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Fractal analysis of seismogenic ULF emissions

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Abstract

Fractal analysis has been performed on ultra-low-frequency (ULF) geomagnetic field data observed at the Izu peninsula. The first attempt of using fractal analysis for ULF geomagnetic field data during the large Guam and Biak earthquakes has been based on FFT. In order to quantitatively estimate the FFT-based fractal analysis, we first study a few fractal analyses, including the Burlaga–Klein method and the Higuchi method, as compared with the former FFT-based method. The accuracies of these three methods have been evaluated by applying them to the test data-sets of fractional Brownian motion with white noise. We conclude that the Higuchi method is superior to the FFT-based method and the Burlaga–Klein method. The S/N effect was also discussed. Then, we have applied the Higuchi method to the ULF geomagnetic field data during a big earthquake swarm (during June–August 2000) in the Izu peninsula. It is found that the fractal dimension exhibits a significant increase just before earthquakes with a magnitude Mj>6.0 associated with the Izu islands swarm with any of these three fractal analyses. This experimental finding will lend further convincing support to the presence of precursory ULF emissions.

1. Introduction

Electromagnetic phenomena have recently been recognized as a possible candidate for short-term earthquake prediction (Hayakawa and Fujinawa, 1994; Hayakawa, 1999; Hayakawa and Molchanow, 2002). Especially, ULF (ultra-low-frequency, frequency < 10 Hz) electromagnetic emissions are found to take place prior to large earthquakes, including Spitak (Kopytenko et al., 1990; Molchanov et al., 1992), Loma Prieta (Molchanov et al., 1992; Fraser-Smith et al., 1990), Guam (Hayakawa et al., 1996; Kawate et al., 1998; Hayakawa et al., 1999), Biak (Hayakawa et al., 2000) earthquakes, etc. Also, the generation mechanism of seismo-ULF emissions has been proposed by Molchanov and Hayakawa (1995), and then some other mechanisms have also been developed (Hayakawa and Fujinawa, 1994; Hayakawa, 1999; Hayakawa and Molchanow, 2002).

The most serious problem in this ULF study is to increase the number of ULF events associated with earthquakes, and also to show useful examples in a reliable manner. Earlier works have been devoted only to the monitoring the intensity of observed ULF magnetic fields (Kopytenko et al., 1990; Molchanov et al., 1992; Fraser-Smith et al., 1990), but it would be possible only when we were so lucky to be located very close to the epicenter (like in the case of Loma Prieta earthquake (Molchanov et al., 1992; Fraser-Smith et al., 1990)). When we are located far away from the epicenter, the essential point is how to identify the seismogenic ULF emissions in the observed time series of ULF geomagnetic field. First, the analysis of polarization (i.e., the ratio of the vertical to horizontal magnetic field component) is found to be extremely useful in distinguishing the seismogenic ULF emissions from the terrestrial geomagnetic variation (magnetospheric effect) (Hayakawa et al., 1996; Kawate et al., 1998). Hayakawa et al. (1999, 2000) have carried out the first attempt of fractal analysis for ULF data on the basis of self-organized criticality (SOC) concept (Bak and Tang, 1989; Trucotte, 1990), but they have used the method of FFT to estimate the scaling properties. Being stimulated by this work, there were followed by some fractal papers on earthquakes (Hongre et al., 1998; Telesca et al., 1999; Barraclough and De Santis, 1999). Additionally, Gotoh

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et al. (2000) have attempted principal component analysis (PCA) for multi-stationed ULF data to identify the ULF signature of the earthquakes.

This paper deals with the two essential problems. The first is the comparison of different fractal analyses (FFT-based method, Burlaga–Klein method (Burlaga and Klein, 1986) and Higuchi method (Higuchi, 1988; Higuchi, 1990)) for analysis of ULF data by using the simulated fractional Brownian motion (fBm) data set. Then, those fractal methods have been applied to the real data (Izu islands earthquake swarm in July–August, 2000 and the Guam earthquake in August, 1993), and we will show whether this kind of fractal analysis would be useful to elucidate the significant fractal changes before and after earthquake.

