

PII S0016-7037(01)00615-9

Calculation of the structural and NMR properties of the tridecameric $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$ polycation

J. A. TOSSELL*

Department of Chemistry and Biochemistry, University of Maryland, College Park, MD 20742, USA

(Received December 1, 2000; accepted in revised form March 5, 2001)

Abstract—The aluminum tridecameric polyoxocation, $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$ is a major component in partially hydrolyzed Al⁺³(aq) solutions and has been extensively studied experimentally, mainly using NMR techniques. I have calculated the equilbrium geometry of this cation using the Hartree-Fock method and a polarized double-zeta effective core potential basis set, obtaining bond distances which agree well with X-ray crystallographic studies of selenate and sulfate salts of the polycation (Al[4]: 4×1.85 Å calc, 4×1.84 Å exp: Al[6]: 4×1.85 , 2×2.05 Å calc, 2×1.84 , 2×1.88 , 1.91, 2.04 Å exp [where the numbers in brackets indicate the coordination numbers]). I have also calculated electric field gradients and NMR shielding constants at all the atoms using the standard 6-31G* basis set and Hartree-Fock and hybrid Hartree-Fock-density functional (B3LYP) techniques. Using the Hartree-Fock method, the central four-coordinate Al is calculated to be deshielded by \sim 56 ppm, and the six-coordinate Al atoms by \sim 16 ppm, vs. the Al(OH₂)⁺³ reference, compared to experimental shifts of 63 and 12 ppm, respectively. The central Al[4] is thus shielded by ~ 20 ppm with respect to the tetrahedral monomer Al(OH)₄⁻¹. Al-NMR shifts obtained from the B3LYP calculations are very similar. The calculated O-NMR shifts, vs. free gas-phase H₂O, are 17 ppm for the η -OH₂ groups, 30 ppm for the μ -OH and μ OH' groups, and 55 ppm for the μ_4 -O group, which match well with the experimentally assigned shifts of 20, 30, and 55 ppm, respectively (vs. liquid H₂O). The B3LYP method yields O shifts, which are systematically about 40 to 50% larger. It is not clear whether the discrepancies in the calculated O shifts vs. liquid water are a result of deficiencies in the model (neglect of the aqueous environment) or in the method (lack of correlation in the Hartree-Fock method). Studies on the $Al_2(OH)_2(OH)_2^{+4}$ cation with small numbers of explicit waters hydrogen-bonded to it indicate that O shifts can be perturbed strongly by the presence of solvent. The calculated ¹⁷O nuclear quadrupole coupling constants, NQCC (using the 6-31G* Hartree-Fock calibration factor of Ludwig et al., 1996 to relate electric field gradients to NQCC values) are 1.7 MHz for μ_4 -O, 7.8 to 8.0 MHz for the μ OH and μ OH' groups and 10.7 MHz for the η OH₂ group. The μ OH values are substantially higher than experimental values of 5 to 6 MHz observed for Al₂OH groups in Al oxyhydroxides. B3LYP values are very similar. However, hydrogen bonding to water in simple model compounds like $Al_2(OH)_2(OH_2)_8^{+4}$ reduces the -OH NQCC values to ~6 MHz. Copyright © 2001 Elsevier Science Ltd

1. INTRODUCTION

The aqueous solution chemistry of Al(III) is an old topic within solution chemistry, but the nature of the chemical species involved is still unresolved and of considerable interest. The chemical behaviour of Al(III) as well as its toxicity are dependent on its chemical speciation, which is characterized generally by its tendency to form strong oligomeric hydroxo complexes (Bottero et al., 1980; Tossell, 1998). The most familiar aqueous oligomer of Al is the "Al13" polyoxocation, $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$, which bears a resemblance to the structure of Al (hydr)oxides (Furrer et al., 1992). This "Al13" cation is an example of an ɛ-Keggin structure, comprised of a central AlO₄ tetrahedron surrounded by 12 octahedrally coordinated Al sharing edges. Each Al octahedron contains one water molecule and six of these deprotonate above pH = 6, leading to agglomeration and precipitation (Furrer et al., 1992). Four different oxygens can be distinguished in $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$: a single internal μ_4 -O, the 12 terminal η -OH₂, and two different types of μ_2 -OH within the total of 24 (Johansson, 1960). Previous ²⁷Al-NMR studies have easily

