Multi-Scale Seismicity Model for Seismic Risk

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Abstract For a general use of the frequency-magnitude (FM) relation in seismic risk assessment, we formulate a multi-scale approach that relies on the hypothesis that only the ensemble of events that are geometrically small, compared with the elements of the seismotectonic regionalization, can be described by a log-linear FM relation. It follows that the seismic zonation must be performed at several scales, depending upon the self-similarity conditions of the seismic events and the linearity of the log FM relation, in the magnitude range of interest. The analysis of worldwide seismicity, using the Harvard catalog, where the seismic moment is recorded as the earthquake size, corroborates the idea that a single FM relation is not universally applicable. The multi-scale model of the FM relation is tested in the Italian region.

Introduction

In the last decades, increasing attention has been paid to seismic hazard (Cornell, 1968; Working Group, 1995) and seismic risk assessment (Molchan *et al.*, 1970; Caputo *et al.*, 1974; Keilis-Borok *et al.*, 1984). The generally accepted methodology for risk assessment includes the following interrelated steps: (1) seismic zoning, that is, the identification of potential earthquake source zones; (2) construction of a seismicity model; (3) construction of a spatial model of strong-motion effects as a function of event location and size; (4) risk assessment based on models (1) through (3), that is, estimation of the probability for a given effect (peak ground acceleration at a site, total economic losses, or the number of injured people in an area), to exceed a fixed threshold during a time interval T.

We will deal with the second step, that is, with the construction of a model for the sequence of main events (not aftershocks). The usual description of long-term seismicity ($T = 50$ to 100 yr) is based on the assumption that, in a given region, the earthquakes follow a random distribution (Poisson hypothesis) and the Gutenberg–Richter (GR) law. More specifically, it is assumed that the numbers $N(\Delta)$ of main events in the elementary cells $\Delta = dg\, dM\, dt$ (g is spatial coordinate, M is magnitude, t is time) are statistically independent and follow the Poisson distribution with mean $\langle N(\Delta)\rangle = n(g,M)\Delta$, where

$$\log n = a - bM, \quad M \in (M_-, M_+) \qquad (1)$$

and each of the parameters (a,b,M_+) has a different space variable dependence. The quantity M_- represents the threshold for damaging earthquakes, while M_+ is treated as the maximum magnitude.

This model is generally considered satisfactory for small and moderate earthquakes, while for the largest events, some authors actually suggest a formal smoothing of relation (1), truncated in the high magnitudes range. For example, the Kulbak principle of maximum entropy leads to the model (Main and Burton, 1984; Kagan, 1991, 1994)

$$\log n = a - bM - 10^{\theta(M-M_0)}, \quad \theta = {}^3\!/_2, \qquad (2)$$

with parameters a, b, and M_0. In (2), log n rapidly decreases around M_0, so that it can be treated as the *effective* maximum magnitude (an analog of M_+).

In other approaches, the seismicity rate $n = n(M)$ around M_+ is transformed into one or several peaks that are supposed to describe the rate of characteristic earthquakes (Schwartz and Coppersmith, 1984) or that of their "cascades," that is, multiple segment earthquakes (Working Group, 1995). The time behavior of the characteristic events is modeled as a nonpoissonian renewal process, and, very importantly, the events are related to a whole fault rather than to a point, as it is usually assumed in application of (1).

The question of what model is preferable has been the subject of lively debate; for example, see the discussion in Wesnousky (1994, 1996) and Kagan (1994, 1996). The debate seems to us to reflect the contradiction between two paradigms that recently appeared in seismology. One paradigm, formulated by Bak and Tang (1989), is related to a new insight into the dynamics of the Earth's crust. They treat the seismic process as a dissipative dynamic system that follows the mechanism of Self-Organized Criticality (SOC), and this implies that (1) is valid in a wide range of M, with a possibly universal parameter **b**. The other paradigm is based on observations and is related to the Characteristic Earthquake (CE) concept (Schwartz and Coppersmith, 1984). The CE has, by definition, the largest possible magnitude for the fault considered and a significantly higher rate of occurrence than the one predicted by (1).

A compromise is adopted in the recent, conceptually important work on seismic hazard for southern California (Working Group, 1995). Each zone is assumed to have randomly distributed earthquakes with a universal **b**-value, **b** = 1, plus characteristic earthquakes on specific faults; additionally, "cascades" of characteristic events can happen as well. Unfortunately, the time dependence of the large earthquakes in this model is described by a large number of parameters.

The leading idea of this article is that in a specific seismogenic zone the SOC paradigm can be applied with some limitations, depending upon the zone itself. Roughly speaking, the (scaling) relation (1) holds for those events whose linear size (l_M) is small compared with the "physical linear size" of the zone (L), that is, $l_M \ll L$. It follows that the description of the recurrence of events, with size $M \in \Delta M$, in a point g, based on the GR law, should be made by finding a zone (containing the point) with the appropriate dimensions and by relating the considered events to that zone rather than to the point. Therefore, depending upon the physical features of the area under study, several levels (scales) of seismic zoning may be needed rather than a single one. Each zoning scale must match the size range of the considered seismic events, ΔM. This scale is due, not to certain features in the spatial distribution of earthquakes of different magnitudes (Woo, 1996), but to event self-similarity conditions within a unit (an elementary area) of the regionalization and to the linear representation of log $n(M)$ in the range ΔM. This idea is illustrated schematically in Figure 1: on a small (detailed) scale (L_1), the area is divided into 10 seismogenic parts where the GR law is satisfied in the range $[M_-, M_1]$, while on a larger scale (L_2), there are three macrozones where the GR law is satisfied in the range $[M_1, M_2]$. The **b**-value depends on the zoning unit and on the magnitude range only, while the differences in the **b**-value in a point for different zoning scales must indicate a change in the self-similarity conditions for events of different size. This approach largely overcomes the contradictions between the two paradigms, in fact, the earthquakes in the range $[M_1, M_2]$ may turn out to be characteristic events for zones of the first level (L_1), while following the GR law in zones of the next larger level (L_2). However, in the framework of the SOC paradigm, we cannot usually predict the shape of the tail in the magnitude distribution; therefore, the occurrence rate for the largest magnitudes may remain unknown. The log $n(M)$ relation in Figure 1, for the maximum scale, (L_3) or ($M > M_2$), can have an unknown nonlinear shape.