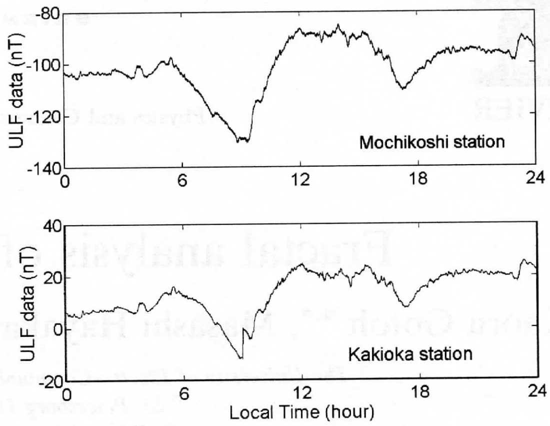

Fig. 2. An example of one day data of the ULF field observed at Mochikoshi station (top panel) and Kakioka station (bottom panel) on June 1, 2000 (H-component).

2. ULF geomagnetic observation and earthquakes

Here we show the details on our stations of observing ULF waves and earthquakes. Observation stations for observing ULF geomagnetic fields by means of the advanced ULF measurement technique (MVC-2DS instrumentation) are at three locations: Mochikoshi, Kamo and Seikoshi on the Izu peninsula, and also at the three stations: Kiyosumi, Uchiura and Unobe on the Boso peninsula (the spacing between the neighboring two stations being 5–10 km) (Kopytenko et al., 1994). The locations of these stations are shown in Fig. 1. This ULF network was established by the collaboration between the Riken and NASDA groups. Three magnetic field components: H (North–South), D (East–West) and Z (Vertical) are measured at each station with the sampling rate 12.5 Hz with GPS time stamp, and are stored on a PC. Data gathered at the Kakioka and Memambetsu stations are taken by the Magnetic observatories of Japan Meteorological Agency. One

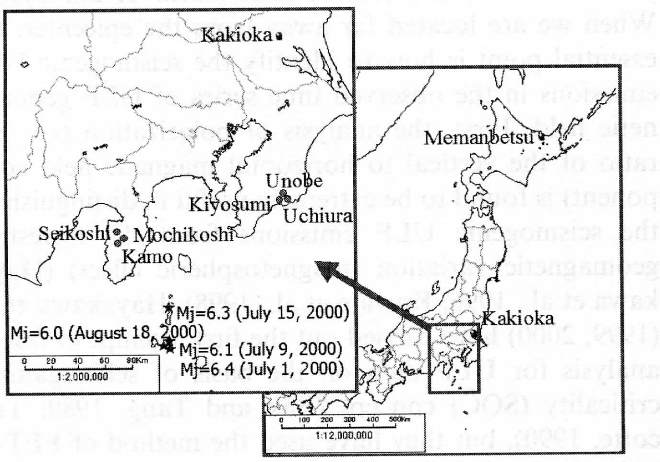

Fig. 1. The locations of ULF observing stations in Japan. The left map shows the ULF network in the Tokyo area, consisting of the Izu, Boso and Kakioka stations. The right panel shows a far station of Memambetsu in Hokkaido.

example of observed data (Mochikoshi, H-component, June 1, 2000) is illustrated in the top panel of Fig. 2. If we compare this ULF geomagnetic record at the Mochikoshi station with the corresponding ULF record at Kakioka observatory presented in the bottom panel of Fig. 2, we find that those records look quite similar. This gives us a common basis for analysis of both sets of data. For the case of the Izu Peninsula, the effective sampling rate (0.5 Hz) and selected duration of each time series to avoid the effect of man-made noise (1 h at midnight, 1800 points) allow us to analyze the properties of ULF emissions in a frequency range less than $f = 0.25$ Hz.