distinguished the Al[4] and Al[6], although the NMR feature

I therefore felt that it would be useful to calculate some of the properties of this polycation at a reasonable quantum mechanical level. Although the molecular ion considered is certainly very large, it represents a feasible target for quantum mechanical methods of moderate rigour. I anticipated that such methods would be able to semiquantitatively match the experimental NMR data if the atomistic models proposed for this

corresponding to Al[6] is quite broad (Kunwar et al., 1984). Similarly, ¹⁷O-NMR studies have distinguished the η -OH₂, μ_2 -OH, and μ_4 -O signals (Thompson et al., 1987; Phillips et al., 2000). Recently, the rate of exchange with ¹⁷O-labeled H₂O has been determined for the different O sites. The fastest exchanging site is the η -OH₂, while the two different μ_2 -OH sites show significantly different water exchange rates and the μ_4 -O does not seem to exchange (Phillips et al., 2000). There has also been some study of both the mechanism of formation of the related Ga₁₃ polyoxocation from simpler species (Michot et al., 2000) and of the agglomeration of the Al₁₃ species (Vogels et al., 1995). There have also been efforts to prepare "substituted" Al₁₃ species (Parker et al., 1997). Analogies have also been drawn between Al₁₃ and the surfaces of various Al(hydr)oxides (Phillips et al., 2000).

^{*(}tossell@chem.umd.edu).

2550

species were qualitatively correct. Such calculations would also provide a foundation for later work on the acid-base and other surface properties of the material and its interaction with the aqueous environment.

2. COMPUTATIONAL METHODS

I use the techniques of Hartree-Fock theory, Hartree-Fock perturbation theory, and density functional theory. The theoretical foundations and capabilities of these techniques are discussed in Hehre et al. (1986) and Jensen (1999). The basis sets used for the calculation of the equilibrium geometry were of the relativistic effective core potential type, as implemented by Stevens et al. (1992) (which I designate SBK), with polarization functions added on the non-H atoms. These basis sets neglect core electrons but properly describe their effects upon the valence electrons, with relativistic effects incorporated.

Geometry optimizations and calculations of electric field gradients at the nuclei and NMR shieldings were carried out using the quantum chemistry software packages GAMESS and GAUSSIAN 94 (Schmidt et al., 1993; Frisch et al., 1994). For calculation of the electric field gradients, EFGs (q) at the nuclei, and the NMR shieldings (σ), I use the all-electron 6-31G* basis set (Hehre et al., 1986) and both the Hartree-Fock method and a hybrid Hartree-Fock density functional method known as B3LYP (Becke, 1992). This method has yielded good results for EFGs in a number of different systems (Bailey et al., 2000). In most cases the B3LYP values of q are only a few percent smaller than the HF values. NMR shieldings were calculated at the Hartree-Fock or B3LYP level using a 6-31G* basis set and the GIAO method as implemented in GAUSSIAN 94 (Wolinski et al., 1992). The accuracy of the B3LYP approach to NMR shieldings is presently under debate. Although some researchers have obtained good results with B3LYP (e.g., Cheeseman et al., 1996; Casanovas et al., 1999), a comprehensive study of numerous gas-phase molecules using large basis sets has concluded that DFT shieldings are not as accurate as those obtained using correlated methods like MP2 and do not even offer a significant improvement over HF results (Wilson et al., 1999). ¹⁷O-NMR shifts (δ) were calculated relative to the calculated shielding for a single H2O molecule optimized in the gas-phase using the same methodology, ignoring the gas-liquid shift of \sim 36 ppm. Our rationale for using this gas-phase reference is discussed more fully in the "Results" section. ²⁷Al-NMR shifts were calculated with respect to $Al(OH_2)_6^{+3}$, as in our earlier work (Tossell, 1998). EFGs expressed in atomic units (au), were converted to NQCC values using nuclear quadrupole moments from Ludwig et al. (1996) for O and Kello et al. (1999) for Al. The molecular plots were drawn using the program, MacMolplt (Bode and Gordon, 1998).

