A quantitative model for the time-size distribution of eruptions

Warner Marzocchi[1] and Lucia Zaccarelli[1]

Received 2 March 2005; revised 13 January 2006; accepted 26 January 2006; published 28 April 2006.

[1] The modeling of the statistical distribution of eruptive frequency and volume provides basic information to assess volcanic hazard and to constrain the physics of the eruptive process. We analyze eruption catalogs from volcanoes worldwide in order to find "universal" relationships and peculiarities linked to different eruptive styles. In particular, we test (1) the Poisson process hypothesis in the time domain, looking for significant clustering of events or the presence of almost regular recurrence times, (2) the relationship between the time to the next eruption and the size of the previous event (the "time predictable" model), and (3) the relationship between the size of an event and the previous repose time (the "size predictable" model). The results indicate different behavior for volcanoes with "open" conduit regimes compared to those with "closed" conduit regimes. Open conduit systems follow a time predictable model, with a marked time clustering of events; closed conduit systems have no significant tendency toward a size or a time predictable model, and the eruptions follow mostly a Poisson distribution. These results are used to build general probabilistic models for volcanic hazard assessment of open and closed conduit systems.

Citation: Marzocchi, W., and L. Zaccarelli (2006), A quantitative model for the time-size distribution of eruptions, *J. Geophys. Res.*, *111*, B04204, doi:10.1029/2005JB003709.

1. Introduction

[2] The statistical modeling of the time-size distribution of volcanic eruptions is a fundamental tool to understand better the physics of the eruptive process, and to make reliable forecasts [*Newhall and Hoblitt*, 2002; *Connor et al.*, 2003; *Marzocchi et al.*, 2004a; *Sparks and Aspinall*, 2004]. Eruption forecasting is commonly associated to different timescales (short-, intermediate-, and long-term; see definition by *Newhall and Hoblitt* [2002]). Regardless of the time frame, the statistical modeling of the past behavior of a volcano is a key ingredient for quantitative forecasting (usually, but not necessarily, over long time intervals) when the volcano has an almost stationary state (for instance, it is dormant). In this case, monitoring data are not particularly informative of the future evolution of the system, at least until the volcano becomes restless and/or changes its stationary state. Hereinafter, the terms "eruption forecasting" and "volcanic hazard" refer to this stationary case.

[3] The main difficulties in providing a general model of eruptive activity are linked to the existence of different types of volcanic activity, to the paucity of eruptive data for most volcanoes, and to the intrinsic complexity of eruptive processes. As a consequence, most of the past papers devoted to this issue are focused on single (or very few) volcanoes [e.g., *Wickman*, 1976; *Klein*, 1982; *Burt et al.*, 1994; *Bebbington and Lai*, 1996; *Marzocchi*, 1996; *Connor et al.*, 2003; *Gusev et al.*, 2003; *Sandri et al.*, 2005] where detailed eruptive catalogs exist. This approach limits the generality of the results. We cannot know if the behavior of the volcano analyzed represents a generic feature of a specific type of volcanism, or if it is peculiar of the volcano itself. Under this perspective, part of the different statistical distributions found by analyzing single eruptive catalogs can be explained by the existence of some peculiarities in volcanic activity.

[4] One way to overcome this drawback, which we use here, is to perform a common analysis on data from several volcanoes. In particular, we test the Poisson hypothesis in the time domain, and the reliability of time-size distributions such as the time predictable model and size predictable model. The results obtained are then used to build a quantitative model of the statistical time-size distribution for some classes of volcanic activities that can be used for volcanic hazard assessment.

2. Data Sets

2.1. Eruptive Catalogs and Variables for the Statistical Analyses

[5] In order to achieve our goal, we analyze different volcanic data sets that potentially may include different classes of eruptive activities. In particular, we consider two data sets extracted from the chronology of worldwide eruptions reported in Volcanoes of the World [*Simkin and Siebert*, 1994], updated until the end of 2003 (L. Siebert, personal communication, 2004). Then, we consider the eruptive catalogs of few volcanoes that report well documented sequences of eruptive activity, Mount Etna, Vesuvius, Kilauea, and Piton de la Fournaise.

[1]Istituto Nazionale di Geofisica e Vulcanologia, Bologna, Italy.

Copyright 2006 by the American Geophysical Union.
0148-0227/06/2005JB003709$09.00

Table 1. N4VEI4 Catalog[a]

Volcano	Latitude	Longitude	Number of Eruptions	μ_{IET}, years
Campi Flegrei	40.827 N	14.426 E	4	2068
Vesuvio[b]	40.821 N	14.426 E	7	1307
Okataina	38.120 S	176.500 E	6	1694
Taupo	38.820 S	176.000 E	15	716
Raoul Island[b]	29.270 S	177.920 W	9	465
Rabaul	4.271 S	152.203 E	4	1181
Kelut[b]	7.930 S	112.308 E	6	81
Taal[b]	14.002 N	120.993 E	4	83
Suwanose-Jima[b]	29.530 N	129.720 E	4	96
Ibusuki volcanic field[b]	31.220 N	130.570 E	15	255
Sakura-Jima[b]	31.580 N	130.670 E	7	1511
Fuji	35.350 N	138.730 E	5	690
Asama	36.400 N	138.530 E	4	1445
Towada	40.470 N	140.920 E	6	1694
Oshima[b]	34.730 N	139.380 E	6	160
Komaga-Take[b]	42.070 N	140.680 E	4	96
Usu[b]	42.530 N	140.830 E	4	63
Shikotsu	42.700 N	141.333 E	4	2898
Ksudach[b]	51.800 N	157.530 E	5	589
Tolbachik[b]	55.830 N	160.330 E	6	385
Bezymianny	55.978 N	160.587 E	4	802
Shiveluch[b]	56.653 N	161.360 E	5	375
Augustine[b]	59.370 N	153.420 W	4	149
St. Helens[b]	46.200 N	122.180 W	8	617
Colima volcanic complex[b]	19.514 N	103.620 W	7	714
Orizaba, pico	19.030 N	97.268 W	5	2006
Fuego[b]	14.473 N	90.880 W	7	65
Bravo, cerro[b]	5.092 N	75.300 W	8	858
Ruiz	4.895 N	75.323 W	4	815
Cotopaxi[b]	0.677 S	78.436 W	4	114
Pelee[b]	14.820 N	61.170 W	18	537
Katla[b]	63.630 N	19.050 W	11	98
Hekla[b]	63.980 N	19.700 W	13	583
Unnamed	37.870 N	25.780 W	4	1499
Furnas	37.770 N	25.320 W	4	860

[a]Volcano name, latitude, longitude, number of eruptions with VEI $\geq$ 4, and average of IETs.
[b]Volcanoes belonging also to the data set N4VEI4§.

[6] The first catalog (N4VEI4; see Table 1) contains the chronology (volcano, start date, and VEI) of 231 eruptions with VEI $\geq$ 4 from the 35 volcanoes that experienced at least four such events in the time interval 10,000 B.C. to 2003 A.D. The second catalog (N4VEI2; see Table 2) contains the chronology (volcano, date, and VEI) of 520 eruptions with VEI $\geq$ 2 from 20 volcanoes that experienced at least four such events, and for which it is possible to estimate a feasible time-size window for the completeness of the sequences. The time-size window of completeness for each volcano is based on subjective choices (L. Newhall, personal communication, 2005) taking into account possible lack of small eruptions and/or the history of European colonization. Later, we will check quantitatively this hypothesis, as well as the completeness of each data set used. The Etna catalog (ETNA) is taken from *Behncke et al.* [2005]. For all the 40 eruptions in the time interval 1950–2005, the catalog reports date, duration, eruptive vent (the spatial point where the magma comes out), type (flank or summit eruption), and volume of the lava erupted. Actually, the catalog published by the authors starts before, but before the fifties the frequency of eruptions is much smaller, denoting an incompleteness of the catalog or a marked change in the eruptive style [*Behncke et al.*, 2005]. In order to avoid possible bias due to these factors (see also section 2.2), we analyze only the catalog after the fifties, where the frequency of events is almost constant. The Vesuvius catalog (VES) is taken from *Scandone et al.* [1993], and reports the chronology of eruptions since 1631 with associated VEI. The Kilauea catalog (KIL) has been taken from the Web page http://hvo.wr.usgs.gov/kilauea/history/historytable.html (modified from *Macdonald et al.* [1986]. It contains dates, durations, and volumes of eruptions since 1918. The Piton de la Fournaise catalog (PdlF) consists of 72 interevent times (defined as the time intervals between the onset of consecutive eruptions) and the duration of the relative eruptions given by *Sornette et al.* [1991].

