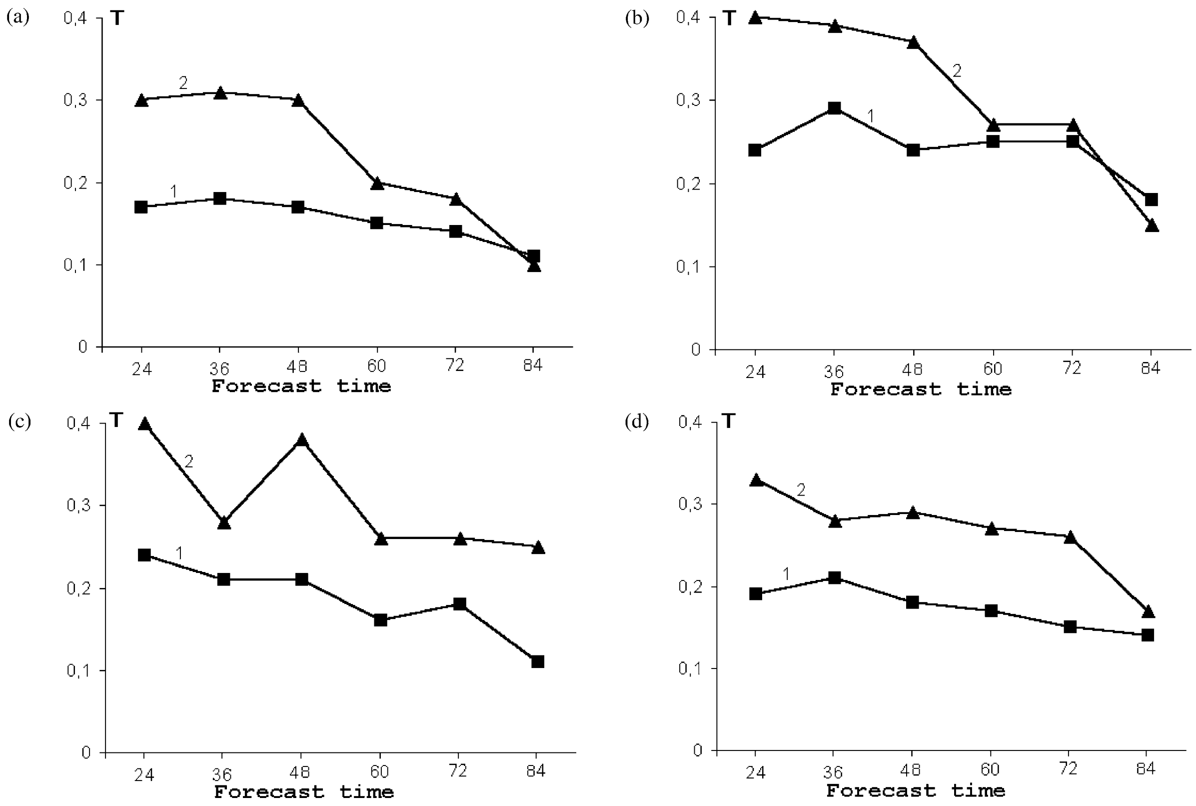

Fig. 2. Pearcy-Obychov criterion values (T) calculated for July 2002 in four regions using two different model versions: 1- old version, 2- version including new method of precipitation calculation. Regions of estimations: European part of Russia (a), Central Europe (b), West Europe (c), North America (d).