Fig. 1 shows also the epicenters of earthquakes with magnitude greater than $\mathrm{Mj} = 6.0$ caused by a Miyake volcano eruption, which started from the end of June 2000. The first massive earthquake occurred on July 1 ($\mathrm{Mj} = 6.4$), and the other strong earthquakes were registered on July 8, 15, 30 and August 18. All earthquakes in the swarm were shallow; their depths being less than 10 km. The distances from the epicenter to the observation stations are about 80, 130, 160 and 1160 km, respectively, for the Izu stations, Boso stations, Kakioka station and Memambetsu station.

3. Analysis method and evaluations

3.1. Analysis method with ULF geomagnetic data

The FFT method allows us to obtain the spectral exponent β for a fractal time series, which is characterized by the power-law spectrum; $S(f) \propto f^{-\beta}$. If we plot the spectrum of the ULF signals in the bi-log form, we can obtain β from the slope of the straight line, which is the least mean square fitting to the spectrum. Then the fractal dimension of the ULF time series can be calculated by using the Berry's equation (Berry, 1979): $D_0 = (5 - \beta)/2$.

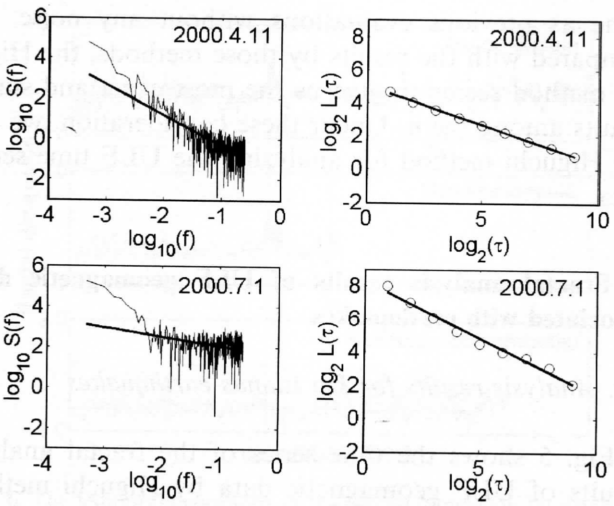

Fig. 3. An example of the comparison of fractal analysis: The left panels refer to the results by the FFT method, while the right panels to those by the Higuchi method. The upper two refer to a seismically quiet day, and the lower two to a seismically active day.

We take Fig. 3 as an example. The left-top panel is the spectrum of ULF geomagnetic data in the seismic-quiet period (Mochikoshi, H-component, April 11, 2000) and the left-bottom one also shows the spectrum in a seismic-active period (Mochikoshi, H-component, July 1, 2000). The straight lines in both panels indicate the least mean square fitting to each spectrum. It is important to notice a difference in the linearity of the spectra between these two panels. The spectrum in the seismo-active period has a nonlinear part; this effect is distinguishable especially in the high frequency range in comparison with the spectrum of a quiet period. It means that the fractal dimension would be seemingly high in the seismic-active period and it has no contradiction with our previous study related to the Guam earthquake (Hayakawa et al., 1999). However, we are concerned about whether the spectral slope changes associated with an earthquake are real or not, because fitting error increases much at the high frequency range by its non-linearity. In this sense, the fractal property of ULF waves seems to be difficult to be estimated during seismo-active periods.