[7] From each catalog, we extract a data set consisting of four variables, namely τ_i, $t_i^\star$, $v_i^{(1)}$, and $v_i^{(2)}$, where $i = 1, .., M$, $M = N - 1$, and N is the number of eruptions in the catalog. The random variable τ is the interevent time (IET hereinafter) between eruptions divided by the average of all IETs (μ_{IET}) for the same volcano. This is a typical normalization for exponentially distributed families [*Cox and Lewis*, 1966], that is necessary to merge IETs coming from volcanoes with different eruptive rates as in N4VEI2 and N4VEI4. In practice, this transformation leads to an eruptive rate equal to one for each volcano ($\mu_\tau = 1$). For N4VEI2, N4VEI4, and KIL, each IET is measured between the onset of consecutive eruptions, because it is usually the most accurately reported and available estimation of the time of occurrence of a volcanic eruption given by these catalogs. Despite the Web page where we take the data for Kilauea lists also the duration of the eruptions, we do not consider them because of the lack of their homogeneity; for instance, for the two longest eruptions before 1980 (occurred in 1972 and 1969), the catalog reports durations that are associated with a lava lake activity rather than to represent real duration of eruptions like for the other events in the same catalog. The choice to consider the onset implicitly leads to the assumption that

Table 2. N4VEI2 Catalog[a]

Volcano	Latitude	Longitude	Start of Catalog	Number of Eruptions	μ_{IET}, years
Rabaul	4.271 S	152.203 E	1875	7	20.8
Kelut	7.930 S	112.308 E	1700	18	16.1
Taal	14.002 N	120.993 E	1600	28	13.7
Suwanose-Jima	29.530 N	129.720 E	1800	20	9.9
Sakura-Jima	31.580 N	130.670 E	1600	24	13.6
Fuji	35.350 N	138.730 E	800	14	69.9
Asama	36.400 N	138.530 E	1500	107	4.4
Oshima	34.730 N	139.380 E	1600	24	17.0
Komaga-Take	42.070 N	140.680 E	1850	10	15.8
Usu	42.530 N	140.830 E	1850	5	36.8
Shikotsu	42.700 N	141.333 E	1850	17	5.2
Tolbachik	55.830 N	160.330 E	1900	20	3.7
Bezymianny	55.978 N	160.587 E	1950	37	1.3
Shiveluch	56.653 N	161.360 E	1950	11	3.5
Colima v.compl.	19.514 N	103.620 W	1700	29	10.2
Fuego	14.473 N	90.880 W	1550	62	7.4
Ruiz	4.895 N	75.323 W	1700	9	22.6
Cotopaxi	0.677 S	78.436 W	1700	54	3.8
Katla	63.630 N	19.050 W	1500	9	42.3
Hekla	63.980 N	19.700 W	1500	15	35.0

[a]Volcano name, latitude and longitude, start year of complete eruption catalog, number of eruptions, and average of IETs.

the erupted volume is associated to the onset of the eruption, or, in other words, that most of the volume is expected to be erupted at a time interval from the onset that is much shorter than the characteristic IET. Note that this seems reasonable, overall for explosive eruptions. For ETNA, VES and PdlF, the catalogs reports also the duration of each eruption, therefore for them we define a IET as the time from the onset of an eruption and the midpoint between the start and the end of the previous eruption.

[8] The random variable $t^\star$ is the time elapsed since the present time to the end of each τ ($t^\star$ increases backward in time), and it is necessary to give a chronological order to the sequence τ_i. In other words, τ_i precedes τ_j in the sequence if $t_i^\star$ is larger than $t_j^\star$. The random variables $v^{(1)}$ and $v^{(2)}$ represent an estimation of the size of the eruptions at the beginning and at the end of each τ, respectively. These variables will be used in the next sections to check possible relationship between IETs and size of the previous/following eruption. In particular, $v^{(1)}$ and $v^{(2)}$ are the logarithm of the erupted volume where available, otherwise we consider the Volcanic Explosivity Index (VEI [*Newhall and Self*, 1982]) that is a widely used classification scheme to describe the size of explosive eruptions. VEI uses an integer scale from 0 to 8 to describe both the volume and plume height of any given eruption. This index is based on both the magnitude (erupted volume) and intensity (eruption column height) information. In practice, for old eruptions VEI is estimated principally on the erupted mass or volume of the deposits. Such an index is rather rough, but it is commonly used as a proxy of the order of magnitude of the erupted volume of the explosive volcanic event. In the following, we also use the symbol V to indicate the volume of erupted material.

2.2. Completeness of the Catalogs

[9] In order to extract unbiased information from a catalog we need to check its completeness. This issue is well known in seismology where the completeness of the catalog is checked mainly by analyzing the Gutenberg-Richter law, and the time evolution of the rate of occurrence of events (λ hereinafter). As regards the first point, it is usually assumed that the magnitude of the events follows a power law distribution (the Gutenberg-Richter law); in this case, the incompleteness of a seismic catalog produces a bending on the Gutenberg-Richter law caused by the omission (a lower number compared to the expected one) of small magnitude events. For what concerns the second point, the earthquake generating process is assumed to be stationary over the time period investigated, therefore λ (in this case, λ is the rate of main shocks because aftershocks are usually removed) has to be almost constant. Here, the incompleteness of a seismic catalog is revealed by a time variation of λ, with higher rates for more recent time intervals.

[10] In volcanology, the incompleteness of a catalog may show similar features, but, conversely, these features cannot be necessarily associated to an incompleteness of the catalog, because the two assumptions reported above (i.e., a well known power law distribution for the magnitude of events, and a constant λ) are much more questionable than in seismology. First, a power law distribution for the magnitude of eruptions of a single volcano is yet to be proved. Until now, there is only some indication that a general power law can hold for global catalogs containing several volcanoes [*Simkin and Siebert*, 1994]. Second, we know that many explosive volcanoes have two distinct regimes, i.e., open and closed conduit regimes, characterized by very different λ. For instance, Vesuvius in the closed conduit regime experienced repose times that lasted centuries, while it was characterized by an average λ of 0.3 event per year during an open conduit regime (in the time period 1631–1944). This behavior is certainly nonstationary (i.e., there is a significant variation of λ), but it is not linked to any incompleteness of the catalog. In other words, a stationary behavior may be considered a sufficient, but not a necessary condition for the completeness of an eruptive catalog.

[11] From a quantitative point of view, for a general stochastic process we can write [cf. *Coles and Sparks*, 2006]

$$\lambda_{obs}(t, \Delta t, V_{min}) = \xi(t, V_{min}) \cdot \lambda_{true}(t, \Delta t, V_{min}), \qquad (1)$$

where λ_{obs} is the expected number of eruption per unit time that we estimate from a catalog, t is the absolute time, Δt is the elapsed time since the previous event, V_{min} is a measure representing the minimum size of eruption reported in the catalog, ξ is a function defined in the interval [0, 1] that mimics the completeness of the observations, and λ_{true} is the real value of the expected eruption rate. The dependency on $\Delta\ t$ mimics general renewal processes [*Cox and Lewis*, 1966]; for the following discussion this dependency is irrelevant and it will be dropped. In equation (1), if the catalog is complete we have $\xi(t, V_{min}) = 1$. If underreporting (i.e., the omission of events due to underrecording of historical data) is uniform for t and V_{min}, then $\xi(t, V_{min}) = k$ $(0 < k < 1)$, that means that λ_{obs} is a fraction (usually unknown) of the real value. In reality, historical incompleteness emerges through an increasing underreporting of small events backward in time. From a statistical point of view, this means that $\xi(t, V_{min})$ is usually an increasing function of t for each fixed value of V_{min} [*Coles and Sparks*, 2006].

[12] The study of incompleteness can be approached in different ways; since we only estimate directly the variable $\lambda_{obs}(t, V_{min})$, we can try to make inference on $\xi(t, V_{min})$ or $\lambda_{true}(t, V_{min})$ by doing some assumptions on the behavior of $\lambda_{true}(t, V_{min})$ or $\xi(t, V_{min})$, respectively. For instance, *Coles and Sparks* [2006] assume a Poisson process for $\lambda_{true}(t, V_{min})$, in order to make inferences on the specific form of $\xi(t, V_{min})$. Here, since the estimation of the form of $\lambda_{true}(t, V_{min})$ (and of its related probability function) is the ultimate goal of this paper, we prefer to follow a different strategy. In particular, we check the completeness of $\lambda_{obs}(t,V_{min})$, through some mild assumptions on the behavior of $\xi(t, V_{min})$ and $\lambda_{true}(t, V_{min})$; as mentioned before and in accordance with *Coles and Sparks* [2006], we assume that underreporting is characterized by a nonstationary process $\xi(t, V_{min})$ that increases as a function of t for a fixed V_{min}, and that $\lambda_{true}(t, V_{min}) \equiv \lambda_{true}(V_{min})$, i.e., the real (unknown) process is stationary (not necessarily Poisson). These assumptions imply that a specific nonstationarity of the observed sequence of events (i.e., an increasing $\lambda_{obs}(t, V_{min})$ with time) is usually indicative of an incomplete catalog. As final consideration, we

Table 3. Regression Analysis for the Completeness of the Catalogs

Data Set	b	t Test α	Runs Test α	Sign Test α
ETNA	−0.20 ± 0.09	0.03	0.79	>0.10
VES	−0.02 ± 0.02	0.21	0.43	0.62
KIL	−0.06 ± 0.06	0.16	0.24	>0.10
PdlF	−0.09 ± 0.03	<0.01	0.02	0.13
PdlF§	−0.2 ± 0.2	0.15	0.54	>0.10
N4VEI2	−0.001 ± 0.002	0.32	0.49	0.15
N4VEI4	−0.013 ± 0.005	<0.01	0.72	0.73
N4VEI4§	0.01 ± 0.01	0.75	0.58	0.06

remind once more that the second assumption relative to stationarity of the real process $\lambda_{\text{true}}(t, V_{\min})$ is maybe more questionable respect to seismology. This leads to a conservative completeness analysis, where a specific nonstationarity of the observed process (i.e., an increasing trend of λ_{obs}) is always ascribed to underreporting.