The condition $l_M \ll L$ (possibly in the weaker form $l_M \leqq L$) is not new in seismology (Caputo *et al.*, 1973), and it appears in the recent articles by Pacheco *et al.* (1992), Romanowicz (1992), and Rundle (1993) who examine, for the global seismicity, the departure of log $n(M)$ from a straight line, in the range of large magnitudes. The departure from linearity for spatially unbounded seismogenic zones has been explained by the saturation effect of the transverse dimension of the fault, as a result of the finite thickness of the brittle zone. If one recalls the usual difficulties with catalogs—small amount of homogeneous data and saturation of all magnitude scales—a statistical substantiation of the above effect is not easy and has been disputed by Kagan (1997).

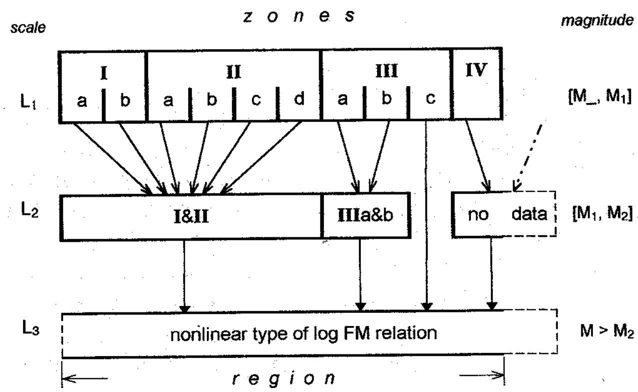

Figure 1. Diagram of the multi-scale representation of the FM relation: log FM is linear in each zone with scale L_1 or L_2 in the range of M: $[M_-, M_1]$ or $[M_1, M_2]$, respectively. **a**- and **b**-values depend on the zone; at the same time, **a** depends on the position g in the zone. Dotted lines indicate the possibility to extend the zone. The zone identification is related to the Italian region (see Figs. 2 and 3).

Here (1) we discuss the idea behind the multi-scale model $n(g, M)$; (2) we present statistical arguments against the universality of the parameter **b**, using the Harvard global centroid moment tensor catalog of earthquakes; and (3) we illustrate our multi-scale model on Italy, where a unique 1000-yr historical catalog is available. A full multi-scale analysis of Italian seismicity can be found in Molchan *et al.* (1996).

Multi-Scale Representation of the Frequency-Magnitude Relation

Seismic risk estimation requires that the seismicity within a set of seismogenic zones is modeled in the best possible way (strictly speaking, the term "best" cannot be defined, since the risk problem has many targets and a model shows an integral effect). So far, the commonly accepted tool to deal with the problem is the GR law and the Poisson hypothesis. The latter assumption permits to consider separately individual seismogenic zones. The choice of a zone is influenced by seismotectonic and geological considerations that are used to provide evidence that the zone is homogeneous with respect to a number of parameters, in particular, the **b**-value. Fixing a zone implicitly introduces a characteristic scale (L) related to the spatial structure of the dominant fault system in the zone and to the physical conditions there, L may be determined by one of several quantities that ultimately control the earthquake size in that zone: for example, the fault length, the thickness of the brittle layer, and the typical distance between rare (compared to the timescale T) events in the zone.

Crustal dynamics is frequently treated as a nonlinear process close to a critical state [e.g., Turcotte (1995) and the references therein]. The idea finds its theoretical support in the Bak *et al.* (1988) model in which, starting from any initial state, a dynamic system with many degrees of freedom will, by itself, attain a critical state—the SOC phenomenon. Usually these processes involve phenomena such as fractality, self-similarity, and power-law relations with "universal" scaling exponents. The GR law belongs to these laws when the size of the earthquakes is expressed in terms of the seismic moment or of the energy. Therefore, in seismology, the SOC paradigm is essentially based on the GR law. However, the appearance of a characteristic scale, L, and the consideration of a limited and fixed zone can violate the self-similarity and the universality of the scaling exponents, at least for $l_M \geqq L$.

Since a satisfactory mathematical modeling for the lithosphere dynamics is missing—we are still in the pre-equation era—we can support our previous considerations only using some analogies, taken from other fields of science.

In a deterministic framework, narrowing the observation of a homogeneous fractal set to a restricted volume S, we can merely get an accidental idea of the statistics of large clusters, since the characteristic cluster (comparable in size to the volumes) can be found in- or outside of the observation volume.

In a probabilistic framework (fractal random set), there is a difference between conditional and unconditional distributions (see Palm measures in the theory of point processes). Namely, the conditional distribution of a cluster K, under the condition that K belong to a fixed area or that the convex hull of K contains a fixed point g_0, and the unconditional one are different. For example, when the time interval between two subsequent Poissonian events contains a point fixed beforehand, the mean length of this interval is twice the mean length of the unconditional case [a well-known paradox for Poissonian processes (Feller, 1966)].

