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Express Letter

Predictability of global surface temperature by means of nonlinear analysis

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Abstract

The time series of annually averaged global surface temperature anomalies for the years 1856–1998 is studied through nonlinear time series analysis with the aim of estimating the predictability time. Detection of chaotic behaviour in the data indicates that there is some internal structure in the data; the data may be considered to be governed by a deterministic process and some predictability is expected. Several tests are performed on the series, with results indicating possible chaotic behaviour.

Keywords: prediction; global change temperature

1. Introduction

Dynamical systems theory has provided a new quantitative perspective on the predictability of weather and climate processes. Takens [1] proved that under fairly general conditions it is possible to deduce the unknown attractor of a physical system from a sufficiently long time series of just one state variable. Once the attractor is properly reconstructed, several predictability measures can be estimated [2,3]. Among them, three are most currently used: Lyapunov exponents, the metric entropy and various fractal dimensions.

During the last few years, several researchers have tried to determine these measures for the atmosphere from observed ‘climatic time scale’ data. The pioneering work was the study by Nicolis and Nicolis [4] using the oxygen isotope record obtained from the V-28-238 equatorial Pacific deep-sea core of N.J. Shackleton and N.D. Opdyke. They concluded that there exist an about three-dimensional attractor and a predictability time of about 30.0 years. Fraedrich [5] also used oxygen isotope records and found a climate attractor with dimensionality 4.4–4.8 and predictability 10.0–15.0 years. In a shorter time scale, Fraedrich [6] found a predictability of 1.5 years in the ENSO occurrence using annual time series of ENSO and Abarbanel and Lall [7] concluded that there exist an approximately three-dimensional attractor and a predictability time of a few hundred days, i.e. about one year, using the

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1848–1992 biweekly time series of the Great Salt Lake water volume.

The aim of this paper was to study the predictability of the annually averaged global surface temperature anomaly through techniques of nonlinear analysis. Two different approaches are used: (a) an attempt to look for a possible attractor, and (b) computation of the metric entropy and the Lyapunov exponents. The first technique requires embedding of the data series in an adequate d-dimensional vector space, while the second can be undertaken with the original data.

2. Data

The analysis was performed on the time series of annually averaged global surface temperature anomalies between 1856 and 1998 produced by the Climatic Research Unit of the University of East Anglia (http://www.cru.uea.ac.uk). The series is displayed in Fig. 1.

3. Global embedding dimension and correlation dimension

In the original data an apparently linear growing trend is obvious. The series was detrended by subtracting the linear approximation, giving the data represented in Fig. 2. Then, the series was analysed through nonlinear techniques.

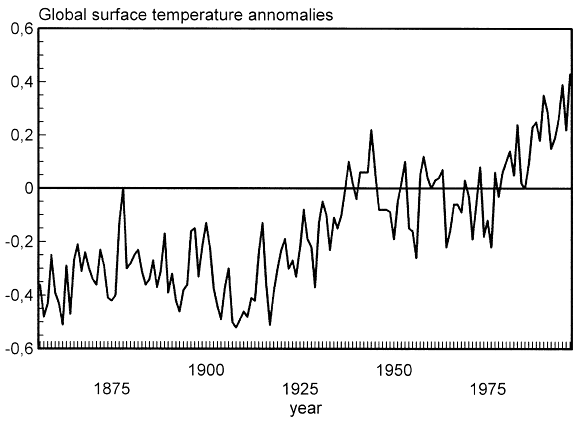

Fig. 1. Series of the annually averaged global surface temperature.

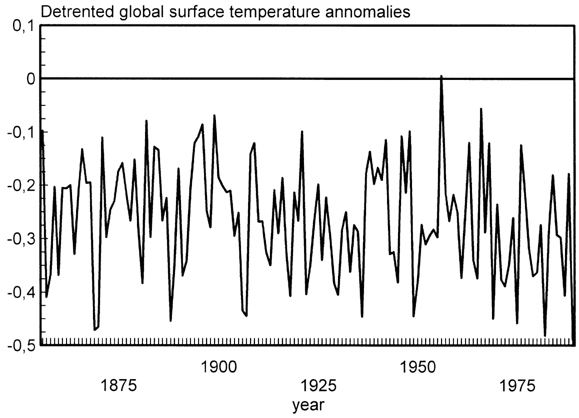

Fig. 2. Detrended data.

To start with we use the time-delay coordinates to reconstruct the phase space of the observed dynamical system. In d dimensions, and with a lag T that can be estimated e.g. from the first minimum of the autocorrelation function of the data [8], the vector:

$$Y(n) = \{s(n),\ s(n-T),\ s(n-2T),\ \ldots\ s(n-(d-1)T)\}$$

describes an equally spaced set of values in the time history prior to the value $s(n)$ (from now on we shall simply write 'time histories'). The set of all d-dimensional time histories contains the same information as the original series, but can be explored with more powerful tools. The choice of the 'embedding dimension' d is not elementary. Several techniques are available for the choice of the most adequate d. Here the method of global false nearest neighbours described by Abarbanel et al. [9] was used. Roughly speaking, the method starts by looking for the nearest histories to every vector $Y(n)$ in dimension d, in the sense of some norm (usually the Euclidean distance is employed, but there are some others). The next step upgrades the search to dimension d+1, i.e. the same procedure is applied to the (d+1) history:

$$Y*(n) = \{s(n),\ s(n-T),\ s(n-2T),\ \ldots\ s(n-dT)\}$$

If a nearest d-dimensional history (for $Y(n)$) moves far away from $Y^*(n)$, then it is called a false nearest neighbour (FNN). When the number of FNNs drops to 0 from some d down, we consider that the structure of the data is most properly described by the set of d-dimensional histories. Fig. 3 displays the percentage of FNNs as d ranges from 1 to 10. The percentage of false neighbours is minimal when $d=3$, so the embedding dimension is taken as 3 according to the FNN method. In this case a small local minimum of the percentage of FNNs can be observed at $d=6$. It was not considered significant enough to deserve a separate study. The lag T is computed by observing that the values of the autocorrelation function:

$$A(t) = \langle s(n)\, s(n-t) \rangle$$

are less than $1/e=0.36$ for every $t>1$ (this linear criterion is alternative to the first zero crossing one as far as the interesting fact is simply how fast $A(t)$ decays to zero [8]). Therefore we chose a lag $T=1$.

