

Modeling of nonlinear passage of acoustic waves caused by underground fracturing through the lithosphere

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Abstract

The nonlinear passage of the acoustic waves through the lithosphere to the surface of the Earth during earthquakes and strong underground explosions is analyzed in this report. The underlying mechanism for this is the nonlinear elastic modulus. The waves are excited at the underground source of the earthquakes. The passage of the acoustic waves propagating almost vertically upward leads to a change of the spectrum. The wide spectrum of the acoustic waves up to the radio wave range is assumed to be produced by the fracturing of the rock at the surface. This has been observed by means of satellite measurements and radio telescope investigation of meteor bombing of the Moon. If the fracture occurs at depth corresponding to high frequencies, the waves transform, through nonlinear interactions, into low and super low frequency waves. Low and super low elastic displacement waves reach the surface and produce a seismograph response. In the report the nonlinear excitation of ultra-low frequency (ULF) acoustic waves caused by low frequency (LF) seismic acoustic burst is also discussed. An analysis of the nonlinear transformation of LF acoustic waves ($f \sim 100$ Hz) into ULF ($f \leq 1$ Hz) waves is presented. The LF wave is excited as the burst-like envelope of a finite transverse scale by the underground seismic motion caused seismic activity of the Earth. Then, it propagates upwards and is subject to both a nonlinear process and dissipation. The nonlinearity leads to the generation of higher harmonics and, thus, to a saw-like wave structure, and also to an increase of the ULF part of the wave spectrum. This process takes place underground at a depth of about 50–30 km.

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1. Introduction

It is very important to investigate the mechanisms of energy flow from the lithosphere into the atmosphere and the ionosphere caused by natural hazards (seismic and volcano activity, for example experiment MASSA (Galperin, 1985)). All mechanisms have different precursors (Warwick, 1963) due to three basic channels of the lithosphere-ionosphere connection, namely, electromagnetic, geochemical, and acoustic channels (Gokhberg et al., 1995). The electromagnetic, optic, acoustic and geochemical precursors are the most common, but

there are also perturbations of the density of plasma in F and E-layers; anomaly in the absorption of cosmic radio emission; strong changes of the ionosphere parameters and moving of E and F-layers; variations of electric E and magnetic H fields and its perturbations; perturbations of the electromagnetic waves, first of all, ELF (extremely low frequency) and VLF (very low frequency); perturbation (increasing) of the intensity of the luminescence of the ionosphere in the main emission of the atomic oxygen at $\lambda = 5577$ and 6300 Å. Finally, other phenomena were observed such as anomalies by the propagation of radio waves above volcanoes and epicenters of strong seismic active sites; perturbation of the sporadic Es-layer emission in the ionosphere; geochemical and biological processes in seas and ocean, and so on. The same appears before the volcanic activity

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Fig. 1. Vulcan Popocatepetl. Mexico. Explosion in December 2000.

takes place (see example in Fig. 1). Also, before volcano explosions, there have been satellite observations (Warwick, 1963) of some big particles and chemical gases like H_2O , CO_2 , S, SO_2 .

The acoustic–electromagnetic phenomenon consists in the transformation, in the ionosphere, of atmospheric acoustic waves caused oscillations of Earth’s surface. In the process, small magnetic field perturbations are excited. This results in the appearance of a periodical structure of the electronic concentration in the ionosphere, and changes in the transparency and displacements of the E and F-layers of the ionosphere. These effects become very effective because of the property of the atmospheric acoustic waves to increase their amplitudes as they propagate upwards.

The geochemical channel consists of the release of various gases, including radioactive ones, aerosols, metal ions, etc., from those areas where adjacent tectonic plates interact. The chemical channel is involved in the formation of clouds, the spatial distribution of polar lights, the cross distribution of density of the ionospheric plasma in D, E, and F-layers, the appearance of bands of abnormal propagation of radio waves, etc. One of the basic results supporting the existence of this mechanism is the observed conductivity increase of the atmosphere and ionosphere above the places of tectonic plate rupture (Gokhberg et al., 1995).