In physics, the Kolmogorov theory of well-developed turbulence (Landau and Lifshitz, 1959) is based on the hypothesis of self-similarity for turbulent pulsations and successive transmission of energy from larger pulsations to smaller ones, and it defines the so-called inertial range of scales, r, in which turbulence scaling is assumed; namely,

$$\mathrm{Re}^{-3/4} \ll r/L \ll 1, \tag{3}$$

where L is the external scale, $\mathrm{Re} = LV/\nu$ is the Reynolds number (V is the characteristic velocity and ν the molecular viscosity).

Consequently, keeping to the standpoint of nonlinear dynamics and treating an earthquake as a spatial object rather than a point, one may assume that the GR law is valid for the earthquakes with linear dimensions, l_M, much smaller than the characteristic scale, L, of the seismic zone considered (hypothesis **A**).

Risk analysis is concerned with damaging ($M \geqq 3.5$) and therefore relatively large earthquakes, thus hypothesis **A** is the analog of the right-hand side of (3). On the other end, a seismological analog of the left-hand side of (3) is of theoretical interest, as described by Aki (1987), who shows a significant departure of log $n(M)$ from a straight line for $M < 1.5$.

In a seismic zone, several characteristic scales can be identified. Caputo *et al.* (1973) distinguish three critical magnitudes ($M_1 < M_2 < M_3$) that can be observed in the statistical properties of earthquake occurrence:

- up to M_1, the source area of an event is small compared with the geometrical dimensions of the main tectonic faults in the zone;
- for $M > M_2$, a rupture penetrates the entire crust, so that the earthquake size can only be increased by increasing the source length, l (the earthquake source has lost one degree of freedom because of the saturation with depth); and
- for $M > M_3$, an earthquake can occur within a single isolated zone only by simultaneous slippage on several faults.

Pacheco *et al.* (1992) and Okal and Romanowicz (1994) give estimates for the saturation of the earthquake size with depth: $M_2 = 6.0$ for mid-ocean ridges (MOR) where the down-dip width of the seismic source zone, w, varies from 10 to 15 km, and $M_2 = 7.5$ for shallow earthquakes in subduction zones (S) where w is about 60 km.

The above-mentioned critical magnitudes indicate the existence of different conditions for the self-similarity of source zones. This may affect the scaling laws, that is, the **b**-values. For this reason, if a zone has several characteristic scales, one would expect the log frequency-magnitude (FM) relation to be piecewise linear, and then the parameters $M_\pm$ in (1) control the size range of the events, ΔM, for which the self-similarity conditions are fulfilled. For instance, if $l \gg w$, then two ranges of linearity of log FM are possible: (M_-, M_1) and (M_2, M_3). In practice, the interval between the two ranges may degenerate to a point, in order to fit the log FM relation when few data are available. Starting from some magnitude, say M_3, the self-similarity conditions are no longer valid for a single seismic zone, and then relation (1) can break down for large M because of the hypothesis **A**.

The idea of the characteristic earthquake (Schwartz and Coppersmith, 1984) is an important attempt to forecast the form of the FM relation for a fault segment when the linear relation (1) is no longer applicable. This idea, as advocated in its orthodox form (Wesnousky, 1994), runs into serious difficulties (Working Group, 1995; Kagan, 1996). On the other hand, the alternative solution (2), based on a formal device, borrowed from information theory, rather than on the physics of the phenomenon, involves an arbitrary choice of a function ψ of the energy E, $\psi(E)$, or of the magnitude

M, $\psi = \psi(10^{\theta M})$ with $\theta = 3/2$. If we fix the mean value of ψ, the principle of maximum entropy in combination with the GR law leads to a new form of the FM relation:

$$\log n(M) = a - bM - \lambda\psi(10^{\theta M}), \tag{4}$$

where a, b, and λ are parameters. If one takes into account the relation of the earthquake energy with the size of the rupture zone, the case $\psi(E) = E^{\rho}$ ($\rho = 2/3$) can be considered as well, and since the statistical estimation of the exponent ρ has a very weak resolution (Kagan, 1991), many other models like (4) can be claimed to fit observed data.

As a rule, to predict the frequency of earthquakes using a linear log FM relation, the larger the event, the greater must be the geometrical dimension of the zone, so that it is of the appropriate scale level for (1) to be valid in the ΔM of interest. However, the zone-broadening process has a limit, since physical factors will interfere with the self-similarity conditions for large events (e.g., Caputo *et al.*, 1973). A hierarchical analysis is then reasonable in which the seismicity is described by several GR laws for several scales and magnitude ranges ΔM (see Fig. 1 and the last section). In this hierarchical analysis, the smaller events may be less useful for the prediction of the occurrence of the larger events.

There are serious obstacles to the use of conventional statistical techniques in the estimation of the maximum magnitude. The statistical technique (Pisarenko *et al.*, 1996) is based on the parametrization of $n(M) = n(g, M)$ for large M, but the SOC paradigm does not permit to predict the recurrence of very large events in the same zone used to predict the smaller events. The multi-scale representation of the FM relation gives, in the best case, a piecewise linear representation of $\log n(g, M)$, which is not universal and is dependent on point g.

Using theoretical arguments, we can infer that the variability of the estimated b-value should not necessarily be explained by appealing to criticisms of the magnitude scales involved and to the poor quality of the data (Kagan, 1994). There is merely a drawback in the current interpretation of the **b**-value determined considering restricted areas. The parameter **b** is representative only for a definite scale range. It is therefore necessary to show that the parameter **b** is not universal also from a statistical point of view.

Variation of the b-value: Global Seismicity

Kagan (1994), from the study of global seismicity, assumes that the **b**-value is universal: **b** = 1 for all events and **b** = 0.75 for mainshocks (excluding aftershocks). Any variation of the **b**-value is treated by Kagan (1994) as an artifact due to the small size of the samples considered and to the fact that the magnitudes used in regional studies are inhomogeneous and have not a clear physical meaning. On the other end, using the same data analyzed by Kagan (1997), we provide here several examples of statistical comparison in major seismic provinces that indicate that the b-value is not universal.