[13] A strategy to check the nonstationarity of a time series has been proposed by *Cox and Lewis* [1966], by analyzing λ_{obs} through the analysis of variance of standard regression [e.g., *Draper and Smith*, 1998]. We proceed as follows: let y_1 be the sum of the first ℓ IETs (i.e., $y_1 = \sum_{j=1}^{\ell} \tau_j$), y_2 the sum of the second group of ℓ IETs ($y_2 = \sum_{j=\ell+1}^{2\ell} \tau_j$), and so on. Here ℓ is an integer such that no appreciable change in λ_{obs} arises in any set of ℓ IETs; an optimal choice is $\ell = 4$ [cf. *Cox and Lewis*, 1966]. Note that it is necessary to omit a few events (preferably from the center of the data set) if the total number M of IETs is not a multiple of ℓ. We thus obtain a series of intervals $y_1, y_2, \ldots, y_L$, where L is the integer part of M/ℓ. Suppose that the underreporting of the catalog induces a λ_{obs} slowly increasing with time. We make the approximation that λ is effectively a constant λ_i within the period covered by y_i and that an independent variable z_i can be attached to each y_i such that

$$\log \lambda_i = a + b z_i \qquad (i = 1, \ldots, L), \tag{2}$$

where

$$z_i = i - \frac{\sum_{j=1}^{L} j}{L}. \tag{3}$$

In other words, z_i is the index of the sequence at which we subtract the average of all values of z. In practice, z_i mimics the time elapsed since the present time and it increases backward in time [*Cox and Stuart*, 1955]. We anticipate that this definition of z_i is not particularly critical here. In fact, all the results reported below are stable if we define z_i as the time from the present to the center of y_i.

[14] Now, we have the linear model

$$\log y_i = -(a' + b z_i) + \varepsilon_i, \tag{4}$$

where $a' = a - e_\ell$ and b are unknown parameters, $e_\ell = 1 + \{1/2\} + \ldots + 1/(\ell - 1) - C$, C is Euler's constant, and ε is a random variable that follows a Gauss distribution with zero mean and variance σ_ε^2. Note that the application of the log transformation to the intervals (see equation (4)) serves two purposes: first, it ensures a constant variance, and, second, it introduces an exponential relation which is the most "natural" simple relationship for rates of occurrence. In the presence of a more complex trend there would be a correlation between different log (y_i) values, but as long as these are not too large, we should get reasonable answers from the regression analysis. Therefore we have a dependent variable with independently distributed successive values, constant variance and a moderately nonnormal probability distribution. Consequently, the properties associated with the first and the second moment of the regression analysis hold exactly and the usual t and F tests are valid to a good approximation.

[15] From this model we can (1) obtain the standard least squares estimates of a' and b; (2) test approximately the null hypothesis H_0: $b \geq 0$ (the underreporting is characterized by $b < 0$) by standard regression methods; (3) test the validity of the model by analyzing the statistical distribution of the residuals. As regards the last point, a reliable application of the model of equation (4) demands, as basic requirements, that the residuals of the model are independently distributed with a constant variance [*Draper and Smith*, 1998]. In this context, we check if the residuals are autocorrelated, i.e., if they tend to occur as "clumps" of adjacent deviations on the same side of the regression line or to have some significant trend through a nonparametric runs test [*Gibbons*, 1971]. The hypothesis of constant variance is checked by examining if the residuals form an approximately uniform band around the regression line; we use a sign test [*Cox and Stuart*, 1955] to ascertain whether the scatter about the regression curve changes as the independent variable increases. To summarize, a reliable model requires that the two null hypotheses (no residuals correlation, and constant variance) are not rejected.

[16] In Table 3 and in Figure 1, we report the results of the regression analysis. As a general result, we can see from Table 3 that only N4VEI4 and PdlF catalogs show a significant nonstationarity with $b < 0$ (at a significance level < 0.01). The trend could be due to an incompleteness of the catalogs. Another possible explanation for N4VEI4 is that the trend may be due to the presence of a larger number of volcanoes in a closed conduit regimes in the past (see, for instance, Usu, Tarumai-Skikotsu and Komaga-Take volcanoes). For PdlF, instead, the trend found could be also due to some real change in the eruptive activity. In order to check the stability of the results obtained by the following analyses, we also use subset of these catalogs. For N4VEI4 we consider a subset composed by volcanoes that have at least 4 eruptions with VEI $\geq$ 4 in the last 2,000 years. This catalog, called N4VEI4§, contains 123 eruptions from 22 volcanoes. For PdlF we consider a subset (PdlF§) composed by the last 31 IETs of the catalog. For both subsets, the regression analysis (see Table 3 and Figure 1) does not show any significant trend (i.e., λ_{obs} is constant). In Table 3, we also report the results of the residuals analysis for all data sets that confirms the validity of the model described by equation (4).

3. Data Analysis and Results

[17] The analyses carried out on the data sets consist of three steps: (1) test of the Poisson process in the time

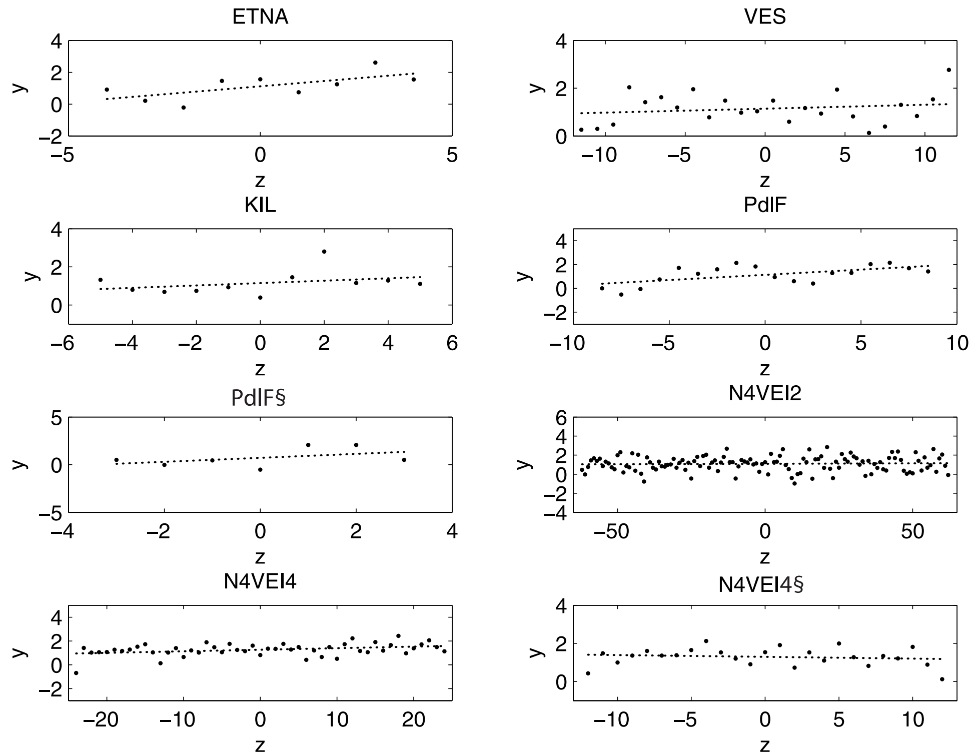

Figure 1. Regression analysis to check the completeness of the data sets. The abscissa reports the variable z_i (see equation (3)) that mimics the time elapsed since the present. The ordinate reports the variable y_i that represents a grouping of IETs. See text for more details.

domain; (2) test of the time predictable model (TPM); and (3) test of the size predictable model (SPM). The rationale and the physical/volcanologic meaning of these choices are discussed in depth in sections 3.1, –3.4 together with the results of the analyses.

3.1. Test of the Poisson Process in the Time Domain

[18] The analysis in the time domain gives information on the characteristics of the marginal distribution $f(\tau) = \int f(\tau, V)\, dV$. In particular, in order to verify the nonrandom distribution of eruptive cycles and better quantify their recurrence time, we define a coefficient of variation, η, given by

$$\eta = \frac{\sigma}{\mu_\tau}, \qquad (5)$$

where μ_τ and σ are, respectively, the average and standard deviation of τ. Note that, in our case, $\mu_\tau = 1$ because of the normalization described before, therefore $\eta \equiv \sigma$. The coefficient η indicates if and how the statistical distribution of τ differs from a Poisson process. For a Poisson process (i.e., exponential distribution) $\eta = 1$, while $\eta > 1$ characterizes statistical distributions more clustered than an exponential one, and $\eta < 1$ is typical for more "regular" time occurrence [cf. *Cox and Lewis*, 1966]; for instance, $\eta = 0$ indicates a perfect periodicity of the process, τ being a constant.

