

## Short Note

# One-dimensional random patchy saturation model for velocity and attenuation in porous rocks

Tobias M. Müller<sup>1</sup> and Boris Gurevich<sup>1</sup>

### INTRODUCTION

Porous rocks encountered in hydrocarbon reservoirs are often saturated with a mixture of two or more fluids. Generation of synthetic seismograms as well as interpretation of in-situ attenuation measurements require a theoretical understanding of the relation between the heterogeneous distribution of fluid patches and the acoustic properties of rocks. Thus, the problem of calculating acoustic properties of rocks saturated with a mixture of two fluids has attracted considerable interest (White, 1975; Murphy, 1982; Gist, 1994; Mavko and Mukerji, 1998; Pride et al., 2004). At the same time, this problem is also interesting from the theoretical point of view because partially saturated rocks represent a particularly interesting situation when the effects of dynamic poroelasticity may be significant at seismic or sonic frequencies. Indeed, it is a radical departure from the situation with a porous material fully saturated with a single fluid. Such a fully saturated material exhibits frequency-dependent effects only at frequencies comparable with Biot's characteristic frequency (Bourbié et al., 1987)  $\omega_c = \eta\phi/\kappa\rho_f$ , where  $\rho_f$  is the fluid density,  $\eta$  is fluid viscosity,  $\kappa$  is permeability, and  $\phi$  is porosity. For frequencies much lower than  $\omega_c$ , the dynamic effects can be ignored and Gassmann theory applies. According to Gassmann theory (Bourbié et al., 1987), bulk ( $K^{sat}$ ) and shear ( $\mu^{sat}$ ) moduli of the porous material made up of a solid grain material with bulk modulus  $K_s$  fully saturated with a fluid with the bulk modulus  $K_f$  are

$$K^{sat} = K + \alpha^2 M, \quad (1)$$

$$\mu^{sat} = \mu, \quad (2)$$

where

$$\alpha = 1 - K/K_s, \quad M = 1/[(\alpha - \phi)/K_s + \phi/K_f], \quad (3)$$

and  $K$  and  $\mu$  are bulk and shear moduli of the dry rock, respectively.

The reason that partially saturated rocks respond to acoustic waves differently from fully saturated rocks is fluid diffusion between pockets of different fluid saturation. A fundamental assumption of the Gassmann theory is that frequency is sufficiently low so that the fluid everywhere in the rock can be considered to be in pressure equilibrium. In a homogeneous, fully saturated rock this is the case for frequencies below  $\omega_c$ . However, in partially saturated rocks, fluid pressure induced by the passing wave in pockets of rock saturated by different fluids will be different. Analysis of acoustic wave propagation in such a material must be based on Biot's equations of dynamic poroelasticity with spatially varying coefficients. Such an analysis (Norris, 1993) shows that the distribution of pressure is governed by the diffusion equation, with the diffusion length

$$\lambda_d = (\kappa N/\omega\eta)^{1/2}. \quad (4)$$

Here,  $\omega$  is angular frequency, and

$$N = ML/H, \quad (5)$$

where  $L = K + 4\mu/3$  and  $H = K^{sat} + 4\mu/3$  are the P-wave moduli of the dry and saturated rock, respectively. If frequency is low enough so that the characteristic heterogeneity length  $d$  is much smaller than  $\lambda_d$ , then pressure equilibration is achieved, and Gassmann theory can be used with fluid bulk modulus calculated by Wood's formula (e.g. Mavko et al., 1998). The P-wave modulus is then  $H_w = H(K_w)$ , where the argument of  $H$  equals the effective fluid bulk modulus,

$$K_w = [S_1/K_{f1} + S_2/K_{f2}]^{-1}, \quad (6)$$

with  $S_i$  and  $K_{fi}$  denoting the saturation and modulus of the  $i$ th fluid. The situation where the pockets of different fluids are sufficiently small is referred to as homogeneous saturation. In the opposite case, when  $d \gg \lambda_d$ , there is no pressure communication between different pockets. In this case, fluid-flow effects

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<sup>1</sup>Curtin University of Technology, Department of Exploration Geophysics, GPO Box U1987, Perth, Western Australia 6845, Australia. E-mail: tobias.muller@geophy.curtin.edu.au; boris.gurevich@geophy.curtin.edu.au.

