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ROCK FAILURE

ON PREDICTION OF ROCK FAILURE ON THE BASIS OF RECORDING THE PULSES OF ELECTROMAGNETIC RADIATION

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The solution is considered for the problem on synthesis of optimal algorithm of failure prediction on the basis of recording the pulses of electromagnetic radiation. Poisson's nonstationary process is taken as the model for radiant flow of pulses. Algorithms obtained ensure automatic trimming for the characteristics of background radiation. The characteristics of algorithm efficiency and results of practical approbation are given.

Failure prediction, electromagnetic radiation, crack accumulation model, Poisson's event flow, threshold function, nonrandomized algorithm

The effect of electromagnetic emission from the focus of failure is widely used by modern scientists to study and predict rock failure [1 – 3]. Different models for crack accumulation were developed [4, 5]. Definition of the concentration criterion [6] was of great importance for failure prediction. However, in scientific publications, there is practically no solution for the problem on synthesis of optimal algorithm of failure prediction on the basis of recording the pulses of electromagnetic radiation. The study in question makes up for this deficiency.

1. INTRODUCTION

It is established that the number of electromagnetic radiation pulses appearing during rock failure is determined by the number of cracks arising in this process [2, 5]. Consequently, the intensity of such pulse flow characterizes the intensity of cracking in the focus of failure, the peculiar feature of which is development by stages [7 – 9]. In the first stage, microcracks accumulate. This stage is the longest and determines the durability of sample under load [8]; it is characterized by approximately constant intensity of cracking. In the second stage of failure, the avalanche increase in number of cracks occurs (according to terminology adopted in [7] the main crack forms in this stage), but the sample is not completely destroyed yet. In [7] the third "post-failure" stage, where the avalanche failure of sample takes place, is distinguished as well. The duration of the second and third stages is much less than the duration of the first one. Therefore, when revealing the first stage, it is important to make a decision within the shortest period of time.

We assume the model of Poisson's event flow as the initial model of crack accumulation. It is known that in the first stage, the process of cracking can be considered approximately stationary, while transition to the second and third stages is connected with the sharp change in intensity and nonstationarity of the process [4, 9], which, in its turn, leads to the sharp change in intensity of electromagnetic radiation pulse flow. Therefore, the instant of time, when the intensity of electromagnetic radiation pulse flow sharply increases, is taken as the prediction characteristic.

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In connection with the fact that in the first stage the process of cracking is stationary only approximately, deviations of its intensity from the mean value are possible at different intervals. To take account of this feature, we divide the observation interval ΔT into n subintervals $\Delta T_1,..., \Delta T_n$, within which the intensity of cracking and, consequently, the flow of electromagnetic radiation pulses recorded are characterized by the constant intensity λ_i, i=1,..., n; in this case, they can differ from each other at different intervals. We consider the process in question as the realization of Poisson's vector process consisting of n components. In the first stage of failure, the values of intensities of all components do not almost differ from each other and have the "background" value. In going to the second stage, the intensity of the nth component λ_n will greatly exceed the weighted-mean (background) value with respect to the rest components. This can be conditionally interpreted as appearance of the "signal component" in the nth component of Poisson's flow of pulses. Thus, the problem of failure prediction on the basis of recording the electromagnetic radiation pulses can be formulated as the problem of signal component detection in many-dimensional Poisson's flow of pulses.

Below is given the solution of this problem for two variants: *the first variant*, when there are no time limitations for observations, and the number of events recorded in each component is fixed; and *the second variant*, when the time of observation is fixed.

2. SOLUTION FOR THE PROBLEM OF FAILURE PREDICTION AS THE PROBLEM OF SIGNAL COMPONENT DETECTION IN MANY-DIMENSIONAL FLOW OF PULSES WITH UNKNOWN INTENSITY

Variant 1. Let each of n components of the process observed represents homogeneous Poisson's flow of pulses with the intensity λ_i, i=1,..., n. The values of $\lambda_1,...,\lambda_n$ are indefinite a priory and can differ from each other. The signal component can appear in the nth component of the process observed, then the value of λ_n will exceed the weighted-mean value with respect to other components λ_j, $j < n$. Thus, the problem of signal-component detection can be formulated as the problem of statistical hypothesis testing

$$\begin{aligned} H_0: \quad & \lambda_n \le \frac{\gamma}{n-1}\sum_{k=1}^{n-1}\lambda_k, \quad \text{there is no signal}, \\ H_1: \quad & \lambda_n > \frac{\gamma}{n-1}\sum_{k=1}^{n-1}\lambda_k, \quad \text{there is a signal}. \end{aligned} \tag{1}$$

The coefficient γ in (1) shows by what factor the intensity of the nth component with the signal component must exceed the mean value of intensity with respect to the rest components.