[19] The Poisson hypothesis is tested by comparing η of a real catalog with the ones obtained from 1000 synthetic catalogs generated by assuming that the eruptive activity is governed by a Poisson process (i.e., $\eta = 1$). In particular, for each volcano of the N4VEI2, N4VEI4, and N4VEI4§ data sets, we calculate η_i from N_i eruptions ($i = 1, .., K$, where K is the number of volcanoes in the data set). Then, we calculate $\eta_i^{(s)}$ for each synthetic catalog (where s stands for the sth synthetic catalog) composed by the same number of volcanoes (K) and the same number of eruptions for each volcano (N_i; $i = 1, \ldots, K$). For these catalogs, the Poisson hypothesis is tested by comparing the medians of η_i and $\eta_i^{(s)}$ (namely, $\tilde{\eta}_i$ and $\tilde{\eta}_i^{(s)}$) by using a Wilcoxon rank sum (two tails) test [*Gibbons*, 1971]. In Table 4 we report the results of the test for N4VEI4, N4VEI4§, and N4VEI2. For these catalogs we show the observed frequency of rejection of the 1000 tests as a function of different significance levels (α) adopted. The Poisson hypothesis is rejected if the frequency of rejection is much larger than α. As regards N4VEI4, for $\alpha = 0.01$ and $\alpha = 0.05$, the expected (α) and observed frequency are almost coincident; for $\alpha = 0.10$, the observed frequency tends to be slightly higher than the corresponding α, with a tendency of $\tilde{\eta}_i < \tilde{\eta}_i^{(s)}$ (i.e., $\tilde{\eta} < 1$). In practice, this means that we cannot reject the Poisson hypothesis at a significance level of 0.05; for larger α, we observe a very weak departure from the Poisson hypothesis, with a more regular time occurrence of eruptions. For N4VEI4, the results do not show any significant departure from a Poisson process for any α considered. For N4VEI2 the Poisson hypothesis is clearly rejected at a significance level < 0.01 with $\tilde{\eta} > \tilde{\eta}^{(s)}$ (i.e., $\tilde{\eta} > 1$).

[20] For ETNA, VES, KIL, and PdlF catalogs, we have a single estimation of η, therefore the significance level of the test is given by $\alpha = 2 \times \min(C_-, C_+)$, where C_- is the

Table 4. Test Results for the Poisson Hypothesis Applied to the Data Sets N4VEI2, N4VEI4, and N4VEI4§

Data Set	$\tilde{\eta}$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
N4VEI2	1.25	0.99	0.94	0.45
N4VEI4	0.94	0.19	0.08	0.01
N4VEI4§	0.82	0.03	0.01	0.001

percentage of cases in which $\eta < \eta^{(s)}$, and C_+ is the percentage of cases in which $\eta \geq \eta^{(s)}$. In Table 5, we report α for ETNA, VES, KIL, and PdlF. The low values reported stand for a rejection of the Poisson hypothesis. The rejection is due to a significant "clustering" of events, i.e., $\eta > \eta^{(s)}$ (i.e., $\eta > 1$).

3.2. Time Predictable Model (TPM)

[21] The term TPM is widely used in the seismological literature [e.g., *Lay and Wallace*, 1995]. It refers to the main practical implication of the model, i.e., the size of the ith eruption (or earthquake) ($V^{(1)}{}_i$) is useful to forecast the time to the next event (τ_i). In probabilistic terms, $f(\tau_i|V^{(1)}{}_i)$ is more appropriate than $f(\tau_i)$ to estimate the probability of the next eruption.

[22] Usually, TPM implies a linear relationship between size and time (see the "pressure-cooker model" of *Burt et al.* [1994]). Here, we consider a more general definition of TPM, by using a power law relationship between $V_i^{(1)}$ and τ_i

$$\tau_i \propto \left[V_i^{(1)}\right]^{\beta}. \tag{6}$$

Only if $\beta = 1$, the relationship between size and time is linear. In our general case, TPM relies on two main assumptions: 1) eruptions occur when a threshold of the magma volume in the storage system is reached, and 2) the magma input in the storage system is a well defined function of the reservoir to be filled to reach that threshold (if $\beta = 1$ the input rate is constant).

[23] These two assumptions deserve more explanation. It could be argued that pressure in a magma reservoir, rather than volume of magma, is more effective to trigger an eruption. Under this perspective, the assumptions behind the conceptual model imply the existence of a direct relationship between the increase of the volume of accumulated magma and the increase of the pressure inside the reservoir [e.g., *Huppert and Woods*, 2002]; in other words, the threshold in magma volume (see assumption 1) would imply a threshold in pressure that has to be overcome to have an eruption. Intuitively, we argue that this model may hold for volcanoes that are unable to sustain significant overpressure like the ones with an open conduit system (see below). Note that an increase of magma volume is not the only way to increase the pressure in the magma chamber, and, conversely, an increasing pressure is not the only triggering of an eruption. The detection of significant evidence supporting TPM means that the magma accumulated in the reservoir is one of the most important factors to trigger an eruption and to improve the forecast of the next eruption. To summarize, the time to the next eruption depends on the time required for the magma entering the storage system to reach the eruptive level. In this model, the size of an eruption, $V^{(1)}$, is a random variable that binds the distribution of τ through equation (6), and $f(\tau) = \int f(\tau/V)f(V)\ dV$.

[24] A reliable application of a TPM requires that $v^{(1)}$ (as previously defined) has to be significantly correlated to the logarithm of the time to the next eruption; we check this hypothesis through the regression analysis of the model

$$\log(\tau_i) = a + b\ v_i^{(1)} + \varepsilon_i, \tag{7}$$

where ε is a random variable normally distributed with zero mean and variance σ_ε^2, i.e., $N(0, \sigma_\varepsilon^2)$. Such a variance can be due to different factors, such as the grouping of the volumes in a discrete set of values (i.e., the VEIs), any departure from a pure TPM, and the effect of a size threshold (see below). Note that b is not usually an unbiased estimator of β for many reasons: (1) the use of normalized IETs instead of real values; (2) the use of completeness threshold that as we show in the following, introduces a bias in the β estimation; and (3) the presence of a large uncertainty in the volume estimation that is not considered in the model described by equation (7). The generalization of the model of equation (7) to account for these factors is beyond the scope of the present paper; such a generalization does not pose any conceptual difficulty, but the solution may be not trivial from a technical point of view.

[25] Despite the factors described before prevents the possibility to test the hypothesis $\beta = 1$, from the regression analysis of the model (7) we can test the existence of a significant correlation between the size of an eruption and the following repose time (the null hypothesis to be tested is H_0: $b \leq 0$), and verify the validity of the model. As regards the last aspect, in the cases where $v^{(1)}$ is VEI, we have repeated measures of $\log(\tau_i)$ relative to each VEI. In this case, the validity of the model is checked by comparing the pure error and the lack of fit through a F test [*Draper and Smith*, 1998]. In the other cases where we do not have repeated measures, the validity of the model is checked by analyzing the distribution of the residuals as described before.

[26] In Figure 2 we plot the regression for the different data sets. We have also checked the stability of the results removing the data with the lowest and highest values of the independent variable. In Table 6 we report the slope with the relative statistical significance level (α), the dispersion around the regression line for the different data sets, and the results of the check of the model. The results show that: (1) a statistically significant relationship exists for all data sets, except N4VEI4§; (2) the slope appears significantly lower than 1 in all the significant cases; (3) the analysis of the residuals confirms the validity of the model; and (4) there is a

Table 5. Test Results for the Poisson Hypothesis Applied to Individual Volcano Data Set for Etna, Vesuvius, Kilauea, and Piton de la Fournaise

Data Set	η	α
ETNA	1.22	0.05
VES	1.27	0.01
KIL	1.87	<0.01
PdlF	1.30	0.01
PdlF§	1.49	<0.01

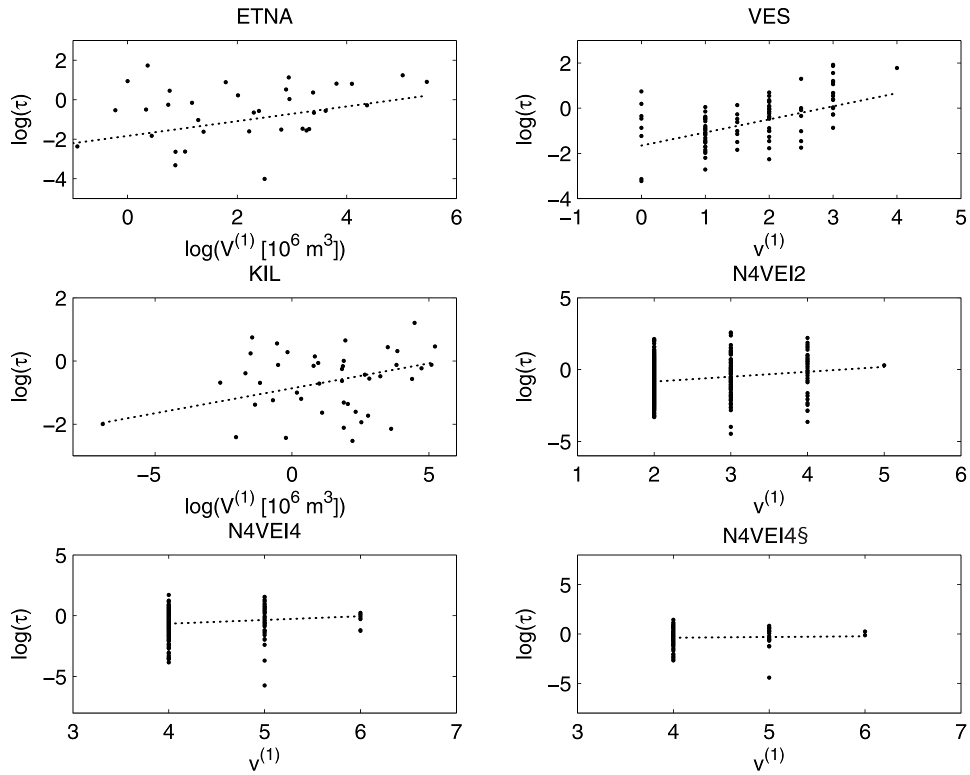

Figure 2. Regression analysis for TPM. The abscissa reports $v^{(1)}$, and the ordinate reports the variable $\log(\tau)$. The variable $v^{(1)}$ does not have any units if VEI is used.

large dispersion around the regression lines that limits the forecasting ability of the model. The volcanological implication of these results will be discussed in section 4.

