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Search for direct empirical spatial correlation signatures of the critical triggering earthquake model

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SUMMARY

We propose a new test of the critical earthquake model based on the hypothesis that precursory earthquakes are 'actors' that create fluctuations in the stress field which exhibit an increasing correlation length as the critical large event becomes imminent. Our approach constitutes an attempt to build a more physically based time-dependent indicator (cumulative scalar stress function), in the spirit of, but improving on, the cumulative Benioff strain used in previous works documenting the phenomenon of accelerating seismicity. Using a simplified scalar space and time-dependent viscoelastic Green's function in a two-layer model of the Earth's lithosphere, we compute spatiotemporal pseudo-stress fluctuations induced by a series of events before four of the largest recent shocks in southern California. Through an appropriate spatial wavelet transform, we then estimate the contribution of each event in the series to the correlation properties of the simplified pseudo-stress field around the location of the mainshock at different scales. This allows us to define a cumulative scalar pseudo-stress function which reveals neither an acceleration of stress storage at the epicentre of the mainshock nor an increase of the spatial stress–stress correlation length similar to those observed previously for the cumulative Benioff strain. The earthquakes we studied are thus either simple 'witnesses' of a large-scale tectonic organization, or are simply unrelated, and/or the Green's function describing interactions between earthquakes has a significantly longer range than predicted for standard viscoelastic media used here.

Key words: critical earthquake, accelerated seismicity, precursors, earthquake forecast.

1 INTRODUCTION

Numerous reports of precursory geophysical anomalies preceding earthquakes have fuelled hope for the development of forecasting or predicting tools. The suggested anomalies take many different forms and relate to many different disciplines such as seismic wave propagation, chemistry, hydrology, electromagnetism and so on. The most straightforward approach consists of using patterns of seismicity rates to attempt to forecast future large events (see for instance Keilis-Borok & Soloviev 2002, and references therein).

Spatiotemporal patterns of seismicity, such as anomalous bursts of aftershocks, quiescence or accelerated seismicity, are thought to betray a state of progressive damage or of organization within the Earth's crust preparing the stage for a large earthquake. There is a large literature reporting that large events have been preceded by anomalous trends of seismic activity both in time and space. Some works report that seismic activity increases as an inverse power of the time to the main event (sometimes referred to as an inverse Omori law for relatively short time spans) (see e.g. Helmstetter *et al.* 2003b, and references therein), while others document a quiescence (Wyss 1997), or even contest the existence of such anomalies at all.

There has been an almost general consensus that those anomalous patterns, if any, are likely to occur within days to weeks before the mainshock and probably not at larger timescales (Jones & Molnar 1979). With respect to spatial structures, the precursory patterns are very often sought or observed in the immediate vicinity of the mainshock, i.e. within distances of a few rupture lengths from the epicentre. The most famous observed pattern is the so-called (not necessarily local) doughnut pattern (Mogi 1969; Eguchi 1998). Thus, in any case, both temporal and spatial precursory patterns were usually thought to take place at short distances from the upcoming large event.

In the last decade a different concept has progressively emerged according to which precursory seismic patterns may occur up to decades preceding large earthquakes and at spatial distances many times the main shock rupture length. This concept is rooted in the theory of critical phenomena (see Sornette 2000, 2004 for an introduction and a review adapted to a general geophysical readership) and has been documented and advocated by the Russian school (Keilis-Borok 1990; Keilis-Borok & Soloviev 2002). Probably the first report by Keilis-Borok & Malinovskaya (1964) of an earthquake precursor (the premonitory increase in the total area of the ruptures in the earthquake sources in a medium-magnitude range) already featured very long-range correlations (over 10 seismic source lengths) and worldwide similarity. More recently, Knopoff *et al.* (1996) have also discovered a surprising long-range spatial dependence in the increase of seismicity of intermediate-magnitude earthquakes prior to large earthquakes in California.

From a theoretical point of view, this ‘critical earthquake’ concept has its seismological roots dating back to the branching model of Vere-Jones (1977). A few years later, Allègre *et al.* (1982) proposed a percolation model of damage/rupture prior to an earthquake, emphasizing the multiscale nature of rupture prior to a critical percolation point. Their model is very similar to the real-space renormalization group approach to a percolation model performed by Reynolds *et al.* (1977). Similar ideas were also explored in a hierarchical model of rupture by Smalley *et al.* (1985). Sornette & Sornette (1990) proposed an observable consequence of the critical point model of Allègre *et al.* (1982) with the goal of verifying the proposed scaling rules of rupture. Almost simultaneously, but following apparently an independent line of thought, Voight (1988, 1989) introduced the idea of a time-to-failure analysis in the form of an empirical second-order non-linear differential equation, which for certain values of the parameters leads to a time-to-failure power law of the form of an inverse Omori law. This was used and tested later for predicting volcanic eruptions. Then, Sykes & Jaumé (1990) performed the first empirical study reporting and quantifying with a specific law an acceleration of seismicity prior to large earthquakes. They used an exponential law to describe the acceleration and did not use or discuss the concept of a critical earthquake. Bufe & Varnes (1993) reintroduced a time-to-failure power law to model the observed accelerated seismicity quantified by the so-called cumulative Benioff strain. Their justification of the power law was a mechanical model of material damage. They did not refer to nor discuss the concept of a critical earthquake.

Sornette & Sammis (1995) published the first paper to reinterpret the work of Bufe & Varnes (1993) and all the previous ones reporting accelerated seismicity within the model of a large earthquake viewed as a critical point in the sense of the statistical physics framework of critical phase transitions. The work of Sornette & Sammis (1995) extended significantly the previous results of Allègre *et al.* (1982) and Smalley *et al.* (1985) in that their proposed critical point theory does not rely on irreversible damage but refers to a more general self-organization of the stress field prior to large earthquakes. In addition, using the insight of critical points in rupture phenomena, Sornette & Sammis (1995) proposed enriching the power-law description of accelerated seismicity by considering complex exponents (i.e. log-periodic corrections to scaling) (Newman *et al.* 1995; Saleur *et al.* 1996a,b; Johansen *et al.* 1996; 2000; Ouillon & Sornette 2000). This concept has been elaborated theoretically to accommodate both the possibility of critical self-organization (SOC) and the critical earthquake concept (Huang *et al.* 1998).

Bowman *et al.* (1998) gave empirical flesh to these ideas by showing that all large Californian events with magnitude larger than 6.5 are systematically preceded by a power-law acceleration of seismic activity in time during several decades, in a spatial domain about 10 to 20 times larger than the impending rupture length (i.e. of a few hundred kilometres). The large event could thus be seen as a temporal singularity in the seismic history time-series. Such a theoretical framework implies that a large event results from the collective behaviour and accumulation of many previous smaller-sized events. Similar results were also obtained by Brehm & Braile (1998, 1999) for other earthquakes. Jaumé & Sykes (1999) have reviewed the critical point concept for large earthquakes and the data supporting it. The additional results of Ouillon & Sornette (2000) on mining-induced seismicity, and Johansen & Sornette (2000) in laboratory experiments, brought similar conclusions on systems of very different scales, in good agreement with the scale-invariant phenomenology reminiscent of systems undergoing a second-order critical phase transition.

In this picture, the system is subjected to an increasing external mechanical load. As the external stress increases, microruptures occur within the medium which locally redistribute stress, creating stress fluctuations within the system. As damage accumulates, fluctuations interfere and become more and more spatially and temporally correlated, i.e. there are more and more, larger and larger domains that are significantly stressed, and thus larger and larger events can occur at smaller and smaller time intervals. This accelerating spatial smoothing of the stress field fluctuations eventually culminates in a rupture, the size of which is of the order of the size of the system. This is the final rupture of laboratory samples, or earthquakes breaking through the entire seismotectonic domain to which they belong. Note that in the case of laboratory sample failures, the system can be easily defined, while for earthquakes no such ‘domain’ has yet been unambiguously identified at the surface of the Earth. We will come back below to this important problem of how to define the relevant system undergoing a critical transition.

This critical rupture concept was verified in numerical experiments led by Mora *et al.* (2000, 2002), who showed that the correlation length of the stress field fluctuations increases significantly before a large shock occurs in a discrete numerical model. More recently, Bowman & King (2001a,b) have shown with natural earthquake data that, in a large domain including the impending major event similar to the critical domain proposed in Bowman *et al.* (1998), the maximum size of natural earthquakes increased with time up to the main shock. If one assumes that the maximum rupture length at a given time is given by (or related to) the stress field correlation length, then this last work shows that this correlation length increases before a large rupture. Sammis & Sornette (2002) summarized the most important mechanisms creating the positive feedback at the possible origin of the power-law acceleration. They also introduced and solved analytically a novel simple model, based on Bowman & King (2001a,b), of geometrical positive feedback in which the stress shadow cast by the last large earthquake is progressively fragmented by the increasing tectonic stress. Keilis-Borok (1990) has also used repeatedly the concept of a ‘critical’ point, but in a broader and looser sense than the restricted meaning of the statistical physics of phase transitions (see also Keilis-Borok & Soloviev 2002, for a review of some of the Russian research in this area).