Let m_i+1 pulses appear in the ith component of the process observed within the time of observation. The interval of time between the appearances of the jth and (j – 1)th pulses in the ith component of the process observed is denoted as $t_i^{(j)}$ $(j=1,..., m_i)$. The joint probability density for the values of $t_i^{(j)}$, $j=1,...,m_i$, and $i=1,..., n$ is subject to exponential law [10] and has the form

$$w(t_1^{(1)},..., t_n^{(1)},..., t_n^{(m_n)}) = \prod_{i=1}^{n}\lambda_i^{m_i}\exp\left\{-\sum_{k=1}^{n}\lambda_k\sum_{j=1}^{m_k}t_k^{(j)}\right\}. \tag{2}$$

Transform (2) to the form suitable for the solution:

$$w(t_1^{(1)},\ldots,t_n^{(1)},\ldots,t_n^{(m_n)})=\prod_{i=1}^{n}\lambda_i^{m_i}\exp\left\{-\left(\lambda_n-\tfrac{\gamma}{n-1}\sum_{k=1}^{n-1}\lambda_k\right)\sum_{j=1}^{m_n}t_n^{(j)}-\right.$$
$$\left.-\sum_{k=1}^{n-1}\lambda_k\left(\sum_{j=1}^{m_k}t_k^{(j)}+\tfrac{\gamma}{n-1}\sum_{j=1}^{m_n}t_n^{(j)}\right)\right\}. \tag{3}$$

The density of probability distribution (3) is characterized by one-dimensional useful parameter $\vartheta=\lambda_n-\dfrac{\gamma}{n-1}\sum_{k=1}^{n-1}\lambda_k$ with sufficient statistic $T_n=\sum_{j=1}^{m_n}t_n^{(j)}$ and nuisance parameters $\lambda_1,\ldots,\lambda_{n-1}$ with sufficient statistics $T_k=\sum_{j=1}^{m_k}t_k^{(j)}+\dfrac{\gamma}{n-1}\sum_{j=1}^{m_n}t_n^{(j)}$, $k=1,\ldots,n-1$.

The problem of signal-component detection in the *n*th component of the process observed is equivalent to the problem of testing the complicated statistical hypotheses relative to the parameters of probability distribution (3)

$$\begin{aligned}&H_0:\ \vartheta\le 0,\quad \lambda_1,\ldots,\lambda_{n-1}\quad \text{any (there is no signal)},\\&H_1:\ \vartheta>0,\quad \lambda_1,\ldots,\lambda_{n-1}\quad \text{any (there is a signal)}.\end{aligned} \tag{4}$$

The sufficient statistics $T_1,\ldots,T_n$ bear all information about useful and nuisance parameters. Therefore, the transfer from the initial sampling $t_i^{(j)}$, $j=1,\ldots,m_i$ to the sufficient statistics in synthesis of the detection algorithm does not lead to the loss of information and optimal algorithm. The joint density of probability distribution for the sufficient statistics is determined by the expression [10, 11]

$$w(T_1,\ldots,T_n)=\frac{\left(\lambda_n-\tfrac{\gamma}{n-1}\sum_{k=1}^{n-1}\lambda_k\right)^{m_n}T_n^{m_n-1}}{(m_n-1)!}\left[\prod_{k=1}^{n-1}\frac{\lambda_k^{m_k}\,(T_k-\tfrac{\gamma}{n-1}T_1)^{m_k-1}}{(m_k-1)!}\right]\exp\left\{-\vartheta T_n-\sum_{k=1}^{n-1}\lambda_k T_k\right\}. \tag{5}$$

The problem on detection of signal component is formulated relative to distribution parameters (5) and has the same form (4) as for the initial problem. In addition to it, distribution (5) belongs to the exponential family and is characterized by $(n-1)$-dimensional parameter $\mu=\{\lambda_1,\ldots,\lambda_{n-1}\}\in(0,\infty)\times(0,\infty)\times\ldots\times(0,\infty)$ containing $(n-1)$-dimensional interval at the hypothesis H_0. By completeness theorem [11], it will be complete. Therefore, the power function of any detection algorithm is continuous relative to its parameters; and according to [11], uniformly the most powerful unbiased algorithm of detection will take the form

$$\varphi(T_1,\ldots,T_n)=\begin{cases}1, & T_n<C(\alpha,T_1,\ldots,T_{n-1}),\\ 0, & T_n\ge C(\alpha,T_1,\ldots,T_{n-1}).\end{cases} \tag{6}$$

Here, $C(\alpha,T_1,\ldots,T_{n-1})$ is the threshold function depending on the level of the false alarm probability α and sufficient statistics $T_1,\ldots,T_{n-1}$ and determined from the equation

$$E[\varphi(T_1,\ldots,T_n)\,|\,\vartheta=0,\ T_1,\ldots,T_{n-1}]=\alpha, \tag{7}$$

where $E[\cdot\,|\,\vartheta=0,T_1,\ldots,T_{n-1}]$ is the conditional expectation at $\vartheta=0$.