Data. For the purposes of seismic risk determination, it is natural to consider shallow earthquakes (with focal depth less than 70 km), and a homogeneous catalog, reporting a physically meaningful measurement of the earthquake size, is needed. The available possibilities are rather limited. There is a short global centroid moment tensor (CMT) catalog (CMTS, 1995) that reports the scalar seismic moment M_0 (dyne-cm) or the moment magnitude $M_w = 2/3$ (log M_0 − 16.1). As of 30 April 1995, the catalog contains 12,417 events, with depth $H \leqq 70$ km, and is, in our estimate, complete for $M_w \geqq 5.75$, $\geqq 5.55$, $\geqq 5.45$ starting from 1977, 1982, and 1987, respectively.

The absence of smaller events in this catalog does not permit the use of refined techniques of aftershock identification (Molchan and Dmitrieva, 1992); therefore, the aftershocks have been identified by the window method. The spatial radius, R, and the time duration of the aftershocks sequence, T, are as follows (Molchan and Dmitrieva, 1992):

M_w	5.5–6.5	6.5–7.0	7.0–7.5	7.5–8.0	≧8.0
R (km)	50	60	70	100	200
T (years)	1	2	2	2	2

Technique. The elimination of aftershocks lends more credence to the Poisson hypothesis in the estimation and comparison of the parameters **a** and **b** in the GR law. The solution of this problem for arbitrary grouping of the data, over magnitude and time, is given in Molchan and Podgaetskaya (1973) and summarized in the Appendix, where the hypothesis H_b of equality of the **b**-value in several samples that obey (1) is tested using the generalized Pearson test, π. The probability ε of excedence of the observed value π_{obs} under the hypothesis H_b gives the significance level of H_b. The hypothesis H_b is doubtful, when ε is small (i.e., $\varepsilon \leqq 5\%$).

Examples. In the time–magnitude intervals in which it is complete, the CMT catalog contains 6776 events, of which 4832 are mainshocks (71%). With this amount of data, we can analyze credibly the **b**-value for the major seismotectonic features only. In this case, we have $l \gg w$, so that the critical characteristic scale is the down-dip width of the fault zone, i.e., $L = w$ for small events and $L = l$ for the large ones. We begin with a well-known example.

1. *Subduction zones and mid-ocean ridges.* According to Okal and Romanowicz (1994), widely different values of w characterize subduction (S) and MOR zones: 60 and 10 km, respectively. The earthquakes with M_w between 5.8 and 6.5 are "small" (there is no saturation along the down-dip width of the zone) for the S zones, and "large" for the MOR zones (there is saturation). The differences in self-similarity conditions for the two source zones do affect the **b**-value.

Table 1 (row 1) contains the **b**-value estimates based on all events (b_+) and on the mainshocks (**b**) for S and MOR zones. The differences in the **b**-value are so large that neither any estimation techniques nor various methods of aftershock identification can remove the effect. The difference is expressed quantitatively by the significance, ε, of the hypothesis H_b.

2. *Subduction zones: two magnitude ranges.* Events with $M_w \geqq 7.5$ are large for an S zone (Pacheco *et al.*, 1992); therefore, the equality of the **b**-value in the ranges $M_w \leqq 7.5$ and $M_w \geqq 7.5$ is doubtful. The conclusion is corroborated statistically in Pacheco *et al.* (1992), who used a combined worldwide catalog for the period 1900 to 1989. The CMT data also indicate a significant change in **b** for $M_w \geqq 5.55$ [see Table 1 (row 2)].

The significant difference in the **b**-value between the S and MOR zones is not exceptional; in fact, we show that both zones are internally inhomogeneous with respect to the **b**-value. In the next two examples, we identify subzones by employing strictly seismotectonic arguments without any preliminary data analysis.

3. *Mid-ocean ridges: two subzones.* The MOR zones are segments of rift zones that are cut by transform faults and dominated by pure strike-slip movement. There are two seismogenic transform faults in the Mid-Atlantic Ridge (MAR) that are abnormal for their linear size ($L \approx 2000$ km in both cases): the Azores-Gibraltar (AG) ridge and the Equatorial (E) fault. We compare the **b**-value for the union (Σ) of the transform faults in AG and E and for its complement (MAR\Σ) in the Mid-Atlantic Ridge zone. Table 1 (row 3) shows that the **b**-values are different at the significance level $\varepsilon \approx 5\%$: $\mathbf{b} \approx 1$ for AG and E and $\mathbf{b} \approx 1.3$ for its complement in the MAR zone. $\varepsilon < 5\%$ if we compare the Σ zone ($\mathbf{b} = 0.97$) with its complement in the MOR zone ($\mathbf{b} = 1.25$), where the number of data is 547 instead of 107.

4. *Island arcs: two subzones.* Following Kronrod (1985), we consider the island arcs in the Northwest Pacific, from Alaska to Taiwan, divided into two sets by their tectonic characteristics: volcanic arcs (V): Aleutians-Commander Is., Kuril Is., Ryu-Kyu, Izu-Bonin, Marianas; geosynclinal arcs (GS): Alaska, Calgary coast, Kamchatka, Japan, and Taiwan. The CMT catalog corroborates an earlier inference by Kronrod (1985), based on the pre-1975 worldwide catalog data, about the existence of a significant difference in the **b**-value for these subduction zones [see Table 1 (row 4)].