The situation is, however, more complicated when the strain (rather than the stress) rate is imposed; in that case, the system may not evolve towards a critical point. The unifying viewpoint is to ask whether the dissipation of energy by the deteriorating system slows down or accelerates. The answer to that question depends on competition between the nature of the external loading, the evolution of the deterioration within the system and how the resulting evolving mechanical characteristics of the system feed back on the external loading conditions. For a constant applied stress rate, the dissipated energy rate diverges in general in finite time leading to a critical behaviour. For a constant strain rate, the answer depends on the damage law (Sornette 1989a). For a constant applied load, Guarino *et al.* (2002) find a critical behaviour of the cumulative acoustic energy both for wood and fibreglass, with an exponent ≈ -0.26 which does not depend on the imposed stress and is the same as for a constant stress rate. Similar results are found in a simple viscoelastic fibre bundle model (Kun *et al.* 2003).

For the Earth's crust, the situation is in between the ideal constant strain and constant stress loading states, and the critical point may emerge as a mode of localization of a global input of energy to the system. The critical point approach leads to an alternative physical picture to the so-called seismic cycle. From the beginning of the cycle, small earthquakes accumulate and modify the stress field within the Earth's crust, making it correlated over larger and larger scales. When this correlation length reaches the size of the local seismotectonic domain, a very large rupture may occur, which, together with its early aftershocks, destroys correlations at all spatial scales. This is the end of the seismic cycle and the beginning of a new one, leading to the next large event. As earthquakes are distributed in size according to the Gutenberg–Richter law, small to medium-sized events are negligible in the energetic balance of the tectonic system, which is dominated by the largest final event. However, they are 'seismo-active' (actors) in the sense that their occurrence prepares for that of the largest one. The opposite view of the seismic cycle is to consider that it is the Large-scale tectonic plate displacements which dominate the preparation of the largest events, which can be modelled to first order as a simple stick-slip phenomenon. In that case, all smaller-sized events would be 'seismo-passive' (witnesses) in the sense that they would reflect only the boundary loading conditions acting on isolated faults without much interaction from one event to the other.

Notwithstanding these works, the critical earthquake concept remains a working hypothesis (Gross & Rundle 1998): from an empirical point of view, the reported analyses possess deficiencies and a full statistical analysis establishing the confidence level of this hypothesis remains to be performed. In this vein, Zoller *et al.* (2001) and Zoller & Hainzl (2001, 2002) have recently performed novel and systematic spatiotemporal tests of the critical point hypothesis for large earthquakes based on the quantification of the predictive power of both the predicted accelerating moment release and the growth of the spatial correlation length. These works give fresh support to the concept.

In order to prove (or refute) that a boundary between tectonic plates is really a critical system, one should perform a direct measure of stress (instead of cumulative Benioff strain) to check the existence or absence of a build-up of cooperativity preparing for a large event. Indeed, one should measure the evolution of the stress field in space and time in such a region, compute its spatial correlation function, deduce the spatial correlation length and show that it increases with time as a power law which defines a singularity when the mainshock occurs. Unfortunately, such a procedure is far beyond our technical observational abilities.

First, it is well-accepted that large earthquakes nucleate at a depth of about 10–15 km, so it is likely that stress field values and correlations would have to be measured at such a depth to get an unambiguous signature. Moreover, the tensorial stress field would have to be measured with a high resolution in order to show evidence of a clear increase in the correlation length. As those measurements are clearly out of reach at present, we propose here a simplified method to approach such a goal.

We consider the most recent events over magnitude 6.5, which have occurred in southern California (Superstition Hills (1987), Landers (1992), Northridge (1994) and Hector Mine (1999)), and test if such a critical scenario is likely to have taken place prior to their occurrence. The choice of these four events is motivated by the critical earthquake concept, which proposes that criticality is better revealed by the largest events (Huang *et al.* 1998). However, we still have no access to the specific geometry of the possible critical systems that these events may belong to. Neither the spatial correlation function of the stress field nor its associated correlation length can be computed directly, as was done in the numerical model of Mora & Place (2002). Instead, we propose below a wavelet-based approach to overcome this lack of knowledge. Our approach constitutes an attempt to build a more physically or mechanically based cumulative function in the spirit of the cumulative Benioff strain used in previous works documenting the phenomenon of accelerated seismicity.

2 GENERAL METHODOLOGY

As direct stress measurements of sufficient extent for the purpose of estimating a correlation length are clearly out of reach, our goal is to estimate indirectly the stress distribution and its evolution with time within the crust through a numerical procedure based on instrumental seismicity.

Estimating the spatial stress history within a tectonic domain requires three different kinds of data: the first consists of knowledge of the far-field stress and/or strain imposed on the system. The second consists of accurate knowledge of the structure and rheology of the Earth's crust. The third consists of knowledge of the sources of internal stress fluctuations, which are mainly related to earthquake occurrence, whatever their size. The time evolution of the spatial structure of the stress field is thus created by the superposition of both far-field and internal contributions, coupled with the rheological response of the system (which can be quite complex). Despite its apparent simplicity, the first type of data are still largely under debate. For example, very different scenarios are still proposed for the tectonic loading of the San Andreas fault system. Moreover, the determination of the precise boundaries of the system remains a subject of controversy and research due to the complexity associated with the fractal hierarchical organization of tectonic blocks (Sornette & Pisarenko 2003). Fortunately, the critical point theory ensures that one need only consider the correlation function of internal fluctuations, which are the ones related to the occurrence of earthquakes, and not the large-scale effects of the boundary conditions (which play the role of control parameters), as long as they vary slowly on the timescale of the available catalogue of events. This is why we will not consider boundary conditions anymore here.

We shall thus use earthquake catalogues as the source of information available to qualify and quantify stress field fluctuations. The usual catalogues contain parameters such as earthquake location (longitude, latitude, depth), origin time and magnitude. For example, the SCSN catalogue that we use here is reasonably complete since 1932 for magnitudes larger than about 3.5. Moreover, no significant time gaps are reported for this catalogue (http://quake.geo.berkeley.edu/anss/cnss-caveats.html). Even if there are some events missed, we expect that the effect of such missed events will be to bias the signal towards a larger acceleration rather than the reverse, due to the fact that it is the early part of the catalogue which is most likely to be incomplete. Small and intermediate events can also be missed in the modern parts of the catalogue because they occurred immediately before or after a larger shock (Kagan 2003). Since we are quantifying processes occurring over months to years, this has no effect on our results.

Unfortunately, the information in seismic catalogues is not sufficient for quantifying the spatial stress perturbations due to a given seismic event. Two major ingredients are lacking. First, we must know the details of the rupture mechanism. This includes size (length and width), strike and dip of the fault plane, as well as the slip distribution upon it (in amplitude and direction). This information is usually only available for spatially and temporally restricted catalogues (but which can cover a large magnitude interval), or for more extended catalogues but only for shocks of large magnitude (for example the Harvard catalogue for shocks of magnitude larger than 5.5). As there are so few such events diluted in a very large spatial and temporal domain, it is clear that in this way we will only get information on the stress field structure at very large scales. If we consider all events in a catalogue, we should be able to gain insight into smaller scales (as such events are much more numerous and have shorter rupture lengths), but would lack the information on the source parameters. We shall opt for the option of using all the observed and complete seismicity, and will define in the next section a simplified Green's function giving the spatial structure of the internal stress fluctuations due to an event of any size occurring anywhere at any time within our system. A drastic consequence will be that this Green's function will be a scalar rather than the correct tensorial structure which would be accessible if we knew the details of the rupture. Our hope is that if the critical nature of rupture is a strong property it should be detectable even with such an approximation. Indeed, sums of random scalars exhibit in general stronger signals of correlations than sums of higher-dimensional objects such as moment tensors due to the presence of dispersion along several possible directions in the latter. The existence (if any) of an increasing correlation of the stress field should thus be detectable more easily, even if not exactly quantitatively.

In order to estimate reliably the stress fluctuations and their evolution with time, we also need an accurate rheological model of the local lithosphere, including knowledge of elastic constants and relaxation times for the viscous layers. These latter ingredients can be deduced from geophysical investigations, at least on a large scale. Of course, the more accurate this model is, the more difficult and lengthy will be the estimations of the stress field perturbations, which would necessitate the use of a finite-element or boundary-element codes. As the rheological behaviour of the material of the Earth's crust and lithosphere can be quite complex, we shall use in the following a simplified rheological model which captures the essential features of stress transmission and relaxation within a viscoelastic layered medium.

The methodology used in this work is the following: we first choose a recent large event (to ensure a sufficiently large catalogue of possible precursor events, both in time and number), occurring at time T_0 and location P_0. We read every event in the catalogue which precedes it, and compute the spatiotemporal stress fluctuations it induces in the whole space. We also estimate, through an appropriate wavelet transform (see below), the contribution of each event to the correlation properties of the stress field around location P_0 at different scales. This will provide us with the correlation length of the stress field around P_0 and its evolution with time, up to the time of occurrence of the large event.