To solve equation (7), it is required to calculate the conditional probability density

$$w(T_n \mid \vartheta = 0, T_1, ..., T_{n-1}) = \frac{w(T_1, ..., T_n \mid \vartheta = 0)}{w(T_1, ..., T_{n-1} \mid \vartheta = 0)} . \tag{8}$$

Let us find denominator in the right part of expression (8):

$$w(T_1, ..., T_{n-1}) = \int_D w(T_1, ..., T_n) dT_n , \tag{9}$$

where D is the domain of determining the density $w(T_1, ..., T_n)$ with respect to the variable T_n. Starting from the fact that $\sum_{j=1}^{m_k} t_j^{(k)} \geq 0$ at any $k = 1, ..., n$, we find that the domain D is assigned by the system of inequalities

$$0 \leq T_n < \frac{1}{\gamma}\left[\min_{k=1,...,n-1}(T_k)\right] . \tag{10}$$

From (8) with regard to (9) and (10) we obtain

$$w(T_n \mid \vartheta = 0, T_1, ..., T_{n-1}) = \frac{T_n^{m_n - 1} \prod_{k=1}^{n-1}\left(T_k - \frac{\gamma}{n-1} T_n\right)^{m_k - 1}}{\int_0^{\frac{1}{\gamma}\min(T_1, ..., T_{n-1})} T_n^{m_n - 1} \prod_{k=1}^{n-1}\left(T_k - \frac{\gamma}{n-1} T_n\right)^{m_k - 1} dT_n} .$$

We determine the threshold function from the equation

$$\int_0^{C(\alpha, T_1, ..., T_{n-1})} w(T_n \mid \vartheta = 0, T_1, ..., T_{n-1})\, dT_n = \alpha$$

which can be presented in the following form:

$$\int_0^{C(\alpha, T_1, ..., T_{n-1})} T_n^{m_n - 1} \prod_{k=1}^{n-1}\left(T_k - \frac{\gamma}{n-1} T_n\right)^{m_k - 1} dT_n = \alpha \int_0^{\frac{n-1}{\gamma}\min(T_1, ..., T_{n-1})} T_n^{m_n - 1} \prod_{k=1}^{n-1}\left(T_k - \frac{\gamma}{n-1} T_n\right)^{m_k - 1} dT_n . \tag{11}$$

Equation (11) can be solved by numerical methods, however, in special cases, the solutions can be analytical as well. As an example, consider the case $m_1 = 1, ..., m_{n-1} = 1$ corresponding to the situation when prior to the beginning of failure, the separate cracks and electromagnetic radiation pulses appear rarely. For this case, the threshold function has the form:

$$C(\alpha, T_1, ..., T_{n-1}) = \frac{n-1}{\gamma} \alpha^{1/m_n} \min(T_1, ..., T_{n-1}),$$

and detection rule expressed in terms of the observed initial values of $t_i^{(j)}$, $j = 1, ..., m_i$, $i = 1, ..., n$

$$\varphi[t_1^{(1)}, ..., t_{n-1}^{(1)}, t_n^{(1)}, ..., t_n^{(m_n)}] = \begin{cases} 1, & \sum_{j=1}^{m_n} t_n^{(j)} < \dfrac{\alpha^{1/m_n} (n-1)\min[t_1^{(1)}, ..., t_{n-1}^{(1)}]}{\gamma(1 - \alpha^{1/m_n})} ; \\ 0, & \sum_{j=1}^{m_n} t_n^{(j)} \geq \dfrac{\alpha^{1/m_n} (n-1)\min[t_1^{(1)}, ..., t_{n-1}^{(1)}]}{\gamma(1 - \alpha^{1/m_n})} . \end{cases} \tag{12}$$

The analysis of algorithm efficiency (12) is reduced to the determination of dependence for the probability β of correct solution to the problem on signal-component presence upon the parameters $\lambda_1, ..., \lambda_n$:

$$\beta = P\left\{\frac{\alpha^{1/m_n}(n-1)\min[t_1^{(1)},..., t_{n-1}^{(1)}]}{\gamma(1-\alpha^{1/m_n})} - \sum_{j=1}^{m_n} t_n^{(j)} > 0\right\} = 1 - F_Z(0) , \tag{13}$$

where $F_Z(\cdot)$ is the integral function of random variable distribution

$$Z = \frac{\alpha^{1/m_n}(n-1)\min[t_1^{(1)},..., t_{n-1}^{(1)}]}{\gamma(1-\alpha^{1/m_n})} - \sum_{j=1}^{m_n} t_n^{(j)} .$$