5. *Subduction zone: three ranges of depth.* The distribution of centroid depths, H_c, in the CMT catalog has two distinct peaks at 10 and 15 km, and a fuzzier one at 33 km. Because of the difficulties inherent in the determination of H_c, the values $H_c = 10$ km or $H_c = 15$ km are just markers (used during different time periods) of shallowness. For this reason, we consider the following division of the H_c scale: up to 15 km, from 16 km to 33 km, and from 34 km to 70 km. This grouping divides all data into three roughly equal parts. In view of the effect of saturation along the down-dip

Table 1
Global Seismicity: **b**-Value Comparison

	Zone		Magnitude Range, M_w	All Events N	All Events b_+	Mainshocks N	Mainshocks $\mathbf{b} \pm \Delta b$*	ε†
1	Subduction zone, S‡		≧5.88	1761	0.98	1233	0.88 ± 0.05	
								<0.05%
	Mid-ocean ridges, MOR‡		≧5.88	313	1.49	298	1.47 ± 0.16	
2			5.55–7.56	3012	0.95	1927	0.80 ± 0.04	
	Subduction zone, S‡							<4.9%
			7.57–8.90	36	1.72	32	1.50 ± 0.70	
	Mid-Atlantic							
3	Ridge,	Σ = AG & E¶	5.45–8.00	71	1.0	61	0.97 ± 0.25	
								<5.4%
	MAR§	MAR\Σ	5.45–8.00	111	1.38	107	1.30 ± 0.22	
4	Island	V	5.45–8.90	519	1.08	350	0.97 ± 0.10	
								<0.5%
	arcs‖	GS	5.45–8.90	329	0.89	212	0.75 ± 0.10	
5	Subduction	$H_c \leqq 15$	≧5.45	965	1.05	639	0.93 ± 0.10	
		$16 \leqq H_c \leqq 33$	≧5.45	792	0.80	487	0.63 ± 0.10	<0.05%
	zone‖	$34 \leqq H_c \leqq 70$	≧5.45	689	0.92	459	0.83 ± 0.11	

*($\mathbf{b} - \Delta b$, $\mathbf{b} + \Delta b$) is the 95% confidence interval for the **b** value. The estimate b_+ is not supplied with a confidence interval because the data are correlated.

†Significance level of the hypothesis H_b: all **b** are equal.

‡S and MOR zones, as in Kagan (1997), include the following Flinn–Engdahl seismic regions (Young *et al.*, 1996): *S* (1, 5–8, 12–16, 18–24, 46); MOR (4, 32, 33, 40, 43–45).

§MAR is zone 32 in Flinn–Engdahl seismic regions (Young *et al.*, 1996).

¶Geographical limits of the zones. AG = (35.6° N, 40.0° N) × (−60° W, −29.7° W), E = (2.1° N, −3.4° E) × (−12.0° W, −31.4° W).

‖See text for the subzone symbols.

width of the zone, we eliminate large (for an S zone) events, that is, events with $M_w > 6.5$, and compare the **b**-values for the three ranges of depth. Table 1 (row 5) shows that the confidence level for the hypothesis of equal **b**-values in the three ranges of H_c is extremely low, $\varepsilon < 0.05\%$. The difference is mainly due to the lower **b**-value obtained for $H_c \in$ [16 to 33] km. This fact is difficult to interpret from a physical point of view, because the depths of shallow earthquakes in the CMT catalog are occasionally incorrect; however, the assumption of a universal **b**-value permits any formal grouping of the data. Therefore, these results can be viewed as another confirmation that in the S zone the **b**-value is not constant.

These five examples show statistically significant variations in the **b**-value for $M_w \geqq 5.55$. The actual **b**-values can depend on the magnitude type used; however, these variations are generally consistent with similar conclusions reached using other magnitude scales. Our results contradict the statement by Kagan (1997) that "there is no statistically significant variation of the **b**-value for all seismic regions except for the mid-ocean ridge systems." The Kagan's conclusion can be explained by the fact that the choice of the CMT catalog dramatically lowers the resolution capability of the tests. Therefore, we use here a technique of great flexibility (see Appendix), and we consider an expanded data set—the starting date of the catalog considered depends on magnitude, and the final date is 8 months later than in Kagan (1997).

FM Relation for Italy

To illustrate the multi-scale representation of the FM relation, we chose Italy as a test region, since there is available a unique historical mainshock catalog that covers a time interval of about 1000 yr (Stucchi *et al.*, 1993) and that is particularly suitable for the analysis of the recurrence of the large events ($M > 5$). The other difficulties inherent with regional earthquake catalog remain; thus, while having a gain in the time span and energy range, compared with the CMT data, we lose in quality, since the earthquake size is usually represented by different magnitudes of different accuracy and representativity. This puts stringent requirements on the data-processing techniques and may hamper the interpretation of any result.

For the period 1900 to 1995, we use a catalog that we have labeled the *Current Catalog of Italy* (CCI, 1994), where the dominant magnitudes are local, M_L, duration, M_D, and macroseismic, M_I. The magnitude used in the analysis is the first available in the sequence: M_L, M_D, M_I. Magnitude $M = M_L$ is grouped in intervals of 0.2 to 0.3, while magnitudes $M \neq M_L$ are grouped in intervals of 0.5. The time boundaries of complete reporting for M are adopted depending upon the location, the value, and type of M. The aftershocks are identified using a flexible minimax approach developed by Molchan and Dmitrieva (1992). We use aftershock areas (of confidence level 95%) to identify different, interacting seismogenic zones, and the technique for the estimation and comparison of **a** and **b** in the GR law is described in the Appendix. The comparison of the **b**-value determined in several different areas is employed as an extra argument in favor of zone broadening or narrowing.