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can be ignored, and overall rock may be considered equivalent to an elastic composite material consisting of homogeneous parts, whose properties are given by Gassmann theory. Since those parts differ only by the saturating fluid, they all have the same shear modulus, and Hill's equation for the bulk modulus (Mavko et al., 1998) can be applied, resulting in

$$H_H = [S_1/H(K_{f1}) + S_2/H(K_{f2})]^{-1}. \quad (7)$$

This situation is referred to as patchy saturation and results in a much more gradual increase of P-wave velocity with saturation than in the case of homogeneous saturation (Figure 1).

The elastic moduli in both homogeneous and patchy saturation are given by real numbers and are independent of frequency. However, while these moduli do not depend on frequency, the ranges of applicability of these limiting moduli do. Homogeneous saturation moduli represent the low-frequency limit and patchy moduli the high-frequency limit. For any nonzero water saturation ( $0 < S < 1$ ), homogeneous moduli are always smaller than those for patchy saturation. This means that at intermediate frequencies the partially saturated medium exhibits P-wave velocity dispersion and attenuation. We are aware of only few models proposed to model frequency-dependent effects of patchy saturation, including the periodic flat-slab model (Norris, 1993) and the spherical-shell model (White, 1975; Dutta and Seriff, 1979). Johnson (2001) developed a more general theory for elastic properties of rock containing regular distribution of pockets of two fluids assuming that the bulk modulus must obey Gassmann theory at low and high frequencies and must also obey causality constraints. He obtained excellent agreement of his "scaling" model with the theoretical solutions for the periodic flat-slab and spherical-shell models. The P-wave velocity and attenuation for the periodic slab model based on Johnson's results is shown in Figure 2 as functions of saturation and frequency. Pride and Berryman (2003) developed a theory for frequency-dependent attenuation due to mesoscale inhomogeneities for a double-porosity

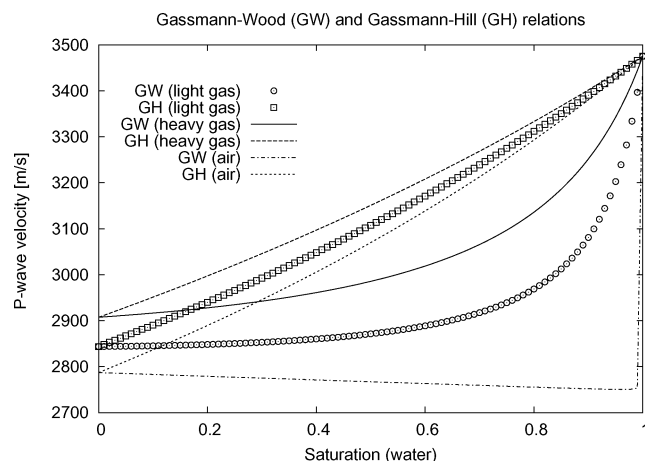


Figure 1. P-wave velocity of the Gassmann-Wood and Gassmann-Hill model as a function of water saturation. The properties of the porous rock are those of a typical reservoir sandstone ( $K_S = 35$  GPa,  $K = 7$  GPa,  $\mu = 9$  GPa,  $\rho_S = 2.65$  g/cm<sup>3</sup>,  $\phi = 8\%$ ,  $K_f = 0.25$  GPa,  $\rho_f = 0.4$  g/cm<sup>3</sup>). For light gas,  $K_f = 0.1$  GPa and  $\rho_f = 0.1$  g/cm<sup>3</sup>. For air,  $K_f = 10^5$  Pa and  $\rho_f = 0.1$  g/cm<sup>3</sup>. For water,  $K_f = 2.25$  GPa and  $\rho_f = 0.99$  g/cm<sup>3</sup>.

medium. A companion paper (Pride et al., 2004) shows that the predictions of their model coincide with those of Johnson (2001) if the fluid patches are interpreted as mesoscale inhomogeneities. A common feature of all these models is that the low-frequency asymptotics of attenuation scales like  $Q^{-1} \propto \omega$ . This is also true for the model of Dvorkin et al. (2003), which is based on the concept of the standard linear solid.