Denote $Z_1 = \frac{\alpha^{1/m_n}(n-1)\min[t_1^{(1)},..., t_{n-1}^{(1)}]}{\gamma(1-\alpha^{1/m_n})}$ and $Z_2 = \sum_{j=1}^{m_n} t_n^{(j)}$. Then [12],

$$F_Z(x) = \int_{-\infty}^{\infty} w_{Z_2}(x_1) F_{Z_1}(x + x_1)\, dx_1 , \tag{14}$$

where $w_{Z_2}(\cdot)$ and $F_{Z_1}(\cdot)$ are the probability density of Z_2 and the integral function of distribution of Z_1, respectively. Taking into account that $Z_1 \geq 0$ and $Z_2 \geq 0$,

$$w_{Z_2}(x_1) = \frac{\lambda_n(\lambda_n x_1)^{m_n - 1}}{(m_n - 1)!}\exp\{\lambda_n x_1\},$$

$$F_{Z_1}(x + x_1) = 1 - P\left\{\min[t_1^{(1)},..., t_{n-1}^{(1)}] \geq \frac{\gamma(1-\alpha^{1/m_n})(x + x_1)}{(n-1)\alpha^{1/m_n}}\right\} =$$

$$= 1 - \prod_{k=1}^{n-1} P\left\{t_k^{(1)} \geq \frac{\gamma(1-\alpha^{1/m_n})(x + x_1)}{(n-1)\alpha^{1/m_n}}\right\} =$$

$$= 1 - \exp\left\{-\frac{\gamma\left(\sum_{k=1}^{n-1}\lambda_k\right)(1-\alpha^{1/m_n})(x + x_1)}{(n-1)\alpha^{1/m_n}}\right\} .$$

From (14) we obtain

$$\beta = \int_0^{\infty}\frac{\lambda_n(\lambda_n x_1)^{m_n - 1}}{(m_n - 1)!}\exp\left\{-\lambda_n x_1 - \frac{\gamma\left(\sum_{k=1}^{n-1}\lambda_k\right)(1-\alpha^{1/m_n})}{(n-1)\alpha^{1/m_n}} x_1\right\} dx_1 = \tag{15}$$
$$= \frac{\alpha q^{m_n}}{[\alpha^{1/m_n}(q-1)+1]^{m_n}} ,$$

where $q = \dfrac{\lambda_n}{\dfrac{\gamma}{(n-1)}\sum_{k=1}^{n-1}\lambda_k}$ is the ratio of the value of pulse flow intensity in the first component to the weighted-mean value of intensity with respect to the rest components of the process observed. It is evident from (15) that the probability of correct solution for the problem on signal-component presence depends only on the parameter q, i.e., on the fact how much times the intensity of pulse flow in the component containing the signal component exceeds the weighted-mean value of intensity with respect to the rest components.

Figure 1 demonstrates the graphs of correct detection versus the parameter q at different values of m_n. It is seen that as m_n increases, the probability of correct solution raises.

Variant 2. Let the observation time $\Delta T_1 = ... = \Delta T_n = \Delta T$ is fixed and identical for each component; the rest initial data and hypothesis formulation are the same as for *Variant 1*.

We assume that in the ith component of the process observed, m_i pulses i = 1, ..., n were recorded within the observation time ΔT. Their joint distribution of probabilities can be given in the form:

$$P(m_1, ..., m_n) = \frac{\exp\left\{-\sum_{i=1}^{n}\lambda_i \Delta T\right\}\exp\left\{\sum_{i=1}^{n} m_i \log(\lambda_i \Delta T)\right\}}{\prod_{i=1}^{n} m_i !}. \qquad (16)$$

It is obvious from expression (16) that the parameters of probability distribution for the values of $m_1, ..., m_n$ depend nonlinearly on $\lambda_1, ..., \lambda_n$. Therefore, it is difficult to obtain the precise solution for problem (1) when the observation time is fixed and n is arbitrary. As will be shown below, such solution can be found for the case, when n = 2 and γ is arbitrary. Nevertheless, the solution to the problem of signal-component detection in many-dimensional flow of pulses on the basis of the initial data $m_1, ..., m_n$ (where m_i is the number of pulses within the observation interval ΔT) is possible if we formulate it with regard to the properties of (16) (formulation of (1) took into account the properties of probability distribution of statistics for the times of arising the separate pulses).