The electromagnetic channel can be explained in terms of alternating currents $\delta\vec{j}(\vec{r}, t)$ occurring at the epicenters of forthcoming earthquakes. In accordance with the Maxwell’s equations, this currents are sources of variable electromagnetic fields reaching the ionosphere practically instantaneously and creating an appropriate local electromagnetic response. The main problem is to specify the $\delta\vec{j}$ connected with the seismic processes. It is possible to define some mechanisms of

occurrence in places of tectonic plate raptures, namely, piezoelectric effect, electro kinetic effect, movement of charged dislocations, movement of underground Earth water with increased electric conductivity, etc. With the knowledge of the spatial–temporal distribution of currents and using the Maxwell’s equations and the conductivity tensor of the ionosphere, it is possible to determine the electromagnetic response of the ionosphere to the current pulse originating in the lithosphere. This problem has already been solved (Molchanov, 1991; Molchanov et al., 1995).

The acoustic channel of the lithosphere-ionosphere coupling seems quite effective. It is due to atmospheric acoustic waves excited by fluctuations of the Earth’s surface (Tarantsev and Birfeld, 1973). This channel is illustrated through related phenomena (Kotsarenko et al., 1997) like the exciting of plasma waves and periodic structure in the ionosphere, the increase of transparency to radio waves, the linear and nonlinear generation of magnetic perturbations, the oscillation of E–F-levels in the ionosphere caused by acoustic and acoustic-gravity waves, the nonlinear change of spectrum of waves in the atmosphere, the ionosphere and the lithosphere. The last case is analyzed below.

2. Model and equations

The geometry of the model is shown in Fig. 2. It consists of a cylindrical surface around the place in which there is a source of underground seismic explosion (or plate deformation). After that, a seismic acoustic burst-like envelope of finite transverse scale begins its passage through the Earth. It is possible to apply elastic theory with nonlinear modulus to the real situation by taking into account the damping of the waves and its diffraction. The elastic theory for the case of the Earth’s crust, considered here as an isotropic uniform medium, has the following Newton’s equation for displacement \vec{U}

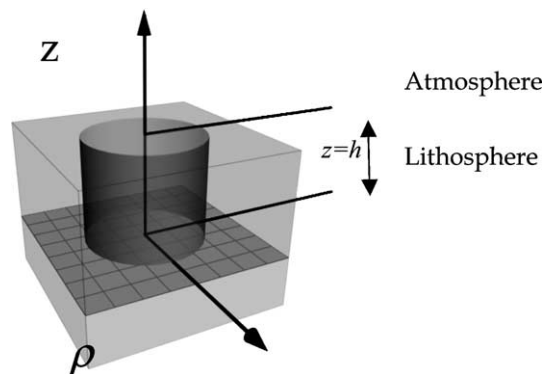


Fig. 2. The geometry of the nonlinear passage of acoustic waves caused by underground fracturing through the lithosphere $A_1(z=0, \rho, t) = (A_{01} \exp(i\Delta\omega \cdot t) + A_{02} \exp(-i\Delta\omega \cdot t)) \exp(-(\rho/\rho_0)^2)$.

$$\frac{\partial^2 \vec{U}}{\partial t^2} = \nabla \{ (S_l^2 - S_t^2) \cdot \text{div} \vec{U} \} + S_t^2 \cdot \Delta \vec{U} + \Gamma(z) \frac{\partial}{\partial t} \Delta \vec{U} + B(z) \nabla \{ (\text{div} \vec{U})^2 \} \quad (1)$$

where the coefficients $\Gamma(Z)$ and $B(z)$ describe the viscosity and nonlinearity of the elastic isotropic uniform medium in a cylindrical system of coordinates, for the acoustic waves, with longitudinal and transversal velocities $S_{l,t}$, respectively. If it is possible to neglect the derivatives of $S_{l,t}$, since we take into account only the dependency of the diffraction's coefficient $\Gamma(z)$ with the coordinate in which the waves pass, we can rewrite Eq. (1) in components as follows. The first is the direction along the Z-axis:

$$\begin{aligned} \frac{\partial^2 U_z}{\partial t^2} \cong & (S_l^2 - S_t^2) \cdot \left\{ \frac{\partial^2 U_z}{\partial z^2} + \frac{1}{\rho} \frac{\partial^2 (\rho U_\rho)}{\partial z \partial \rho} \right\} \\ & + S_t^2 \cdot \left\{ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial U_z}{\partial \rho} \right) + \frac{\partial^2 U_z}{\partial z^2} \right\} \\ & + \Gamma(z) \cdot \frac{\partial}{\partial t} \left(\frac{\partial^2 U_z}{\partial z^2} \right) + B(z) \cdot \frac{\partial}{\partial z} \left(\frac{\partial U_z}{\partial z} \right)^2 \end{aligned} \quad (2)$$

It should be noticed that the transverse profile is quite smooth. The second component is the transversal direction having only the ρ -component:

$$\frac{\partial^2 U_\rho}{\partial t^2} \cong (S_l^2 - S_t^2) \cdot \frac{\partial^2 U_z}{\partial \rho \partial z} + S_t^2 \cdot \frac{\partial^2 U_\rho}{\partial z^2} \quad (3)$$

Let us use the variables z, ρ and $\eta = t - \int_0^z \frac{dz'}{S_l(z')}$. The last variable helps in taking into account the diffraction of waves. We use also the method of slow varying profile since the investigation is devoted to the nonlinear evolution of a seismic acoustic burst-like envelope of finite transverse scale in a cylindrical system of coordinates. This method gives the possibility to obtain a simple modeling of Eqs. (2) and (3).

For the ρ -component we have

$$\frac{\partial^2 U_\rho}{\partial \eta^2} \cdot \left(1 - \frac{S_t^2}{S_l^2} \right) \cong - \frac{(S_l^2 - S_t^2)}{S_l} \cdot \frac{\partial^2 U_z}{\partial \rho \partial \eta} \quad (4a)$$

or

$$\frac{\partial U_\rho}{\partial \eta} \cong -S_l \cdot \frac{\partial U_z}{\partial \rho} \quad (4b)$$