Both polarized SBK and 6-31G* basis sets are relatively small by contemporary quantum chemical standards, but for the $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$ polycation they are the largest presently feasible. Using the 6-31G* basis set, the calculation on $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$ utilized 943 basis functions. Certainly such a highly charged cation would be strongly hydrated in aqueous solution, but for the present, we consider only the free gas-phase cation. However, we have studied the effect of hydration upon some of the calculated properties for the much smaller $\mathrm{Al}_2(\mathrm{OH})_2(\mathrm{OH}_2)_8^{+4}$ molecule, a dimeric model for the -OH and the -OH₂ groups and the Al[6] of the tridecameric cation. In general, it is found that ¹⁷O-NMR shieldings and NQCC values are sensitive to solvent effects, particularly if the solvent is H-bonding. Recently, such solvent effects have been treated using supermolecule or microsolvation techniques with small numbers of explicit water molecules. For example, the recent results of Kowalewski et al.(2001) show large changes in the ¹⁷O shifts for the O atoms of methyl formate when a small number of explicit water molecules are included in the calculation.

3. RESULTS

Calculated and experimental geometries are compared in Table 1. The experimental distances are taken from a fairly old X-ray crystallographic study on the selenate salt of Al_{13} (Johansson, 1960). The calculated and experimental distances compare well, but there is some discrepancy for the Al[6] bond

Table 1. Calculated and experimental bond distances (in Å) in $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$.

| Site | R(Al-O) _{calc.} | R(Al-O) _{exp.} ^a |
|-------|--------------------------|--------------------------------------|
| A1[4] | 4×1.85 | 4×1.84 |
| A1[6] | 4×1.85 | $2 \times 1.84, 2 \times 1.88$ |
| | 2×2.05 | 1.91, 2.04 |

^a From Johansson, 1960.

distances to -OH2. These bonds are weak and are on the periphery of the cluster, so they could be more strongly influenced by hydration or crystal packing forces. A plot of the molecular geometry of the Al13 polyoxocation is given in Figure 1, with the different Al and O sites labeled. Calculated values of Al-NMR shielding and shift, EFG, and NQCC are given for the Al[4] and Al[6] sites in Table 2, along with experimental values of the shifts (Kunwar et al., 1984). Agreement is reasonably good but it appears that the shift is slightly overestimated for Al[6] and somewhat underestimated for Al[4]. If the error were mainly in the theoretical method, we might expect the shifts of both sites to be either under- or overestimated. Different errors for the two sites may suggest that hydration effects are the main source of error. The central Al[4] is also calculated and observed to be strongly shielded with respect to monomeric Al[4], e.g., Al(OH)₄⁻¹, whose experimentally observed shift is ~ 80 ppm. In a previous study (Tossell, 1998), using the HF GIAO method with a 6-31G* basis set, we calculated a shift of between 89 and 84 ppm for $Al(OH)_4^{-1}$ compared to $Al(OH_2)_6^{+3}$, depending on whether we corrected the calculated bond distances for Born hydration energy effects. Increased shielding of the central Al[4] in the Al13 tridecamer cation is expected since it is highly polymerized, that is, each O is shared with three other Al cations. The calculated values for the NQCC at Al are also consistent with experiment, that is, the Al[4] signal is an extremely narrow line with NQCC ~ 0.8 MHz while for Al[6], the NQCC is estimated as ~ 10.2 MHz, based on results for basic Al sulfate and selenate (Kumar et al., 1984).

Calculated values of O-NMR shielding and shift, EFG, and NQCC are given for the different O sites in Table 3, along with experimental values for the shifts (Thompson et al., 1987; Phillips et al., 2000) and measured NQCC values for related materials (Thompson et al., 1987; Walter and Oldfield, 1989). Calculated values are given for both HF and B3LYP methods. As is generally observed (e.g., Wilson et al., 1999; Xue and Kanzaki, 2000), the B3LYP values are systematically deshielded compared to the HF. However, both methods give the same trend, although the differences in shift are considerably larger using B3LYP. While the calculated relative shifts for the four different O sites in the Al13 tridecamer cation agree well with experiment (using the assignment of Phillips et al., 2000), the shifts with respect to the water reference match experiment only if we adopt free gas-phase H₂O as the reference. ¹⁷O in liquid water is deshielded compared to that in gas-phase water by \sim 36 ppm. Such use of a gas-phase water reference would be correct only if the deshielding of the various O atoms in the Al₁₃ cation by the aqueous environment was of similar magnitude to the deshielding of water in the bulk liquid. I find that when six water molecules are allowed to