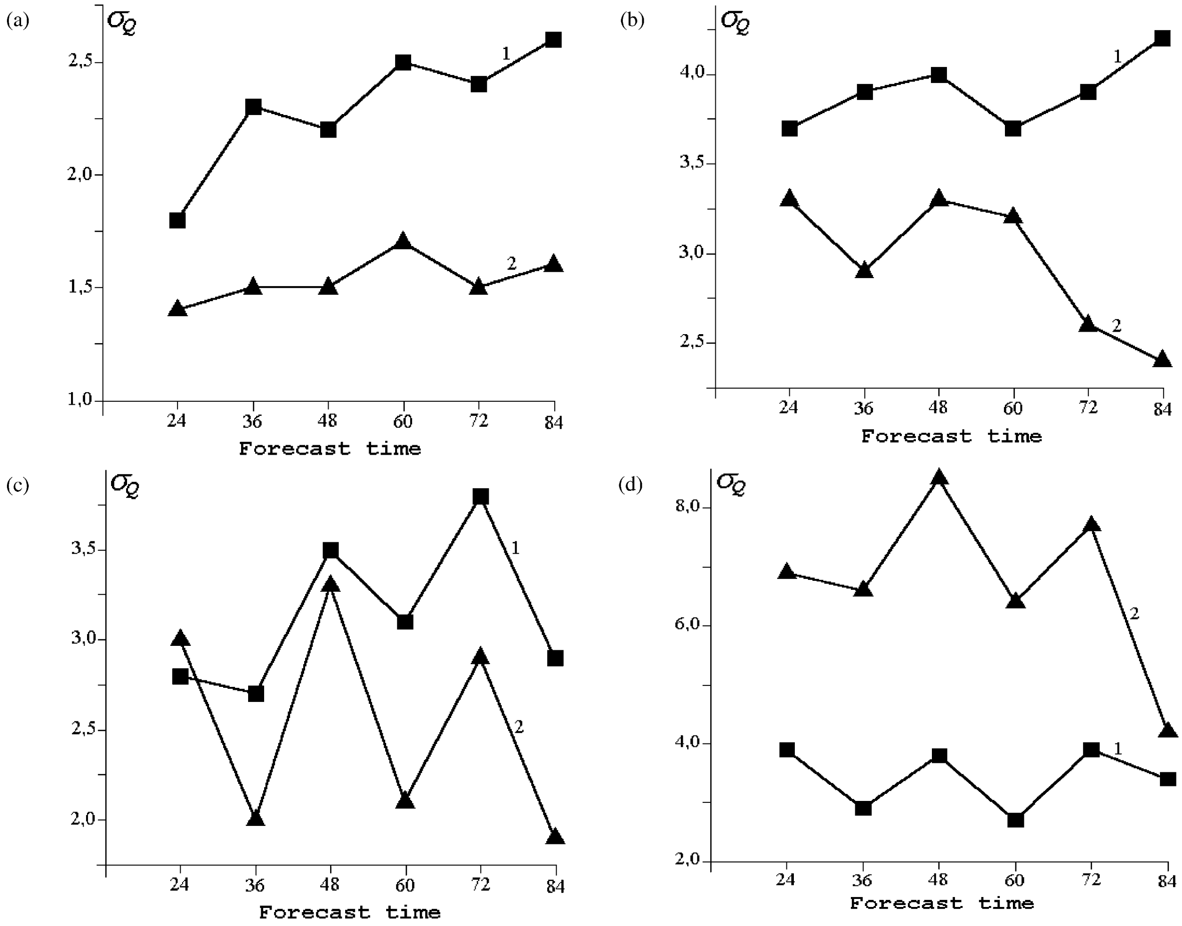

Fig. 3. Precipitation forecast root-mean-square error (σ_Q) calculated for July 2002 in four regions using two different model versions: 1- old version, 2- version including new method of precipitation calculation. Regions of estimations: European part of Russia (a), Central Europe (b), West Europe (c), North America (d).

error in the new version is 1.5 times greater than the old version error. The possible reasons of such results in this region will be expressed below.

The comparison between two versions of model was conducted during time period March–December 2002. The results of estimations showed that new precipitation calculation method gives better precipitation forecast results than the one obtained from the old version of model, especially at lead times 24 and 36 h. Pearcy–Obychov criterion values T and root-mean-square forecast error σ_Q calculated for all estimation period for lead times 24 and 36 h are presented in Figs. 4 and 5. The figures show that criterion T values obtained from the new version of model forecast is greater than T values obtained from the old version during all estimation period. Especially for lead time 24 h in period April–July 2002 criterion T values rises from level 0.2–0.3 to level 0.3–0.5. The main impact in this is improvement in forecast of light rain or snow areas which was wrongly represented by the old version of model.

Figs. 4 and 5 also show that the usage of new precipitation calculation method reduces the root-mean-square forecast error in all regions except North America. The values of σ_Q became smaller by 10–20%, which shows that the new method gives better precipitation quantity values. In North America, σ_Q values obtained from the new version of model is also smaller with the exception June–August period, where forecast errors overcome the errors resulted