Transform (16) to the form suitable for the solution to the problem of signal-component detection

$$P(m_1, ..., m_n) = \frac{\exp\left\{-\sum_{i=1}^{n}\lambda_i \Delta T\right\}\exp\left\{m_n\left[\log(\lambda_n \Delta T) - \frac{1}{n-1}\sum_{i=1}^{n-1}\log(\gamma_1 \lambda_i \Delta T)\right]\right\}}{\prod_{i=1}^{n} m_i !}\times$$

$$\times\exp\left\{\sum_{i=1}^{n-1}\log(\lambda_i \Delta T)\left(m_i + \frac{m_n}{n-1}\right)\right\}\exp\{m_n \log(\gamma_1)\} =$$

$$= \frac{\exp\left\{-\sum_{i=1}^{n}\lambda_i \Delta T\right\}}{\prod_{i=1}^{n} m_i !}\gamma_1^{m_n}\exp\left\{m_n \log\left[\frac{\lambda_n}{\gamma_1\left(\prod_{i=1}^{n-1}\lambda_i\right)^{\frac{1}{n-1}}}\right] + \sum_{i=1}^{n-1}\left(m_i + \frac{m_n}{n-1}\right)\log(\lambda_i \Delta T)\right\}. \qquad (17)$$

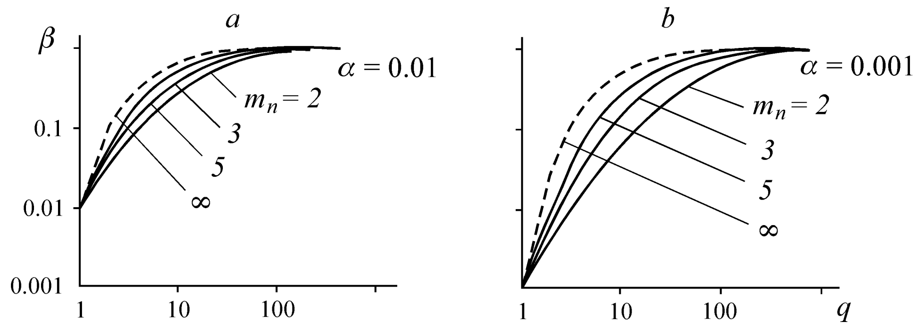

Fig. 1. Characteristics of the efficiency of algorithm for the signal-component detection in Poisson's flow of pulses

Distribution (17) is characterized by one useful parameter $\vartheta=\log\left(\lambda_n \Big/ \left(\gamma_1 \sqrt[n-1]{\prod_{i=1}^{n-1}\lambda_i}\right)\right)$ and $(n-1)$ nuisance parameters $\mu_i = \log(\lambda_i \Delta T)$, $i = 1, ..., n-1$. The problem of signal-component detection can be formulated as the problem of testing the complicated statistical hypotheses relative to (17):

$$\begin{aligned} &H_0: \quad \vartheta \le 0, \quad \mu_i \in (-\infty,+\infty), \ \ i=1,...,n-1, \ \ \text{there is no signal}, \\ &H_1: \quad \vartheta > 0, \quad \mu_i \in (-\infty,+\infty), \ \ i=1,...,n-1, \ \ \text{there is a signal}. \end{aligned} \tag{18}$$

We compare the formulations of detection problem in the form of (18) and (1). Expression (1) assumes the comparison of the intensity λ_n with the arithmetic mean value of the intensity with respect to the components $\lambda_1,..., \lambda_{n-1}$. The coefficient γ shows how much times the intensity of the nth component must exceed the mean value of the intensity with respect to the rest components, if the signal is present. According to expression (18), it is required to compare the value of the intensity λ_n with the geometric mean value of the intensity with respect to the components $\lambda_1,..., \lambda_{n-1}$ in order to make a decision on signal presence; the coefficient γ_1 shows how much times the intensity of the nth component must exceed the geometric mean value of the intensity with respect to the rest components, if the signal is present. When $n = 2$ and $\gamma_1 = \gamma$, the formulations of hypotheses (1) and (18) coincide.

Distribution (17) belongs to the exponential family and has sufficient statistics $U = m_n$ and $T = \{t_1,..., t_{n-1}\}$, $t_i = m_i + \frac{m_n}{n-1}$, $i = 1, ..., n-1$; with the hypothesis H_0, it depends on $(n-1)$-dimensional parameter $\{\mu_i\}$ whose domain of values contains $(n-1)$-dimensional interval. Therefore, by completeness theorem [11], it is complete. The power function of any algorithm is continuous, and according to [11], uniformly the most powerful unbiased algorithm of detection has the form

$$\varphi(U,T) = \begin{cases} 1, & U > C(\alpha,T), \\ \psi, & U = C(\alpha,T), \\ 0, & U < C(\alpha,T). \end{cases} \tag{19}$$

Here, $C(\alpha, T)$ is the threshold function depending on the level of the false alarm probability α, sufficient statistic T, and $0 \le \psi \le 1$. The value of $C(\alpha, T)$ and the constant ψ are determined from the equation

$$E[\varphi(U, T) \mid \vartheta = 0, T] = \alpha, \tag{20}$$

where $E[\cdot \mid \vartheta = 0, T]$ is the conditional expectation at $\vartheta = 0$.