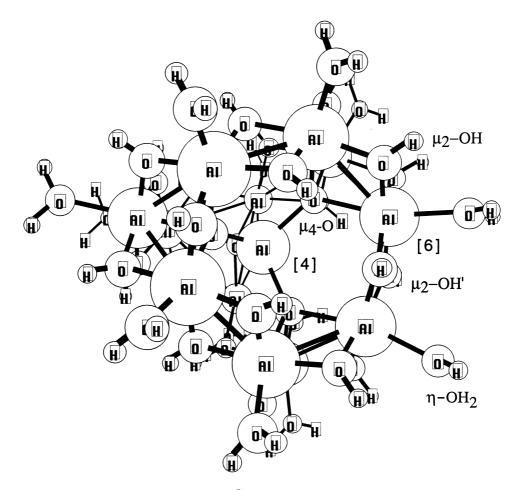


Fig. 1. Plot of the geometry of $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$ optimized at the Hartree-Fock level using a polarized SBK basis. No symmetry was assumed, but the X-ray crystal structure of the selenate salt was used as the starting geometry. Examples of the various Al and O atom types are labeled on the right hand side of the figure.

interact with the simpler model molecule $Al_2(OH)_2(OH_2)_8^{+4}$, some of the $-OH_2$ groups are deshielded by as much as 23 ppm. Clearly, additional studies of hydration effects upon the ¹⁷O shieldings of the Al_{13} tridecameric cation are needed to establish the sensitivity to hydration effects of the different types of O atoms.

Note that such problems in calculating ¹⁷O-NMR shifts are not at all unusual. In their study of NMR shieldings in SiO_2 polymorphs, Xue and Kanzaki (2000) also adopted gas-phase water as the reference molecule for their computations of ¹⁷O shifts but found fairly good agreement with experiment shifts, which were actually referenced to liquid H_2O . For electronegative atoms, nonlocal deshielding effects from distant atoms appear to be generally large as previously shown for N shieldings (Tossell, 1997) and O shieldings (Tossell, 1999). Others have shown that calculated ¹⁷O shieldings consistently converge more slowly with increasing cluster size than do ²⁹Si shieldings (e.g., Bull et al., 2000; Xue and Kanzaki, 2000), confirming the importance of such nonlocal effects.

NQCC values are not available for the O atoms of the Al_{13} cluster, but the calculated values are in fair agreement with experimental results (Walter and Oldfield, 1989) for other Al

Table 2. Calculated and experimental ²⁷Al NMR properties (shieldings and shifts^a in ppm, EFGs in atomic units, nuclear quadrupole coupling constants in MHz).

| | σ | | δ | | | q^{Al} | | NQCC | |
|----------------|----------------|----------------|--------------|--------------|------------|------------|-------|------------|-------------------|
| Site | HF | B3LYP | HF | B3LYP | exp. | HF | B3LYP | HF | B3LYP |
| A1[4] | 580.6 | 554.6 | 55.8 | 55.8 | 63 2:12 | 0 | 0 | 0 | 10.9 |
| Al[4] Al[6] | 580.6 620.0 | 554.6 591.7 | 55.8 16.4 | 55.8 18.7 | 63 ≈12 | 0 0.369 | | 0 0.356 | 0 0 0.356 11.3 |

^a The shift reference is $Al(OH_2)_6^{+3}$, with a calculated shielding of 636.4 ppm for HF and 610.6 ppm for B3LYP, using the polarized SBK optimized geometry, the 6-31G* basis and the GIAO method.

J. A. Tossell

Table 3. Calculated and experimental ¹⁷O-NMR properties (as in Table 2). Shifts are calculated with respect to free gas-phase H₂O. Experimental shifts are given in parentheses. Experimental values of NQCC given are for $AlO_4Al_{12}(OH)_{24}(H_2O)_{12}^{7+}$ - selenate and Al oxyhydroxides.

| Site | σ | | δ^{a} | | | q^{o} | | NQCC ^{b,c} | |
|--------------------------|--------------------|---------|-----------------------|-------|------|-----------------------|----------|---------------------|------|
| | HF | B3LYP | HF | B3LYP | exp. | HF | B3LYP | HF | exp. |
| η - OH_2 | 310–312 av. 311 | av. 299 | 17 | 23.5 | 20 | 1.83 | 1.73 | 10.7 | |
| μ - OH | 291–293 av. 292 | av. 271 | 36 | 51 | ≈30 | 1.34 | 1.28 | 7.8 | 5–6 |
| μ - ΟΗ' | 290–294 av. 292 | av. 271 | 36 | 51 | ≈30 | 1.35–1.39 av. 1.37 | av. 1.33 | 8.0 | 5–6 |
| μ ₄ - Ο | 273 | 235 | 55 | 87.3 | 55 | 0.29 | 0.28 | 1.2–2.2 | |

^a Shift reference is free gas-phase water, with ¹⁷O shieldings of 328.0 ppm for HF and 322.2 ppm for B3LYP, using the polarized SBK optimized geometry, the 6-31G* basis, and the GIAO method.