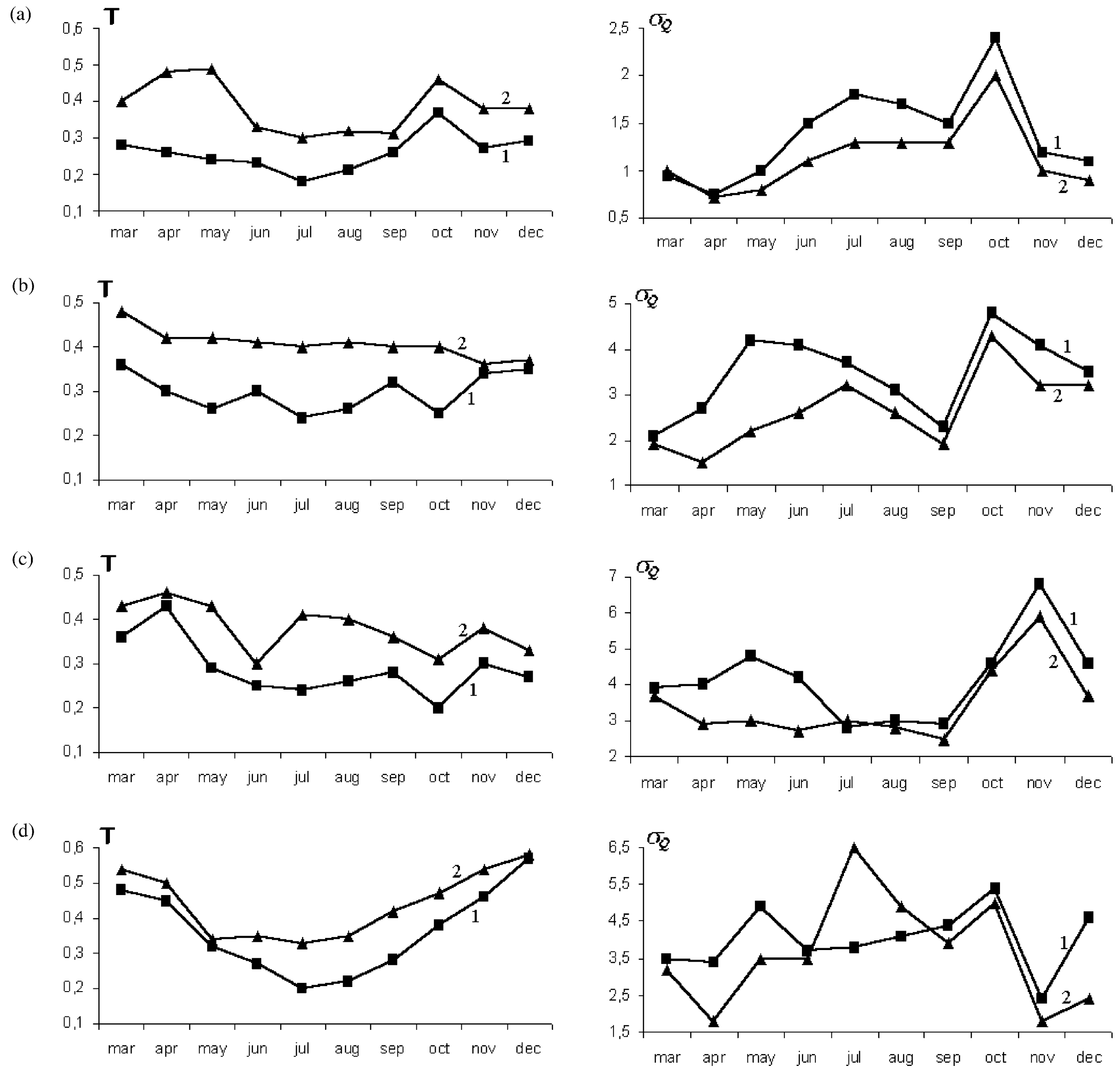

Fig. 4. Pearcy-Obychov criterion values (T) and forecast root-mean square error (σ_Q) for 24 hours precipitation forecast, calculated for verification period March - December 2002. Versions of model 1- old version, 2- version including new method of precipitation calculation. Regions of estimations: European part of Russia (a), Central Europe (b), West Europe (c), North America (d).

from the old version forecast. The reason for this fact is that new precipitation calculation method uses empirical relations and values of parameters (relations (1) and (2) and r_{CR}, $\bar{E}_{\mathrm{d}}$, ε values) which were obtained from aircraft measurements data performed in the middle latitudes region. But the weather conditions in South Coast of North America in summer are closer to tropical weather. Therefore, the usage of such relations in this season is not quite correct. The possible way to eliminate this is to use different values and relations for cloudy layer parameters for different hemisphere climatic regions.

The second stage of comparison was calculation of precipitation forecast estimations for observation stations on the European part of Russia. The 10 stations related to main Russian cities were chosen.