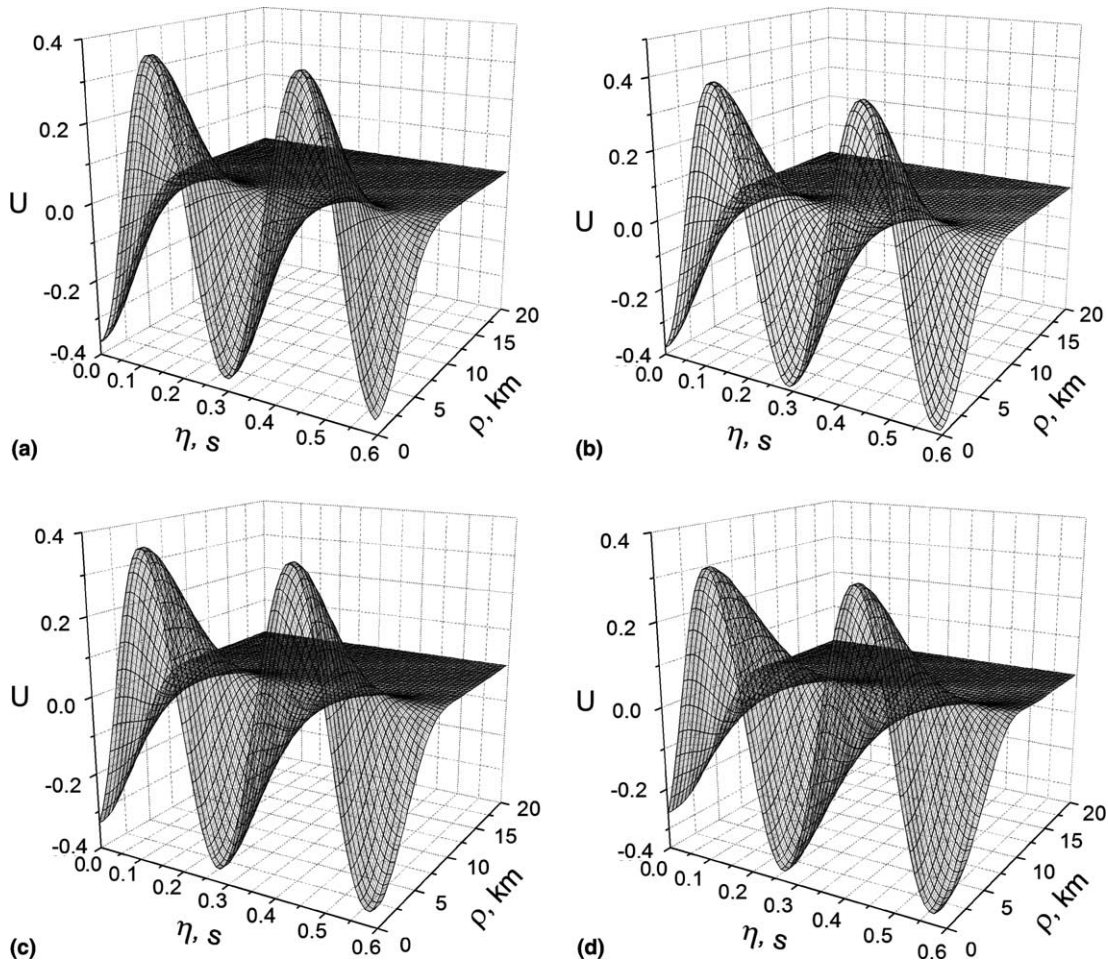


Fig. 3. Evolution of low frequency component: (a) is at $z = 5$ km; (b) $z = 10$ km; (c) $z = 20$ km; (d) $z = 30$ km. $\rho_0 = 6$ km; $G = 0.25$ km⁻¹.

Eq. (5) shows that the most important component is the z-component, the transverse profile is quite smooth, and in the modeling it possible to neglect of the transversal component.

For the z-component we have

$$\frac{\partial U_z}{\partial z} - \frac{\Gamma(z)}{2S_l^3} \cdot \frac{\partial^2 U_z}{\partial \eta^2} + \frac{B(z)}{2S_l^4} \cdot \left(\frac{\partial U_z}{\partial \eta}\right)^2 - \frac{S_l}{2} \cdot \int_{-\infty}^{\eta} U_z(\eta') \cdot d\eta' = 0 \tag{5}$$

This is the equation of Rudenko (1995). Below we use it in one-dimensional form with the coefficient $\Gamma(z) \rightarrow G(z)$ and $B(z) \rightarrow N(z)$, and use the diffraction coefficient D :

$$\frac{\partial U}{\partial z} - G(z) \cdot \frac{\partial^2 U}{\partial \eta^2} + N(z) \cdot \left(\frac{\partial U}{\partial \eta}\right)^2 - D \int_{-\infty}^{\eta} U_z(\eta') \cdot d\eta' = 0 \tag{6}$$

like in (Rudenko, 1995). For the modeling it is necessary to make a fast Fourier transforms by $\eta(\infty \exp(-jk_\eta \eta))$.

In this approximation we have to divide in two parts: high frequency (HF) and low frequency (LF).

For the HF part we have the equation

$$\frac{\partial U}{\partial z} - G(z) \frac{\partial^2 U}{\partial \eta^2} + N(z) \left(\frac{\partial U}{\partial \eta}\right)^2 = 0 \tag{7}$$

while for the LF parts we have the equation

$$\frac{\partial U}{\partial z} - G(z) \frac{\partial^2 U}{\partial \eta^2} + N(z) \left(\frac{\partial U}{\partial \eta}\right)^2 - D \Delta_p \int_{-\infty}^{\eta} U(\eta') d\eta' = 0 \tag{8}$$

We take into account the diffraction only for the case of ULF part.

