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Polar wander caused by the Quaternary glacial cycles and fluid Love number

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Abstract

Perturbations of the Earth's rotation caused by the Quaternary glacial cycles provide an important constraint on the viscosity of the deep mantle because they represent a long-wavelength response of the Earth to surface load redistribution. The predicted present-day polar wander speed (PWS) is, however, sensitive to both the lower mantle viscosity (η_{lm}), the density jump at 670 km depth, and the lithospheric thickness and viscosity (e.g., Sabadini and Peltier, Geophys. J. R. Astron. Soc. 66 (1981) 553–578; Yuen et al., J. Geophys. Res. 87 (1982) 10745–10762; Peltier and Wu, Geophys. Res. Lett. 10 (1983) 181–184; Wu and Peltier, Geophys, J. R. Astron. Soc. 76 (1984) 753–791; Peltier, J. Geophys. Res. 89 (1984) 11303–11316; Vermeersen et al., J. Geophys. Res. 102 (1997) 27689–27702; Mitrovica and Milne, J. Geophys. Res. 103 (1998) 985–1005; Johnston and Lambeck, Geophys. J. Int. 136 (1999) 537–558; Nakada, Geophys. J. Int. 143 (2000) 230–238). For earth models with $\eta_{\mathrm{lm}} < 5\times10^{21}$ Pa s and an elastic lithosphere, the present-day PWS is very sensitive to the M1 mode (buoyancy mode) related to the density jump at 670 km depth [Mitrovica and Milne, J. Geophys. Res. 103 (1998) 985–1005]. The contribution of the M1 mode, however, is less significant for earth models with a viscoelastic lithosphere [Nakada, Geophys. J. Int. 143 (2000) 230–238]. This is due to the fact that this contribution depends on the relative strength of the M1 mode, $\Delta k_2^{\mathrm{T}}(\mathrm{M1})/k_{\mathrm{f}}^{\mathrm{T}}$, where $\Delta k_2^{\mathrm{T}}(\mathrm{M1})$ is the magnitude of tidal Love number (k_2^{T}) of the M1 mode and $k_{\mathrm{f}}^{\mathrm{T}}$ is the value of k_2^{T} in the fluid limit (fluid Love number). The magnitude of $k_{\mathrm{f}}^{\mathrm{T}}$ for earth models with a viscoelastic lithosphere is larger than that for an elastic lithosphere, and it is smaller for a thicker elastic lithosphere than for a thinner one. Thus, for earth models with a viscoelastic lithosphere, the PWS is mainly sensitive to the lower mantle viscosity regardless of the behavior of the 670 km density discontinuity. This relation also explains why the predicted PWS increases with increasing thickness of an elastic lithosphere. That is, since the value of $\Delta k_2^{\mathrm{T}}(\mathrm{M1})/k_{\mathrm{f}}^{\mathrm{T}}$ with a thicker elastic lithosphere is larger than that with a thinner elastic lithosphere, the M1 mode will have a higher contribution in the case of a thicker elastic lithosphere.

Keywords: polar wandering; viscoelasticity; lithosphere; viscosity; isostasy

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1. Introduction

Polar wander, wander of the rotation pole with respect to the surface geography, caused by the Quaternary glacial cycles has been extensively examined in order to investigate the rheological structure of the Earth's mantle (e.g., [1–9]). These studies indicated that the present-day polar wander speed (PWS) is sensitive to the lower mantle viscosity ($\eta_{\rm lm}$), the density jump at the 670 km discontinuity ($\Delta\rho_{670}$) and the thickness of an elastic lithosphere (H). Nakada [9] also pointed out that the PWS is sensitive to a lithospheric viscosity.

Polar wander studies for the glacial cycles have generally been based on earth models with an elastic lithosphere (e.g., [1–8]). For earth models with an elastic lithosphere, the predicted present-day PWS for $\eta_{\rm lm} < 5\times10^{21}$ Pa s is very sensitive to the M1 mode associated with the density jump at 670 km depth (e.g., [7–9]). The M1 mode arises from the deflection of the density discontinuity, and is due to the buoyancy between the upper and lower mantle. The PWS also significantly increases with increasing thickness of an elastic lithosphere for $\eta_{\rm lm} < 5\times10^{21}$ Pa s [3,5,7,9]. In earth models with $\eta_{\rm lm} < 5\times10^{21}$ Pa s, however, the contribution of the M1 mode is less significant for a viscoelastic lithosphere [9]. Then the PWS mainly depends on the lower mantle viscosity only, similar to the sensitivity of the rate of change of the Earth's rotation ($\dot{m}_3$) associated with the glacial cycles (e.g., [4,7,8]). (The rate of change of the degree two zonal harmonic of the geopotential ($\dot{J}_2$), which is proportional to $\dot{m}_3$, has been shown to be predominantly sensitive to the lower mantle viscosity.) Perturbations of the Earth's rotation will be sensitive to the viscosity of the deep mantle because they represent a long-wavelength response of the Earth to surface load redistribution. However, the reason why the PWS is highly sensitive to the lithospheric thickness and viscosity is not understood. A main purpose of this study is to address this problem. This will be important when we put some constraints on the rheological structure of the Earth by examining observed and predicted perturbations of the Earth's rotation. The earth models adopted in this study are compressible.