The value $M_I = 7.3$ defines the maximum magnitude observed in Italy during the last 1000 yr. For this reason, the representation of the FM relation for risk purposes is relevant for shallow events with $M \in [3.5$ to $7.5]$. Therefore, a nontrivial FM representation can involve no more than two or three scale levels, and a possible multi-scale model of this type is presented formally in Figure 1. In what follows, we limit our attention to space scale l, rather than to physical scale L, since the available data do not allow its definition.

The largest space scale (l_3) is controlled by plate tectonics and by the size of the area under study. According to Lort (1971), the Adriatic region is regarded as an African promontory at the plate boundary between Eurasia and Africa. The boundary is well marked in Figure 2b by the largest earthquakes ($M > 6.3$), and it justifies to keep the region as a whole, with $l_3 \approx 1500$ km.

Since the data ($M > 6.3$) in Figure 2b span about 1000 yr, and moderate events are well dispersed over the entire region (Fig. 2a), the alignment of epicenters (at least for central and southern Italy) cannot be casual. Hence, the kernel standardized technique recently suggested by Woo (1996) to smooth maps of earthquake activity **a**(g, M) calls for careful handling.

The intermediate space scale ($l_2 = 400$ to 500 km) is in part controlled by the geometry of the plate boundary. From a tectonic point of view, Italy is conventionally divided into four zones (Boriani *et al.*, 1989): (I) Alpine compression zone, (II) Northern Apennine Arc, (III) Calabrian arc, and (IV) Sicily with a possible continuation toward Tunisia. The **b**-zones of space scale level l_2 are represented in Figure 2b where **b**-zones I and II are lumped in a single one, because the **b**-values for the larger events are similar. From now on, **a**-zone and **b**-zone will indicate zones with a postulated constant value of **a** or **b**, respectively.

The lowest space scale ($l_1 = 200$ to 300 km) for zones having constant **b**-values is determined by hypothesis **A**: $l_M \ll l_1$. We use the aftershock zone linear size (L_{aft}) to estimate l_M. According to our analysis, for $M \in [4$ to $6]$, the typical values of L_{aft} are 20 to 60 km, while for $M \in [5$ to $7]$, a few observations give $L_{aft} = 100$ to 140 km; for $M \geqq 3.5$, a further splitting of the **b**-zones generally leads to a poorer resolution of the **b**-value. Some seismotectonic and statistical considerations have led us to define 10 **b**-zones of level 1 (Fig. 3), composed by elements of the seismotectonic regionalization of Italy, recently developed by GNDT (1992). Each element of the GNDT zoning includes seismogenic structures of definite kinematics type (see Fig. 3) and has typical dimensions of 40 to 130 km by 20 to 30 km, comparable with L_{aft}. Most GNDT zones contain a small number of instrumental data with $M \geqq 3.5$; it is therefore impossible

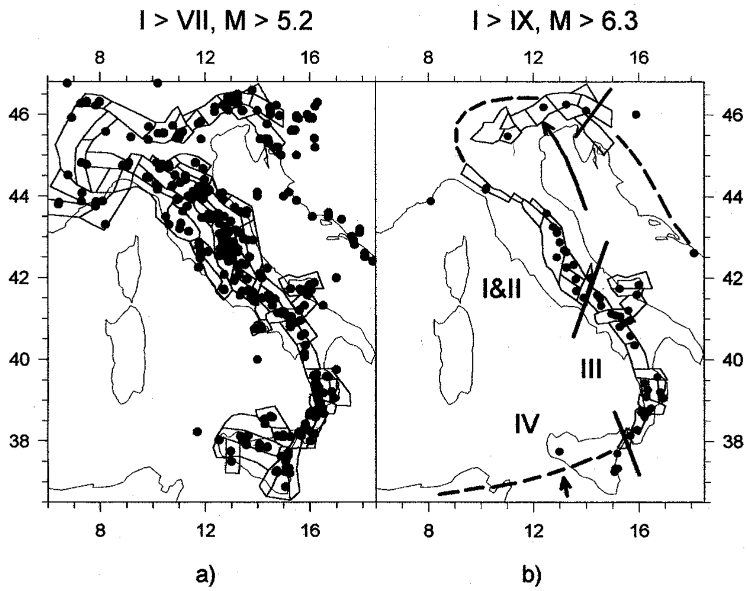

Figure 2. Space distribution of mainshocks (solid circles) for the period 1000 to 1980 according to Stucchi *et al.* (1993) and seismogenic zones after GNDT (1992). Right panel: solid segments mark the boundaries between **b**-zones of scale l_2; the indicated GNDT zones forms the hypothetical seismogenic area for events with $M > 6.3$; dashed line is the sketch of the plate boundary between Eurasia and Africa in the Italian region (Lort, 1971); arrows show the direction of motion relative to Eurasia.

to subdivide them, when they are used as **a**-zones of scale level 1.

Table 2 presents the estimated **b**-values for two levels: level 1 is appropriate for $M \in$ [3.5 to 5], while level 2, for $M \in$ [5 to 7]. The historical data are used for the **b**-value estimates of level 2 only. The data indicate that the magnitude range [5 to 6] can be considered as an intermediate one. Although the events with $M \in$ [5 to 6] essentially control the **b**-value in the entire range [5 to 7] (see Appendix), we have statistically significant difference in the **b**-value for the two scale levels. The physical and statistical nature of the variation in the **b**-value, reported in Table 2, calls for a special study.