This approach has some similarity with models of triggered seismicity in which future seismicity (quantified by the seismic rate probability) is the sum of time- and-space-dependent kernels contributed by all past events (Kagan & Knopoff 1981; Ogata 1988; Console & Murru 2001; Helmstetter & Sornette 2002a). The contribution of each past event to the future seismic rate probability is similar to a Green's function and presents a functional dependence in space and time similar to that used here. Notwithstanding the obvious similarity of the quantities that are computed, our aim and method are, however, rather different. Our goal is to test for the relevance of the critical earthquake model, so that the source of information is not contained in the absolute value of such a weighted sum at P_0 but in the shape of its time evolution: the computed parameters should display a singular behaviour when approaching the occurrence time of a large event, such that they (or one of their derivatives) should diverge at the critical time. This specific aspect has not been previously addressed, nor have the spatial correlations that can arise and betray a critical behaviour close to the mainshock been quantified.

3 CONSTRUCTION OF THE GREEN'S FUNCTION

We will consider an approximation of the stress field (or pseudo-stress) due to a seismic source in a 3-D elastic, infinite and isotropic medium. As catalogues do not provide us with all the parameters needed to accurately compute the exact elastic solution, we will make the following assumptions:

(1) We will consider that each source is isotropic and that the stress perturbation is positive with radial symmetry around the source.

(2) This stress perturbation $\sigma_L(r)$ is assumed to decay from the source as

$$\sigma_L(r) = \frac{(L/2)^3}{(L/2)^3 + r^3}, \tag{1}$$

where L is the linear size of the source (which plays the role of the rupture length in real events) and r is the distance from the source.

The size L is determined empirically using a statistical relationship between magnitudes and rupture lengths established for strike-slip faults in California (Wells & Coppersmith 1994):

$$\log(L) = -2.57 + 0.62 \times M_l, \tag{2}$$

where M_l is the local magnitude and L is expressed in kilometres. To ensure that all earthquakes are treated on the same footing, this statistical relationship is also used for the events for which the information on the rupture plane is available. Note that the computed stress $\sigma_L(r)$ (which has a cylindrical symmetry) is always positive, so that it does not define a genuine stress. It can, however, be interpreted as a kind of influence function, with L playing the role of the size of the area in which a shock will possibly influence following events.

We now take into account that the source does not occur in a purely homogeneous elastic medium but in a two-layer viscoelastic one. The upper layer is considered as a viscoelastic medium with relaxation time τ_1. The lower layer is also taken as a viscoelastic medium (possibly semi-infinite) with relaxation time $\tau_2 < \tau_1$. We assume that earthquakes are localized within the upper (more brittle) layer, and that the quantity of interest is the scalar stress field measured in this layer, taken as constant in the vertical dimension so as to ensure that the stress field is 2-D within the horizontal plane. The thicknesses of the layers and the existence of free surfaces are embodied in the phenomenological constants defined below. The depths of the events are taken to be identical and we neglect any vertical variation. This amounts to calculating the stress field at this nucleation depth.

The rupture and relaxation of the stress field in the two-layer system is modelled as follows. Once an event occurs in the upper layer, the instantaneous elastic solution for the stress field is given by expression (1). Then, both layers begin to flow by viscous relaxation. The lower layer flows faster, due to a smaller relaxation time associated with a less viscous rheology. The effect of this viscous relaxation is to progressively load the upper layer thus creating a kind of post-seismic rebound. This loading effect computed in the upper layer is assumed to be described by a simple function of the type

$$f(r, t) = \sigma_L(r)[1 - C \exp(-t/\tau_2)]H(t), \tag{3}$$

where $\sigma_L(r)$ is the elastic isotropic solution given by eq. (1) and C is a constant which quantifies the maximum quantity of stress which is transferred in the upper layer, and which depends on the geometry of the problem. If $C = 0$, no transfer occurs. $H(t)$ is the Heaviside function which ensures that the stress fluctuation becomes non-zero once the event has occurred. Here, t is the time elapsed since the seismic event. At the same time, the stress also relaxes in the upper layer at a rate which varies as $\exp(-t/\tau_1)$. This relaxation takes into account the usual viscous processes as well as the effect of micro-earthquakes which dissipate mechanical energy.

As both relaxations occur simultaneously, the evolution of the stress field in the upper layer is given by the sum of two contributions: (1) the direct relaxation $\sigma_L(r)\exp(-t/\tau_1)$ of the instantaneous elastic stress load in the upper layer due to the event and (2) the convolution of the time derivative of $f(r, t)$ with the exponential relaxation function $\exp(-t/\tau_1)$ in the upper layer. This second contribution sums over all incremental stress sources $df(r, t)/dt$ per unit time in the upper layer stemming from the relaxation of the lower layer. After some algebra, we get the stress perturbation induced by an earthquake of the form

$$\sigma(r, t) = \frac{(L/2)^3}{(L/2)^3 + r^3}\left(\exp(-t/\tau_1) + B\frac{\tau_1}{\tau_1 - \tau_2}(\exp(-t/\tau_1) - \exp(-t/\tau_2))\right), \tag{4}$$

where r and t are respectively the horizontal distance from the source and the time since the occurrence of the earthquake. The constant B represents the relative contribution to the stress field in the upper layer due to the delayed loading by the slow viscous relaxation of the lower layer that has been loaded by the instantaneous elastic stress transfer at the time of the earthquake compared with the direct relaxation of the elastic stress created directly in the upper layer. The numerical value of B is difficult to ascertain as it depends strongly on the geometry of the layers as well as on their rheological contrast. We expect both contributions to be of the same order of magnitude and, in the following, we shall take $B = 1$. Taking $B = 0$ amounts to considering only the relaxation in the upper layer and to neglecting the effect of the lower layer. If the inter-event time is on average larger than τ_2 in the neighbourhood of P_0 (at a scale larger than the minimum event size in the catalogue), this neglect has a minor influence on the final results.

The Green's function defined here is a rough approximation of what really takes place within the crust and the lithosphere, but it nonetheless captures qualitatively the overall evolution of the stress field. One could raise the criticism that it does not feature any azimuthal dependence of the stress field perturbation, but, as we have already stated, this is done in view of the absence of detailed information on the source mechanisms of the events. On the other hand, as stated above, the use of an isotropic stress field is expected to lead to an overestimation of the correlation length, and thus possibly to an amplification of the signal we are searching for. While we cannot provide a rigorous proof of this statement, it is based on the analogy between percolation and Anderson localization (Souillard 1987; Sornette 1989b,c): the first phenomenon (percolation) describes the transition of a system from conducting to isolating by the effect of the addition of positive-only contributions; The second phenomenon (Anderson localization) refers to the transition from conducting to isolating when taking into account the 'interferences' between the positive, negative and more generally phase-dependent amplitudes of the electronic quantum wavefunctions. In this latter case, the transition still exists but is much harder to obtain and to observe. In the future, it may nevertheless be interesting to check this point and test a generalization of the present model in which a random or better constrained source orientation is chosen for each event and the angular dependence of the associated double-couple stress is taken into account.

The Green's function we propose also assumes a complete decoupling between space and time, so that viscous relaxation does not exhibit any diffusive pattern. The presence of a diffusion process would imply an increase of the size L of the area of influence with time. Recent works on the diffusion of aftershocks after large events shows that such possible growth of $L(t)$ with time is at best very weak (Helmstetter & Sornette 2002b; Helmstetter *et al.* 2003a). As the amplitude of the stress signal decreases exponentially with time, we believe that the effect of diffusion is not crucial (because it is too slow and too weak in amplitude) in order to obtain and measure an increase of the stress field correlation length, if any. We thus fix L as constant with time for each event. Another assumption of our rheological model is that the viscoelastic component is linear, allowing an unambiguous definition of relaxation times. This allows us to define a simple and convenient computation procedure to estimate a correlation length, as discussed in the next section.

The simplified Green's function $\sigma(r, t)$ given by (4) has several interesting properties catching the overall physics of the stress evolution in the upper layer after an event. The elastic pre-factor $\sigma_L(r)$ given by (1) implies that the stress perturbation is initially of order unity within a circle of radius $L/2$, and sharply decreases as r^{-3} outside this circle. Note that the maximum amplitude of the stress perturbation is independent of the size L, as the stress drop is thought to be constant, whatever the size of an event. At any point in the upper layer, the stress will first be given by the elastic solution. As $\tau_1 > \tau_2$, the stress at any point in the upper layer will first increase due to the relaxation of the lower layer, reach a maximum and then decrease with time as the upper layer is relaxing too, but with a longer relaxation time.

Fig. 1 shows such a scenario with $\tau_1 = 10$ yr and $\tau_2 = 1$ yr. The maximum amplitude depends on the distance between the event and the point where this stress is measured (as well as on B).

If we now superimpose the contributions of all successive earthquakes in a catalogue, the stress history at any given point will be a succession and/or superposition of such fast-growing and slowly decaying stress pulses. We thus construct the cumulative stress function $\Sigma(t)$ at point P_0 defined as

$$\Sigma(t) = \sum_i \sigma(r_i, t_i), \tag{5}$$

where $\sigma(r_i, t_i)$ is given by (4) and r_i and t_i are the distance and the time of event i to the main shock. For example, Fig. 2(a) shows the stress history measured at the location of the Landers epicentre due to the succession of all previous events in the catalogue, assuming $\tau_1 = 1$ and $\tau_2 = 6$ months. Fig. 2(b) shows the same computation for $\tau_1 = 10$ yr, while Fig. 2(c) assumes $\tau_1 = 100$ yr. Increasing τ_1 widens the stress pulses, which leads them to overlap and produces a more continuous stress history.