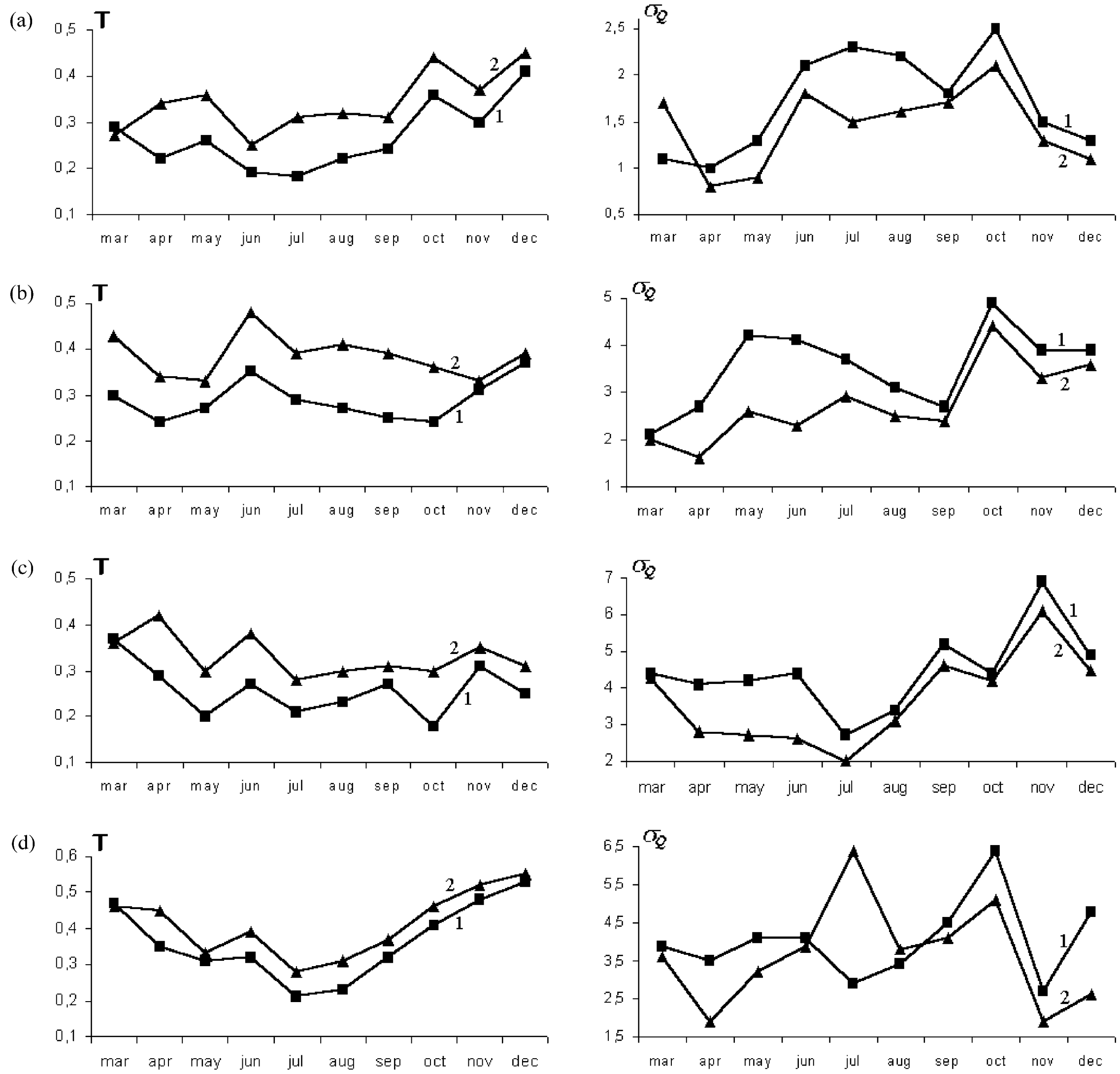

Fig. 5. Pearcy-Obychov criterion values (T) and forecast root-mean square error (σ_Q) for 36 hours precipitation forecast, calculated for verification period March - December 2002. Versions of model 1- old version, 2- version including new method of precipitation calculation. Regions of estimations: European part of Russia (a), Central Europe (b), West Europe (c), North America (d).

As an example, in Table 1, the estimation criteria values calculated for precipitation forecasts at station Kirov for March 2001 are presented. The table shows that at a rather high level of old version scores ($T = 0.6$), the new method gives higher values ($T = 0.66$). The results of estimations calculated for precipitation forecasts at station Moscow for June 2001 are presented in Table 2. This table confirms the conclusions, which were made from Fig. 1 analysis, about precipitation forecast improvement for summer ($T = 0.3$–0.4) in comparison with the old version results ($T = 0.2$). In general, the calculation of estimations in different stations shows that values of scores have greater variability than results of estimations obtained from the whole European Part of Russia region. But the tendency in criterion T improvement in the case of the new version of model usage was seen at different stations. Forecast error analysis gives more complex results. Sometimes, new version of

Table 1
Predictability estimations in March 2002, Kirov

Forecast time (h)	Version of model	T	U	U_P	U_{NP}	Π_P	Π_{NP}	δQ	σ_Q	$P_{<3}$
24	Operative	0.60	80	79	81	79	81	−0.2	1.5	90
	New	0.66	83	79	87	85	81	−0.2	2.0	86
36	Operative	0.57	77	92	67	65	92	0.3	2.5	80
	New	0.58	79	81	77	81	77	1.5	2.9	83

Table 2
Predictability estimations in June 2002, Moscow

Forecast time (h)	Version of model	T	U	U_P	U_{NP}	Π_P	Π_{NP}	δQ	σ_Q	$P_{<3}$
24	Operative	0.25	62	35	84	64	62	−0.2	1.8	83
	New	0.31	66	31	89	65	66	−0.1	0.6	100
36	Operative	0.19	60	54	65	59	60	−1.3	1.5	80
	New	0.46	72	59	84	76	70	−1.9	2.2	67

model gave greater values of precipitation forecast errors (δQ and σ_Q) and lower values of forecast insurance $P_{<3}$. Therefore, from station data analysis, it was difficult to decide what version of model gain better results. This fact is the consequence of local precipitation features, which is different from station to station and could not be described in large-scale model. Therefore, despite the success in precipitation fields forecast in the new version of model on the scale of region, there are some problems in precipitation forecast improvement at different stations. The possible way to eliminate these problems is statistical correction of calculated precipitation values at different stations.

5. Conclusions

Thus, the method of precipitation calculation based on parameterization of the evolution of distribution function is presented in this paper. The method takes into account the processes of cloud particle growth due to gravitational and turbulent coalescence, and sublimation in mixed type clouds. Despite the complexity of physical processes, the realization of method requires only information about the vertical distribution of temperature and humidity. Thus, the suggested method can be used in any global atmospheric model, which contains heat influx and humidity transfer equations. The hypothesis used to describe precipitation formation in this method have more realistic physical basis, than standard description of autoconversion process. Therefore, the precipitation values obtained from this method should be more realistic. In total, the results of numerical experiments confirmed a preliminary assumption that the new method will improve the spatial distribution of precipitation simulated by the global spectral model T85L31 of the Hydrometcentre of Russia.

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Appendix A

A.1. Forcing of precipitation intensity in underlying cloudy layer

In order to parameterize forcing of precipitation intensity in underlying cloudy layer, it is necessary to describe distribution function of precipitation particles $N_{PR}(R)$. Following the work of Shlesinger et al. (1988), function $N_{PR}(R)$ was chosen as a gamma distribution with two parameters R_0 and μ

$$N_{PR}(R) = N_0 R^{\mu} \exp\left(-\frac{R}{R_0}\right) \tag{A1}$$

where N_0 is the concentration of precipitation particles.