3. Simulation and discussing of the results

For the computer modeling of the nonlinear passage of an acoustic waves caused by underground fracturing through the lithosphere, we take into account our approximation and Eqs. (7) and (8). This computer

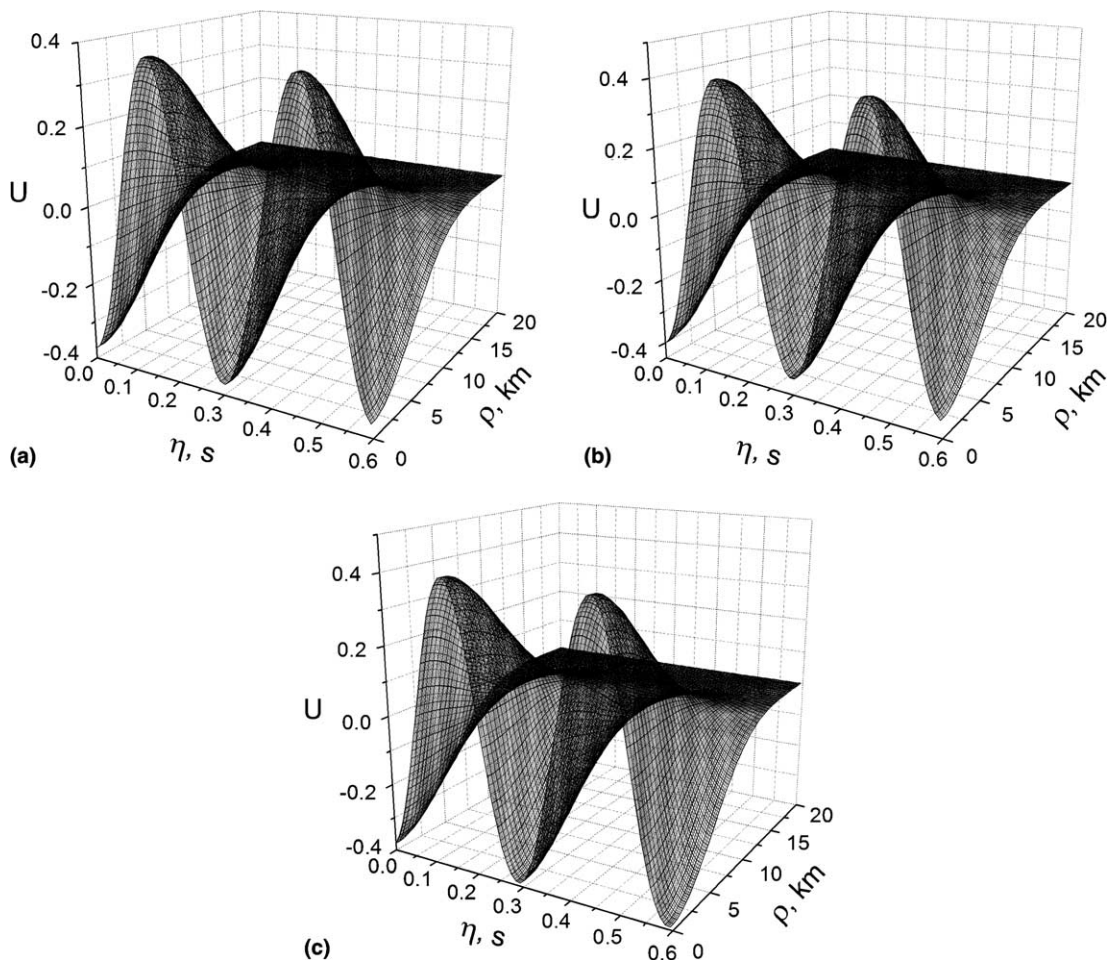


Fig. 4. Evolution of low frequency component: (a) is at $z = 5$ km, (b) $z = 10$ km; (c) $z = 30$ km. $\rho_0 = 10$ km; $G = 0.25$ km⁻¹.