The constructions of $\Sigma(t)$ shown in Figs 2(a)–(c) are analogous to the cumulative Benioff strain studied by Bufe & Varnes (1993),

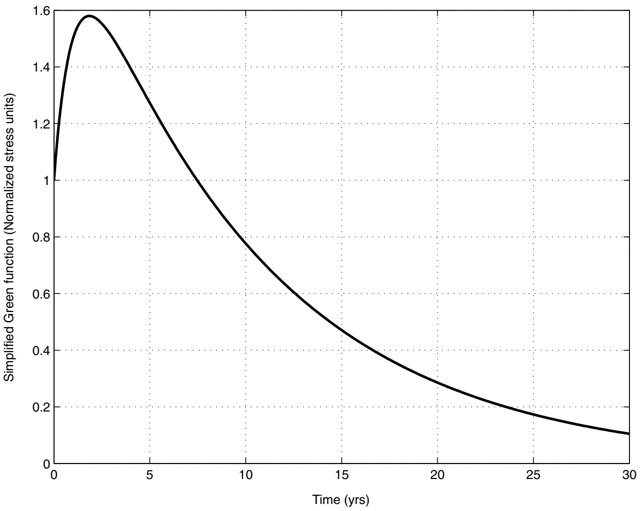

Figure 1. Evolution with time of the time-dependent part of the normalized stress field showing the loading phase induced by the relaxing lower layer and the large time relaxation phase in the upper layer. The parameters are $\tau_1 = 10$ yr, $\tau_2 = 1$ yr and $B = 1$.

Sornette & Sammis (1995), Bowman *et al.* (1998), Brehm & Braile (1998, 1999), Jaumé & Sykes (1999) and Ouillon & Sornette (2000), and are an attempt to improve upon them, as we now explain. They are analogous because they can be seen as similar to the sums of the type

$$M_q(t) = \sum_{i|t_i<t} [M_0(i)]^q, \tag{6}$$

where $M_q(t)$ is a moment-generating function of order q, t_i and $M_0(i)$ are the time and seismic moments of the ith earthquake and q is an exponent usually taken between 0 and 1. The cumulative Benioff strain is obtained as $M_{q=1/2}(t)$ where the sum is performed over all events above a magnitude cut-off in a pre-defined spatial domain. Taking $q = 1$ corresponds to summing the seismic moments, while taking $q = 0$ amounts to simply constructing the cumulative number of earthquakes. The constructions shown in Figs 2(a)–(c) can be seen as equivalent to $M_{q=0}(t)$ when the two following limits hold: (1) all earthquakes in the catalogue are so close to each other that they are all within a distance less than their rupture length from the point where the stress is calculated (in this case, the elastic stress perturbation brought by each event is equal to the constant stress drop); and (2) the time difference between the occurrence of each event and the main shock is significantly less than τ_2, or much larger than τ_2 and much lower than τ_1, such that the time dependence in eq. (4) can be neglected.

A significant advantage in our construction of the cumulative stress function $\Sigma(t)$ defined by eq. (5) compared with the cumulative Benioff strain resides in the fact that we do not need to specify in advance a spatial domain, a delicate and not fully resolved issue in the construction of cumulative Benioff strain functions. The definition of the relevant spatial domain is automatically taken into account by the spatial dependence of the Green's function.

Two ingredients are going to modify the observed acceleration of the Benioff strain when studying the cumulative stress function

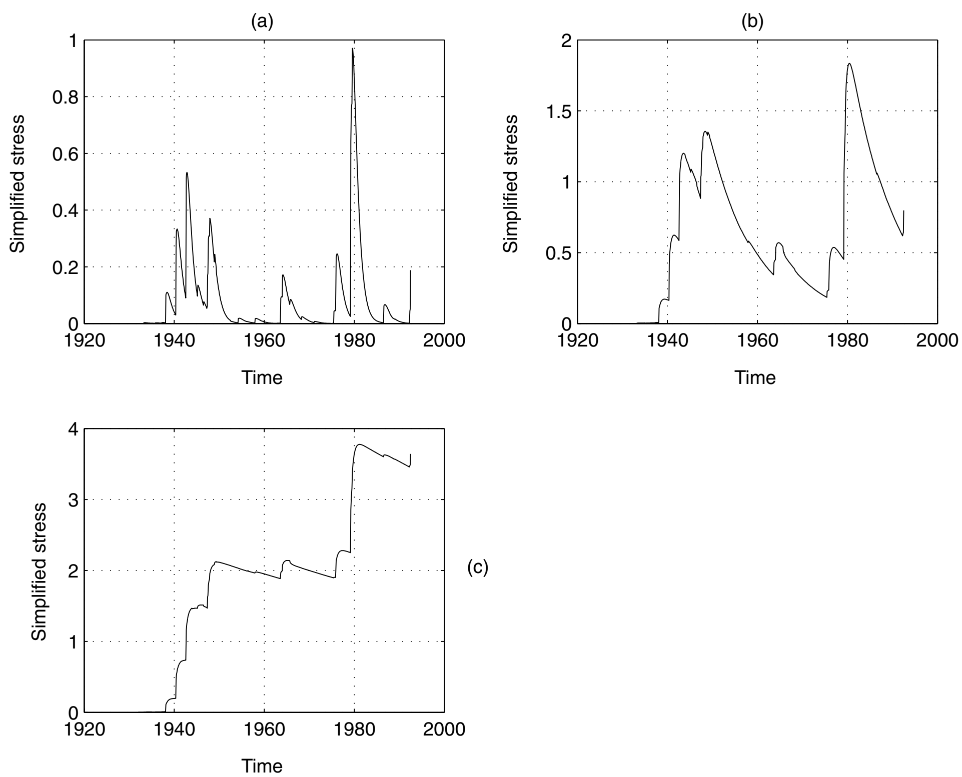

Figure 2. Cumulative stress function as a function of time at the location of the Landers epicentre calculated by summing the contributions $\sigma(r_i, t_i)$ given by (4) of the Green's functions generated by all previous events i, that occurred at times t_i prior to the Landers earthquake taken at the origin of time and at distances r_i from the Landers epicentre. (a) $\tau_1 = 1$ yr and $\tau_2 = 6$ months; (b) $\tau_1 = 10$ yr and $\tau_2 = 6$ months; (c) same as (a) with $\tau_1 = 100$ yr and $\tau_2 = 6$ months.

$\Sigma(t)$ defined by eq. (5). The first one is that each event contributes a maximum stress perturbation equal to the stress drop. In contrast, large events contribute significantly more in the cumulative Benioff strain as the square root of their seismic moment and independently of their distance. There is, however, a size effect in our calculation of $\Sigma(t)$ that reveals itself at large distances $r_i \gg L_i$, stemming from the magnitude dependence of the range L_i of the stress perturbation. According to eq. (2) and using the standard relationship between magnitude M_1 and seismic moment M_0, $M_1 = (2/3)$ [log $M_0 - 9$], we obtain $L_i \sim [M_0(i)]^{0.4}$ and thus $\sigma(r_i, t_i) \sim L_i^3 \sim [M_0(i)]^{1.2}$ for $r_i \gg L_i$. This size effect has, however, an almost negligible contribution in generating such an acceleration because the stress field becomes small at large distances. The second ingredient limiting the acceleration of the cumulative stress function $\Sigma(t)$ defined by (5) is the relaxation in time which is responsible for the decay observed in Figs 2(a)–(c). The longer τ_1 is, the smaller is the amplitude of this decay, until $\Sigma(t)$ is replaced by a 'staircase' in the limit $\tau_1 \to +\infty$. The largest values of τ_1 that we have explored are significantly larger than the total duration of the catalogue and larger values will not change our results quantitatively.

Another important issue is the contribution of the small events not taken into account in the sum (5). Indeed, the typical area $S(L)$ over which the stress redistribution after an event is significant is of the order of the square $S(L) \propto L^2$ of the size L of the rupture. If the earthquake seismic moments M are distributed according to a density Pareto power law $\propto 1/M^{1+\beta}$ with $\beta \approx 2/3$ (which is nothing but the Gutenberg–Richter law for magnitudes translated into moments), using the fact that $M \propto L^3$, the density distribution of the areas $S(L)$ is also a power law $\propto 1/S^{1+(3/2)\beta}$ with an exponent $(3/2)\beta \approx 1$. Thus, the contribution of each class of earthquake magnitude is an invariant: small earthquakes contribute as much as large earthquakes to the sum (5). Therefore, it seems *a priori* very dangerous to ignore them in our sum (5) which attempts to detect a build-up of correlation. However, if we assume that the physics of self-organization of the crust prior to a critical point is self-similar, then so will be the structure of the stress field, and the critical behaviour should be observable at all the different scales. Thus, neglecting the contribution of small events should not lead to a destruction of the signal nor to a modification of its relative variations, only to a change in its absolute amplitude. Of course, as in any other natural phenomenon, strict self-similarity does not hold over an infinite range of scales but is truncated by upper and lower cut-offs. The existence of these cut-offs ensures that our simplified stress field remains finite. Some works (see for instance Keilis-Borok & Soloviev 2002, and references therein) have found that the β value is not stationary, which seems to invalidate our argument. However, Helmstetter *et al.* (2003b) have shown that such variations of the measured β value for precursors may not reflect a genuine change of β but may result from the effect of conditioning on measurements performed in finite time windows associated with increased seismicity prior to large earthquakes.