Let us consider that parameter R_0, which characterizes mean radius of precipitation particles, is a variable. Second parameter μ is assumed as constant value $\mu = 0.41$ obtained from approximation of experimental data (Shlesinger et al., 1988).

Precipitation intensity, $I_{\downarrow}$, resulted from cloudy layer, can be calculated from the following relation

$$I_{\downarrow} = I^{\downarrow} + \Delta I + I_P \tag{A2}$$

where $I^{\downarrow}$ is the intensity of precipitation falling from the upper layer, ΔI is precipitation intensity variance due to collection of small cloud particles by precipitation particles which have come from the upper layer, I_P is precipitation intensity which formed in this layer, defined by the formula (3). Formula (A2) can be applied consequently for multilevel cloudiness assuming, that for each subsequent layer value, $I_{\downarrow}$ becomes $I^{\downarrow}$. For upper cloudy layer, $I_{\downarrow} = I_P$.

In order to define precipitation intensity variance ΔI, let us consider precipitation water content variance $\Delta\delta_{PR}$ due to coalescence between precipitation and cloudy particles

$$\Delta\delta_{PR} = \int_0^{\infty} \Delta m N_{PR}(R)\mathrm{d}R \tag{A3}$$

where $\Delta m = 4\pi\rho_W R^2 \Delta R$, ΔR is the variation of precipitation particles radii.

The value of ΔR can be calculated using formula (15) by taking into account the fall velocity of precipitation drops $U(R)$, which is set by the following relation (Rogers, 1988)

$$U(R) = k_2\sqrt{R} \tag{A4}$$

where $k_2 = 2\times 10^2\ \mathrm{m}^{1/2}/\mathrm{s}$ is a constant.

Substituting Eq. (15) in Eq. (A3) with $N_{PR}(R)$ defined according to Eq. (A1) after solving the integral, the next formula for $\Delta\delta_{PR}$ is resulted

$$\Delta\delta_{PR} = \left(\pi \bar{E}_R \delta k_2 N_0 (R_0^{\downarrow})^{\mu+3.5} \Gamma(\mu+3.5) + \frac{64}{3}\pi\delta N_0 \sqrt{\frac{2\varepsilon}{15\pi\nu}} (R_0^{\downarrow})^{\mu+4} \Gamma(\mu+4)\right)\Delta\tau \tag{A5}$$

where δ is the water content of cloudy layer, $\Delta\tau$ is the fall time of precipitation through the layer, $\bar{E}_R$ is the mean coefficient of cloud drops collection by precipitation particles.

Precipitation water content can be defined from microphysical parameters according to the next relation:

$$\delta_{PR}^{\downarrow} = \frac{4}{3}\pi\rho_W \int_0^{\infty} R^3 N_{PR}(R)\mathrm{d}R \tag{A6}$$

Substituting Eq. (A1) in Eq. (A6) after solving the integral, the next formula was obtained:

$$\delta_{PR}^{\downarrow} = \frac{4}{3}\pi\rho_W N_0 (R_0^{\downarrow})^{\mu+4} \Gamma(\mu+4) \tag{A7}$$

Excluding concentration N_0 from system of Eqs. (A5) and (A7), the next expression can be received:

$$\Delta\delta_{PR} = \delta_{PR}^{\downarrow}\left(\frac{3\bar{E}_R\delta}{4\rho_W}\frac{k_2}{\sqrt{R_0^{\downarrow}}}\frac{\Gamma(\mu+3.5)}{\Gamma(\mu+4)} + \frac{16\delta}{\rho_W}\sqrt{\frac{2\varepsilon}{15\pi\nu}}\right)\Delta\tau \tag{A8}$$

Assuming that $\Delta I/I^{\downarrow} = \Delta\delta_{PR}/\delta_{PR}^{\downarrow}$ and substituting Eq. (A8) in Eq. (A2), the next formula for precipitation intensity forcing was obtained:

$$I_{\downarrow} = I^{\downarrow}\left[1 + \left(\frac{3\bar{E}_R\delta}{4\rho_W}\frac{k_2}{\sqrt{R_0^{\downarrow}}}\frac{\Gamma(\mu+3.5)}{\Gamma(\mu+4)} + \frac{16\delta}{\rho_W}\sqrt{\frac{2\varepsilon}{15\pi\nu}}\right)\Delta\tau\right] + I_P \tag{A9}$$

Fall time of precipitation $\Delta\tau$ can be defined from the following relation

$$\Delta\tau = \Delta H/(U(R_0^{\downarrow}) - W_A) \tag{A10}$$

where ΔH is the thickness of cloudy layer, W_A is the vertical velocity in the atmosphere.

The variation of mean radius of precipitation particles after passing through the layer can be written as:

$$R_{0\downarrow} = R_0^{\downarrow} + \Delta\bar{R}_0 \tag{A11}$$

where $\Delta\bar{R}_0$ defined as

$$\Delta\bar{R}_0 = \frac{\int_0^\infty \Delta R N_{PR}(R)\mathrm{d}R}{\int_0^\infty N_{PR}(R)\mathrm{d}R}$$

After substituting ΔR in Eq. (A11) accordingly to Eq. (8) with $N_{PR}(R)$ defined by formula (A1) after solving the integral, the next formula for variation of mean radius R_0 was obtained:

$$R_{0\downarrow} = R_0^{\downarrow} + \left(\frac{\Gamma(\mu+1.5)}{\Gamma(\mu+2)} \frac{\bar{E}_R \delta}{4\rho_W} k_2 \sqrt{R_0^{\downarrow}} + \frac{16}{3} \frac{\delta}{\rho_W} \sqrt{\frac{2\varepsilon}{15\pi\nu}} R_0^{\downarrow} \right) \Delta\tau \tag{A12}$$

Thus formulas (A9), (A10), (A12) define precipitation intensity forcing for multilevel cloudiness.