To solve (20), it is required to calculate the conditional probability

$$p(U \mid \vartheta = 0,\ T) = \frac{p(U, T \mid \vartheta = 0)}{p(T \mid \vartheta = 0)} . \tag{21}$$

Joint distribution of the sufficient statistics $P(U,T)$ can be obtained from expression (17) if we take into account that Jacobian of transformation $\{m_1,...,\ m_n\} \to \{U,T\}$ is equal to unit, and $m_n = U$, $m_i = t_i - \frac{U}{n-1}$, $i = 1,...,\ n-1$:

$$p(U,T) = \frac{\exp\left\{-\exp\left[\vartheta + \frac{\log \gamma_1}{n-1} + \frac{\sum_{i=1}^{n-1} \exp(\mu_i)}{n-1}\right] - \sum_{i=1}^{n-1} \exp(\mu_i)\right\}}{U! \prod_{i=1}^{n-1} \left(t_i - \frac{U}{n-1}\right)!} r_1^U \exp\left\{\vartheta U + \sum_{i=1}^{n-1} \mu_i\, t_i\right\} . \tag{22}$$

Let us find denominator in the right part of expression (21):

$$p(T \mid \vartheta = 0) = \sum_D p(U, T \mid \vartheta = 0) , \tag{23}$$

where D is the domain of admissible values of the variable U. Summation in (23) is performed with respect to all m_n from D. Starting from the fact that $m_k \ge 0$ at any k = 1, ..., n, we determine that D is assigned by the system of inequalities

$$0 \le U < (n-1)\min(t_1,...,\ t_{n-1}) . \tag{24}$$

From (22) with regard to (23) and (24) we obtain

$$w(U \mid \vartheta = 0, T) = \frac{\gamma_1^{\,U}}{U! \prod_{i=1}^{n-1} \left(t_i - \frac{U}{n-1}\right)! \left[\sum_{k=0}^{(n-1)\min(t_1,...,t_{n-1})} \frac{\gamma_1^k}{k! \prod_{i=1}^{n-1} \left(t_i - \frac{k}{n-1}\right)!}\right]} . \tag{25}$$

Substituting (25) into (20), we have the equation for determining $C(\alpha,T)$ and ψ:

$$\frac{\psi \frac{\gamma_1^{C(\alpha,T)}}{C(\alpha,T)! \prod_{i=1}^{n-1} \left(t_i - \frac{C(\alpha,T)}{n-1}\right)!} + \sum_{U=C(\alpha,T)+1}^{(n-1)\min(t_1,...,t_{n-1})} \frac{\gamma_1^U}{U! \prod_{i=1}^{n-1} \left(t_i - \frac{U}{n-1}\right)!}}{\sum_{k=0}^{(n-1)\min(t_1,...,t_{n-1})} \frac{\gamma_1^k}{k! \prod_{i=1}^{n-1} \left(t_i - \frac{k}{n-1}\right)!}} = \alpha . \tag{26}$$

To determine the probability β of correct detection, it is required to find the expectation of the decision function $\varphi(U,T)=\varphi(U,t_1,...,t_{n-1})$ of algorithm (19), using the joint distribution of probabilities $P(U,T)$:

$$\beta=\sum_{t_1=0}^{\infty}\cdots\sum_{t_{n-1}=0}^{\infty}\sum_{U=C(\alpha,T)}^{(n-1)\min(t_1,...,t_{n-1})}\left\{\exp\left\{-\exp\left[\vartheta+\frac{\log\gamma_1}{n-1}+\frac{\sum_{i=1}^{n-1}\exp(\mu_i)}{n-1}\right]-\sum_{i=1}^{n-1}\exp(\mu_i)\right\}\times\right.$$

$$\left.\times\frac{\gamma_1{}^U\exp\left\{\vartheta U+\sum_{i=1}^{n-1}\mu_i t_i\right\}}{U!\prod_{i=1}^{n-1}\left(t_i-\frac{U}{n-1}\right)!}\varphi(U,t_1,...,\ t_{n-1})\right\}. \tag{27}$$

Consider in detail the structure and characteristics of algorithm (19) at *n* = 2. Its structure expressed in terms of the components of the initial sampling has the form:

$$\varphi(m_1,m_2)=\begin{cases}1, & m_2>C(\alpha,m_1+m_2)\ ,\\ \psi, & m_2=C(\alpha,m_1+m_2)\ ,\\ 0, & m_2<C(\alpha,m_1+m_2)\ .\end{cases} \tag{28}$$