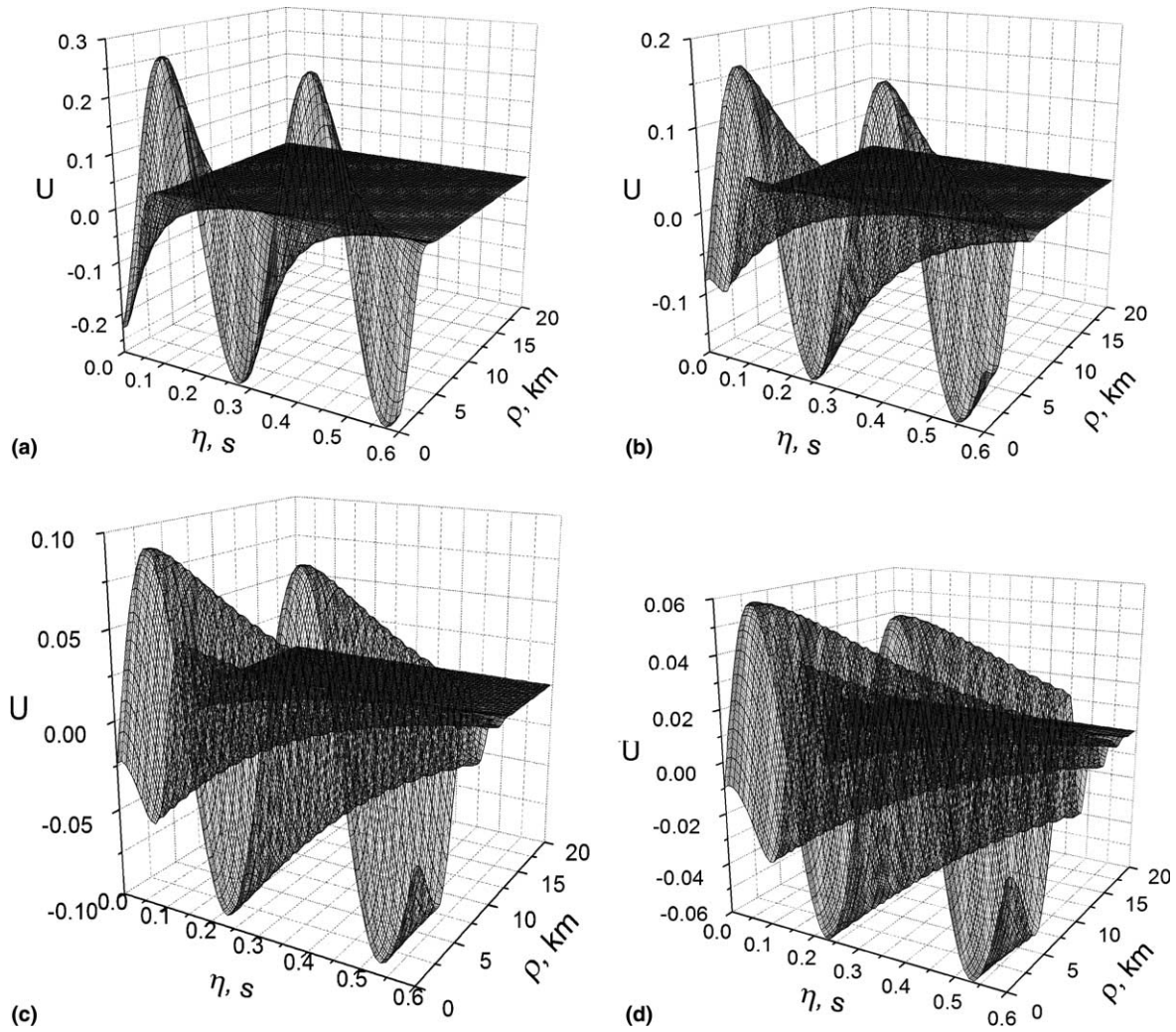


Fig. 5. Evolution of low frequency component: (a) is at $z = 5$ km, (b) $z = 10$ km; (c) $z = 20$ km; (d) $z = 30$ km. $\rho_0 = 2$ km; $G = 0.25$ km⁻¹.