All above-mentioned solutions assume regular distribution of fluids in a sense that the size of the fluid pockets is the same throughout the medium. In reality, of course, fluids are distributed throughout the medium somewhat randomly and form pockets of different sizes and shapes. Experiments show that the geometry of fluid patches strongly depends on the experimental setup (see, e.g., Cadoret et al., 1995, 1998) and is also linked to rock heterogeneity (Knight et al., 1998). It is the purpose of this Short Note to establish a relation between wavefield attributes and heterogeneous distribution of fluids for the case the fluid patches form a realization of a statistically homogeneous random medium. This implies that only simple statistical measures are required in order to characterize the patch distribution. In particular, we propose a 1D random model based on the theory of statistical wave propagation which includes the effect of interlayer flow. That is to say, the diffusive fluid flow between the patches degenerates into a 1D geometry where fluid flows from one slab into another. It is a simplistic model in the sense that it does not account for the origin of fluid patches nor for local flow effects occurring on the pore scale. It also assumes that the fluids in the rock are immiscible. However our model allows the analysis of the frequency characteristics of patchy saturation which are (as shown below) different from those assuming a periodic distribution of patches. This difference has important implications for the observability of the patchy saturation effect in field situations and the determination of the scales of fluid distribution in the earth. The proposed model can be used to compare the patchy saturation effect observed at different frequency ranges such as sonic logs and surface seismic data.

In the next section, we extend an existing theory for velocity dispersion and attenuation in a poroelastic medium to the case of partial saturation. We are particularly interested in the frequency dependence of attenuation and its asymptotic behavior. It follows a comparison of theoretical and reported experimental results. Finally, we discuss the difference between the 1D random model and those based on periodicity assumptions.

## A 1D RANDOM PATCHY SATURATION MODEL

If the distribution of fluids in a rock is random, wave propagation can be analyzed using Biot's equations of poroelasticity whose coefficients are now piecewise constant random functions of position. The concept of a randomly inhomogeneous porous medium was introduced by Lopatnikov and Gurevich (1988). For the 1D case, Gurevich and Lopatnikov (1995) developed a model for attenuation and velocity dispersion. This model, which employs a so-called Bourret approximation for the ensemble-averaged (coherent) wavefield, accounts for conversion scattering from fast P-waves into Biot's slow wave but neglects elastic scattering. These approximate results were later validated theoretically [using the generalized O'Doherty-Anstey formalism by Gelinsky et al. (1998)] and numerically (Gurevich et al., 1997).

In the 1D model of Gurevich and Lopatnikov (1995), all the material parameters of the porous medium are assumed to be stationary random functions of one coordinate  $z$  with given correlation properties. From the geometrical point of view, it is a randomized version of the periodic flat slab model, with slab thickness randomly varying from slab to slab. In the Bourret approximation, the effective complex P-wave modulus  $\tilde{H}$  for waves propagating perpendicular to layering ( $z$ -direction) in a random system of porous layers as a function of frequency is given by

$$\tilde{H}(\omega) = H_0 \left[ 1 - isk_2 \int_0^\infty \psi(\xi) \exp(ik_2\xi) d\xi \right], \quad (8)$$

where

$$k_2 = \sqrt{i\omega} \frac{\left\langle \sqrt{\frac{\eta}{\kappa} N} \right\rangle}{\langle N \rangle} \quad (9)$$

is the effective wavenumber of Biot's slow wave [ $\sqrt{i} = (i+1)/\sqrt{2}$ ] and

$$\psi(\xi) = \langle \varepsilon(z)\varepsilon(z-\xi) \rangle / \langle \varepsilon(z)^2 \rangle \quad (10)$$

is the normalized autocorrelation function of the fluctuating parameter  $\varepsilon(z) = \alpha(z)M(z)/H(z) - \langle \alpha(z)M(z)/H(z) \rangle$ . The an-

gle brackets denote ensemble averaging. The degree of inhomogeneity of the medium is characterized by the dimensionless coefficient

$$s = \left\langle \frac{1}{H} \right\rangle^{-1} \left( \left\langle \frac{\alpha^2}{L^2} N \right\rangle - \left\langle \frac{\alpha}{L} \right\rangle^2 \left\langle \frac{1}{N} \right\rangle^{-1} \right). \quad (11)$$

Expression 11 for  $s$  is the corrected version of equation 81 in Gurevich and Lopatnikov (1995). Parameter  $H_0$  denotes the P-wave modulus for an effective homogeneous porous medium, i.e.,  $\tilde{H}(0) \equiv H_0$ . We note that in the presence of permeability fluctuations one has to use an averaged permeability value to obtain  $k_2$ . As shown in Shapiro and Müller (1999), the arithmetic average of  $\kappa$  provides a good approximation to  $\langle \kappa \rangle$ . Note that the coefficient  $s$  does not include fluctuations of permeability but those of porosity (via  $M$ ).