The threshold function $C(\alpha,\ m_1+m_2)$ and randomization parameter ψ depend on the prescribed level of the false alarm probability α, value of sum m_1+m_2 and are determined as the solution of equation (26) which at *n* = 2 has the form:

$$\sum_{y=C(\alpha,\ m_1+m_2)+1}^{m_1+m_2}\binom{m_1+m_2}{y}\left(\frac{\gamma_1}{1+\gamma_1}\right)^y\left(\frac{1}{1+\gamma_1}\right)^{m_1+m_2-y}+$$

$$+\psi\binom{m_1+m_2}{C(\alpha,m_1+m_2)}\left(\frac{\gamma_1}{1+\gamma_1}\right)^{C(\alpha,\ m_1+m_2)}\left(\frac{1}{1+\gamma_1}\right)^{m_1+m_2-C(\alpha,\ m_1+m_2)}=\alpha\ , \tag{29}$$

where $\binom{m_1+m_2}{y}$, $\binom{m_1+m_2}{C(\alpha,m_1+m_2)}$ are the binomial coefficients and α is the prescribed level of the false alarm probability. If we examine only nonrandomized algorithms of detections (assuming that $\psi=0$), then to determine the threshold function, it is required to solve the inequality

$$\max_{C(\alpha,\ m_1+m_2)}\sum_{y=C(\alpha,\ m_1+m_2)+1}^{m_1+m_2}\binom{m_1+m_2}{y}\left(\frac{\gamma_1}{1+\gamma_1}\right)^y\left(\frac{1}{1+\gamma_1}\right)^{m_1+m_2-y}\le\alpha\ . \tag{30}$$

Since the initial sampling has the integral character, then the threshold function found leads to some oscillations of conditional probability of false alarm when the values of statistic m_1+m_2 change (Fig. 2).

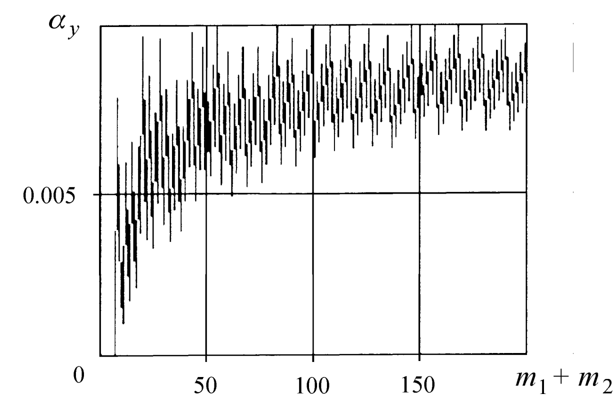

Fig. 2. Dependence of the conditional probability of false alarm on the value of statistic $m_1 + m_2$ at $\alpha = 0.01$; $\gamma_1 = 1$ (nonrandomized algorithm of detection)

We emphasize that the conditional probability of false alarm depends only on the sampling volume $m_1 + m_2$ prescribed level of the false alarm probability α, and the parameter γ_1, but it does not depend on the parameters of distribution of the initial sampling λ_1, λ_2.

If $(m_1 + m_2)\left(\frac{\gamma_1}{1+\gamma_1}\right)\left(\frac{1}{1+\gamma_1}\right) > 9$ and $\frac{1}{m_1 + m_2 + 1} < \frac{\gamma_1}{1+\gamma_1} < \frac{m_1 + m_2}{m_1 + m_2 + 1}$, then to calculate $C(\alpha, m_1 + m_2)$ and ψ, we can use approximation of binomial distribution (30) by normal distributions with the mean $(m_1 + m_2)[\gamma_1/(\gamma_1 + 1)]$ and dispersion $(m_1 + m_2)[\gamma_1/(\gamma_1 + 1)]\cdot$ $\cdot[1/(\gamma_1 + 1)]$ [12]:

$$C(\alpha, m_1 + m_2) = \left[(m_1 + m_2)\left(\frac{\gamma_1}{1+\gamma_1}\right) + F^{-1}(1-\alpha)\sqrt{(m_1 + m_2)\left(\frac{\gamma_1}{1+\gamma_1}\right)\left(\frac{1}{1+\gamma_1}\right)} \right];$$