To sum up, our physically based definition of the cumulative stress function adding up the contribution of stress loads by all earthquakes preceding a main shock seems to be unable to reproduce a critical acceleration similar to those observed previously for the cumulative Benioff strain (see Bufe & Varnes 1993; Sornette & Sammis 1995; Bowman *et al.* 1998; Brehm & Braile 1998, 1999; Jaumé & Sykes 1999, for examples of such accelerations). This is due to the fact that, conditioned on the hypothesis of a magnitude-independent stress drop and using standard elasticity, the impact of the largest events is not significantly larger than that of smaller events. In view of this failure, we now attempt another hopefully more robust characterization of the critical point model.

Until now, we have studied the time behaviour of (a simplified measure of) stress at the locus of an impending large event, trying to track a critical acceleration with time. In the case of a simple static to dynamic frictional instability this point of view is certainly relevant. However, there is *a priori* no reason to observe a strong stress increase considering the evidence that the stress drop during a major event is very small compared with the stress that would be expected from the value of the lithostatic pressure and from the values of friction coefficients established in the laboratory. In the case of a state- and-rate-dependent frictional behaviour, a major event may occur without any stress increase at all. Indeed, for a large event to occur, one certainly needs a large amount of energy, but this energy can be stored in a large connected domain around the hypocentre. In this picture, approaching a critical event should thus enhance the spatial correlation length of the stress field around the future hypocentre, not necessarily the average stress level.

4 ANALYSIS OF THE STRUCTURE OF THE STRESS FIELD

Our objective is to determine the correlation length of the computed stress field in the neighbourhood of four large shocks in California as a function of the time before their occurrence. To achieve this goal, we are going to analyse the structure of the stress field around each main shock epicentre to check whether the stress fluctuations are increasing or decreasing in size. In order to extract a robust estimation of the correlation length of the stress field reconstructed from a limited number of events, we investigate what spatial scales or wavelengths are developing around each main shock epicentre, that is, what is the characteristic scale of the roughness of the computed stress field.

An efficient way to achieve such a goal is to perform a 2D wavelet transform of the stress field, which acts as a microscope allowing us to focus on separate scales. As we are interested only in the spatial structure surrounding the upcoming mainshock (defined as point P_0), we compute wavelet coefficients centred at location P_0. We consider the following wavelet

$$\frac{1}{a}\left(2 - \frac{r_p^2}{a^2}\right) \exp\left(-\frac{r_p^2}{2a^2}\right) \tag{7}$$

centred at point P_0. This 'Mexican hat' wavelet is the second-order derivative of the Gaussian function. By construction, it eliminates signals of constant amplitude or of constant gradient at scale a or larger. It is symmetric around P_0. r_p is the distance to point P_0 and a is the analysing scale (the larger a, the larger the width of the wavelet). Such wavelet analysis is thus well-suited for isolating fluctuations at various chosen scales. Working with a scale a means that the corresponding structures have in fact a size $2.2a$ (Ouillon 1995). The factor 2.2 is obtained as follows. First, the wavelength corresponding to a given wavelet of scale a, obtained by Fourier spectrum analysis, is found equal to 4.4 a (Ouillon 1995; Ouillon *et al.* 1995, 1996). Now, a given fluctuation can be seen as the positive bump of the wavelet, which is surrounded by two negative arches. This implies that the typical scale of such a fluctuation is about half the wavelength, hence the number $2.2a$.

For each time in the stress field history, the wavelet transform is obtained by convolution of this function with the computed spatial stress field, for different values of a. If the resulting wavelet coefficient is close to 0, this means that the stress field is uniform or varies linearly around P_0, at scale a. If the coefficient is strongly negative, this means that P_0 is at or near a local stress minimum, at scale a. If it is strongly positive, this means that P_0 is at or near a stress maximum at scale a, indicating that the stress is both locally high and correlated at that scale. This is exactly the property that we want to check.

Our analytical procedure is thus the following: we consider the first event in the catalogue. We compute the stress field fluctuation due to this event at any time and any location through eq. (4). The wavelet transform provides the contribution of this event at any time to the total wavelet coefficient at any scale a at location P_0. Summing all contributions of successive events (as the rheology we chose is linear) up to the major mainshock at time T_0 provides us with the complete evolution of the scale content of our computed stress field around P_0. From the wavelet coefficient of the cumulative stress field as a function of scale at a fixed time t, we extract the corresponding correlation length $\rho(t)$ as the scale corresponding to the maximum coefficient, multiplied by 2.2. If the critical point hypothesis is correct, $\rho(t)$ should behave as

$$\rho(t) = A + K(T_0 - t)^{-\nu}, \tag{8}$$

where ν is a positive critical exponent. Note that, due the very small rupture size L for small earthquakes, and as the scale a varies from 1 to 100 km, it would be necessary to grid a very large domain (of linear size a few hundred kilometres) with a very small mesh size (of the order of a few tens of metres). This would make computations and data storage practically intractable. This is why we have defined a procedure which computes data only on very small subgrids whose size (and mesh size) depends on the wavelet scale and on the event size. This procedure is made possible because we compute wavelet coefficients at several scales but only at a single location, namely the position of the upcoming large event. Moreover, we do not store the stress history for all locations, but only at the position P_0 of the epicentre of the target main shock.

For the purpose of predicting a future large event, according to the critical point theory one should repeat this computation for many different target points, building correlation maps for each of them and studying the time evolution of each map in order to detect precursory growth of correlations. Before performing this time-consuming work, it is important to check ex-post if large events could have been predicted with the critical earthquake concept. This is our goal here.

5 RESULTS

We have analysed the evolution of the stress field before four large southern Californian shocks: Superstition Hills (1987), Landers (1992), Northridge (1994) and Hector Mine (1999). We restricted our analysis to those four recent events as this ensures that our computed stress field history is the longest possible for this area, so that finite time effects, if any, are the most limited (as these four events are located near the end of the catalogue). The SCSN catalogue we used is thought to be complete since 1932 for events of magnitude larger than 3.5. Computation of the stress field before each of the selected large events included all events of magnitude larger than 4 since 1932.

Three parameters dictate the properties of the Green's function of a seismic event in our computations, namely the relaxation timescales τ_1 and τ_2, and the stress amplification factor B. We made several computations, varying those three parameters. We already checked that the least influential parameter is B. Another parameter which has a rather small influence on the results is τ_2, the relaxation time of the lower less viscous medium. The most influential parameter is τ_1, the relaxation time of the upper layer. If τ_1 is too small, then all events appear as very well individualized temporal stress pulses decaying very fast before the next event takes place. As a consequence, the dominating space scale is never defined, except at the time of occurrence of each event, where it is of the order of the distance between this event and P_0. The dominant space scale (defined as the scale at which the wavelet coefficient is maximum) thus varies very wildly with time.

When increasing τ_1, stress pulses gradually overlap in time. Finally, when τ_1 is infinite, stress pulses become steps without any relaxation. Increasing τ_1 leads to a less erratic behaviour of the dominating spatial scale length obtained from our wavelet analysis. We will here consider a Green's function with relaxation times $\tau_1 = 100$ yr and $\tau_2 = 0.5$ yr. The scalar stress history computed at the location of the Landers shock is shown in Fig. 2(c). It globally increases with time (as in all previous events stress perturbations are positive by definition) but does not exhibit any critical acceleration (acceleration is defined in the usual sense of a non-linear growth with velocity increasing with time; critical acceleration refers specifically to a growth with a diverging velocity at the critical time). Note that stress steps (due to neighbouring events) are followed by a smooth decay, due to the very slow relaxation associated with the high τ_1 value. The time step for the computation of each successive point of the cumulative stress is 1 month. We stress that the procedure we use provides results independent of the time step, thanks to our linear rheology.