2. Mathematical formulation

A Maxwell viscoelastic earth model is adopted here. The Earth's angular velocity ω is given by $\omega = \Omega(m_1,\ m_2\ 1+m_3)$, in which Ω is the mean angular velocity of the Earth [10,11]. Then, Liouville's equation describing the polar wander is given by [1,4,10,11]:

$$\frac{i}{\sigma_r}\dot{\mathbf{m}} + \mathbf{m} = \frac{1}{C-A}[\delta(t) + k_2^{\rm L}(t)]^* \left[\Delta\mathbf{I} - \frac{i}{\Omega}\Delta\dot{\mathbf{I}}\right] + \frac{k_2^{\rm T}(t)}{k_{\rm f}^{\rm T}}{}^*\mathbf{m} \tag{1}$$

where $\sigma_{\rm r} = (C-A)\Omega/A$, $\mathbf{m} = m_1 + im_2$ and $\Delta\mathbf{I} = \Delta I_{13}^{\rm R} + i\Delta I_{23}^{\rm R}$. The asterisk denotes a time convolution. A and C are the equatorial and polar moments of inertia, respectively. $\Delta I_{13}^{\rm R}$ and $\Delta I_{23}^{\rm R}$ are the components of the inertia tensor associated with the surface load distribution on a rigid Earth. $k_2^{\rm L}$ and $k_2^{\rm T}$ are the degree two load and tidal Love numbers, respectively, and $k_{\rm f}^{\rm T}$ is the value of $k_2^{\rm T}$ in the fluid limit. In Eq. 1, the first term of the left-hand side ($\dot{\mathbf{m}}$) and the $\Delta\dot{\mathbf{I}}$ term on the right-hand side are safely neglected in the evaluation of the secular term of polar wander [4,7,12]. Then, Eq. 1 is expressed as:

$$\mathbf{m} = \frac{1}{C-A}[\delta(t) + k_2^{\rm L}(t)]^*\Delta\mathbf{I} + \frac{k_2^{\rm T}(t)}{k_{\rm f}^{\rm T}}{}^*\mathbf{m} \tag{2}$$

The first term of the right-hand side of Eq. 2 represents the polar wander caused by the surface load and its related Earth's deformation. The second term is associated with the deformation induced by the shift of the rotation axis. The sensitivity of the PWS to the M1 mode and lithosphere is more clearly understood if we recognize that the sensitivity of the PWS to $k_2^{\rm T}(t)$ depends on $k_2^{\rm T}(t)/k_{\rm f}^{\rm T}$ rather than $k_2^{\rm T}(t)$ itself, as discussed below.

In general, the $k_2^{\rm L}(t)$ and $k_2^{\rm T}(t)$ for a Maxwell viscoelastic earth model are evaluated using a Laplace transform method, and $k_2^{\rm L}(s)$ and $k_2^{\rm T}(s)$ in the Laplace transform domain are given by [13]:

$$k_2^{\mathrm{L}}(s) = k_2^{\mathrm{L,E}} + \sum_{j=1}^{N} \frac{\gamma_j^{\mathrm{L}}}{s+s_j} \tag{3}$$

and:

$$k_2^{\mathrm{T}}(s) = k_2^{\mathrm{T,E}} + \sum_{j=1}^{N} \frac{\gamma_j^{\mathrm{T}}}{s+s_j} \tag{4}$$

where s is the Laplace transform variable, s_j is the inverse relaxation time of eigenmode j, γ_j^{L} and γ_j^{T} are the strengths of the eigenmode j. $k_{\mathrm{f}}^{\mathrm{T}}$ is evaluated by:

$$k_{\mathrm{f}}^{\mathrm{T}} = k_2^{\mathrm{T}}(s=0) = k_2^{\mathrm{T,E}} + \sum_{j=1}^{N} \frac{\gamma_j^{\mathrm{T}}}{s_j} \tag{5}$$