We now specify the above results for the particular case of partial saturation. Let us assume that the rock frame is homogeneous, while the fluid properties are piecewise constant random functions of  $z$  with values corresponding to "water" or "gas." Such a medium can be also viewed as a system of alternating water- and gas-saturated layers of random thickness. Then the averaging operation for some quantity  $X$  can be written  $\langle X \rangle \equiv X_w S + X_g (1 - S)$ , where subscripts  $w$  and  $g$  refer to the water and gas phase, respectively. Applying this averaging

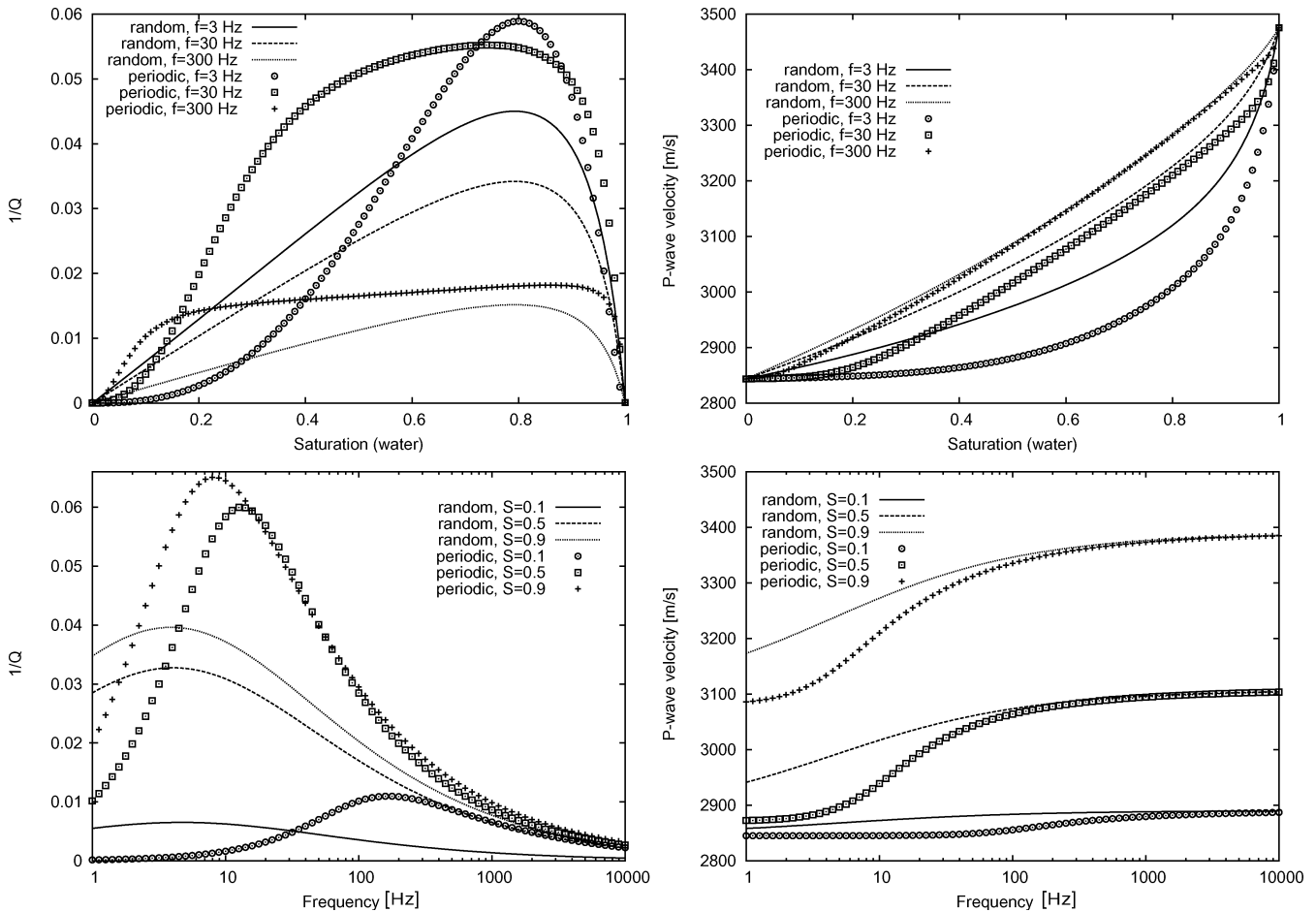


Figure 2.  $Q^{-1}$  and velocity for P-waves as a function of saturation and frequency for random and periodic models.