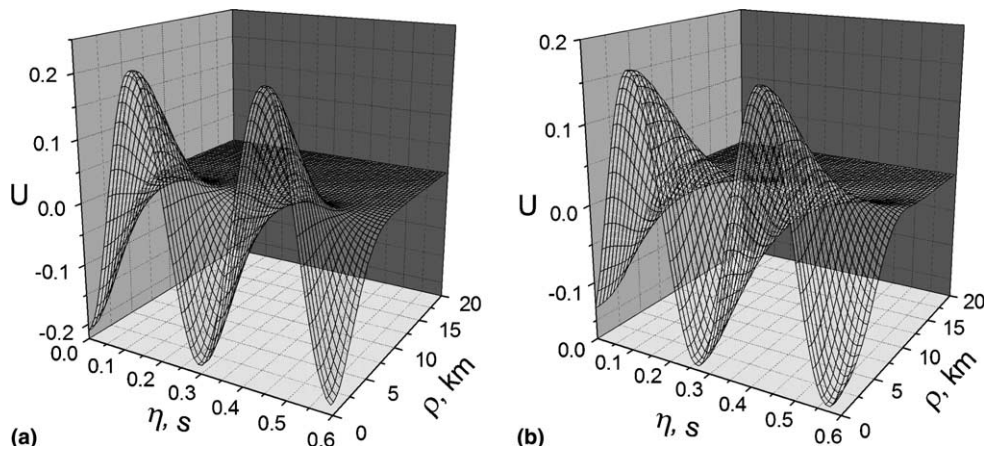


Fig. 6. Evolution of low frequency component: (a) is at $z = 5$ km, (b) $z = 30$ km. $\rho_0 = 6$ km; $G = 0.5$ km⁻¹.

simulation confirms this assumption above (see results in Figs. 3–9). For example, we can see in Fig. 3 the

evolution of the low frequency part as described by Eq. (8). The initial burst has the profile described in Fig. 3.

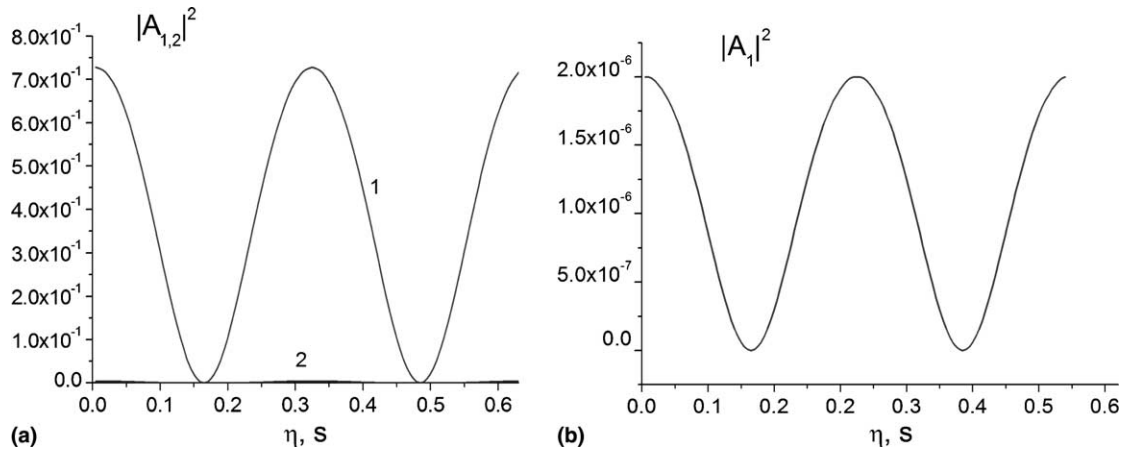


Fig. 7. Evolution of the amplitudes of high frequency component $|A_{1,2}(\rho = 0)|^2$: (curve 1 is $|A_1(\rho = 0)|^2$, curve 2 is $|A_2(\rho = 0)|^2$); (a) $z = 5$ km; (b) $z = 30$ km. $G = 0.25 \text{ km}^{-1}$.