A finite difference method is adopted here in calculating **m** from Eq. 2. In this method, we do not have to evaluate a few hundred complex rotational eigenmodes [7,12] existing in a realistic earth model such as PREM [14]. We compute Love numbers based on an initial value approach developed by Hanyk et al. [15], and solve Eq. 2 using a finite difference method. The Love numbers $k_2^{\mathrm{L,H}}(t)$ and $k_2^{\mathrm{T,H}}(t)$ due to a Heaviside load $H(t)$, which are evaluated by initial value approach [15], can be expressed through the usual Love numbers $k_2^{\mathrm{L}}(t)$ and $k_2^{\mathrm{T}}(t)$ due to a $\delta(t)$ load:

$$k_2^{\mathrm{L,H}}(t) = \int_0^t k_2^{\mathrm{L}}(t-\tau)H(\tau)\mathrm{d}\tau \tag{6}$$

and:

$$k_2^{\mathrm{T,H}}(t) = \int_0^t k_2^{\mathrm{T}}(t-\tau)H(\tau)\mathrm{d}\tau \tag{7}$$

We note $k_{\mathrm{f}}^{\mathrm{T}} = k_2^{\mathrm{T}}(s=0) = k_2^{\mathrm{T,H}}$ $(t=\infty)$ by using Eqs. 4 and 7. The temporal terms of $m_j(t)$ and $\Delta I_{j3}^{\mathrm{R}}(t)$ $(j=1,2)$ are expressed as a series of Heaviside increments:

$$m_j(t) = \sum_{i=0}^{\infty} \delta m_j^i H(t-i\Delta t) \tag{8}$$

and:

$$\Delta I_{j3}^{\mathrm{R}}(t) = \sum_{i=0}^{\infty} \delta \Delta I_{j3}^{\mathrm{R},i} \mathrm{H}(t-i\Delta t) \tag{9}$$

Substituting Eqs. 8 and 9 into Eq. 2, using relations Eqs. 6 and 7, we have:

$$\delta m_j^n = \left[1 - \frac{k_2^{\mathrm{T,H}}(0)}{k_{\mathrm{f}}^{\mathrm{T}}}\right]^{-1} \times \left\{ \frac{\Omega}{A\sigma_r}\left[\Delta I_{j3}^{\mathrm{R}}(n\Delta t) + \sum_{i=0}^{n} \delta\Delta I_{j3}^{\mathrm{R},i} k_2^{\mathrm{L,H}}(n\Delta t - i\Delta t)\right] + \sum_{i=0}^{n-1}\left[\frac{k_2^{\mathrm{T,H}}(n\Delta t - i\Delta t)}{k_{\mathrm{f}}^{\mathrm{T}}} - 1\right]\delta m_j^i \right\} \tag{10}$$

We evaluate δm_j^n by using $\Delta t = 1$ yr.

3. Results

We use a compressible earth model with elasticity and density structure given by the seismological model PREM [14]. A surface load history with 10 saw-tooth load cycles is adopted here, in which each glacial cycle is characterized by a 90 kyr glaciation phase and by a 10 kyr deglaciation phase. The final deglaciation is assumed to have ended 6 kyr BP. $\Delta I_{13}^{\mathrm{R}}$ and $\Delta I_{23}^{\mathrm{R}}$ in Eq. 2 are the same as those by Mitrovica and Milne [7], and the values of $\Delta I_{13}^{\mathrm{R}}$ and $\Delta I_{23}^{\mathrm{R}}$ at glacial maximum are -6.67×10^{31} kg m^2 and 2.31×10^{32} kg m^2, respectively. This simple loading history is adopted here to clearly indicate the effect of the lithosphere and the M1 mode on predictions of PWS.

