Geomechanics and Geotechnical Engineering

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Particles Discrete Element Method for Crack Propagation of rock mass

Abstract: The rock mass can be assumed to homogeneous material from a macroscopic view, it is the heterogeneous material in mesoscopic scale and its physico-mechanical properties are discontinuous in space. The failure of jointed rock mass was usually caused by the initiation, propagation and coalescence of new wing cracks derived from primary joint. For more in-depth study of rock fracture instability, we need to study the expansion of rock cracks under external loads from the macro-meso perspective. This paper, based on the manifold cover concept, proposes a new discrete element numerical method, Manifold Lattice Discrete, combining with the particle contact model, introduced concept of stress boundary. The proposed method can easily simulate the generation, propagation and coalescence of rock crack from the macro-meso perspective. The whole process of rock fragmentation is thereafter reproduced. By analyzing the manifold cover and ball particle model, this paper constitutes the sphere unit cover function of three-dimensional manifold cover, establishes tetrahedron units, and obtains the equilibrium equation and compatible equation of the MLD model. For rock-like brittle material, crack propagation process can be simulated.
Key words: macrostructure and microstructure, MLD model, manifold cover, crack propagation, numerical simulation.

1. Introduction

The rock is a complex mixture composed of various mineral crystals, cements, and pore defects. Usually, a large number of geological faults such as cracks, fracture surfaces, joints, holes, and fillings occur in them and they are randomly distributed in the rock. Under the action of the external load, the nucleation and expansion of rock internal micro defects, and interaction between them determine rock's deformation and fracture characteristics [1]. Along with the rapid development of high-performance computers and the theory of numerical calculations, several models and software based on numerical analysis method has been applied in the field of geotechnical engineering to simulate the mechanical response and failure modes of the material. Some numerical method such as RFPA, DDA, meshless method, manifold method, boundary element method and discrete element method are applied to the study on crack propagation, and good results have been achieved (P.A. Cundall, H.Y. Liu, J.P. Harrison, Weizhong Chen, Shucai Li, Chunan Tang etc. [2–10]).

For complex rock, there is no good numerical simulation method can truthfully describe the actual behavior of the rock mass, and especially for rupture process of complex rock is not effectively analyze. With the combined of Micromechanics and statistical strength theories, and continuum damage mechanics and damage mechanics, in recent years, some of the numerical simulation appeared based on meso-structure considerations in the aspect of rock and concrete fracture analysis（Gianluca Cusatis [11, 12]）. Based on the manifold cover concept, this paper proposes a new discrete element numerical method, Manifold Lattice Discrete, to simulate the generation, propagation and coalescence of rock crack from the macro-meso perspective, combining with the particle contact model, and introduced concept of stress boundary.

2. MLD Model Unit

2.1. Rock structure

The rock is a complex mixture composed of various mineral particles, cements, and pore defects. As shown in Fig. 1, taken out a representative unit from the brittle materials such as rock, and analyze rock structure from the macro meso-scale.

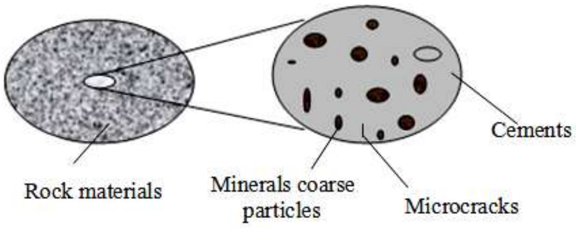


Fig. 1. Distribution of rock macro-meso-structure.

2.2. Generation of the MLD Model Unit

Shown in Fig. 2, the microstructure of the rock, mineral grains will be equivalent to the ball units, and the sphere unit is assumed to be physical cover. Tetrahedral elements formed the force between mineral grains cements define mathematical coverage. Combined with the sphere grain unit and manifold unit, the center and the midpoint of each side of the unit is connected to form the MLD unit shown in Fig. 2.