3.3. Size Predictable Model (SPM)

[27] In SPM, the length of the repose of the volcano, i.e., the time since the last eruption τ_i, is useful to forecast the size of eruption $V_i^{(2)}$. As in section 2, we consider a more general definition of SPM, by using a power law relationship between $V_i^{(2)}$ and τ_i

$$V_i^{(2)} \propto \tau_i^{\beta}. \tag{8}$$

Only if $\beta = 1$, the relationship between time and size is linear [see the "water-butt model" by *Burt et al.*, 1994]. In our general case, SPM relies on two main assumptions: (1) the output of each eruption is determined only by the magma accumulated since the last eruption, i.e., each eruption brings the magma inside the plumbing system to the same initial level, and (2) magma enters in the plumbing system at a rate described by a well defined function of the magma volume in the reservoir (if $\beta = 1$ the input rate is constant).

[28] Also in this case, the assumptions deserve a careful explanation. Eruption size could be described better by the amount of pressure accumulated in the reservoir in a volcanic system able to sustain significant overpressure (like for "sealed" volcanoes). The assumptions behind the conceptual model imply that this pressure is directly related to the magma accumulated. Obviously, many other factors, such as degassing (many dormant volcanoes are not completely "sealed"), and the kind of mechanical failure during the reopening of the conduit, may be important to determine the size of an eruption. The detection of significant evidence of a SPM in real data means that the magma accumulated during the repose period may have a prominent importance in determining the VEI of the eruption. In this model, τ is a random variable that determines the distribution of the size of eruption through equation (8), and $f(V^{(2)}) = \int f(V^{(2)}/\tau) f(\tau) d\tau$.

[29] The reliability of SPM is evaluated by the regression analysis of the model

$$\log(\tau_i) = a + b\, v^{(2)} + \varepsilon_i, \tag{9}$$

where ε is a random variable normally distributed with zero mean and variance σ^2, i.e., $N(0, \sigma)$. Note that in equation (9) we still consider the size ($v^{(2)}$) as "independent" variable; this choice does not have a physical rationale, but it is adopted only for technical convenience [see *Draper and Smith*, 1998] because $v^{(2)}$ is often grouped as for the VEIs. In practice, since we are focused on estimating the statistical significance of the relationship between $\log(\tau)$ and $v^{(2)}$, this decision is not critical. Finally, we remind that also in this case, b is not an unbiased estimator of β for the same reasons described before. The check of the model of equation (9) is carried out as described before for TPM.

[30] In Figure 3 and Table 7, we report the results relative to different data sets, showing the slope and its statistical significance. We have also checked the stability of the results removing the data with the lowest and highest values of the independent variable. The results obtained show that

Table 6. Regression Results for TPM

Data Set	Slope	t Test α	SD	Runs Test α	Sign Test α	F Test α
ETNA	0.4 ± 0.2	0.03	1.9	0.74	0.51	–
VES	0.7 ± 0.1	<0.01	0.84	–	–	0.06
KIL	0.16 ± 0.07	0.01	1.1	0.89	0.55	–
N4VEI4	0.3 ± 0.2	0.04	1.2	–	–	0.42
N4VEI4	0.0 ± 0.2	0.38	1.0	–	–	0.61
N4VEI2	0.34 ± 0.08	<0.01	1.2	–	–	0.97

(1) a statistically significant relationship exists only for N4VEI2 and weaker for N4VEI4 (but not for N4VEI4§); (2) the slope is less than one in all the significant cases; (3) the analysis of the residuals shows the appropriateness of the model; and (4) there is a large dispersion around the regression lines that limits significantly the forecasting ability of the model. The volcanological implication of these results will be discussed later.

3.4. Influence of the Completeness (VEI) Threshold on the Analysis of N4VEI2 and N4VEI4 Catalogs

[31] Catalogs N4VEI2, N4VEI4, and N4VEI4§ have been defined by using a minimum threshold of VEI in order to guarantee their completeness. Therefore the results obtained so far for these catalogs can be interpreted only by assuming that eruptions with VEI smaller than the threshold do not modify significantly the occurrence of larger events. This is an important issue that deserves a careful examination.

[32] Specifically, if the size of an eruption is important to estimate the previous or the following IET, the removal of small events may potentially have a large impact on the results of the analysis. In order to explore in detail this issue we simulate an ideal TPM in the whole VEI range (from 0 to 8), with a theoretical slope of 1 between $\log(\tau_i)$ and $(v_i^{(1)})$ ($\beta = 1$ in equation (6), that means a linear relationship), and the volume with a power law distribution similar to the Gutenberg-Richter law with slope equal to one. For an ideal SPM, the discussion follows the same line.

[33] In Table 8 we report the results of the regression analysis as a function of different VEI thresholds. From the results we can see that the grouping of volumes (in a discrete number of VEI) introduces a dispersion (standard deviation) around the regression line that depends weakly on the VEI threshold. This dispersion is significantly lower than that reported in Tables 6 and 7, suggesting the existence of other sources of variation in real catalogs. The use of a VEI completeness threshold has an important effect. The increase of the threshold leads to a significant reduction of the slope. Remarkably, despite this bias in the slope estimation, we still obtain a significant TPM also for a threshold VEI = 4.

[34] These results have important implications. First, the bias introduced in the slope estimation does not allow the linear relationship between size and time ($\beta = 1$) to be directly estimated from the slope obtained by the data. Second, the use of a VEI threshold for completeness of eruptive catalogs does not necessarily blur the signal of a TPM; conversely, we also argue that the use of a VEI threshold on eruptive catalogs generated by a pure random process with independent statistical distribution for τ and V acts as a random sampling, therefore it cannot induce any pattern (like TPM and SPM) into the data.

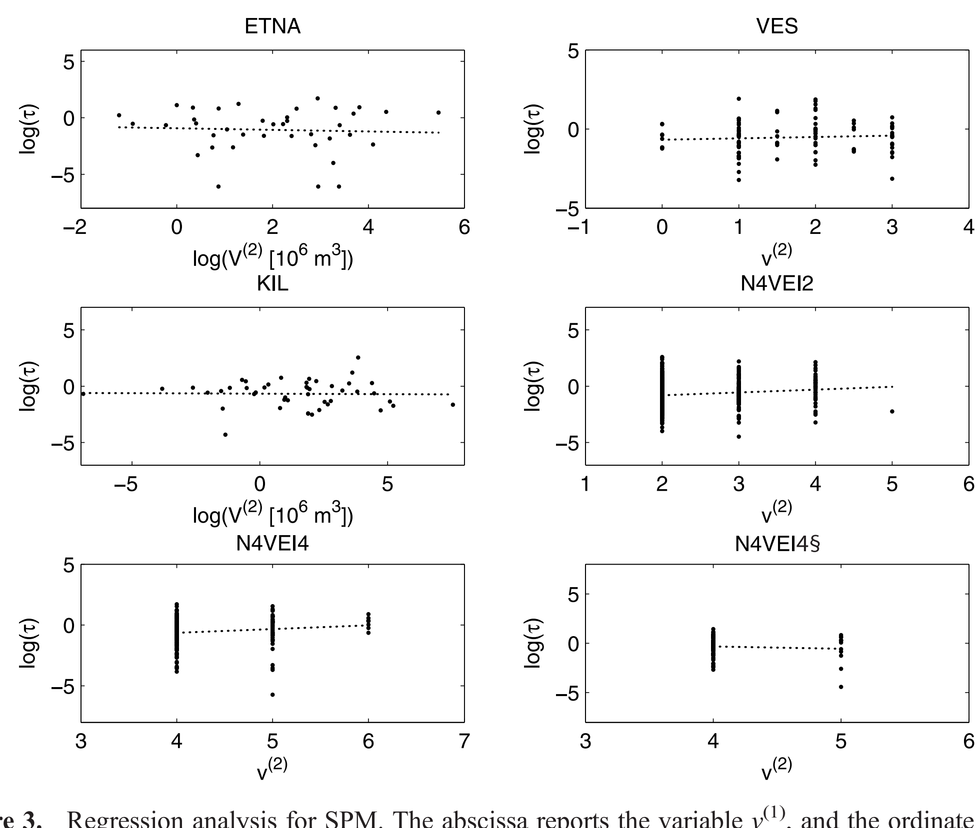

Figure 3. Regression analysis for SPM. The abscissa reports the variable $v^{(1)}$, and the ordinate reports $\log(\tau)$. The variable $v^{(1)}$ does not have any units if VEI is used.