Fig. 1 shows the tidal Love numbers $k_2^{\mathrm{T,H}}(t)$ due to a Heaviside load $H(t)$ for earth models with upper and lower mantle viscosities of 10^{21} Pa s. The notation for these results in Fig. 1 is of the form E($H, \eta_{\mathrm{lith}}, i$), where H and η_{lith} refer to the thickness of lithosphere (km) and lithospheric viscosity (Pa s), respectively. The model result with '$i=c$' corresponds to an earth model with a constant (depth-independent) lithospheric viscosity. That with '$i=d$' corresponds to an earth model with a depth-dependent lithospheric viscosity defined by $\eta(D) = \eta_{\mathrm{lith}}10^{-\Delta\eta D/H}$. D is the depth

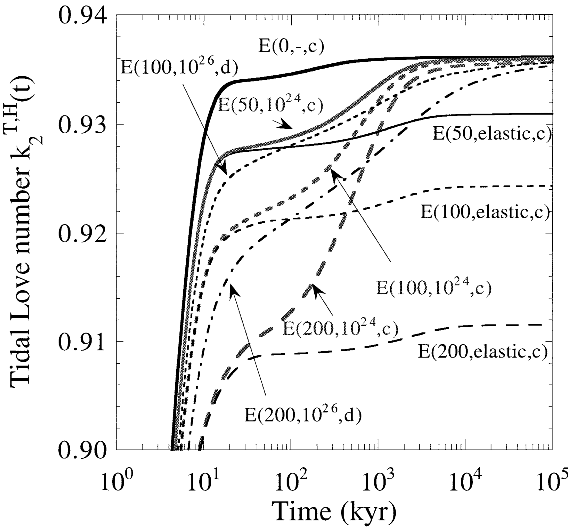

Fig. 1. Tidal Love number resulting from a Heaviside load $H(t)$ for a compressible earth model with elasticity and density structure given by the seismological model PREM [14]. The lower and upper mantle viscosities adopted here are 10^{21} Pa s. The notation for the rheological model is of the form E(H,η_{lith},i). H and η_{lith} represent the lithospheric thickness (km) and lithospheric viscosity (Pa s), respectively. The model result with '$i=c$' corresponds to an earth model with a constant (depth-independent) lithospheric viscosity. That with '$i=d$' corresponds to an earth model with a depth-dependent lithospheric viscosity defined by $\eta(D)=\eta_{\mathrm{lith}}10^{-\Delta\eta D/H}$. D is the depth from the surface, $\Delta\eta=\log(\eta_{\mathrm{lith}}/\eta_{\mathrm{um}})$ and η_{um} is the upper mantle viscosity. The model with an elastic lithosphere is denoted by E(H,elastic,c), and that with no lithosphere is denoted by E(0,−,c).

from the surface, $\Delta\eta=\log(\eta_{\mathrm{lith}}/\eta_{\mathrm{um}})$ and η_{um} is the upper mantle viscosity. η_{lith} is denoted by 'elastic' for an elastic lithosphere. Model results with no lithosphere are referred to as E(0,−,c). This figure indicates that $k_2^{\mathrm{T,H}}(t=10^5$ kyr) depends on the viscosity of the lithosphere and thickness of an elastic lithosphere. Those values for earth models used here are shown in Table 1. Those for E(100,10^{26},d) and E(200,10^{26},d) are similar to those with a viscoelastic lithosphere. That is, $k_2^{\mathrm{T,H}}(t=10^5$ kyr), which corresponds to k_f^{T} in Eq. 1 in our study, takes a similar value in earth models with a finite lithospheric viscosity. Its value for an elastic lithosphere is significantly different from that with a viscoelastic lithosphere. Moreover, its magnitude for an elastic lithosphere significantly decreases with increasing lithospheric thickness, because a thinner elastic lithosphere weakly supports the stresses in the Earth's deformation compared with a thicker lithosphere.

Fig. 2a–c shows the present-day PWS as a function of the lower mantle viscosity for models with $H=50$, 100 and 200 km, respectively. These figures indicate that predictions are less sensitive to the lithospheric viscosity. That is, the only important thing seems to be whether a viscoelastic or elastic lithosphere is adopted. In Fig. 2a, I also show model results for a hypothetical earth model with $k_2^{\mathrm{T,H}}(t)$ for E(50,elastic,c) and $k_f^{\mathrm{T}}=k_2^{\mathrm{T,H}}(t=10^5$ kyr) for E(0,−,c), referred to as EM(50) here. Results for EM(100) with $H=100$ km and EM(200) with $H=200$ km are shown in Fig. 2b,c, respectively. Predictions for EM(50), EM(100) and EM(200) would correspond to those with an extremely high lithospheric viscosity, because isostatic state is attained in an infinite time. For earth models with $\eta_{\mathrm{lm}}<10^{22}$ Pa s, model results for EM(100) and EM(200) are significantly different from those with an elastic lithosphere, but are very similar to those with a finite lithospheric viscosity such as 10^{24} Pa s.