3. Balance and geometric equations of the MLD model

Tetrahedron from the center point can be divided into four regions, and V1 is one of them (shown in Fig. 2). As shown in Fig. 2, the center coordinate matrix of each sphere unit $\mathbf{X}_i = [x_i, y_i, z_i]^{\mathrm{T}}$; each sphere unit has 6 degrees of freedom to define the displacement field using rigid body dynamics:

$$\mathbf{U}(\mathbf{X}) = \mathbf{U_i} + \boldsymbol{\theta}_\mathbf{i} \times (\mathbf{X} - \mathbf{X_i}) = \mathbf{A_i}(\mathbf{X})\mathbf{Q_i} \qquad (1)$$

In the preceding formula, $\mathbf{A_i}(\mathbf{X}) = \begin{bmatrix} 1 & 0 & 0 & 0 & z - z_i & y_i - y \\ 0 & 1 & 0 & z_i - z & 0 & x - x_i \\ 0 & 0 & 1 & y - y_i & x_i - x & 0 \end{bmatrix}$, $\mathbf{Q}_i = \left[\mathbf{U}_i^T \boldsymbol{\theta}_i^T\right]$ and $\mathbf{U}_i^T = [u_{1i}, u_{2i}, u_{3i}]$ are translational displacements of particle i while $\boldsymbol{\theta}_i^{\mathrm{T}} = [\theta_{1i}, \theta_{2i}, \theta_{3i}]$ is the rotational displacement.

The general cover of manifold method is composed of mathematical cover and physical cover. The cover function and related weight function are defined based on the general finite cover and the overlapping portion is transformed into conventional units. Then, solve established global equilibrium equations of general finite cover. Based on manifold cover, the tetrahedron formed by connection between ball particles is regarded as mathematical cover and its related weight function is $N_j(x_1, x_2, x_3)$. Then,

$$\begin{cases} N_{j1}(x,y,z) = f_{11} + f_{12}x + f_{13}y + f_{14}z \\ N_{j2}(x,y,z) = f_{21} + f_{22}x + f_{23}y + f_{24}z \\ N_{j3}(x,y,z) = f_{31} + f_{32}x + f_{33}y + f_{34}z \\ N_{j4}(x,y,z) = f_{41} + f_{42}x + f_{43}y + f_{44}z \end{cases} \quad A\nu\delta \quad \begin{cases} \sum\limits_{x,y,z\in U_j} N_j(x,y,z) = 1 \\ N_j(x,y,z) \geq 0, \quad (x,y,z) \in U_j \\ N_j(x,y,z) = 0, \quad (x,y,z) \notin U_j \end{cases} \tag{2}$$

In the preceding formula, $f_{ij}(i, j = 1,2,3,4)$ are 16 constants related to node coordinates of manifold units.

The displacement function of the manifold unit is the weighted average of displacement functions of 4 physical covers; that is, they are connected through weight function $N_j(x,y,z)$. Then, the overall displacement function of the manifold unit can be expressed by:

$$\begin{aligned} \mathbf{U}^h(\mathbf{X}) &= \sum_i^4 N_{ji}(x,y,z)\left[\mathbf{U_i} + \boldsymbol{\theta}_\mathbf{i} \times (\mathbf{X} - \mathbf{X_i})\right] \\ &= \sum_i^4 N_{ji}(x,y,z)\mathbf{A_i}(\mathbf{X})\mathbf{Q}_i = \sum_i^4 \mathbf{T}_i\mathbf{Q}_i \quad . \end{aligned} \tag{3}$$

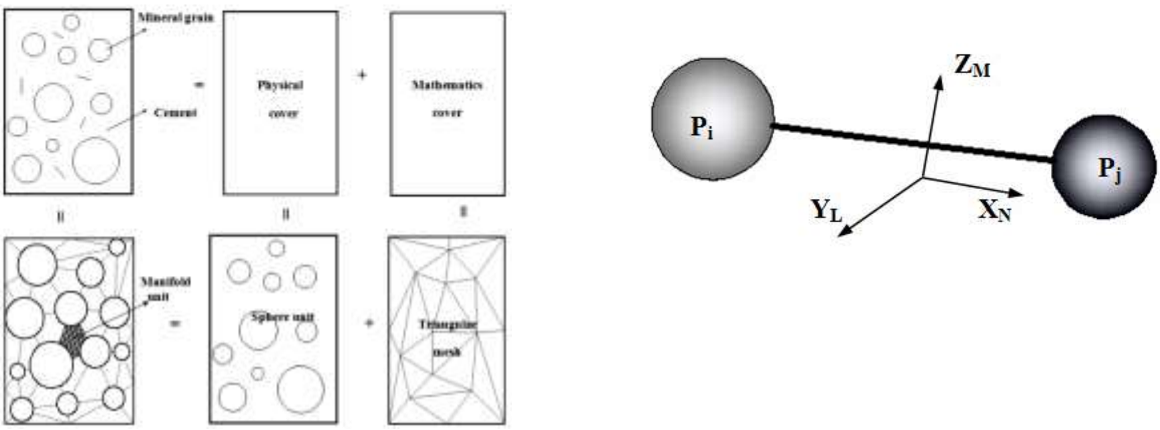


Fig. 2. MLD unit schematic diagram.