Conclusion

This article is an attempt to derive from the SOC paradigm corollaries relevant to seismic risk problems (e.g., Main, 1995; Woo, 1996). Our conclusions are these:

- Given a seismic zone, the conventional description of seismicity puts conditions on the scale and the magnitude range to be considered when representing the FM relation in log linear form. Hypothesis **A** leads to a multi-scale representation of the FM relation preserving the log linear character. In this case, the query of Wesnousky (1994) "The Gutenberg–Richter or characteristic earthquake distribution, which is it?" could be answered: both. Large events can themselves form a statistical population having a GR law in a zone with the appropriate scale.
- The adoption of hypothesis **A** may reduce the number of parameters needed to describe the recurrence of the larger events, but, at the same time, hypothesis **A** reduces our statistical ability to estimate $M_{\max}$.
- Using the CMT catalog, a significant worldwide variation in the **b**-value has been found for $M_w > 5.5$, and this fact justifies the search for geometrical/physical factors that cause the regional variations in **b**.
- The analysis of the seismicity in Italy shows that the multi-scale approach can be used where the size range of damaging earthquakes is large ($\Delta M > 4$) and catalogs with a large amount of historical data are available.
- The multi-scale approach calls for an understanding of seismicity at different space–time scales.

Appendix

Seismic risk studies involve different groupings over magnitude, depending on space and time. This considerably complicates the statistical estimation and comparison of the parameters in the GR law, for a set of nonintersecting volumes in location-magnitude-time space. These problems are considered in the nearly inaccessible article by Molchan and Podgaetskaya (1973), which we summarize below. The technique is essential even for earthquake catalogs containing the scalar seismic moment, given in the form of an exponent and a two-digit prefactor, leading to nonuniform grouping over M_w.

*Estimation of (**a**, **b**).* Let $\{N_i\}$ be the sample of mainshocks in an area G in nonintersecting time–magnitude cells $\Delta T_i \Delta M_i$. Taking into consideration the GR law, we assume that N_i are independent and that they follow the Poisson distribution with parameter Λ_i (mean value of N_i); that is,

$$\Lambda_i = \Delta T_i \int_{\Delta M_i} 10^{\mathbf{a}+\mathbf{b}M} dM.$$

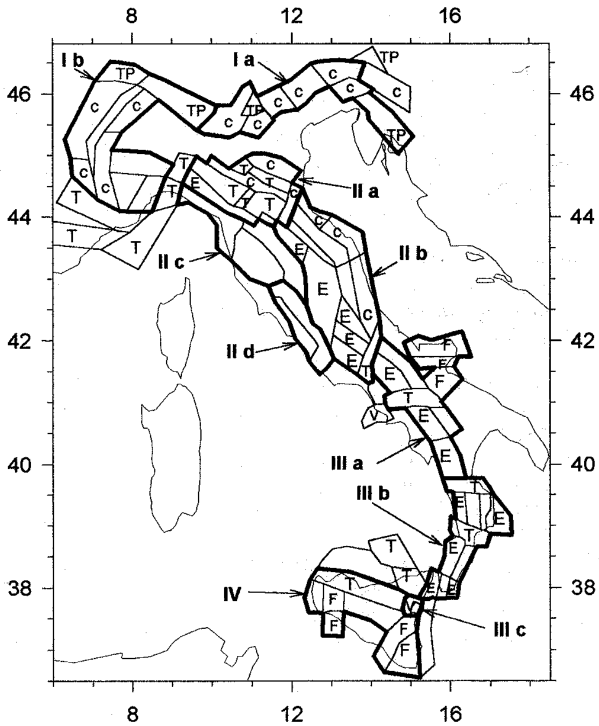

Figure 3. Seismogenic zones (solid line) from GNDT (1992), and **b**-zones of scale l_1 (bold line). Notation for GNDT zones: C, compressional areas; F, areas of fracture in the foreland zones; T, transition areas; TP, areas of transpression; V, volcanic areas. Notation for **b**-zones of scale l_2: Ia, southern branch of Eastern Alps; Ib, Western Alps; IIa, Northern Apennines; IIb, Central Apennines and Ancona zone; IIc, Tuscany; IId, Roma comagmatic zone; III, Calabrian arc [(a) northern branch and Gargano zone, (b) Centre, (c) Etna]; IV, Sicily.

The estimate of (**a**, **b**) is given by the point where the log likelihood of the data, $\mathcal{L}(G)$, reaches the maximum. Here

$$\mathcal{L}(G) = \sum_{i=1}^{k} \mathbf{l}(N_i \mid \Lambda_i),$$

where $\mathbf{l}(n \mid \Lambda) = n \ln\Lambda - \Lambda$.

The conditional log likelihood for N_i given the statistics $N = \Sigma N_i$, namely,

$$\mathcal{L}(G|N) = \mathcal{L}(G) - \mathbf{l}(N \mid \sum \Lambda_i),$$

is a function of **b** only. The statistics $\mathcal{L}(G|N)$ is approximately gaussian. This permits one to define a more exact distribution of $\mathcal{L}(G|N)$ using the Edgeworth expansion and six moments of $\mathcal{L}(G|N)$. Knowing the distribution of $\mathcal{L}(G|N)$, one can find a confidence interval for **b** (Cox and Hinkley, 1974).

Explicit formulas for **b** exist only in the following theoretical situation: $\Delta T_i \equiv T$, $\Delta M_i \equiv \Delta$, $M_+ = \infty$. The formulas are

$$\hat{b} = \begin{cases} \Delta^{-1} lg[1 + N / \sum N_i(i-1)], & \Delta > 0 \quad \text{(Kulldorf, 1961)} \\ Nlge / \sum (M_1 - M_-), & \Delta = 0 \quad \text{(Aki, 1954)} \end{cases} \tag{A1}$$

Table 2

FM Multi-scale Model for Italy (the **b**-value of each zone depends on space scale level and magnitude range)

Zones of Scale Level I (Fig. 3)	T*: 1900–1993 $3.5 \leqq M < 5.0$		Zones of Scale Level II (Fig. 2b)	T: 1000–1993 $5 \leqq M \leqq 7$	
	N*	$\mathbf{b} \pm \Delta b$†		N	$\mathbf{b} \pm \Delta b$
Ia,b; IIa,b,d‡	1491	0.89 ± 0.15			
			I&II	184	1.07 ± 0.13
IIc	169	1.32 ± 0.24			
IIIa	224	0.65 ± 0.16			
			IIIa,b	59	≤0.65§
IIIb	178	1.00 ± 0.20			
IIIc¶	189	1.02 ± 0.20		log FM is not linear	
IV	151	0.76 ± 0.20	IV&?‖	data are not complete	

*T is the time period covered by the catalog; N is the total number of used main events.