The geometric complexity of the embedded series in d-dimensional space is measured by some pseudo-fractal dimension concept. The actual fractal or Hausdorff dimension is an idealisation whose existence is restricted to purely mathematical fractal sets, and these are the result of infinite recursive processes, which is not the case for finite data sets like the embedded series. Several approximations for a fractal dimension concept are available for real-world data, yielding numerical values of roughly the same order. The idea behind all these calculations is to obtain a measure of how the spatial distribution of the set varies according to the measuring scale. A very popular pseudo-fractal dimension is the so-called correlation dimension [10] obtained as an estimate of the variation of the number of points subsequent to any given that can be found in a sequence of d-hyperspheres centred at the point and with variable radii. In this case, with $d=3$ and $T=1$, the estimated correlation dimension is $C=2.62\pm0.63$.

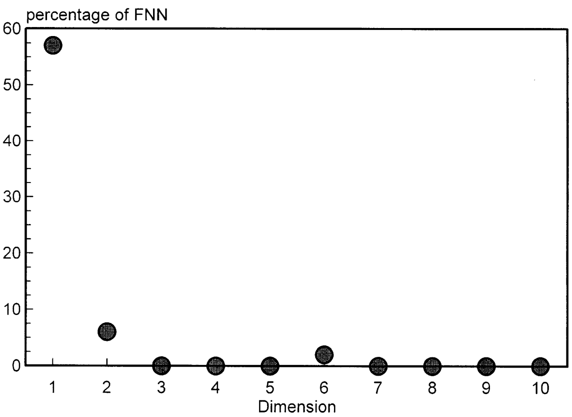

Fig. 3. Percentage of FNNs.

The correlation dimension must be bounded by the embedding dimension. Of course, in some computations the mathematical artifact will yield values higher than d, and they must be rejected as physically meaningless. The closer the correlation dimension to the embedding dimension, the worse for prediction. It is worth commenting on this fact: the computed dimension ranges between 2 and 3. In the best case, $C\approx2$ can be roughly interpreted as the data embedded in 3-space being displayed about some rather plane surface:

$$s(n) = F(s(n-1),\ s(n-2))$$

and this would mean a close-to-deterministic behaviour, where any value could be computed after the two preceding ones. The other extreme case, $C\approx3$, would mean a complete lack of internal structure in the data, thus invalidating any predictive attempt. With the present data set further analyses were performed in order to obtain physically sensible results.

4. Metric entropy, Lyapunov exponents, and predictability

The metric entropy or K-S entropy (after Kolmogorov and Sinai) is a quantity H used to estimate the rate at which information is created by the observed system as it evolves in time. If $H=0$, meaning that no new information appears, the system is completely determined by the initial formulae and data and we face a deterministic system. On the other hand, $H=\infty$ will mean an absolutely random system: all kinds of information can appear at any time, making prediction an impossible task. Whenever some structure is present in the data, $0 < H < \infty$. The closer H is to 0, the better predictions can be formulated. In this case the computed H is 0.137, a very low value. From a geometric viewpoint the creation of information amounts to divergent behaviours between orbits of the system originating at close initial conditions. The measure of this divergence is given by the set of Lyapunov exponents.

Theoretically, the entropy H equals the sum of the positive Lyapunov exponents of the series [8,11,12]: if it is chaotic, at least one of them is positive. Computations show, for an embedding dimension of 3 and a projection time of 7 (i.e. calculations involve seven time steps), that in this case the only positive Lyapunov exponent ranges between 0.255 and 0.395, far from agreement with the metric entropy value.

The unit of H is 1/time, and the inverse of H, or equivalently the inverse of the Lyapunov exponent, is an estimate of the predictability limit. As the experimental agreement between the two magnitudes is bad, predictability estimates range from 7 years from the H value, to some 2.5 years according to the upper bound of the estimate for the positive Lyapunov exponent.

5. Concluding remarks

These results suggest that the predictability of the global temperature anomalies could be estimated through the use of nonlinear dynamics. However, the series presently available is too short, producing uncertainties in the exact value of the predictability limit, according to the applied method (K-S entropy, or Lyapunov exponents). The obtained predictability range (2.5–7 years) for the detrended anomalies series should be considered with great caution, but suggests that typical values for the predictability should be in the interannual scale, close to the El Niño periodiocity band. Thus, the global temperature series could be envisaged as composed of a linear trend plus an anomaly component with some chaotic structure whose predictability limit is in the low frequency range. There are abundant references [13–15] that provide evidence that El Niño is indeed chaotic and possibly a subsystem of a grand complex system. This subsystem may be characterised by a much smaller dimensionality [15–18] and better predictability. The way in which this subsystem is connected to the grand climate system could explain the predictability of global surface temperature anomalies. The investigation of a possible relationship between El Niño and the predictability of the global temperature anomalies series should require further research, which is beyond this preliminary study. ***[AC]***

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