 $^{\rm b}$ Using the 6-31G* calibration factor of Ludwig, et al. (1996) to relate $q^{\rm o}$ to NQCC.

^c Experimental values from Thompson, et al. (1987) and Walter and Oldfield (1989).

(hydrox)oxides in the case of the μ_4 -O site and are somewhat larger than experiment for the μ_2 -OH sites. I have found that for μ_2 -OH sites in the simpler system Al₂(OH)₂(OH₂)⁺⁴₈, that by H-bonding single waters to these sites, the O-H bond is lengthened, reducing the NQCC value (calculated at the Hartree-Fock 6–31G* level) from ~7.0 to 5.7 MHz, thus giving considerably better agreement with experiment.

4. CONCLUSIONS

It is clear that the Al₁₃ polyoxocation can be treated fairly reliably by presently available quantum mechanical methods if sufficient computer resources are allocated (the geometry optimizations and NMR shielding required ~9 d of CPU time on a Compaq DEC XP1000). Al-NMR shieldings can be obtained fairly accurately, but O-NMR shieldings show the same problems of nonlocal contributions recently observed for calculations on silicates (Bull et al., 2000; Xue and Kanzaki, 2000; Tossell, 1999). Nonetheless, the relative shifts of the different types of O are given fairly accurately at the HF level. O NQCC values can also be obtained fairly accurately, particularly if the effects of H-bonding to water are taken into account using simpler model systems.

Acknowledgments—This work was supported by NSF grant EAR-0001031 and DOE grant DE-FG02-94ER14467. Prof. B. Phillips provided an insightful discussion of the results.

Associate editor: G. Sposito

REFERENCES

- Bailey W. C., Gonzalez F. M., and Castiglione J. (2000) Density functional theory and Hartree-Fock-density functional theory calculations of ¹⁷O, ³³S and ⁶³Ge quadrupole coupling constants. *Chem. Phys.* **260**, 327–335.
- Becke A. D. (1992) Density-functional thermochemistry. I. The effect of the exchange only gradient correction. J. Chem. Phys. 96, 2155– 2160.
- Bode B. M. and Gordon M. S. (1998) MacMolPlt: A graphical user interface for GAMESS. J. Mol. Graph. Model. 16, 133–138.
- Bottero J. Y., Cases J. M., Flessinger F., and Poirier J. E. (1980) Studies of hydrolyzed aluminum chloride solutions. 1. Nature of aluminum species and composition of aqueous solutions. J. Phys. Chem. 84, 2933–2939.
- Bull L. M., Bussemer B., Anupold T., Reinhold A., Samoson A., Sauer J., Cheetham A. K., and Dupree R. (2000) A high-resolution ¹⁷O and

²⁹Si NMR study of zeolite siliceous ferrierite and ab initio calculation of NMR parameters. *J. Am. Chem. Soc.* **122**, 4948–4958.