Fig. 3 shows the wavelet coefficients for the cumulative stress function constructed for the 1992 Landers earthquake as a function of scale at various times. We show results for times after 1950, and have plotted only one curve every 6 months for the sake of clarity. The curves with the lowest amplitudes, corresponding to the early years, are flat as the number of shocks is low, so that the stress field is almost zero everywhere, and no specific structure emerges as too few events have been included in the computation. Later, the amplitude of the profile increases (either positively or negatively), but it is worth noting that its shape is almost constant. As time increases, the amplitude of the stress field varies, but its structure remains constant, at least at point P_0. For example, for wavelet scales lower than 10 km (true size lower than 22 km), the 'future' Landers epicentre is found to be located in a local stress deficit. The local correlation length of the stress field, given by the maximum of the wavelet coefficient, occurs for a constant scale of about 25 km (true size of about 55 km). We note that this maximum occurs at the same scale for all times. Fig. 4 shows the evolution with time of the correlation length (defined as the dominating space scale). It first fluctuates widely, as there are too few events to compute a representative stress fluctuation field, but then enters a very stable phase with no noticeable variation with time. We thus show no increase or decrease of this local correlation length, which confirms the fact that the local structure of the computed stress field does not exhibit any major change when approaching failure around P_0. Figs 5 to 7 show the results of the same computations before the Superstition Hills event. The correlation length is found constant from 1958 to 1987, with a value of about 77 km (wavelet scale of 35 km).

Figs 8 to 10 show the results of the same computations before the Northridge event. The correlation length is found constant from 1972 to 1994, with a value of about 66 km.

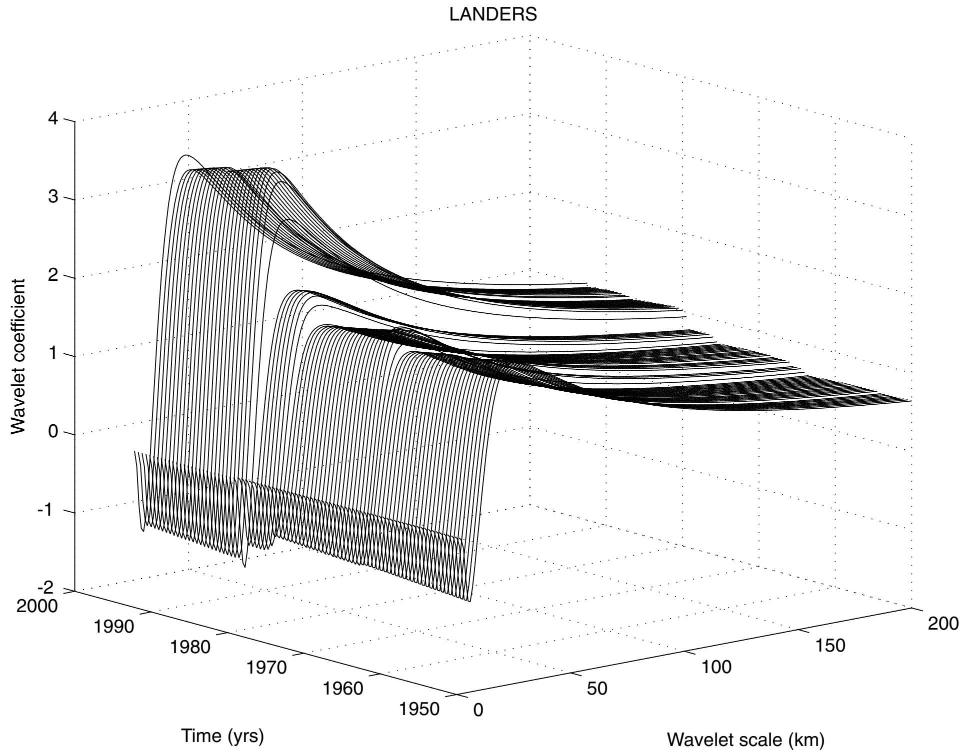

Figure 3. Wavelet coefficients for the cumulative stress function constructed for the 1992 Landers earthquake as a function of scale *a* at various times. Curves are plotted only every 6 months to increase picture quality.

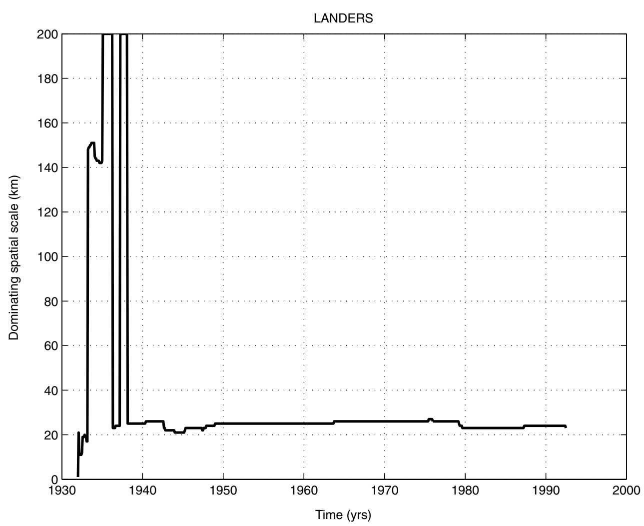

Figure 4. Correlation length estimated at the Landers epicentre of the cumulative stress function for the Landers earthquake as a function of time.

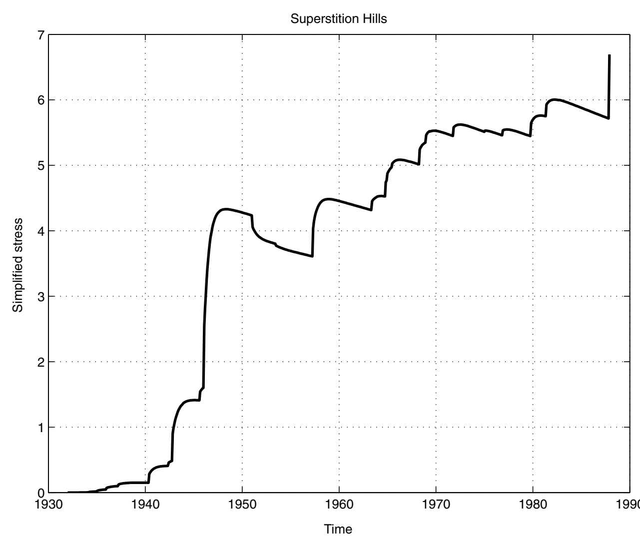

Figure 5. Same as Fig. 2(c) for the 1987 Superstition Hills earthquake.

Figs 11 to 13 show the results of the same computations before the 1999 Hector Mine event. Once again, no clear increase of the correlation length occurs before the large event.

We also performed the same tests considering only catalogue events of magnitude larger than 5. We obtain exactly the same results, except that the wavelet profiles of Figs 3, 6, 9 and 12 are found to be stretched along the scale axis. This just reflects the fact that fewer events are taken into account in the computations, and are thus more diluted in space. We also performed tests using a larger distance of influence of each given event quantified by eq. (1) by doubling the rupture size $L \to 2L$. The results are qualitatively the same.

6 INTERPRETATION AND DISCUSSION

Using simplified models of earthquake elastic stress transfer and of the lithosphere rheology, we have attempted to model the stress field evolution from 1932 up to the time of occurrence of recent large southern Californian events. This allowed us to analyse the

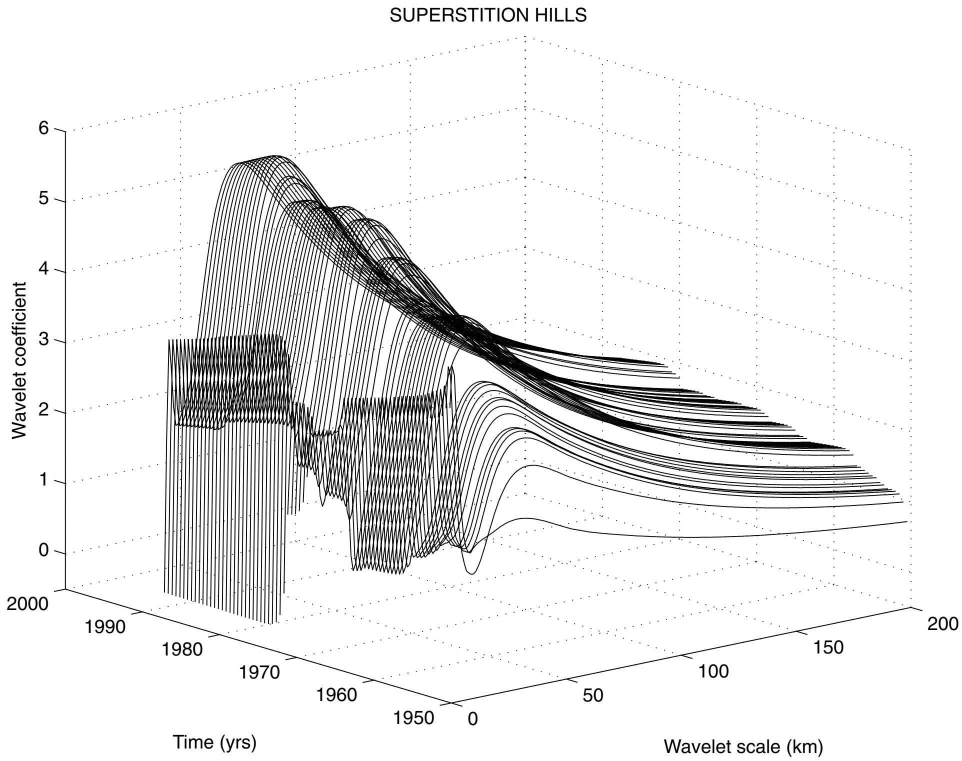

Figure 6. Same as Fig. 3 for the 1987 Superstition Hills earthquake.