operation to equations 9 and 11 yields the explicit results

$$k_2 = \sqrt{\frac{i\omega}{k} \frac{\sqrt{\eta_w N_w S} + \sqrt{\eta_g N_g (1-S)}}{N_w S + N_g (1-S)}}, \quad (12)$$

and

$$s = \alpha^2 \frac{\frac{M_w^2}{H_w^2} S + \frac{M_g^2}{H_g^2} (1-S) - \left[ \frac{M_w}{H_w} S + \frac{M_g}{H_g} (1-S) \right]^2}{[N_g S + N_w (1-S)] \left[ \frac{S}{H_w} + \frac{(1-S)}{H_g} \right]}. \quad (13)$$

The effective P-wave modulus  $H_0$  is then given by the P-wave modulus for homogeneous saturation:  $H_0 = H_w$ . Equation 8 together with equations 12 and 13 describes the velocity dispersion and attenuation of the coherent wavefield for all frequencies and saturations. If the wavelength is much larger than the average patch size (which is plausible for seismic and sonic frequencies), the proposed model can be also used in order to characterize velocity and attenuation in single realizations of the random medium.

The spatial fluid distribution enters equation 8 through the autocorrelation function  $\psi(\xi)$ . The solution can vary greatly depending on the properties of  $\psi(\xi)$ . In particular, by choosing  $\psi(\xi)$  to be a periodic function with a fixed period  $d$ , one can attain the uniform flat-slab model (Gurevich and Lopatnikov, 1995). If, on the other hand, the correlation function  $\psi(\xi)$  is integrable from 0 to  $\infty$  (i.e.,  $\int_0^\infty \psi(\xi) d\xi = A < \infty$ ), then the low-frequency asymptote is given by

$$\tilde{H} = H_W [1 + isAk_2 + \dots], \quad (14)$$

which results in the following scaling for attenuation:

$$Q^{-1} \equiv \frac{\Im\{\tilde{H}\}}{\Re\{\tilde{H}\}} \propto \sqrt{\omega}. \quad (15)$$

In the static limit, the result for homogeneous saturation is recovered, i.e.,  $\tilde{H} = H_W$ . The high frequency asymptotic depends on the behavior of  $\psi(\xi)$  in the vicinity of  $\xi=0$ . For a large class of correlation functions that can be expanded about the origin in the power series  $\psi(\xi) = 1 - 2|\xi|/d + \dots$  with  $d < \infty$ , the high-frequency asymptotics yield

$$\begin{aligned} \tilde{H} &= H_W [1 + s - 2is(k_2 d)^{-1} + \dots], \\ &\simeq H_H [1 - 2is(k_2 d)^{-1} + \dots], \end{aligned} \quad (16)$$

where  $H_H$  denotes the P-wave modulus for patchy saturation. This results in the proportionality

$$Q^{-1} \propto \frac{1}{\sqrt{\omega}}. \quad (17)$$

In particular, for the exponential correlation function  $\psi(\xi) = \exp(-2|\xi|/d)$ , the integral in equation 8 can be evaluated analytically for all frequencies to give

$$\tilde{H}(\omega) = H_W \left[ 1 + \frac{s}{1 + \frac{2i}{k_2 d}} \right]. \quad (18)$$

For the Gaussian correlation function  $\psi(\xi) = \exp(-4\xi^2/d^2)$ , we obtain

$$\tilde{H}(\omega) = H_W [1 - i\sqrt{\pi} s \Omega \exp(-\Omega^2) [1 + \operatorname{erf}(i\Omega)]], \quad (19)$$

where  $\Omega = k_2 d/4$  and  $\operatorname{erf}$  denotes the error function. Result 18 in terms of P-wave velocity  $v_p \equiv \sqrt{\Re\{\tilde{H}\}}/\rho$  and attenuation  $Q^{-1}$  is illustrated in Figure 2 for constant correlation length  $a \equiv d/2 = 0.2$  m as a function of water saturation and frequency (lines labeled “random”). The poroelastic moduli are the same as for Figure 1, and the pore fluids are water and light gas with viscosity  $\eta = 3.0 \times 10^{-5}$  Pa · s. The 1D random patchy saturation model is compared with the solution for the periodic slab model (spatial period is  $d = 0.2$  m) of Johnson (2001). The corresponding curves are denoted “periodic.” It can be observed that for all intermediate values of saturation, the velocity for the random patchy model is larger than that for the periodic solution. This difference becomes larger if frequency decreases. For the high-frequency range ( $f \geq 300$  Hz), both models practically coincide with the Gassmann-Hill curve (see Figure 1). Note also the more gradual variation of velocity with frequency compared to the periodic slab model. This is consistent with the larger width and lower magnitude of the attenuation for the random patchy saturation model.