- Casanovas J., Pacchioni G., and Illas F. (1999) ²⁹Si solid state NMR of hydroxyl groups in silica from first principle calculations. *Mater. Sci. Eng. B.* 68, 16–21.
- Cheeseman J. R., Trucks G. W., Keith T. A., and Frisch M. J. (1996) A comparison of models for calculating nuclear magnetic resonance shielding tensors. J. Chem. Phys. 104, 5497–5509.
- Frisch M. J. et al. (1994) Gaussian 94, Rev. B. 3, Gaussian Inc., Pittsburgh, PA.
- Furrer G., Ludwig C., and Schindler P. W. (1992) On the chemistry of the Keggin Al13 polymer. I. Acid-base properties. J. Colloid Interf. Sci. 149, 56–67.
- Jensen F. (1999) Introduction to Computational Chemistry, Wiley, New York.
- Johansson G. (1960) On the crystal structures of some basic aluminum salts. Acta Chem. Scand. 14, 771–773.
- Kello V., Sadlej A. J., Pyykko P., Sundholm D., and Tokman M. (1999) Electric quadrupole moment of the ²⁷Al nucleus: Converging results from the AIF and AICI molecules and the Al atom. *Chem. Phys. Lett.* **304**, 414–422.
- Kowalewski D. G., Kowalewski V. J., Contreras R. H., Diez E., Casanueva J., San Fabian J., Esteban A. L., and Galache M. P. (2001). Solvent effects on oxygen-17 chemical shifts in methyl formate: Linear solvation shift relationships. *J. Magn. Reson.* 148, 1–10.
- Kunwar A. C., Thompson A. R., Gutowsky H. S., and Oldfield E. (1984) Solid state aluminum-27 NMR studies of tridecameric Aloxo-hydroxy clusters in basic aluminum selenate, sulfate, and the mineral zunyite. J. Magn. Reson. 60, 467–472.
- Ludwig R., Weinhold F., and Farrar T. C. (1996) Effective O-17 quadrupole moments for the calibrated computation of quadrupole coupling parameters at different levels of theory. J. Chem. Phys. 105, 8223–8230.
- Michot L. J., Montarges-Pelletier E., Lartiges B. S., d'Esounise de al Caillerie J. B., and Briois V. (2000) Formation mechanism of the Ga₁₃ Keggin ion: A combined EXAFS and NMR study. J. Am. Chem. Soc. **122**, 6048–6056.
- Parker, Jr. W. O., Millini R., and Kiricsi I. (1997) Substitution in keggin-type tridecameric aluminum-oxo-hydroxy clusters. *Inorg. Chem.* 36, 571–575.
- Phillips B. L., Casey W. H., and Karlsson M. (2000) Bonding and reactivity at oxide mineral surfaces from model aqueous complexes. *Nature* 404, 379–382.
- Schmidt M.W., et al. (1993) General atomic and molecular electronic structure system. J. Comput. Chem. 14, 1347–1363.
- Stevens W. J., Krauss M., Basch H., and Jansen P. G. (1992) Relativistic compact effective core potentials and efficient, shared-exponent basis sets for the third-, fourth- and fifth-row atoms. *Canad. J. Chem.* **70**, 612–630.
- Thompson A. R., Kunwar A. C., Gutowsky H. S., and Oldfield E.

(1987) Oxygen-17 and aluminium-27 nuclear magnetic resonance spectroscopic investigations of aluminium(iii) hydrolysis products. *J. Chem. Soc. Dalton* 2317–2321.

- Tossell J. A. (1997) Second-nearest-neighbor effects upon N NMR shieldings in models for solid Si_3N_4 and C_3H_4 . J. Magn. Reson. 127, 49–53.
- Tossell J. A. (1998) The effects of hydrolysis and oligomerization upon the NMR shieldings of Be⁺² and Al⁺³ species in aqueous solution. J. Magn. Reson. **135**, 203–207.
- Tossell J. A. (1999) Local and long-range effects on NMR shieldings in main-group metal oxides and nitrides. In *Modeling NMR Chemical Shifts: Gaining Insights into Structure and Environment* (ed. J. C. Facelli, J. C. and A. C. De Dios), pp. 314–319, ACS Symp. Series 732.
- Vogels R. J. M., Kloprogge J. T., Buining P. A., Seykens D., Jansen J. B. H., and Geus J. W. (1995) The tridecameric aluminum complex

as an appropriate precursor for fibrous boehmite: A ²⁷Al NMR study on the partial hydrolysis of aluminum sec-butoxide. *J. Non-Cryst. Solids* **191**, 38–44.

- Walter T. H. and Oldfield E. (1989) Magic angle spinning oxygen-17 NMR of aluminum oxides and hydroxides. J. Phys. Chem. 93, 6744-6751.
- Wilson P. J., Amos R. D., and Handy, N. C. (1999) Density functional magnetizabilities and nuclear shielding constants. *Molec. Phys.* 97, 757–768.
- Wolinski K., Hinton J. F., and Pulay P. (1992) Efficient implementation of the gauge-dependent atomic orbital method for NMR chemical shift calculations. J. Am. Chem. Soc. 112, 8251–8260.
- Xue X. and Kanzaki M. (2000). An ab initio calculation of ¹⁷O and ²⁹Si NMR parameters for SiO₂ polymorphs. *Solid State Nucl. Mag.* **16**, 245–259.