A.2. Evaporation of precipitation particles

The variance of precipitation water content due to evaporation $\Delta\delta_E$ is calculated using formula similar to Eq. (A3)

$$\Delta\delta_{PR} = \int_0^\infty \Delta m N_{PR}(R)\mathrm{d}R \tag{A13}$$

where $\Delta m_E = 4\pi\rho_W R^2 \Delta R_E$, ΔR_E is the decreasing of precipitation particles radii due to its evaporation, which is calculated from the next formula (Rogers, 1988)

$$\Delta R_E = -\frac{1}{R} \frac{1-S}{K_1+K_2} F\Delta\tau \tag{A14}$$

where S is the relative humidity in unsaturated layer, $K_1 = (L/R_W T - 1)L\rho_W/kT$, $K_2 = R_W T \rho_W / De(T)$, L is the latent heat of evaporation, R_W is gas constant for water vapour, k is the coefficient of air heat conductivity, $e(T)$ is the saturated pressure for water vapour, $F = 0.78 + 0.31 Sc^{1/3} Re^{1/2}$ is ventilation factor, $Re = 2\rho R U(R)/\eta$ is Reynolds number, $Sc = \eta/\rho D$ is Shmidt number, η is the dynamic viscosity of air.

Substituting Eq. (A14) in Eq. (A13) and using formula (A7) for $\delta_{PR}^{\downarrow}$ after solving the integral, the next formula was received

$$\Delta\delta_E = -\frac{3\delta_{PR}^{\downarrow}}{(R_0^{\downarrow})^2} \frac{1-S}{K_1+K_2} \left(\frac{a}{(\mu+2)(\mu+3)} + b(R_0^{\downarrow})^{3/4} \frac{\Gamma(\mu+2.75)}{\Gamma(\mu+4)} \right) \Delta\tau \tag{A15}$$

where $a = 0.78$ and $b = 0.31 Sc^{1/3}(2\rho k_3/\eta)^{1/2} = 1682.0\ \mathrm{m}^{-3/4}$ are constants.

The formula for precipitation intensity decreasing due to its evaporation resulted from Eq. (A15) in the same way as the definition (A9) from Eq. (A8):

$$I_{\downarrow} = I^{\downarrow}\left[1 - \frac{3}{(R_0^{\downarrow})^2} \frac{1-S}{K_1+K_2} \left(\frac{a}{(\mu+2)(\mu+3)} + b(R_0^{\downarrow})^{3/4} \frac{\Gamma(\mu+2.75)}{\Gamma(\mu+4)} \right) \Delta\tau \right] \tag{A16}$$

The corresponding formula for decreasing of mean radius R_0 is resulted from Eq. (A11) with substitution in it ΔR_E according to Eq. (A14):

$$R_{0\downarrow} = R_0^{\downarrow} - \frac{S-1}{K_1+K_2} \frac{1}{R_0^{\downarrow}} \left(\frac{a}{\mu} + b(R_0^{\downarrow})^{3/4} \right) \Delta\tau \tag{A17}$$

Thus formulas (A16) and (A17) define the decreasing of precipitation intensity due to evaporation in unsaturated layer.

A.3. Melting of precipitation

The variance of precipitation snow content due to its melting $\Delta\delta_M$ can be expressed by formula similar to Eq. (A3):

$$\Delta\delta_M = \int_0^\infty \Delta m_M N_{PR}(R)\mathrm{d}R \tag{A18}$$

where Δm_M is decreasing of precipitation mass due to its melting that can be calculated using next formula (Mason, 1971)

$$\Delta m_M = -4\pi R \frac{k}{L_f} F_M (T - T_S) \Delta\tau \tag{A19}$$

where L_f is the latent heat of melting, $T - T_S$ is the difference between temperatures in the atmosphere and on the surface of melting particle, $F_M = 1.6 + 0.3Re^{1/2}$.

Substituting Eq. (A19) in Eq. (A18) with $N_{PR}(R)$ defined accordingly to Eq. (A1) using formula Eq. (A7) for $\delta^{\downarrow}_{PR}$ after solving, the integral the next formula was received

$$\Delta\delta_M = -\frac{3\delta^{\downarrow}_{PR}}{\rho_W(R^{\downarrow}_0)^2}\frac{k}{L_f}(T - T_S)\left(\frac{c}{(\mu+2)(\mu+3)} + d(R^{\downarrow}_0)^{3/4}\frac{\Gamma(\mu+2.75)}{\Gamma(\mu+4)}\right)\Delta\tau \quad \text{(A20)}$$

where $c = 1.6$ and $d = 0.3(2\rho k_3/\eta)^{1/2} = 1920.0\ \text{m}^{-3/4}$ are constants.