## INTERPRETATION OF LABORATORY ATTENUATION MEASUREMENTS

Patchy saturation is considered as one of the possible causes of intrinsic attenuation at seismic and sonic frequencies. With the help of the 1D random patchy saturation model, we will now try to interpret attenuation measurements where fluid flow between patches is believed to be the main source of attenuation. Reported lab measurements of the dependence of attenuation on the degree of saturation and frequency are often based on the resonant bar technique (e.g. Cadoret et al., 1998) operating at frequencies from several hundred Hz to a few kHz. Instead of P-wave attenuation, the attenuation of extensional waves is measured, and the above-mentioned theories should be modified accordingly. However, measuring attenuation of extensional ( $Q_E$ ) and shear waves ( $Q_S$ ) will also provide an estimate of  $Q_P \equiv Q$  if the material’s Poisson ratio  $\nu$  is known because

$$(1 - \nu)(1 - 2\nu)Q^{-1} = (1 + \nu)Q_E^{-1} - 2\nu(2 - \nu)Q_S^{-1} \quad (20)$$

(e.g. Mavko et al., 1998). The Poisson ratio is also dependent on saturation and can be estimated using the measured P- and S-wave velocities.

In the following, we interpret the experimental data of Cadoret et al. (1995, 1998). In particular, we choose the data set of the drying and depressurized experiments on the “Estailades” limestone, where the observed velocity and attenuation behavior are mainly caused by the wave-induced fluid flow between patches (see Figures 13 and 3 in Cadoret et al., 1995 and 1998, respectively). For this data set, computer tomography (CT) scans of the rock sample are also available. We use these scans in order to infer a saturation-dependent correlation length  $a(S)$ . We find that an exponential decrease of  $a$  with saturation is most consistent with the data. This is also supported by the analysis of Tserkovnyak and Johnson (2002), where an average patch size is directly estimated from

the measured velocity and attenuation data using Johnson's (2001) theory. Figure 3 shows the P-wave velocity (normalized by the dry rock velocity) measured at two frequencies for the drying experiment as a function of water saturation. The theoretical predictions according to the 1D patchy saturation model are also displayed. The parameter  $a_0$  corresponds to a reference correlation length and is estimated from the saturation map with  $S = 0.92$  (Figure 4 in Cadoret et al., 1998). One can observe a qualitative agreement between experimental data and theoretical prediction. The overall shape of the saturation dependency of velocity and attenuation is well reproduced. For the experiment with frequency  $f = 1$  kHz, the random patchy-saturation model overestimates the experimental velocities for almost all saturations. This mismatch is possibly caused by the fact that the diffusion length ( $\lambda_d \approx 3.5$  cm) is of the order of the radius of the cylindrical sample (4 cm) so that the wave-induced pressure is practically equilibrated even at finite frequencies. In fact, the Gassmann-Wood bound provides a better predic-