†($\mathbf{b} - \Delta b$, $\mathbf{b} + \Delta b$) is 95% confidence interval for the **b**-value.

‡All five zones are uniform with respect to the **b**-value; therefore, we show only the **b**-value estimate for one of them (Ia, $N = 275$) plus the total number of events in their union.

§We give a qualitative **b**-value estimate because of lack of data.

¶Etna volcanic zone: observed $M_{\max} = 5.2$. It is an interesting example of a small **b**-zone ($l = 45$ km) that contains the 95%-aftershocks area related to the mainshocks with $M < M_{\max}$.

‖Sicily zone must apparently be extended beyond the boundary of the studied area, toward North Africa.

The distribution of Aki (1954) estimator is known exactly. Namely, $\hat{\mathbf{b}}/\mathbf{b} = 2N/\chi^2_{2N}$, where the random variable χ^2_f has the χ^2 distribution with f degrees of freedom. It follows that $\hat{\mathbf{b}}$ has the bias $\Delta\mathbf{b} = \langle\hat{\mathbf{b}} - \mathbf{b}\rangle \approx \mathbf{b}/N$ and the standard deviation $\sigma_b = \mathbf{b}/\sqrt{N}$. The modified estimator $\hat{\mathbf{b}}^* = (1 - 1/N)\hat{\mathbf{b}}$ reduces the variance σ_b^2 and removes the bias.

The explicit formulas (A1) show that the contribution of the statistics of M_i in **b** estimates is proportional to N_iM_i. Since N_i is proportional to 10^{-bM_i}, the maximum likelihood (MLH) estimate of **b**, based on uniformly sampled data, depend rather weakly on the large events. Consequently, when the zone size and the magnitude range involved in the statistical estimation of a **b** are mismatched, the MLH estimate of **b** may represent the log FM relation correctly only among the smaller events of the range (events of different magnitudes are here assumed to be completely reported for the same time span).

*Comparison of the (**a**, **b**) Parameters.* Let us consider d nonintersecting volumes $V_a = (G_aT_aM_a)$ with their grouped data (G_a is a spatial area, T_a a time period, M_a a magnitude range). In order to compare $\mathbf{b}_a$ or $(\mathbf{a}, \mathbf{b})_a$ for different areas G_a, one usually has to test one of the following two hypotheses: all the $\mathbf{b}$-values for the volumes V_a are equal, whereas the $\mathbf{a}$-values may be arbitrary (hypothesis H_b: $\mathbf{b}_1 = \ldots = \mathbf{b}_d$), or all $(\mathbf{a}, \mathbf{b})$ pairs for volumes V_a are identical (hypothesis $H_{a,b}$: $\mathbf{b}_1 = \ldots = \mathbf{b}_d$, $\mathbf{a}_1 = \ldots = \mathbf{a}_d$). The general alternative to the hypotheses H_b and $H_{a,b}$ is H: $(\mathbf{a}, \mathbf{b})_a$, $\alpha = 1, \ldots, d$ are arbitrary; that is, there are no restrictions on $\mathbf{a}$ and $\mathbf{b}$. The hypothesis H_i, $i = \mathbf{b}$ or $i = (\mathbf{a}, \mathbf{b})$, against H is tested using the generalized Pearson statistics

$$\pi_i = -2\left[\max_{H_i} \mathcal{L}_\Sigma - \max_{H} \mathcal{L}_\Sigma\right],$$

where $\mathcal{L}_\Sigma = \sum_{a=1}^{d} \mathcal{L}(G_a)$ and the maximum is taken over the parameters $(\mathbf{a}, \mathbf{b})_a$ with due account of the relevant hypothesis H_i and H. The values of $(\mathbf{a}, \mathbf{b})_a$ that provide max $\mathcal{L}_\Sigma$ under the hypothesis H_i are maximum likelihood estimates in the general case.

The asymptotic theory of statistical hypothesis testing asserts (Cox and Hinkley, 1974) that H_i should be rejected in favor of H, if $\pi_i > u(f_i, \gamma)$. Here $u(f, \gamma)$ is the quantile of the χ^2_f-distribution of level $\gamma, f_b = d - 1, f_{a,b} = 2(d - 1)$. In this case, the probability of rejecting H_i when it is in fact true is approximately equal to $\varepsilon = 1 - \gamma$.

When the Aki (1954) estimators are valid, a test of the hypothesis H_b for the two regions G_1 and G_2 can be based on the exact distribution of $\hat{\mathbf{b}}_1/\hat{\mathbf{b}}_2$; namely,

$$\hat{\mathbf{b}}_1/\hat{\mathbf{b}}_2 = (\mathbf{b}_1/\mathbf{b}_2)\cdot(N_1/N_2)\cdot(\chi^2_{2N_2}/\chi^2_{2N_1}),$$

where the χ^2 variables are independent and N_i is the total number of events in region G_i. Therefore, if $\mathbf{b}_1 = \mathbf{b}_2$, the ratio $\hat{\mathbf{b}}_1/\hat{\mathbf{b}}_2$ for the two regions follows the F-distribution (Utsu, 1971).

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