Snow intensity, $I_{S\downarrow}$, resulted at the lower boundary of layer, can be calculated from the following relation

$$I_{S\downarrow} = I^{\downarrow}_S + \Delta I_S \quad \text{(A21)}$$

where $I^{\downarrow}_S$ is snow intensity on the upper boundary of layer, ΔI_S is intensity variance in the layer due to melting.

Assuming in Eq. (A20) that $\Delta I_S/I^{\downarrow}_S = \Delta\delta_M/\delta^{\downarrow}_{PR}$, the formula (A21) can be rewritten to the following form:

$$I_{S\downarrow} = I^{\downarrow}_S\left[1 - \frac{3}{\rho_W(R^{\downarrow}_0)^2}\frac{k}{L_f}(T - T_S)\left(\frac{c}{(\mu+2)(\mu+3)} + d(R^{\downarrow}_0)^{3/4}\frac{\Gamma(\mu+2.75)}{\Gamma(\mu+4)}\right)\Delta\tau\right] \quad \text{(A22)}$$

Formula (A22) allows to receive the part of solid particles in precipitation F_P from the following relation:

$$F_P = I_S/I \quad \text{(A23)}$$

Part of solid particles F_P defined by formulas (A22), (A23) is calculated for melting layer in which upper boundary is defined as zero isotherm, where $I_S = I$ and lower boundary is defined as level where $I_S = 0$.

Appendix B

List of scores

1. $T = n_{11}/n_{01} - n_{12}/n_{02}$: Pearcy–Obychov criterion
2. $U = (n_{11} + n_{22})/n_{00}$: total forecast predictability (describes percentage of the successful forecasts).
3. $U_P = n_{11}/n_{10}$: predictability of precipitation existence (describes percentage of the successful forecasts of precipitation occurrence).
4. $U_{NP} = n_{22}/n_{20}$: predictability of precipitation absence (describes percentage of the successful forecasts of precipitation absence).
5. $\Pi_P = n_{11}/n_{01}$: reliability of precipitation existence (describes percentage of the successfully predicted cases of precipitation occurrence).
6. $\Pi_{NP} = n_{22}/n_{02}$: reliability of precipitation absence. (describes percentage of the successfully predicted cases of precipitation absence).

   Here n_{00} is the total number of estimated cases, n_{11} is the number of cases when precipitation was predicted and it was observed, n_{12} is the number of cases when precipitation was predicted, but it was not observed, n_{21} is the number of cases, when no precipitation was predicted, but it was observed, n_{22} is the number of cases, when no precipitation was predicted and no precipitation was observed, $n_{10} = n_{11} + n_{12}$ is the total number of cases when precipitation was predicted, $n_{01} = n_{11} + n_{21}$ is the total number of cases when precipitation was observed, $n_{20} = n_{21} + n_{22}$ is the total number of cases when no precipitation was predicted, $n_{02} = n_{12} + n_{22}$ is the total number of cases when no precipitation was observed.

7. $\delta Q = (1/n_{00})\sum_{i=1}^{n_{00}}(Q_{\text{Observed}}(i) - Q_{\text{Forecast}}(i))$:

   systematic error of precipitation forecast.
8. $\sigma_Q = \sqrt{\frac{1}{n_{00}}\sum_{i=1}^{n_{00}}(Q_{\text{Observed}}(i) - Q_{\text{Forecast}}(i))^2}$:
   root-mean-square error of the precipitation forecast.
9. $P_{<3} = n_{<3}/n_{00}$: insurance of precipitation forecast. It is defined as percentage of forecast cases in which difference between the observed and the predicted precipitation quantity is less than 3 mm. Here $n_{<3}$ is the number of cases, where difference between the observed and the predicted precipitation quantity is less than 3 mm.

References

Akimov, I.V., 2001a. A method of precipitation amount calculation from liquid and mixed phase clouds, based on a microphysical processes parameterization, Proceedings of Hydrometcentre of Russia, vol. 336., p. 149-163 (Russian).

Akimov, I.V., 2001b. A method of precipitation intensification in underlying cloud layers calculation. Definition of evaporation and melting rates of precipitation, Proceedings of Hydrometcentre of Russia, vol. 336., pp. 164-178 (Russian).

Almeida, F.C., 1979. The effects of small-scale turbulent motions on the growth of a cloud droplet spectrum. J. Atmos. Sci. 36 (8), 1557–1563.

Bourke, W., McAvarey, B., Puri, K., Thurling, R., 1977. Meth. Comput. Phys. 17.

Mazin, I.P., Khrgian, A.C. (Eds.), 1989. Cloud and Cloudy Atmosphere. Reference Book, Hydrometeoizdat, Leningrad, pp. 647, (Russian).

Khrgian, A.C. (Ed.), 1961. Cloud Physics. Reference Book, Hydrometeoizdat, Leningrad, pp. 459, (Russian).

Deardorff, J.W., 1978. Efficient prediction of ground surface temperature and moisture with inclusion of a layer of vegetation. J. Geophys. Res. 83, 1889–1903.

Dmitrieva-Arrago, L.R., Akimov, I.V., 1996. Precipitation formation criteria in nonconvective precipitation calculation in weather models. Meteorol. Hydrol. 8, 5–16.(Russian).

Geleyn, J.F., Hollingsworth, A., 1979. An economical analytical method for the computation of the interaction between scattering and line absorption of radiation. Beitr. Phys. Atmos. 52.