4. Numerical results

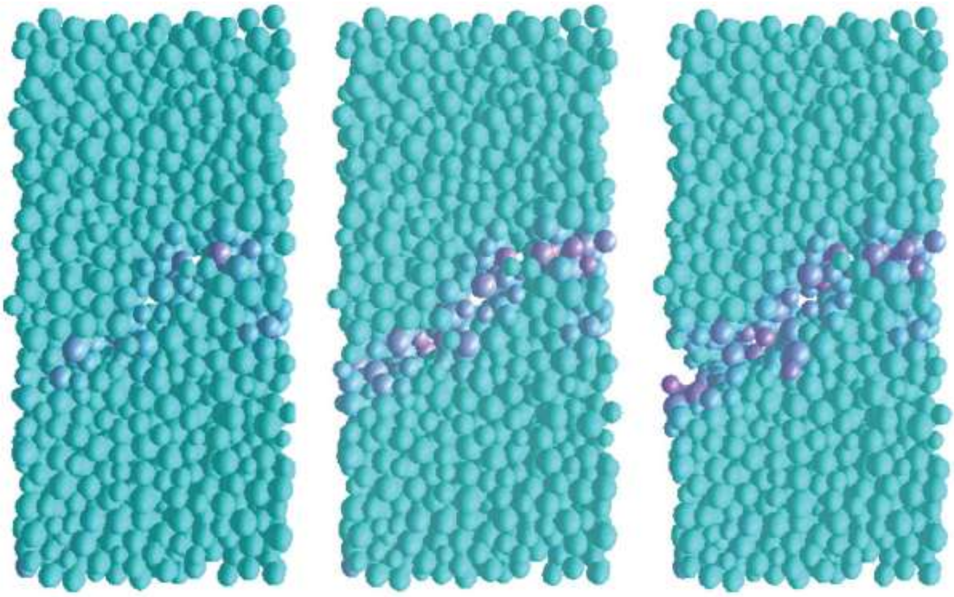
Simulation of the uniaxial tensile test. The sample size is 50 mm× 50 mm ×100 mm. The number of ball particles is 2800, the maximum diameter is 3.5 mm. The model contains a jointed, and its inclination is 45°. The physical computing parameters of the computing model are listed in Table. Compared to the particles, impose force between particles instead of on particles to simulate the acting force of cements. Based YADE platform, using ParaView triangular mesh, the calculation model is as shown in Fig. 3.

Fig. 4 shows crack path of rectangular model under tension for various time steps; (a) Step 135, (b) Step 155, (c) Step 175.

Physical-mechanical properties of mortar sample

Medium	Density ρ /(g.cm-3)	UCS Compressive strength σ_c /MPa	Cohesion /MPa	Elasticity Modulus E_C /GPa	Internal friction angle /°	Poisson's ratio μ
Particle	2.2	36.9	4.95	10.6	11.96	0.21

Fig. 3. Computational model.



(a) Step 135 (b) Step 155 (c) Step 175

Fig. 4. Tensile failure process

Conclusion

This paper, based on the three-dimensional manifold cover, constituted the three-dimensional manifold cover function and proposed a new discrete element numerical method based on manifold cover, Manifold Lattice Discrete, combining with the particle contact model. This method can be applied to the entire process of analyzing rock-like brittle material failure. It also verifies the accuracy of the proposed numerical method and feasibility of crack propagation and rock failure analysis through numerical examples.

MLD method inherited structural response process of discrete particles, and use the modeling and analysis environment of the YADE. It can be performed to analyze stress and strain of each polyhedron unit, reflecting the heterogeneity of the material characteristics. Build three-dimensional model, the static and dynamic fracture process of the material can be analyzed from the perspective of macroscopic and microscopic. This method has good research and application prospects.

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Метод дискретных элементов для расчета распространения трещин в массиве горных пород

Аннотация: Применен численный метод для расчета распространения трещин в массиве горных пород. Показана эффективность его применения для хрупких горных пород.
*Ключевые слов*а: макро- и микроструктура, MLD-модель, теория покрытия многообразий, распространение трещин, цифровое моделирование.