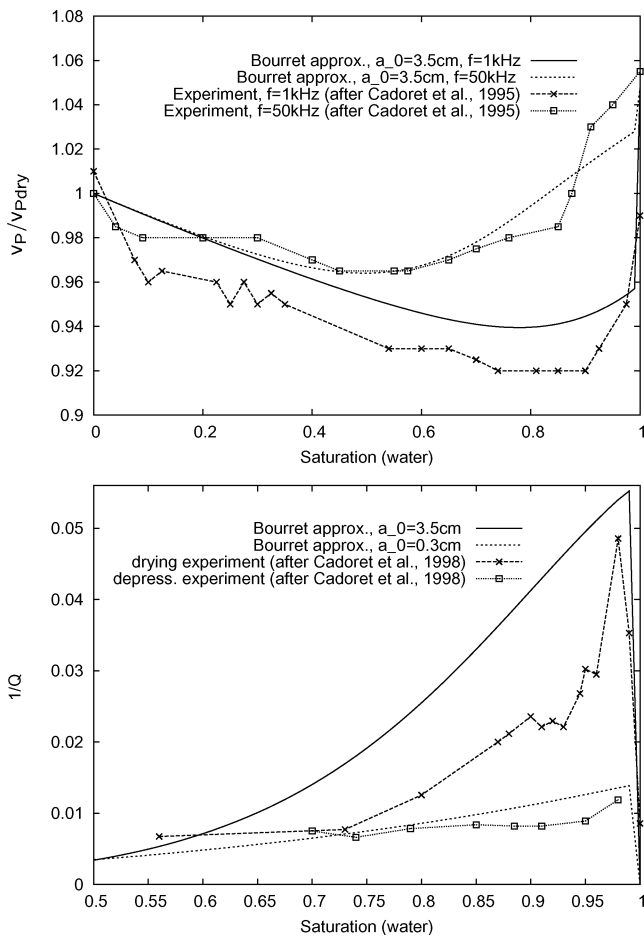


Figure 3. (Top) Dependence of normalized P-wave velocity (normalized by the P-wave velocity of dry rock) on saturation after the drying experiments of Cadoret et al. (1995). Experimental data can be qualitatively described by the random patchy saturation model with an estimated correlation length from the CT scans of the rock sample ( $a_0$  denotes the correlation length estimated for  $S = 0.92$ ). (Bottom) For the drying and depressurized experiments with  $f = 1$  kHz, the P-wave attenuation data can be also qualitatively described by the random patchy-saturation model.

tion. Taking into account the almost constant S-wave velocity (Figure 6 in Cadoret et al., 1995), we compute the Poisson ratio. The computed values of  $\nu$  range from 0.13 for the dry rock to 0.27 for the fully saturated rock. We note that the poroelastic constants inferred from the experimental data are consistent with those determined by Tserkovnyak and Johnson (2002). Based on relationship 20 among the quality factors  $Q$ ,  $Q_E$ , and  $Q_S$ , we compute  $Q^{-1}$  for the drying and depressurized experiments (connected symbols in the bottom part of Figure 3). These experimental data can be qualitatively described by the random patchy saturation model (curves in the bottom part of Figure 3). In particular, the larger attenuation in the drying experiment compared to that in the depressurizing experiment can be consistently modeled using information on the average patch size (we use  $a_0 = 3.5$  cm for drying and  $a_0 = 0.3$  cm for depressurizing experiment). We are aware of the fact that the measured attenuation in the depressurizing experiment may be partially caused by the Biot-Gardner effect, which introduces additional error in the  $Q_P$  estimate (White, 1986). The latter

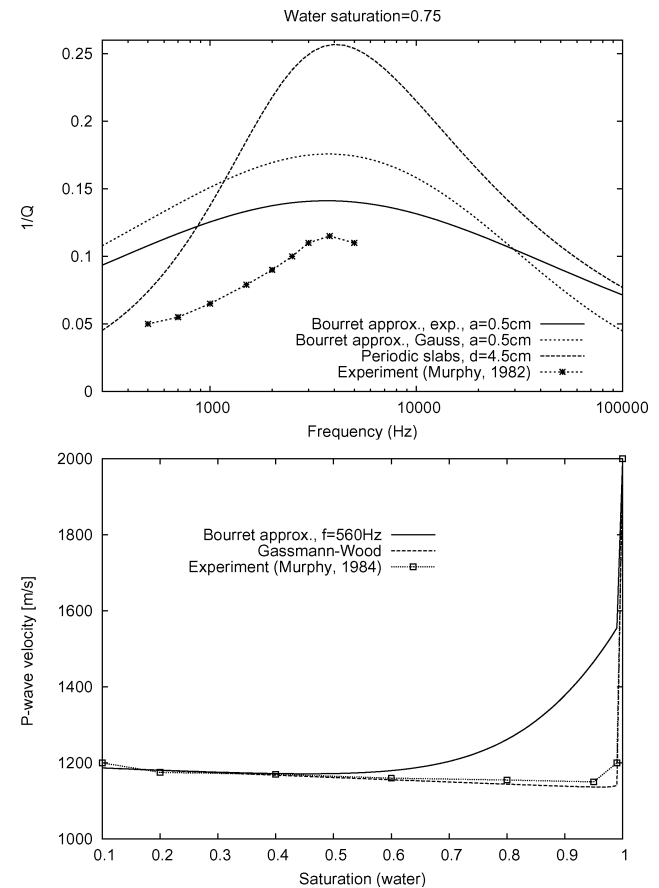


Figure 4. (Top)  $Q^{-1}$  as a function of frequency after the experiment of Murphy (1982). If correlation length is chosen such that experimental and theoretical maxima coincide, the random patchy saturation model is able to predict the magnitude of measured attenuation well. The periodic-slab model, however, overestimates the amount of attenuation. (Bottom) P-wave velocity as a function of saturation measured at frequency  $f = 560$  Hz (after Murphy, 1984). The random patchy-saturation model yields reasonable results only for  $S < 0.8$ . The Gassmann-Wood bound fits the data for all saturation values very well.

error may result in an overestimation of the attenuation caused by fluid flow between the patches. However, it shows that the overall behavior of attenuation is consistent to first order with the 1D random patchy-saturation model.