Heise, E., Roeckner, E., 1990. The performance of physically based cloud schemes in general circulation models. Beitr. Phys. Atmos. 63, 121–134.

Hence, A., Heise, E., 1984. A sensitivity study of cloud parameterizations in general circulation models. Beitr. Phys. Atmos. 57, 240–259.

Kachurin, L.G., Morachaevsky, V.G., 1965. Kinetic of Water Phase Transition in Atmosphere. Leningrad State University, 144 pp (Russian).

Kessler, E., 1969. On the distribution and continuity of water substance in atmospheric circulations. Meteor. Monogr. 10, 84.

Koziol, A.S., Leighton, H.G., 1996. The effect of turbulence on the collision rates of small cloud drops. J. Atmos. Sci. 53 (13), 1910–1920.

Kuo, H.L., 1974. Further studies of the parameterization of the influence of cumulus convection on large-scale flow. J. Atmos. Sci. 31, 1232–1240.

Kurbatkin, G.P., Degtiarev, A.I., Frolov, A.V., 1994. Spectral Model Of Atmosphere, Initialization and Database for the Numerical Weather Forecast, Hydrometeoizdat, Leningrad, 184 pp (Russian).

Litvinov, I.V., 1980. Precipitation in Atmosphere and on the Surface, Hydrometeoizdat, Leningrad, 208 pp (Russian).

Lohmann, U., Feichter, J., Chuang, C.C., Penner, J.E., 1999. Prediction of the number of cloud droplets in the ECHAM GCM. J. Geophys. Res. 104 (D8), 9169–9198.

Mason, B.J., 1971. The Physics of Clouds. Clarendon Press, Oxford.

Methodical Instructions, 1991. Realisation of Operative Validation of the New and Advanced Methods of Hydrometeorological and Geliogeophysical Forecasts. RD 52.27.284-91, Hydrometeoizdat, Leningrad, 149 pp (Russian).

Matveev, L.T., 1981. Cloud dynamics. Hydrometeoizdat, Leningrad, 331 pp (Russian).

Matveev, L.T., 2000. Physics of Atmosphere. S.Pb. Hydrometeoizdat, 778 pp (Russian).

Pinsky, M.B., Khain, A.P., 1997. Turbulence effects on the collision kernel. I. Formation of velocity deviations of drops falling within three-dimensional flow. Q. J. R. Meteorol. Soc. 123, 1517–1542.

Rogers, R.R., 1988. A Short Course in Cloud Physics. Pergamon Press, Oxford.

Rotstayn, L.D., 1997. A physically based scheme for the treatment of stratiform clouds and precipitation in large-scale models. I. Description and evaluation of the microphysical processes. Q. J. R. Meteorol. Soc. 123, 1227–1282.

Rutledge, S.A., Hobbs, P.V., 1983. The mesoscale and microscale structure and organization of clouds and precipitation in midlatitude cyclones. VIII. A model for the 'seeder–feeder' process in warm-frontal rainbands. J. Atmos. Sci. 40, 1185–1206.

Sasamori, T., 1975. A statistical model for stationary atmospheric cloudiness, liquid water content and rate of precipitation. Mon. Wea. Rev. 103 (6), 1037–1049.

Shaw, R.A., Reade, W.C., Collins, L.R., Verlinde, J., 1998. Preferential concentration of cloud droplets by turbulence: effects on the early evolution of cumulus cloud droplet spectra. J. Atmos. Sci. 55, 1965–1976.

Shlesinger, M.E., Oh, J.H., Rosenfeld, O., 1988. A parameterisation of the evaporation of rainfall. Mon. Wea. Rev. 116 (10), 1887–1895.

Smith, R.N.B., 1990. A scheme for predicting layer clouds and their water content in a general circulation model. Q. J. R. Meteorol. Soc. 116, 435–460.

Smoluchowsky, M., 1916. Drei Vortage uber Diffusion Brounische Bewegung und Koagulation von Kolloidteilchen. Phys. Z. Bd 17, 557–585.(German).

Sundqvist, H., 1978. A parameterization scheme for non-convective condensation including prediction of cloud water content. Q. J. R. Meteorol. Soc. 104, 677–690.

Sundqvist, H., 1993. Inclusion of ice phase of hydrometeors in cloud parameterization for mesoscale and large-scale models. Beitr. Phys. Atmos. 66, 137–147.

Tripoli, G.J., Cotton, W.R., 1980. A numerical investigation of several factors contributing to the observed variable intensity of deep convection over south Florida. J. Appl. Meteorol. 19, 1037–1063.

Voloshyk, V.M., Sedynov, Y.S., 1975. Coalescence Processes in Disperse Systems. Hydrometeoizdat, Leningrad, 320 pp (Russian).

Wacker, U., 1995. Competition of precipitation particles in a model with parameterized cloud microphysics. J. Atmos. Sci. 52 (14), 2577–2589.

Wilson, D.R., Ballard, S.P., 1999. A microphysically based precipitation scheme for the UK Meteorological Office Unified Model. Q. J. R. Meteorol. Soc. 125, 1607–1636.

Zhao, Q., Carr, F.H., 1997. A prognostic cloud scheme for operational NWP models. Mon. Wea. Rev. 125 (8), 1931–1953.