Hence, this paper deals with comparisons of different fractal analysis methods to be applied to the ULF field. FFT procedure for the estimation of fractal analysis has already been proposed with significant success (Hayakawa et al., 1999). However, there have been proposed a few alternative methods for fractal analysis. Burlaga and Klein suggested a method (the BK method), which provides a stable estimation of spectral exponents through the calculation of stable values of the fractal dimension D_0 (Burlaga and Klein, 1986). They defined the curve length L of the curve $B(t)$ representing a geophysical time series by dividing blocks that has same width, as follows:

$$L(\tau) = \sum_{i=1}^{n} \left|\overline{B}(t_i + \tau) - \overline{B}(t_i)\right| / \tau, \tag{1}$$

where $n = [L/\tau]$ ([ ] is Gauss' notation) and τ are the maximum division number and the block width, respectively, and $\overline{B}(t_i)$ denotes the average value of $B(t)$ between $t = t_i$ and $t = t_i + \tau$. For statistically self-affine curves, the length is expressed as $L(\tau) \propto \tau^{-D_0}$. This relation enables us to estimate the fractal dimension D_0. There is an additional fractal analysis method proposed by Higuchi (1988). The Higuchi method is very similar to the BK approach, and also it gives us stable values of fractal dimensions as well. Higuchi modified the BK method, suggesting that the calculation for curve length should be taken before the averaging procedure. He constructed a new time series $B_\tau^m(\tau)$ from the original time series $B(t)$ and defined the curve length L of the original time series as an average value of the lengths $L_m(\tau)$ representing each time series $B_\tau^m(\tau)$ as follows:

$$B_\tau^m(\tau); B(m), B(m+\tau), B(m+2\tau), \ldots, B(m+n\cdot\tau) \quad (m = 1, 2, \ldots, \tau) \tag{2}$$

$$L_m(\tau) = \left\{ \left(\sum_{i=1}^{n} |B(m+i\tau) - B(m+(i-1)\cdot\tau)| \times \frac{N-1}{n\cdot\tau} \right) \right\} \Big/ \tau, \tag{3}$$

$$L(\tau) = \langle L_m(\tau) \rangle \propto \tau^{-D_0}, \tag{4}$$

where $n = [(N-m)/\tau]$. The right panels of Fig. 3 show the scaling properties by the Higuchi method, to be compared with the spectrum of the FFT method on the left panels. The block width used in the procedure of the Higuchi method is set to $\tau_{max} = 2^7$. The Higuchi method gives us mostly the linear scaling property over all range. Thus we can consider that the ULF data has a fractal property.

3.2. Evaluation of FFT-based method, Higuchi method and Burlaga–Klein method

In this section, let us examine the three methods: the FFT method, the BK method and the Higuchi method from the general point of view. Here we use the simulated data set generated by a fBm to evaluate those methods before analyzing the real ULF data. The increment between the two successive sampling points of the observed ULF time series (1 h at midnight, for example) is found to follow the Gaussian distribution, which means that the fBm can be used as a test data set for the evaluation in our case. The fBm data set is generated by the Wood–Chan method (Wood and Chan, 1994). The fractal dimension of the test data set is arranged to be in the range of $1.05 < D_0 < 1.95$ with a step of 0.05, and the data length is $N = 1800$ points same as 1 h data observed at the Izu peninsula. We use $\tau_{max} = 2^7$ for the BK and Higuchi methods because of the Higuchi's suggestion (Higuchi, 1990) that a suitable τ_{max} is around $N/10$. For the FFT procedure, the FFT length is 2^7 with taking into account the scaling range with other methods. The results of fractal dimension on average for 20 different fBm data sets estimated by those methods with Hurst exponent variation ($H = 2 - D_0$) indicate clearly that the accuracy of the FFT method is quite low, in particular, for the dataset with an assumed Hurst exponent less than 0.25 (anti-persistent noise) and more than 0.6 (persistent noise). As concerned with the BK method, its estimation is slight higher in comparison with the theoretical value all over the H range. This is the result from the fact that the BK method makes the curve length shorter because of the averaging procedure before the length calculation. This effect is emphasized for an anti-persistent noise. On the contrary, the Higuchi method yields good estimates; mostly fitting with the theoretical.