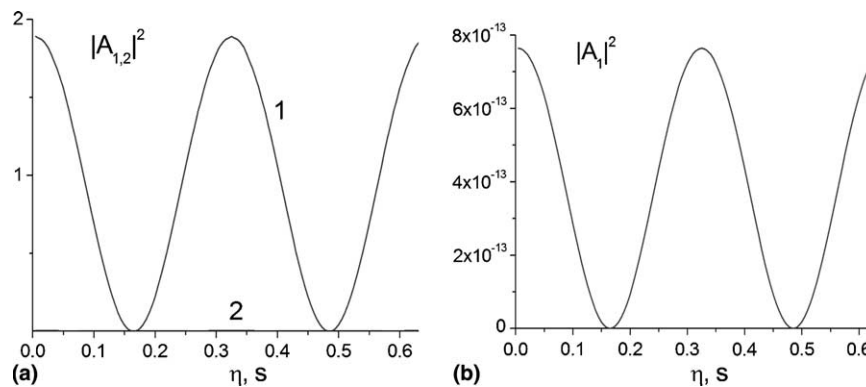


Fig. 8. Evolution of the amplitudes of high frequency component $|A_{1,2}(\rho = 0)|^2$: (a) is at $z = 1.5$ km (curve 1 is $|A_1(\rho = 0)|^2$, curve 2 is $|A_2(\rho = 0)|^2$); (b) $z = 30$ km. $G = 0.5 \text{ km}^{-1}$.

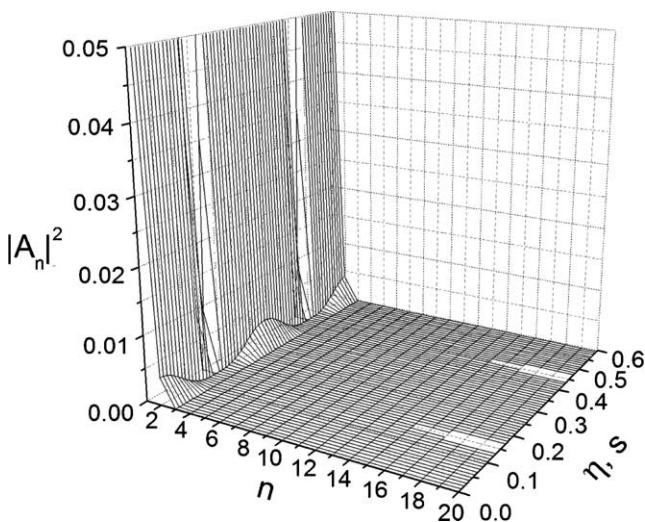


Fig. 9. Distribution of the amplitudes of high frequency component $|A_n(\rho = 0)|^2$ at $z = 5$ km; (d) $z = 30$ km. $G = 0.25 \text{ km}^{-1}$ (it corresponds to the Fig. 5c).

The Figs. 2–8 illustrate that only ULF waves can go through the Earth without big losses and reach the

surface of the Earth. But the diffraction is essential for this burst (compare the bursts in Figs. 3 and 5). Although the coefficients of the nonlinearity and viscosity are the same, the diffraction is essential in the case of Fig. 3. And the burst gas is essential for the deformation of the cylinder.

The HF part (see last figures) get damped very fast. But the diffraction is essential for ULF parts. This is clearly shown in Fig. 4. At the surface of the Earth we see a large diffraction of the waves (impulses in Fig. 5d). Finally, in Fig. 6, it is shown that with that, with the increase of viscosity, the LF and ULF waves decrease at the Earth's surface. This takes place, of course, if the medium has this particular coefficient of viscosity.

4. Conclusion

The modeling of the nonlinear passage of an acoustic waves caused by underground fracturing through the lithosphere shows that there is an effective nonlinear interaction of the burst in the lithosphere. First of all, we

have low frequency (LF) and ultra low (ULF) acoustic waves and an efficient nonlinear process for these waves to be excited. The diffraction increases the value of impulse, but losses of these waves are small. High frequency (HF) waves are excited too, but they present large losses and, at the surface of the Earth, are so small that they cannot be detected at the Earth's surface. Therefore, we have shown that there is a transformation of a seismic burst into LF and ULF acoustic waves, as it passes through the lithosphere, so that it is possible to have the record in a seismograph. The nonlinear interaction of the acoustic burst additionally shows the importance of the acoustic channel as an example of the nonlinear mechanism of energy flow from the lithosphere into the ionosphere.

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