On the estimation of elastic stresses in the mantle at the time moment of a large meteorite fall

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[1] Simple relations are obtained for estimating stresses in the mantle at the time moment of a large meteorite fall from values of the velocity and mass of the meteorite and the sizes of its crater. It is shown that the fall of the Chicxulub meteorite, the largest over the last 100 Myr, produced stresses of the order of 0.1–1 MPa. *INDEX TERMS:* 8164 Tectonophysics: Stresses: crust and lithosphere; 8168 Tectonophysics: Stresses: general; 1236 Geodesy and Gravity: Rheology of the lithosphere and mantle; *KEYWORDS:* stresses in the mantle, geodynamics.

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[2] *Jones et al.,* [2002] proposed a hypothesis according to which the fall of a large meteorite (such as the Chicxulub meteorite 10–16 km in diameter, which fell 65 Myr ago at a velocity of about 11 km s^{-1} and produced a crater 180–300 km in size) can activate convection in the lower mantle and provoke the formation of plumes in the D$''$ layer. An attempt to estimate the order of magnitude of elastic stresses arising at the time moments of such events is made in the given paper.

[3] The main uncertainty involved in the estimation of elastic stresses is related to the question of which part of the kinetic energy of a meteorite is converted into heat and which part, into the energy of seismic vibrations. It is natural to divide the fall process into two phases: (1) the phase of inelastic impact (accompanied by the fracture of material in a region comparable in size with the crater) and (2) the phase of elastic interaction of the resulting fragments with the mantle.

[4] As follows from the law of conservation of momentum, the average velocity of fragments in the first phase is determined by the relation

$$\mathbf{v}_2 = \frac{m_1}{m_2}\mathbf{v}_1 \sim \frac{r_1^3}{r_2^3}\mathbf{v}_1 \ , \tag{1}$$

where (m_1, r_1) and (m_2, r_2) are the mass and average radius of the meteorite and the crater, respectively, and $\mathbf{v}_1$ is the velocity of the meteorite fall. Accordingly, the ration of the total kinetic energy of fragments to the initial energy of the meteorite is

$$\eta = \frac{m_2 v_2^2}{m_1 v_1^2} = m_1/m_2 \sim (r_1/r_2)^3 \ . \tag{2}$$

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[5] The phase of elastic interaction of fragments with the mantle outside the crater can be described by the elastic energy balance

$$\frac{\partial \varepsilon}{\partial t} = \mathrm{div}\mathbf{S} \ , \tag{3}$$

where

$$\varepsilon = \frac{\lambda}{2}(\nabla \cdot \mathbf{u})^2 + \mu e_{ik} e_{ik} \tag{4}$$

is the volume density of elastic energy, λ and μ are the Lamé coefficients, $\mathbf{u}$ is the displacement vector, e_{ik} is the stress tensor, and

$$S_k = \lambda(\nabla \cdot \mathbf{u})\mu + 2\mu e_{ik} \tag{5}$$

is the kth component of the Poynting vector in a Cartesian coordinate system (summation over repeated indices is assumed in (4) and (5)). Neglecting the depth dependence of the elastic moduli, we may assume that the elastic energy in mantle is concentrated in a spherical layer centered at the impact point and bounded by spheres of radii R_1 and R_2; the thickness of the layer $R_2 - R_1$ is determined by (i) the finite time of the interaction of fragments with the lower boundary of the crater and (ii) the diffusion of the wavefront associated with the dispersion of seismic wave velocities. In the case of body waves, the velocity dispersion is only due to the inelasticity of the medium (the velocities depend on the period of vibrations and are independent of the wavelength). Given the quality factor values characteristic of the Earth's mantle ($Q_\mu \sim 10^2 - 10^3$), the velocity dispersion is $\delta V_p/V_p \sim \delta V_s/V_s \sim 10^{-2}$, and the value of the wavefront dispersion due to the velocity dispersion is of the order of $\delta R \sim R_1 \delta V_{(p,s)}/V_{(p,s)} \sim 10^{-2} R_1$. It is easy to show that this effect is negligible compared to the first effect. To estimate the finite time of the interaction of fragments with the inner boundary of the crater δt, note that elastic strains at the lower boundary of the crater assume have ultimate (failure) values typical of rocks: $e_{\max} \sim 10^{-3} \cdot 10^{-4}$. In accordance with the law of conservation of momentum, we have for the interaction process of fragments with the inner boundary of the crater:

$$F\delta t \sim m_2 v_2 = m_1 v_1 \ , \tag{6}$$

where the average interaction force is of the order of

$$F \sim \mu e_{\max} r_2^2 \ . \tag{7}$$

Hence, we have $\delta t \sim m_1 v_1/(\mu e_{\max} r_2^2)$ and

$$R_2 - R_1 \sim V_{(p,s)}\delta t \sim V_{p,s} m_1 v_1/(\mu e_{\max} r_2^2) \ . \tag{8}$$

[6] In accordance with the equation of the energy balance in the second phase of the process, the stresses in the wave-front region e_{fr} are determined by the relation

$$\delta V \mu e_{fr}^2/2 = \eta \ m_1 v_1^2/2 \ , \tag{9}$$

where

$$\delta V \sim 2\pi R_1^2 (R_2 - R_1) \sim 2\pi R_1^2 V_{p,s} m_1 v_1/(\mu e_{\max} r_2^2) \tag{10}$$

is the volume of the spherical layer in which the energy of elastic vibrations is concentrated. Accordingly, the maximum stresses in the mantle at the distance R_1 from the center of the crater can be estimated as

$$\begin{aligned} \mu e_{fr} &\sim (\eta \ m_1 v_1^2/(\mu \delta V))^{1/2} \sim \mu r_2 (\eta v_1 e_{\max}/(2\pi R_1^2 V_{p,s}))^{1/2} \\ &\sim \mu r_1^{3/2} r_2^{-1/2} R_1^{-1} (v_1 e_{\max}/(2\pi V_{p,s}))^{1/2} \ . \end{aligned} \tag{11}$$

Setting $R_1 = r_2$ in this formula and equating the result to the known value of elastic stresses at the lower boundary of the crater $\mu e_{\max}$, we obtain

$$r_1^{3/2} r_2^{-3/2} (v_1/(2\pi V_{p,s} e_{\max}))^{1/2} \sim 1 \ , \tag{12}$$

which yields

$$r_2 \sim r_1 (v_1/(2\pi V_{p,s} e_{\max}))^{1/3} \ .$$

With characteristic values of ultimate tangential stresses of rocks of the order of $10^{-3} - 10^{-4}$ and $v_1 \sim \pi V_{p,s} \sim 10$ km s^{-1}, this formula yields the estimate $r_2 \sim (10 - 25) r_1$ relating the sizes of a meteorite and its crater and agreeing well with data on the Chicxulub meteorite. To obtain numerical estimates of stresses that developed in the D$''$ layer at the time moment of the Chicxulub meteorite fall, one can use either formula (11) giving $r_2/r_1 \sim (10 - 25)$ or the aforementioned geological data according to which $r_2/r_1 \sim (180 - 300)$ km/(10–16) km$\sim$ 10–30. Given $e_{\max} \sim 10^{-3} - 10^{-4}$ and $\mu \sim 10^{11}$ dyn cm^{-2}, we obtain in both cases $\mu e_{fr} \sim 10^6 - 10^7$ dyn cm^{-2} = 0.1–1 MPa.

[7] This value is about an order of magnitude smaller than the tectonic stresses developing in the crust and upper mantle before an earthquake.

References

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