Next we compare the effect of noise on the estimation of fractal dimension by these three methods under external noise by using the same test calculation as before. White noise was added to the simulated data set as an external noise. Fig. 4 illustrates the results on the data set ($D_0 = 1.55$) with the external white noise 10 dB $< S/N <$ 50 dB. As the marks in the Fig. 4, the triangle, square and circle indicate foe the FFT method, BK method and Higuchi method results and thickness line is theoretical ($D_0 = 1.55$). The figure indicates that the method weakest against the noise is the FFT method. The BK and Higuchi methods are found to generally show a more robust estimation than the FFT method, but their estimation error increases remarkably with the decrease in the S/N ratio. The BK method gives a high estimation value even at high S/N ratio, being the same as previous evaluations without any noise. As compared with the results by those methods, the Higuchi method seems to give us the most exact and stable results among them. Under these consideration, we use the Higuchi method for analyzing the ULF time series data.

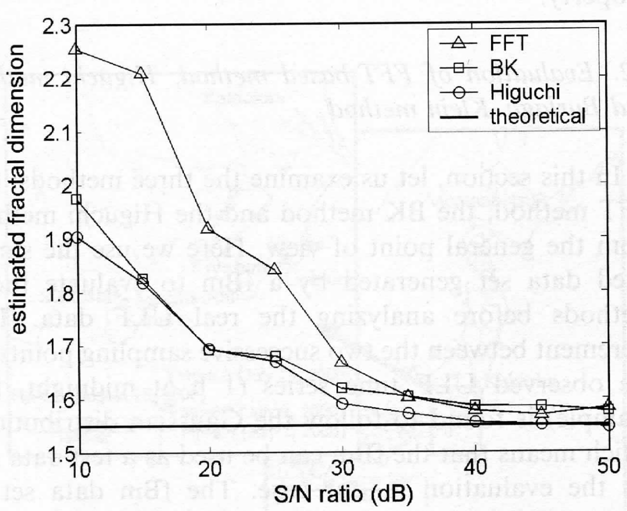

Fig. 4. The effects of S/N ratio on estimating the fractal dimension by the FFT, Burlaga–Klein and Higuchi methods with the fBm time series data set ($D = 1.55$) added with white noise with 10 dB $< S/N <$ 504 dB.

4. Fractal analysis results of ULF geomagnetic data associated with earthquakes

4.1. Analysis results for Izu islands earthquakes

Fig. 5 shows the time series of the fractal analysis results of ULF geomagnetic data by Higuchi method ($\tau_{max} = 2^7$) at midnight (LT 2:00–3:00) on each day. We have used the ULF data (H-component) observed at Izu peninsula during the period of February 2, 2000–February 3, 2001. The vertical lines in Fig. 5 indicate the times of occurrence of earthquakes with magnitudes greater than Mj = 6.0. The data after the earthquakes in August have not been available to us or there are no data because of system trouble by the shock of earthquake except at Seikoshi station. The Mochikoshi station worked properly again in the middle of October. The fractal dimensions at all the stations exhibit clearly a prominent increase from about one month before the first massive earthquake (July 1, Mj = 6.4). The value goes down right after the earthquake, and rises again before the second earthquake (July 9, Mj = 6.1). We cannot find any other increase before the third earthquake on July 15 (Mj = 6.3), but the fractal dimension shows a significant rise before the fourth earthquake (August 18, Mj = 6.0) again. Thus, we have to consider why there is no anomaly only for the third earthquake. It is because to have different characteristics in the generation mechanism only for the third earthquake. It