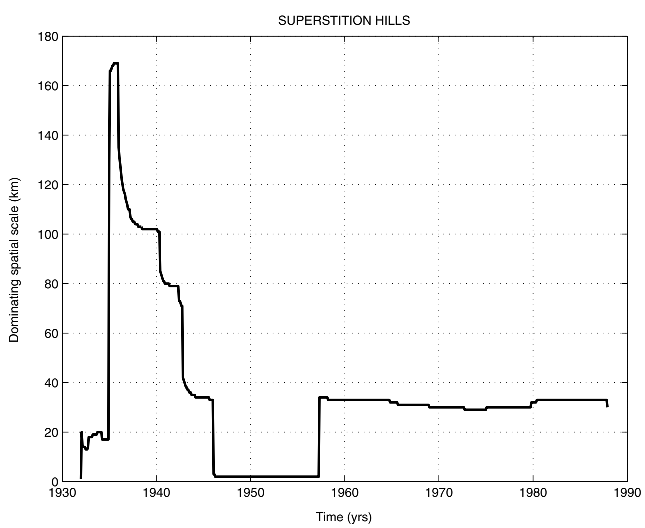

Figure 7. Same as Fig. 4 for the 1987 Superstition Hills earthquake. The correlation length is found constant from 1958 to 1987, with a value of about 77 km (wavelet scale of 35 km).

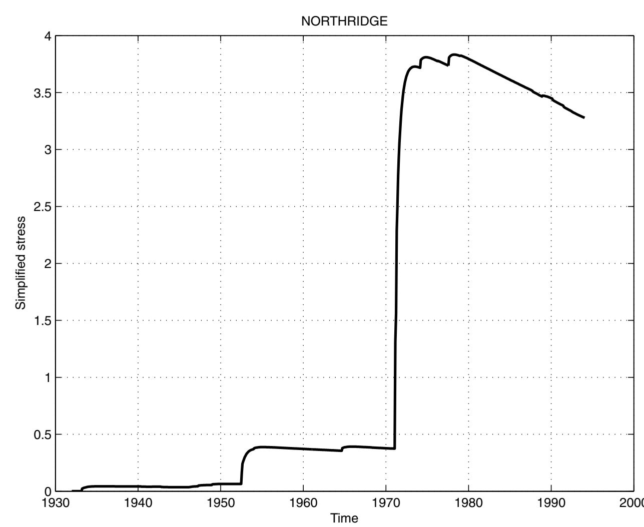

Figure 8. Same as Fig. 2(c) for the 1994 Northridge earthquake.

time evolution of our simplified cumulative stress field at the loci of large impending shocks before their occurrence, and to determine the spatial correlation length of this local stress field. The use of a variety of rheological models did not permit us to find evidence of a strong increase (nor any other peculiar variation) of both the cumulative stress field and the correlation length before any of the four major events studied here. These negative results would not change by replacing the simple exponential decays by power laws of the form of the Omori law for aftershocks, since taking an infinite range correlation $\tau_1 \to +\infty$ does not change our results.

We have observed that all large events occurred in a local minimum of the computed stress field at (true) scales less than 20–25 km, and that this minimum becomes more and more pronounced with time. A magnitude 7 event has an average rupture length of about 70 km. As we have stressed before, such an event certainly nucleates in a zone where the stress field is correlated on long wavelengths. The final length of the rupture will stem from the interplay between this initial static stress field structure and details of rupture dynamics (inertial effects coupled with the specific geometry of the rupture plane). We can reasonably assume that the final extent of the rupture will be larger than the initial correlation length of the stress field. This is why we could expect that this correlation length

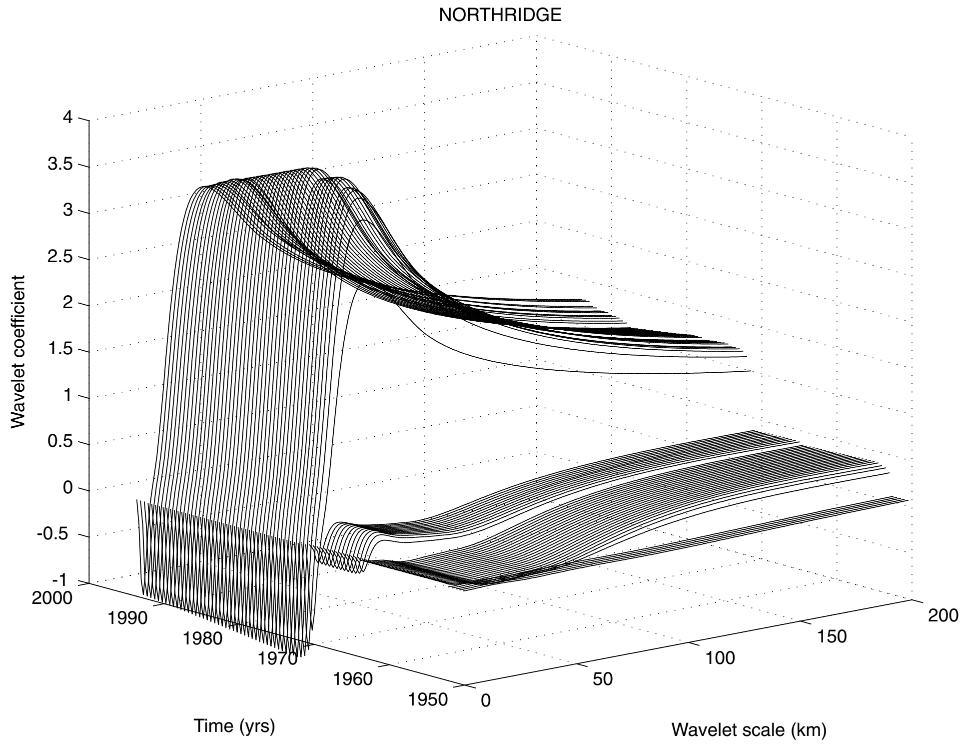

Figure 9. Same as Fig. 3 for the 1994 Northridge earthquake.

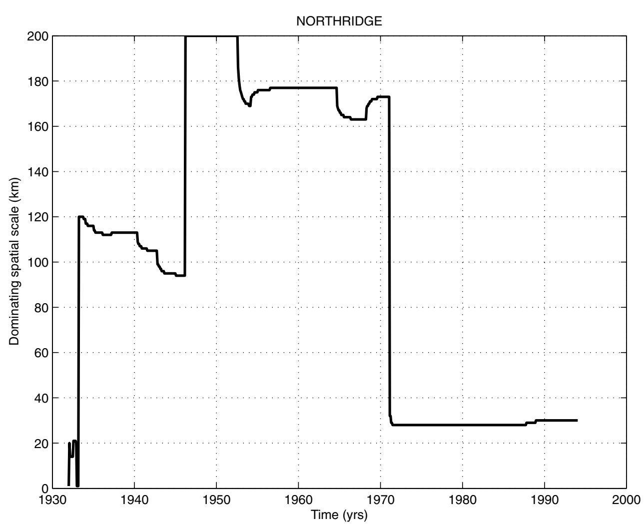

Figure 10. Same as Fig. 4 for the 1994 Northridge earthquake. The correlation length is found constant from 1972 to 1994, with a value of about 66 km.

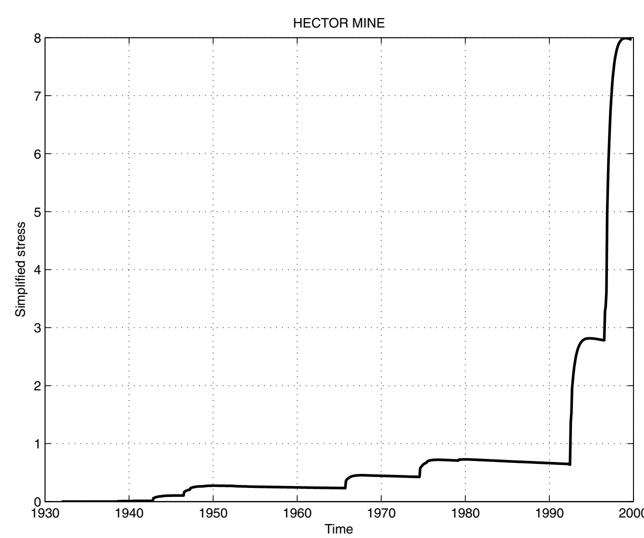

Figure 11. Same as Fig. 2(c) for the 1999 Hector Mine earthquake.

before each of the four major events should have been of the order of a few tens of kilometres. It is thus puzzling to observe that the wavelet coefficients at scales of 10 to 20 km are becoming more and more negative with time. This observation is perhaps due to the naive shape of the Green's function we considered, which is positive everywhere. However, we believe that if this assumption obviously affects the value of the computed stress field, it should certainly lead us to an overestimation of the correlation length, as more space is filled with positive stress. We are thus forced to conclude that there is neither a strong stress field nor a large stress correlation at the scale of a few kilometres. It thus seems that the mechanism of stress transfer due to the occurrence of successive smaller-sized events is not a direct ingredient in building long correlations in the cumulative stress field, which are necessary for the propagation of large future events according to the critical point model.