$$\psi \approx \frac{\alpha - \left\{ 1 - F\left(\dfrac{C(\alpha, m_1 + m_2) + 1 - \dfrac{(m_1 + m_2)\gamma_1}{1+\gamma_1}}{\sqrt{\dfrac{(m_1 + m_2)\gamma_1}{(1+\gamma_1)}\left(\dfrac{1}{1+\gamma_1}\right)}} \right) \right\}}{F\left(\dfrac{C(\alpha, m_1 + m_2) + 1 - \dfrac{(m_1 + m_2)\gamma_1}{1+\gamma_1}}{\sqrt{\dfrac{(m_1 + m_2)\gamma_1}{(1+\gamma_1)}\left(\dfrac{1}{1+\gamma_1}\right)}} \right) - F\left(\dfrac{C(\alpha, m_1 + m_2) - \dfrac{(m_1 + m_2)\gamma_1}{1+\gamma_1}}{\sqrt{\dfrac{(m_1 + m_2)\gamma_1}{(1+\gamma_1)}\left(\dfrac{1}{1+\gamma_1}\right)}} \right)}, \qquad (31)$$

where $[x]$ is the integral part of number x, and $F^{-1}(1-\alpha)$ is the inverse function to the integral function of standard normal distribution of probabilities with the zero mean and unit dispersion.

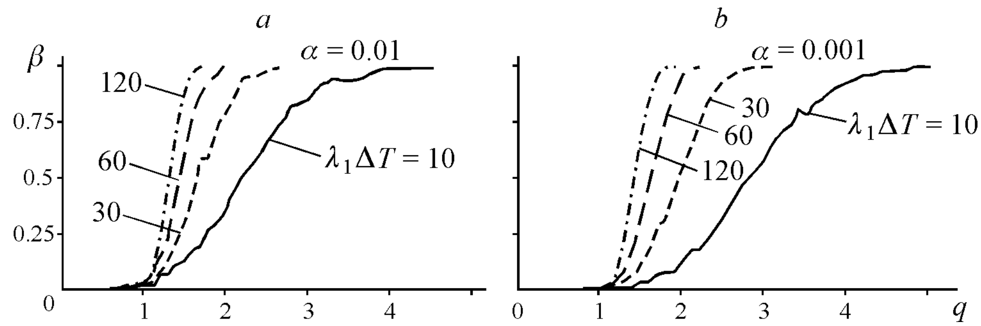

Fig. 3. Dependence of the correct detection probability β on the parameter q when the false alarm probability $\alpha = 0.01$ (a) and $\alpha = 0.001$ (b) and at different values of background component intensity $\lambda_1 \Delta T$

Figure 3 shows the dependence of correct detection probability β of algorithm (28), where the threshold function $C(\alpha,\ m_1 + m_2)$ and randomization parameter ψ are calculated according to (31), on the ratio of flow intensities in the second and first components $q = \lambda_2 / \lambda_1$ at different values of the false alarm probability, and the first component intensity. The dependences are obtained by method of simulation of algorithm (28) on a computer.

As is seen from Fig. 3, the probability of correct detection of algorithm (28), unlike algorithm (12), depends not only on the ratio of the intensities of Poisson's flow components but also on the value of background component intensity. In this case, the false alarm probability (in Fig. 3, it is the domain $q \le 1$) does not exceed the prescribed level α.

3. PRACTICAL APPROBATION OF ALGORITHMS

Practical approbation of algorithms for failure prediction was performed under laboratory conditions during failure of marble samples; in this case, algorithms (12) and (28) were taken for realization (they were tested simultaneously on the same samples). In experiments, parameters for (12) were assumed as follows: $n = 7$, $m_1 = ... = m_6 = 1$, $m_7 = 10$, $\alpha = 10^{-3}$, and $\gamma = 1$. Algorithm (28) was realized at $\alpha = 10^{-3}$, $\gamma_1 = 1$, and $\Delta T = 40$ ms.

All in all, more than 100 experiments were conducted. In order to check the value of false alarm probability, the tested sample was subjected to uniaxial compression by constant load that is certainly less than that one during which the failure occurs. False alarms were not recorded.

In order to check the prediction efficiency, the marble samples were subjected to uniaxial compression with the constant loading velocity equal to 10 N/s. The use of algorithms (12) and (28) ensured the stable detection of variation in cracking intensity by $12-15$ electromagnetic radiation pulses. At this loading velocity, the failure was predicted for $10-20$ s prior to its beginning.

CONCLUSIONS

1. The problem of failure prediction is formulated as the problem of testing the statistical hypotheses relative to the parameter of flow intensity of electromagnetic radiation pulses arising during rock failure.

2. Two optimal prediction algorithms are synthesized — at fixed number of pulses recorded and at fixed observation time. The algorithms determined ensure the maximum probability of correct solution to the problem on variation in pulse flow intensity when the value of false alarm probability is fixed.

3. Practical approbation of algorithms demonstrated the possibility of their use for the rock failure prediction.

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