Murphy (1982) measured attenuation of extensional and shear waves in porous sandstones using a resonant bar technique. Attenuation is measured as a function of frequency and saturation. Because of the very small Poisson ratio of the “Massilon” sandstone ( $\nu \approx 0.02$ ), the extensional wave attenuation can be interpreted as P-wave attenuation (with an error of 5% as estimated by Murphy). The results of Murphy (1982) are replotted in Figure 4, where  $Q^{-1}$  is shown in the frequency interval between 500 and 5000 Hz for an estimated saturation of  $S \approx 75\%$ . Most data points were collected in the vicinity of the maximum (about 4 kHz). All poroelastic moduli are known from the experiment (Murphy, 1982, 1984). However, there is no information available on the average size of the fluid patches. That means we have to choose the correlation length (and also the type of correlation function) one way or another in order to model the experimental results. Assuming that the observed attenuation is only caused by fluid flow between patches, we choose the correlation length in such a way that maximal attenuation occurs at the same frequency in theory and experiment. We think that this is a natural choice because (1) the measurements clearly identify such a maximum and (2) it is the frequency dependence which is specific for the attenuation mechanism under consideration. According to the random patchy-saturation model, attenuation becomes maximal at frequency  $f = \kappa N / (2\pi \eta a^2)$ . The solid line in the upper plot of Figure 4 corresponds to  $Q^{-1}$  based on formula 18 with a correlation length of  $a = 0.5$  cm. Comparing our theoretical estimates and experiment (solid line and connected symbols in Figure 4), we conclude that the 1D random patchy saturation model predicts the magnitude of attenuation reasonably well (within an accuracy of 20% in the maximum region). Using a Gaussian correlation function with the same correlation length produces a slightly larger attenuation. Applying the same criterion (coincidence of attenuation peaks) to the periodic slab solution with spatial period  $d = 4.5$  cm yields the dashed line in Figure 4, which clearly overestimates the amount of attenuation. Obviously, the choice of the correlation function influences the result. A consistency check with the given P-wave velocity measured at  $f = 560$  Hz (Figure 1 in Murphy, 1984) shows that our model yields only reasonable results for  $S < 0.8$  (lower part of Figure 4). For higher saturation (i.e.,  $S = 0.91$ ), the Gassmann-Wood bound (or equivalently our model with  $a \rightarrow 0$ ) yields a satisfactory prediction. This implies a case of homogeneous saturation with zero attenuation which, however, contradicts to the observed attenuation of  $Q^{-1} \approx 0.05$  (see Figure 8 in Murphy, 1982). This means that this attenuation is caused by some physical mechanism other than patchy saturation.

## DISCUSSION AND CONCLUSIONS

The main conclusion from our results is that velocity dispersion and attenuation for waves propagating in rocks with a 1D random fluid distribution change significantly when the fluid is characterized by a 1D or 3D periodic distribution. In particular, for a 1D random fluid distribution with exponential correlation, the inverse quality factor at high frequencies

scales with  $\omega^{-1/2}$ , which is the same as for the scaling model and the periodic models (see Johnson, 2001; Pride et al., 2004). At low frequencies, however,  $1/Q$  in the 1D random patchy saturation model scales with  $\omega^{1/2}$ , while in the scaling model and the periodic models  $1/Q$  scales with  $\omega$ . Consequently the shape of the dispersion and attenuation curves for the random patchy-saturation model is different compared to periodic models in that both the attenuation and dispersion curves exhibit much more gradual variation with frequency. This difference in P-wave velocity dispersion can be clearly observed in Figure 2. This behavior is to be expected since there is no longer a single pocket size or period which defines a single crossover frequency.

There are several important implications from this discrepancy. First of all, it is reasonable to assume that random spatial distribution of fluids is more realistic than the periodic one. Thus, given that the dispersion relationships for the two situations are qualitatively different, the random patchy-saturation model may be more useful in the analysis of patchy saturation effects. Second, the much more gradual variation of velocity and the width of the attenuation peak in this situation imply that effects of dispersion and attenuation may be significant over a much wider frequency range than was previously thought. Third, it is clear that not just the parameters, but the very form of the real dispersion relationship will depend on the statistical properties of spatial fluid distribution.

From the comparison with laboratory attenuation measurements, we conclude that the 1D random patchy-saturation model is able to describe the saturation and frequency dependence as well as the magnitude of attenuation at least qualitatively. The proposed model may be a good starting point for interpretation of in-situ attenuation measurements and estimation of the average size of the fluid patches. A model of patchy saturation with a more realistic 3D random fluid distribution is under development.

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