These results are in contradiction with those reported in the literature (Bufe & Varnes 1993; Sornette & Sammis 1995; Bowman *et al.* 1998; Brehm & Braile 1998, 1999; Jaumé & Sykes 1999; Ouillon & Sornette 2000) based on the cumulative Benioff strain, which showed that large-scale spatial and temporal correlations characterize seismicity before a large event in the same area.

Our results may be reconciled with those previous studies if we acknowledge that medium-sized events are not seismo-active (they

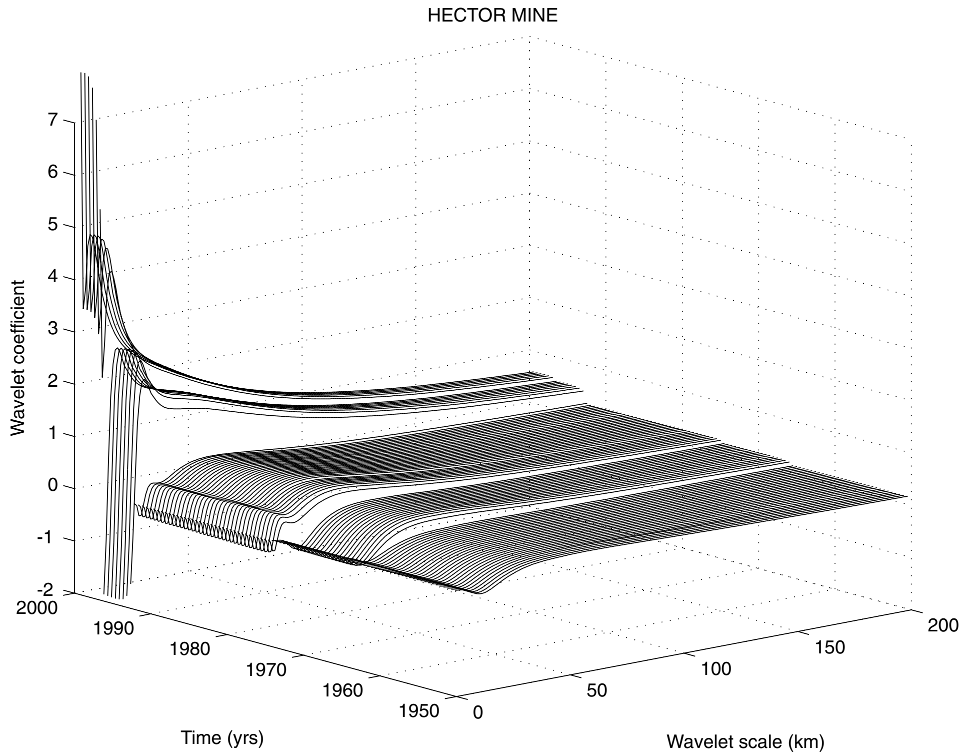

Figure 12. Same as Fig. 3 for the 1999 Hector Mine earthquake.

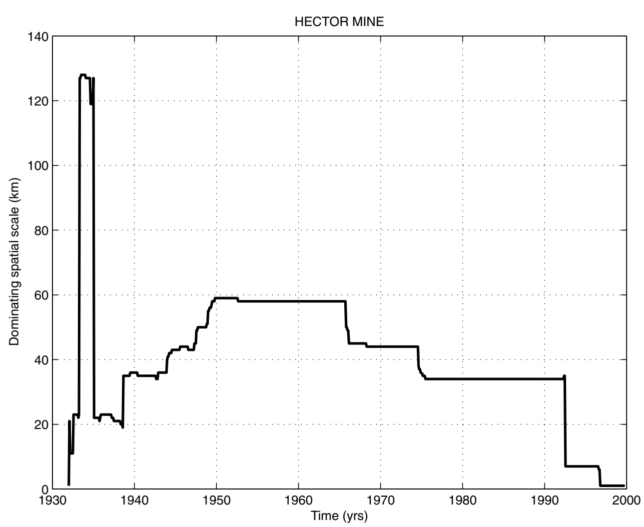

Figure 13. Same as Fig. 4 for the 1999 Hector Mine earthquake.

are not 'actors'). In other words, the temporal singularities defined in (Bowman *et al.* 1998) for instance stem rather from the large-scale geometry of the boundary loading conditions and correlations not directly mediated by the stress field (that were not taken into account in the present work) than from strong interaction between seismic events mediated by the stress field. In this spirit, Bowman & King (2001a,b) and Sammis & Sornette (2002) have developed a model in which the main mode of loading of a previously ruptured major fault occurs by localized viscous flow beneath this fault. The consequence is that the extent of the stress shadow due to the previous mainshock decreases with time, so that seismicity migrates back to the mainshock epicentre in an accelerating manner, the temporal singularity coinciding with a new mainshock on the fault. However, such a model implies that seismicity migrates towards P_0, which cannot reasonably be inferred from our computations either (Figs 3, 6, 9 and 12). If this were the case, the wavelet coefficients should be negative at P_0 (i.e. located in an area with a deficit of seismicity), and the width of the domain around P_0 where coefficients are negative should decrease with time (as events migrate towards P_0). This suggests that the loading mechanism proposed by Bowman & King (2001a,b) and Sammis & Sornette (2002) is not compatible with our results, but that another loading mechanism may explain the temporal singularity coinciding with large events.

Another solution to explain the discrepancy between the large-scale correlations observed in seismic catalogues (Bufe & Varnes 1993; Sornette & Sammis 1995; Bowman *et al.* 1998; Brehm & Braile 1998, 1999; Jaumé & Sykes 1999; Ouillon & Sornette 2000) and our results is to argue that our geometrical/rheological model of the lithosphere is incorrect, which makes our Green's function imperfect. The Green's function we have considered is representative of a linear viscoelastic layered medium, and we checked that our results are not strongly dependent on its various parameters. One possibility is that, if the observed absence of correlations is due to our choice of the Green's function, then the true Green's function must be of a fundamentally different nature. The Earth's crust is a very complex medium, composed of blocks of various sizes separated by fractures or fault zones, subjected to a confining pressure and a temperature which increases with depth. We would be very lucky if such a medium behaved as a perfect linear medium. Indeed, crustal rheology must be of non-linear nature, even in its most superficial 'elastic' part. Some evidence of a non-linear response associated with the anisotropic response of a cracked medium under compression compared with tension has been reported in Peltzer *et al.* (1999). Let us extend this argument: if, for example, the crust behaves as a granular material, then we must expect that tectonic forces propagate over longer distances within much narrower channels than those predicted by standard elastic models. This singular property is due to the hyperbolic nature of differential stress propagation equations in granular media (Bouchaud *et al.* 1995; Bouchaud *et al.* 2001), whereas those equations are of elliptical nature in standard elastoplastic media. The real rheology of the Earth's crust is probably somewhere between that of a granular material and more standard models of (possibly non-linear) viscoelastic plasticity.

More recently, using a cellular automaton model, Weatherly *et al.* (2002) showed that the overall behaviour of seismicity depends strongly on the exponent of the elastic Green's function (that we took equal to 3, that is, corresponding to a rather short interaction range). In particular, they showed that for this value of 3 for the exponent they did not observe any critical behaviour before a large event. Interestingly, they found a critical behaviour for smaller exponents, suggesting that large-scale critical correlation requires long-range interactions in this problem. Bai *et al.* (2003) have shown theoretically that such small Green's function exponents could be justified for materials with non-linear concave stress–strain characteristics as occur for damaged materials, such as the Earth. We have also shown that, if the crust is criss-crossed with faults filled with drained fluids close to the lithostatic pressure, standard elasticity is replaced by an asymmetric non-linear elasticity also leading to smaller exponents of the decay of the stress transfer (strictly, one cannot speak anymore of a Green's function since the medium is non-linear) (Ouillon & Sornette 2003). Roux & Hild (2002) have also given the analytical solution for the stress field and for the dependence of the exponent describing the range of stress redistribution in a non-linear asymmetric elastic medium in the case of antiplane mode III loading and find the same effect. It thus seems important to better understand crustal rheology (and its associated Green's function), in order to check the changes it would imply in the various brittle crustal modes of deformation and in the way earthquakes 'speak to each other'. In this spirit, phenomenological models of earthquake interaction and triggering are quite successful in capturing most of the phenomenology of seismic catalogues (Helmstetter & Sornette 2002a,b; Helmstetter *et al.* 2003b). It remains to derive the triggering Green's function (or its non-linear generalization) from physically based mechanisms, which seem to require much more than just viscoelastic stress transfers.

Finally, we cannot exclude the possibility that neither the stress level nor the correlation length are pertinent signatures of criticality. It could be that other measures, such as a variable-mixing non-linear stress level and correlation length (such as an average stress level over a time-varying correlation length), are more relevant in order to reveal critical properties. Such issues are known to arise in the physics of complex heterogeneous systems in which non-linear susceptibilities seem to provide more relevant clues to criticality than their standard linear counterparts used for non-random critical systems (Mézard *et al.* 1987).

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