Table 7. Regression Results for SPM

Data Set	Slope	*t* Test α	SD	Runs Test α	Sign Test α	F Test α
ETNA	−0.1 ± 0.2	0.87	2.0	0.99	0.67	–
VES	0.1 ± 0.1	0.25	1.0	–	–	0.21
KIL	−0.1 ± 0.7	0.55	1.2	0.45	0.03	–
N4VEI4	0.31 ± 0.16	0.03	1.2	–	–	0.49
N4VEI4§	−0.2 ± 0.3	0.77	0.98	–	–	–
N4VEI2	0.25 ± 0.08	<0.01	0.98	–	–	0.13

4. Discussion of the Results

[35] The results reported above show a marked difference between N4VEI4 and N4VEI4§, and the other catalogs. For N4VEI4 and N4VEI4§, the volcanic events do not show statistically significant departures from a Poisson process; N4VEI4 shows marginally significant support for a SPM. This result is not confirmed by the analysis of N4VEI4§ that does not show any significant pattern between volumes and IETs. On the contrary, the eruptions contained in all the other catalogs are clustered in time, and show a significant TPM. N4VEI2 is the only catalog that shows significant support for both TPM and SPM.

[36] The difference between N4VEI4 and N4VEI4§ and all the other catalogs seems to be linked to the distinct average of the IETs, μ_{IET}. In Figure 4, we plot the μ_{IET} of each volcano considered. Note that the catalogs ETNA, KIL, VES, and PdlF consist only of one point because they are relative to single volcanoes. The catalogs show marked differences, with the maximum separation at few decades. For instance, μ_{IET} for ETNA, VES, KIL, PdlF/PdlF§, and most of volcanoes contained in N4VEI2 are shorter than 30 years, while for N4VEI4 and N4VEI4§ the μ_{IET} are larger. In order to understand the twofold behavior of N4VEI2 (both TPM and SPM are supported by the data), we analyze a subset of N4VEI2 composed by volcanoes with at least four consecutive eruptions having IETs smaller than 30 years (data set N4VEI2*). The results relative to this data set are reported in Table 9 and in Figure 5. In this case, as for the other catalogs characterized by low μ_{IET} (ETNA, VES, KIL and PdlF), the sequence of eruptions statistically supports a TPM, while evidence in favor of SPM are no longer statistically significant. Notably, this result also implies that, for the N4VEI2 catalog, the longest IETs (larger than 30 years) tend to end with higher VEI eruptions (characteristic of SPM).

[37] It could be argued that the largest μ_{IET} for N4VEI4 and N4VEI4§ does not have any real meaning, because it is due only to the highest VEI threshold adopted; in other terms, the largest μ_{IET} does not imply the largest average of real repose time. However, the hypothesis that the largest μ_{IET} is due only to the highest VEI threshold does not explain one important feature found for N4VEI2, specifically, the relation between the longest IETs and the highest VEI (i.e., the existence of a statistically significant evidence supporting SPM). As a consequence, we argue that the differences in μ_{IET} have a physical meaning and they are not simply due only to the difference in VEI threshold. In other words, for N4VEI2, N4VEI4, and N4VEI4§ the inclusion of events with VEI smaller than the threshold does not modify substantially the average of repose time before larger events.

[38] The interpretation of the results in terms of the physics of the eruptive process requires the definition of a conceptual scheme. For this purpose, we suggest that the marked difference of μ_{IET} is associated to a bimodal behavior of a volcanic system. Small μ_{IET} (characteristic of catalogs N4VEI2/N4VEI2*, ETNA, VES, KIL, and PdlF/PdlF§) are typical of an "open conduit system" (OCS), where the high frequency of eruptions is mainly due to the intrinsic incapacity to bear strong overpressures. A large IET (characteristic of N4VEI4 and N4VEI4§ catalogs) favors the closure of the conduit due to viscous relaxation and the cooling of rocks within the conduit [*Quareni and Mulargia*, 1993], that may lead to a transition from OCS to a "closed conduit system" (CCS). Such a transition may be also induced by other factors such as the emptying of the feeding system.

[39] Under this perspective, we find that an OCS is modeled by a TPM and it is characterized by clusters of eruptions that may be described by an upper bounded distribution in the time domain (note that the clusters of eruptions is compatible with an upper bounded power law distribution, but clustering does not imply necessarily a power law distribution). The upper bound of the distribution is due to the fact that after a large eruption, the TPM calls for a long IET that can lead to a transition from OCS to CCS (see above).

[40] The closure of the conduit tends to facilitate the recharging of the system, by increasing the resistance of the volcano to overpressure in the magma chamber, leading to higher (on average) VEI. The range of the size of eruptions in the two regimes probably overlaps. VEI = 3 eruptions are common in both regimes, and such a size could represent the overlapping boundary region of energy between the two regimes. Note that, in this scheme, the twofold behavior of N4VEI2 is explained by the fact that this catalog contains eruptions that occurred in both regimes of activity. The support to TPM is due to the volcanoes in a OCS state, while SPM is supported by the presence of the longest IETs that lead to a transition from OCS to CCS that, in turn, produces an increase of the VEIs. In other words, SPM is supported by the data of N4VEI2 relative to the transition between OCS and CCS.

[41] As regards CCS, our analyses show that the time of eruption is almost random in time (Poisson), probably being due to many independent factors, such as the mechanical failure of the edifice or to the triggering of external events [*Marzocchi*, 2002; *Marzocchi et al.*, 2004b]. The very weak evidence of a SPM only for N4VEI4, and not confirmed by the analysis of N4VEI4§, seems to indicate that the repose time during a CCS does not play a major role in forecasting the size and explosivity of the next eruption.

[42] Other factors, like degassing during repose may play a comparable role. Although degassing is intimately related to closed/open conduit distinction, we can conceive the

Table 8. Regression Results of the Influence of VEI Thresholds on the Eruptive Catalogs

VEI Threshold	Number of Data	Slope	SD
0	300	0.98 ± 0.04	0.28
2	300	0.75 ± 0.05	0.30
4	300	0.63 ± 0.05	0.33

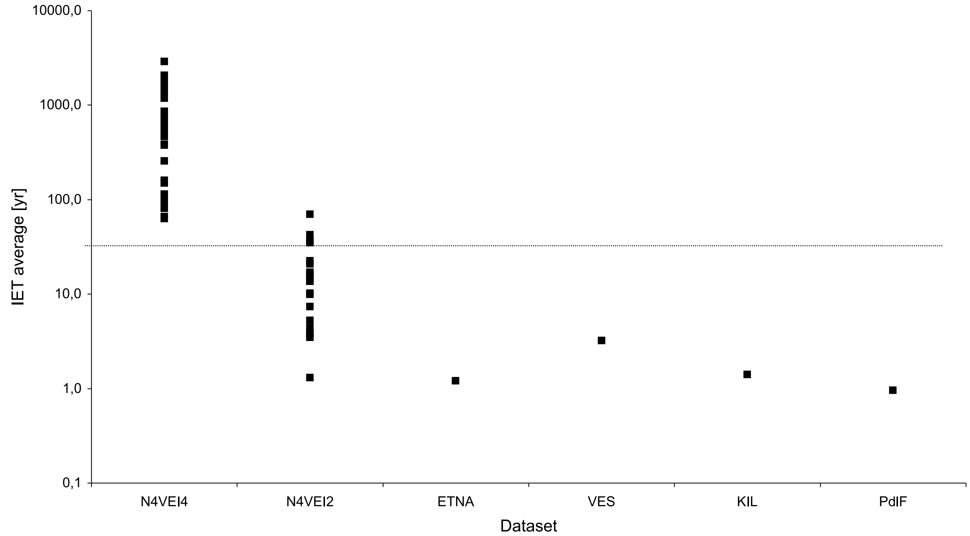

Figure 4. Plot of μ_{IET} for volcanoes contained in the different data sets. The horizontal dotted line represents a time (30 years) that may characterize the differences between OCS and CCS (see text for more details).

presence of different levels of degassing in CCS. In this case, since a strongly degassed magma is apparently less capable of large VEIs [*Newhall et al.*, 1994; *Newhall*, 2003], a significant degassing may allow very long repose times and yet still produce small eruptions. Another relevant aspect in forecasting the size of the next eruption may be the complex geophysical, geochemical and geomechanical factors in the reopening of the conduit, that would lead to a power law distribution of the erupted volumes as for any complex system [see *Bak et al.*, 1988].

[43] As final consideration, we discuss some practical aspects of our analyses. First, we use the terms TPM and SPM because they are commonly used in geophysics, but we emphasize that the word "predictability" may be misleading (see, for instance, discussion by *Marzocchi et al.* [2003]). Eruption forecasting can be only done from a probabilistic point of view. In this respect, the statistically significant evidence supporting a TPM for OCS only indicates that, in this case, conditional probability allows a better eruption forecasting compared to the one obtained by using a probabilistic model only in the time domain. Second, our results have a general validity, because they come from simultaneous analyses of different volcanic systems. The inclusion of detailed studies and quantitative modeling of the peculiar behavior of each single volcano can potentially reduce the variance about the model (i.e., the variance of the residuals) and further improve the forecasting for a specific volcano.

5. Models of Open and Closed Conduit Systems

[44] We now present quantitative general models for the time-size character of Open Conduit (OCS) and Closed Conduit Systems (CCS).