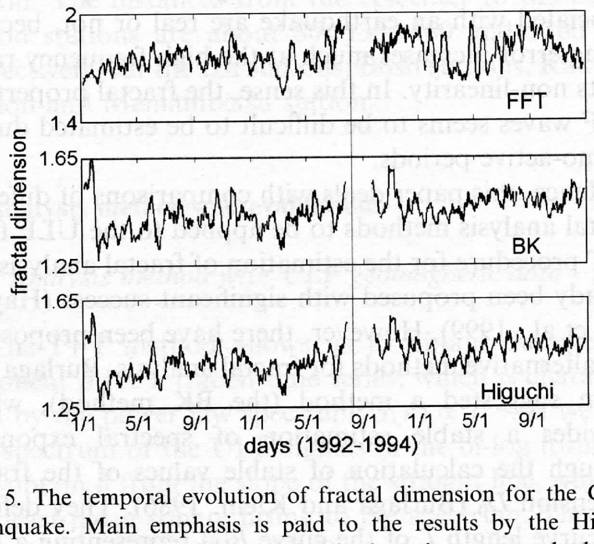

Fig. 5. The temporal evolution of fractal dimension for the Guam earthquake. Main emphasis is paid to the results by the Higuchi method (bottom panel), but the results by other two methods (top panel with FFT method, middle panel with BK method) are also included for the sake of comparison.

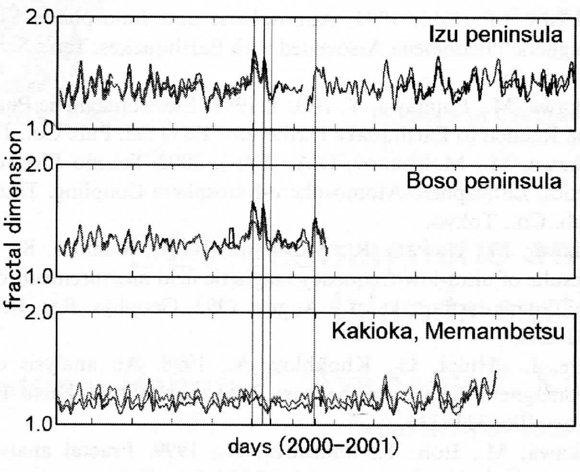

Fig. 6. The temporal evolutions of the fractal dimension by means of the Higuchi method during the period from February 2000 to February 2001. The top panel is for the three stations at Izu peninsula (red, blue and black indicate the Kamo, Seikoshi and Mochikoshi stations, respectively) and the middle panel is for the three stations at Boso peninsula (red, blue and black indicate the Uchiura, Kiyosumi and Unobe, respectively). The bottom panel shows results for rather far stations, Kakioka (red) and Memambetsu (blue). Vertical lines indicate the times of occurrence of earthquakes with magnitude greater than Mj = 6.0.

follows from what has been said thus far that the fractal dimension changes before strong earthquakes. But if it is clarified that these anomalies are a local phenomenon just around the epicenter, we can more strongly claim these transitions are related with a series of earthquakes. The Izu and Boso stations are situated within 130 km from the epicenter, thus we also analyze the data observed at rather far stations, the Kakioka station (160 km from the epicenter) and the Memambetsu station (1160 km from the epicenter). The bottom panels of Fig. 6 illustrate the corresponding fractal analysis results by the same Higuchi method for the Kakioka and Memambetsu data, respectively. The vertical lines have the same meaning in Fig. 5. Any anomalies observed at these far stations are not confirmed to be associated with earthquakes unlike in the top and middle panels those show the results at the stations situated close to the epicenter. Thus there are no doubts about our previous conclusion.

4.2. Analysis results for Guam earthquake

Here we present the fractal analysis for the same Guam data as in our previous study (Hayakawa et al., 1999), but this study is not based on the FFT method but on the Higuchi method. Fig. 7 shows the fractal analysis results by Higuchi method. For the sake of comparison, the results by two other methods are also included for the Guam earthquake (Ms = 8.1, August 8, 1993). We use the H-component data (1 h per each day, LT 14:00–15:00) during three years, from the beginning of 1992 to the end of 1994. The vertical line indicates the