5.1. Model for an OCS

[45] From the regression of equation (7), we can build a general model for an OCS. This relationship allows the conditional probability density function of the times to the next eruption τ given the size of the previous volcanic event to be derived. From equation (7) we have

$$f\left(\log(\tau)|v^{(1)}\right) = N\left(a + bv^{(1)}, \sigma\right), \qquad (10)$$

where a and b are the unknown of the model. Note that here τ represents the real IET (not normalized as before). The conditional probability density function of τ given $v^{(1)}$ is therefore a lognormal distribution

$$f\left(\tau|v^{(1)}\right) = \frac{1}{\tau\sigma\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{\log(\tau) - \left(a + bv^{(1)}\right)}{\sigma}\right)^2\right]. \qquad (11)$$

The probability of eruption in a given forecasting time window Δt is estimated by

$$P = \frac{\int_{t_0}^{t_0+\Delta t} f\left(\tau|v^{(1)}\right) d\tau}{\int_{t_0}^{\infty} f\left(\tau|v^{(1)}\right) d\tau}, \qquad (12)$$

where t_0 is the time elapsed since the last eruption. As regards the volume of the next eruption, the model of equation (10) does not give any clue that may help to forecast it. From the discussion made before, it may be

Table 9. Regression Results for TPM and SPM Relative to the Data Set N4VEI2*

Model	Slope	t Test α	SD	Runs Test α	Sign Test α	F Test α
TPM	0.26 ± 0.11	0.01	0.98	–	–	0.72
SPM	0.0 ± 0.1	0.60	0.99	–	–	0.90

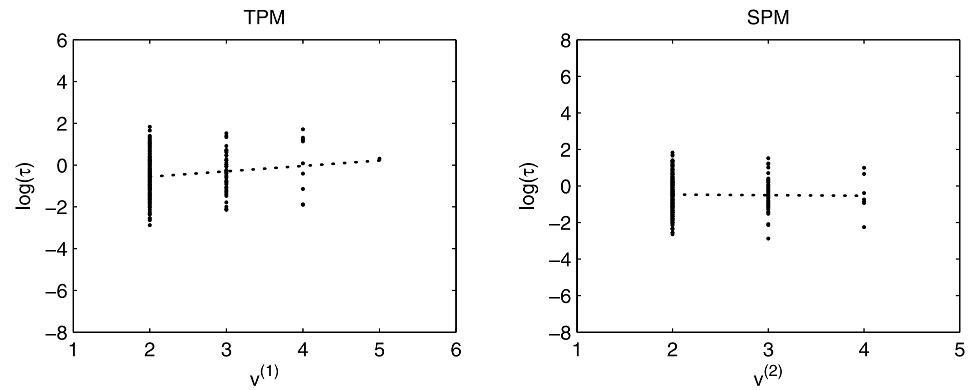

Figure 5. Plot of the regression analysis for TPM and SPM relative to the data set N4VEI2*. The axes report the same variables as in Figures 2 and 3.

reasonable to assume that the distribution of such a volume is a generic upper bounded power law.

[46] The model given by equation (11) has three parameters to be estimated from the data, a, b and σ. These parameters can be estimated through the Maximum Likelihood Estimation. The Likelihood function is

$$L = \prod_{i=1}^{M} f\left(\tau_i | v_i^{(1)}\right) = \prod_{i=1}^{M} \frac{1}{\tau_i \sigma \sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{\log(\tau_i) - \left(a + b v_i^{(1)}\right)}{\sigma}\right)^2\right], \quad (13)$$

and the log likelihood is

$$\ell = \log(L) = \sum_{i=1}^{M}\left[-\log(\tau_i) - \log(\sigma) - \frac{1}{2}\log(2\pi) - \frac{1}{2}\left(\frac{\log(\tau_i) - \left(a + b\, v_i^{(1)}\right)}{\sigma}\right)^2\right]. \quad (14)$$

The parameters can be derived by equalizing the first derivative to zero [*Kalbfleisch*, 1985]. The parameters are

$$b = \frac{M \sum\left[v_i^{(1)} \log(\tau_i)\right] - \sum \log(\tau_i) \sum v_i^{(1)}}{M \sum\left(v_i^{(1)}\right)^2 - \left(\sum v_i^{(1)}\right)^2}, \quad (15)$$

$$a = \frac{\sum \log(\tau_i) - b \sum v_i^{(1)}}{M}, \quad (16)$$

[47] The normal approximation allows the standard deviation s of the parameters to be estimated by means of the second derivatives of the likelihood. In particular,

$$I(\theta) - = \frac{d^2\ell}{d\theta^2} \quad (18)$$

and

$$s_\theta = \frac{1}{\sqrt{I(\theta)}}, \quad (19)$$

where θ is a generic parameter. In our case we obtain

$$s_a = \frac{\sigma}{\sqrt{M}}, \quad (20)$$

$$s_b = \frac{\sigma}{\sqrt{\sum\left(v_i^{(1)}\right)^2}}, \quad (21)$$

$$s_\sigma = \frac{\sigma}{\sqrt{2M}}. \quad (22)$$

[48] Here, the parameter estimation of the model of equation (10) is useful mainly for forecasting purposes. The interpretation of the parameters in terms of the physics of the process is not trivial because of the issues mentioned before related to the test of $\beta = 1$ in equation (6). We also suggest not using regression analysis to estimate the parameters of the model, because regression analysis can introduce a further significant bias. This issue is discussed in depth by *Bender* [1983] for the parameters of the Gutenberg-Richter law.

[49] For the sake of example, we test the eruption catalog of Mauna Loa, that has not been used in the previous

$$\sigma = \left[\frac{-2a \sum \log(\tau_i) + \sum \log^2(\tau_i) + Ma^2 + b^2 \sum\left(v_i^{(1)}\right)^2 + 2ab \sum v_i^{(1)} - 2b \sum\left[v_i^{(1)} \log(\tau_i)\right]}{M}\right]^{\frac{1}{2}}. \quad (17)$$

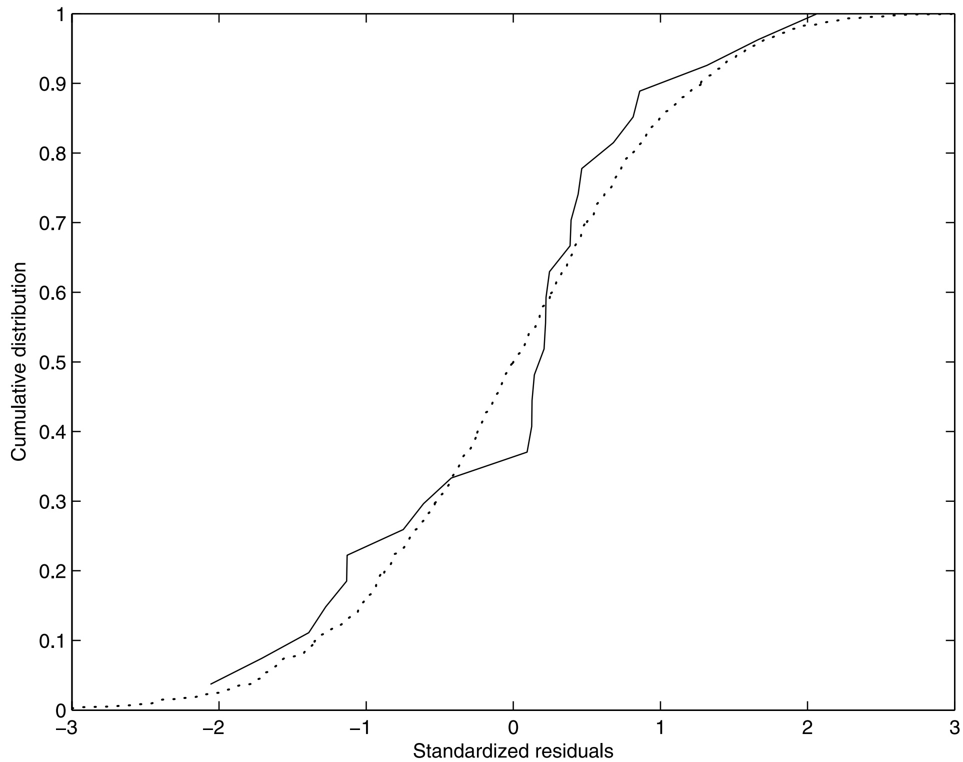

Figure 6. Plot of the empirical cumulative function (solid line) of residuals of the model, i.e., the difference of the observed IETs and the predicted IETs by using the model of equation (10), for the Mauna Loa eruptive catalog. The dotted line represents the theoretical cumulative function for a Gaussian distribution.

analyses. The catalog is taken from the Web page http://hvo.wr.usgs.gov/maunaloa/history/historytable.html (modified from *Lockwood and Lipman* [1987]) and contains 29 eruptions. The parameters and uncertainties are $a = 8.1 \pm 0.1$, $b = 0.31 \pm 0.04$, $\sigma = 0.65 \pm 0.09$. We evaluate the goodness of the fit of the model by testing if the residuals, i.e., the difference between the observed IETs and the values estimated by the model of equation (10), have a normal distribution. This hypothesis is not rejected at a significance level of 0.05 by using a chi square test [*Gibbons*, 1971]. In Figure 6 we show the empirical cumulative function and the theoretical Gauss cumulative function.

5.2. Model for a CCS

[50] The results reported above do not show strongly significant evidence in favor of an SPM or a TPM for CCS. As regards the time distribution of eruptions, we find no significant difference from a Poisson distribution. Therefore the time-size distribution can be described by

$$f\left(\tau, v^{(2)}\right) = f_1(\tau) f_2\left(v^{(2)}\right), \tag{23}$$

where $f_1(\tau)$ is the distribution of IETs and $f_2(v^{(2)})$ represents the distribution of the erupted volumes. Our analyses have shown that $f_1(\tau)$ is reasonably an exponential distribution (the distribution of IETs generated by a Poisson process)

$$f_1(\tau) = \lambda \exp(-\lambda\tau), \tag{24}$$

where λ is the rate of occurrence of events. Therefore the probability of eruption in a given forecasting time window Δt is given by

$$P = \frac{\int_{t_0}^{t_0+\Delta t} f_1(\tau) d\tau}{\int_{t_0}^{\infty} f_1(\tau) d\tau} = 1 - \exp(-\lambda \Delta t), \tag{25}$$

where t_0 is the time elapsed since the last eruption. The parameter λ and its uncertainty can be estimated as [*Cox and Lewis*, 1966]

$$\lambda = \frac{M}{\sum_{i=1}^{M} \tau_i} \tag{26}$$

and

$$\sigma_\lambda = \frac{\lambda}{\sqrt{M}}. \tag{27}$$

[51] As regards $f_2(v^{(2)})$, this distribution is still mostly unknown, and the analysis carried out above does not provide any constraint on it. At the present state of knowl-

edge we can only speculate that (1) the distribution may be truncated, because it may need a minimum level of energy to reopen a closed system; and (2) above the minimum threshold, the size of eruption may follow a power law distribution, being due to many factors in a complex system [e.g., *Bak et al.*, 1988].

6. Conclusions

[52] The main goal of the paper was to provide a generic quantitative model of the time-size distribution of eruptions. The results of the analysis show different behaviors as a function of the length of the interevent times. Volcanoes with short interevent times show a time clustering of events and follow a time predictable model [cf. *Klein*, 1982; *Burt et al.*, 1994; *Sandri et al.*, 2005]. On the other hand, volcanoes with long interevent times show a complete randomness of the occurrence of events (Poisson process) and no significant evidence supporting time or size predictable models. We interpret these two classes of volcano behaviors as open and closed conduit systems, respectively. Our general probabilistic models of the time-size distribution can potentially be used to improve volcanic hazard assessment for each volcanic system belonging to these two regimes.

[53] **Acknowledgments.** We thank Chris Newhall, Steve Sparks, Louise Burt, and Geoff Wadge for their reviews of this paper; their comments have improved the quality and readability of the paper. We also thank Lee Siebert for providing us updates of Smithsonian catalog. This work was partially funded by the Italian Dipartimento della Protezione Civile in the frame of the 2004–2006 Agreement with Istituto Nazionale di Geofisica e Vulcanologia - INGV (V4 project).

References

Bak, P., C. Tang, and K. Wiesenfeld (1988), Self-organized criticality, *Phys. Rev. A*, *38*, 364–374.

Behncke, B., M. Neri, and A. Nagay (2005), Lava flow hazard at Mount Etna (Italy): New data from a GIS-based study, in *Kinematics and Dynamics of Lava Flows*, edited by M. Manga and G. Ventura, *Spec. Pap. Geol. Soc. Am.*, *396*, 187–205.

Bebbington, M. S., and C. D. Lai (1996), Statistical analysis of New Zealand volcanic occurrence data, *J. Volcanol. Geotherm. Res.*, *74*, 101–110.

Bender, B. (1983), Maximum likelihood estimation of b values for magnitude grouped data, *Bull. Seismol. Soc. Am.*, *73*, 831–851.

Burt, M. L., G. Wadge, and W. A. Scott (1994), Simple stochastic modelling of the eruption history of a basaltic volcano: Nyamuragira, Zaire, *Bull. Volcanol.*, *56*, 87–97.

Coles, S. G., and R. S. J. Sparks (2006), Extreme value methods for modeling historical series of large volcanic magnitudes, in *Statistics in Volcanology*, Geol. Soc., London, in press.

Connor, C. B., R. S. J. Sparks, R. M. Mason, C. Bonadonna, and S. R. Young (2003), Exploring links between physical and probabilistic models of volcanic eruptions: The Soufriere Hills volcano, Montserrat, *Geophys. Res. Lett.*, *30*(13), 1701, doi:10.1029/2003GL017384.

Cox, D. R., and P. A. W. Lewis (1966), *The Statistical Analysis of Series of Events*, Methuen, New York.

Cox, D. R., and A. Stuart (1955), Some quick sign tests for trend in location and dispersion, *Biometrika*, *42*, 80–95.

Draper, N. R., and H. Smith (1998), *Applied Regression Analysis*, John Wiley, Hoboken, N. J.

Gibbons, J. D. (1971), *Non-parametric Statistical Inference*, 306 pp., McGraw-Hill, New York.

Gusev, A. A., V. V. Ponomareva, O. A. Braitseva, and I. V. Melekestsev (2003), Great explosive eruptions on Kamchatka during the last 10,000 years: Self-similar irregularity of the output of volcanic products, *J. Geophys. Res.*, *108*(B2), 2126, doi:10.1029/2001JB000312.

Huppert, H. E., and A. W. Woods (2002), the role of volatiles in magma chamber dynamics, *Nature*, *420*, 493–495.

Kalbfleisch, J. D. (1985), *Probability and Statistical Inference*, 2nd ed., 2 vols., Springer, New York.

Klein, F. W. (1982), Patterns of historical eruptions at Hawaiian volcanoes, *J. Volcanol. Geotherm. Res.*, *12*, 1–35.

Lay, T., and T. C. Wallace (1995), *Modern Global Seismology*, Elsevier, New York.

Lockwood, J. P., and P. W. Lipman (1987), Holocene eruptive history of Mauna Loa volcano, in *Volcanism in Hawaii*, edited by R. W. Decker, T. L. Wright, and P. H. Stauffer, *U.S. Geol. Surv. Prof. Pap.*, *1350*, vol. 1, 509–535.

Macdonald, G. A., A. T. Abbott, and F. L. Peterson (1986), *Volcanoes in the Sea: The Geology of Hawaii*, 2nd ed., pp. 80–81, Univ. of Hawaii Press, Honolulu.

Marzocchi, W. (1996), Chaos and stochasticity in volcanic eruptions: the case of Mt. Etna and Vesuvius, *J. Volcanol. Geotherm. Res.*, *70*, 205–212.

Marzocchi, W. (2002), Remote seismic influence on large explosive eruptions, *J. Geophys. Res.*, *107*(B1), 2018, doi:10.1029/2001JB000307.

Marzocchi, W., L. Sandri, and E. Boschi (2003), On the validation of earthquake-forecasting models: the case of pattern recognition algorithms, *Bull. Seismol. Soc. Am.*, *93*, 1994–2004.

Marzocchi, W., L. Sandri, P. Gasparini, C. Newhall, and E. Boschi (2004a), Quantifying probabilities of volcanic events: The example of volcanic hazard at Mt. Vesuvius, *J. Geophys. Res.*, *109*, B11201, doi:10.1029/2004JB003155.

Marzocchi, W., L. Zaccarelli, and E. Boschi (2004b), Phenomenological evidence in favor of a remote seismic coupling for large volcanic eruptions, *Geophys. Res. Lett.*, *31*, L04601, doi:10.1029/2003GL018709.

Newhall, C. G. (2003), "Restless" isn't always dangerous and "quiet" isn't always reassuring!, paper presented at Cities on Volcanoes 3, Univ. of Hawaii, Hilo, 14–18 July.

Newhall, C. G., and R. P. Hoblitt (2002), Constructing event trees for volcanic crises, *Bull. Volcanol.*, *64*, 3–20, doi:10.1007/s004450100173.

Newhall, C. G., and S. Self (1982), The volcanic explosivity index (VEI): An estimate of the explosive magnitude for historical eruptions, *J. Geophys. Res.*, *87*, 1231–1238.

Newhall, C. G., R. S. Punongbayan, and T. M. Gerlach (1994), Tight and leaky volcanoes: Implications for forecasting explosive eruptions, in *Large Explosive Eruptions*, *Atti Conv. Lincei*, *112*, 13–21.

Quareni, F., and F. Mulargia (1993), Modeling of closure of volcanic conduits with an application to Mount Vesuvius, *J. Geophys. Res.*, *98*, 4221–4229.

Sandri, L., W. Marzocchi, and P. Gasperini (2005), Some insights on the occurrence of recent volcanic eruption of Mount Etna volcano (Sicily, Italy), *Geophys. J. Int.*, *163*, 1203–1218, doi:10.1111/j.1365-246X.2005.02757.x.

Scandone, R., L. Giacomelli, and P. Gasparini (1993), Mount Vesuvius: 2000 years of volcanological observations, *J. Volcanol. Geotherm. Res.*, *58*, 5–25.

Simkin, T., and L. Siebert (1994), Volcanoes of the World, Geosciences, Tucson, Ariz.

Sornette, A., J. Dubois, J. L. Cheminee, and D. Sornette (1991), Are sequences of volcanic eruptions deterministically chaotic?, *J. Geophys. Res.*, *96*, 11,931–11,945.

Sparks, R. S. J., and W. Aspinall (2004), Volcanic activity: Frontiers and challenges in forecasting, prediction, and risk assessment, *State of the Planet: Frontiers and Challenges in Geophysics*, *Geophys. Monogr. Ser.*, vol. 150, edited by R. S. J. Sparks and C. J. Hawkesworth, pp. 359–374, AGU, Washington, D. C.

Wickman, F. E. (1976), Markov models of repose-period patterns of volcanoes, in *Random Process in Geology*, pp. 135–161, Springer, New York.

W. Marzocchi and L. Zaccarelli, INGV-Bologna, Via D. Creti 12, I-40128 Bologna, Italy. (marzocchi@bo.ingv.it; zaccarelli@ov.ingv.it)