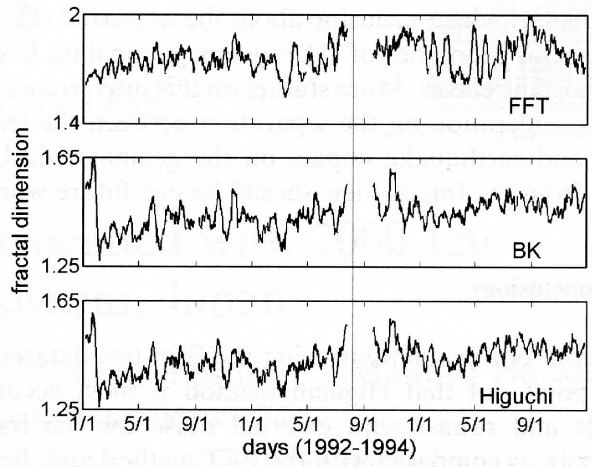

Fig. 7. The temporal evolution of fractal dimension for the Guam earthquake. Main emphasis is paid to the results by the Higuchi method (bottom panel), but the results by other two methods (top panel with FFT method, middle panel with BK method) are also included for the sake of comparison.

earthquake time as usual. The blank in the figure shows the period of no data because of system trouble. A significant increase is found in Fig. 7 to appear before the earthquake exactly in the same way as the Izu earthquake case. The important point is that is obtained not only by the FFT method, but also by the more reliable Higuchi method. A comparison of the results by all of these three methods suggests that the fractal behavior by the Higuchi and BK methods show distinctive increase before the earthquake, as compared with FFT methods. It leads us to believe that the fractal properties of geomagnetic ULF data were affected by seismic activity before the earthquake.

We can distinguish in Fig. 7 some oscillations especially in the period around 1994. Now we compare it with the Ap-index (Mayaud, 1980) corresponding to the activity of terrestrial magnetism in Fig. 8. When the Ap index has a maximum, the fractal dimension takes a minimum point conversely. This means that there is a negative correlation between the two. Thus we can

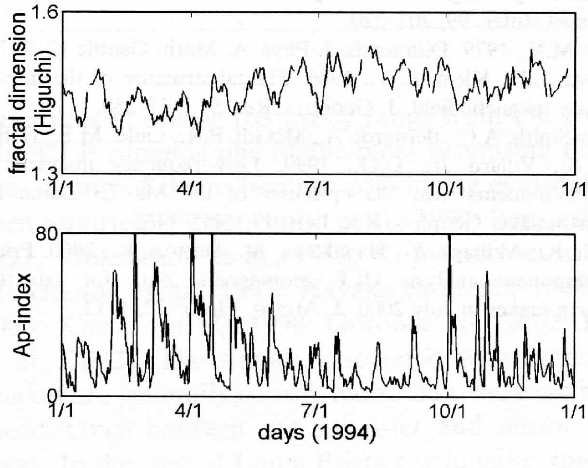

Fig. 8. The comparison with temporal evaluation of fractal dimension and Ap-index.

obtain one more conclusion about the terrestrial effect on the fractal properties of ULF geomagnetic data besides seismogenic effects. More studies on this interference and the consideration on the separation between the terrestrial and earthquake impact on the geomagnetic ULF data from the fractal view should be our future works.

5. Conclusions

From our evaluations using the fBm test dataset, we can point out that Higuchi method is most accurate, stable and robust with external noise for the fractal analysis, as compared with the FFT method and the BK method. Under this consideration, fractal analysis of ULF geomagnetic data at the Izu and Boso stations has been performed during the period before and after the Izu earthquakes in 2000 by the Higuchi method. It has been found that the fractal properties change before earthquakes and the fractal dimension increases prior to strong earthquakes. In addition we can argue with much confidence by a comparison of the analysis results with those gathered at the stations far from the epicenters, the Kakioka and Memambetsu stations.

The analysis results for the Guam earthquakes in 1993 by the Higuchi method do not contradict the Izu case. Although the FFT method is not suitable for this case, the conclusion of our previous study that the fractal dimension increases before strong earthquakes was confirmed. Furthermore, the possibility of other effects on the fractal property has been pointed out by this examination. The fractal property of geomagnetic ULF data was found to be influenced by the geomagnetic effect caused by solar activity.

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