Fig. 3 depicts the $m_1(t)$ and $m_2(t)$ with a function of time for models with $H=100$ km. The upper mantle viscosity is 10^{21} Pa s, and model results with $\eta_{\mathrm{lm}}=10^{21}$ and 10^{22} Pa s are shown in these figures. These figures also point out that model results with an elastic lithosphere are significantly different from those with a viscoelastic lithosphere and those with an extremely high

Table 1
Values of $k_2^{\mathrm{T,H}}(t=10^5$ kyr) for earth models used in this study

Name	$k_2^{\mathrm{T,H}}$ ($t=10^5$ kyr)
E(0,−,c)	0.9362
E(50,elastic,c)	0.9310
E(100,elastic,c)	0.9244
E(200,elastic,c)	0.9115
E(50,10^{24},c)	0.9360
E(100,10^{24},c)	0.9359
E(200,10^{24},c)	0.9357
E(100,10^{26},d)	0.9357
E(200,10^{26},d)	0.9354

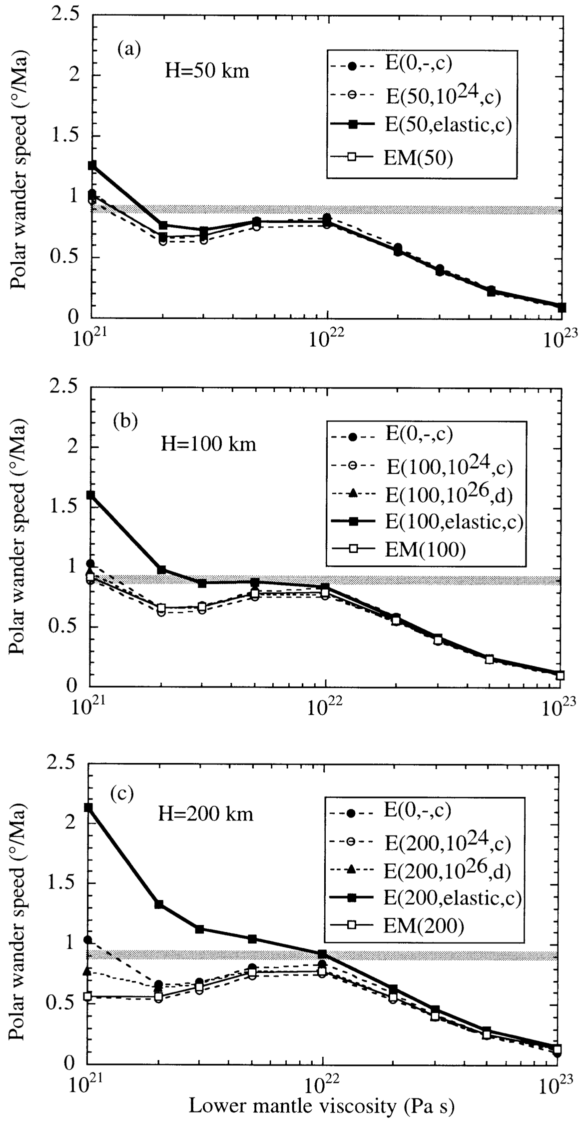

Fig. 2. Predicted PWSs for the seismological model PREM [14] as a function of the lower mantle viscosity. The upper mantle viscosity is 10^{21} Pa s. The notation for the model results is the same as that in Fig. 1, except for the case of EM(H). (a) $H=50$ km. (b) $H=100$ km. (c) $H=200$ km. Model results denoted by EM(H) represent the predictions for earth models with $k_2^{T,H}(t)$ for E(H,elastic,c) and $k_f^T = k_2^{T,H}(t=10^5$ kyr) for E(0,−,c). The shaded region represents the observed PWS reported by McCarthy and Luzum [18].

lithospheric viscosity (model EM). Thus, the predicted polar wander due to the Quaternary glacial cycles is very sensitive to the lithospheric viscosity. In Section 4, I will discuss the reason for this sensitivity by considering the relative strength of $k_2^{T,H}$ of the M1 mode to k_f^T, $\Delta k_2^{T,H}$ (M1)/k_f^T.

4. Discussion and concluding remarks

For earth models with both $\eta_{lm} < 5\times10^{21}$ Pa s and an elastic lithosphere, predicted present-day PWS due to the Quaternary glacial cycles increases with increasing lithospheric thickness, and the M1 mode contributes a dominant portion of the predicted PWS (e.g., [7–9]). On the other hand, the PWS for a viscoelastic lithosphere is very different from that with an elastic lithosphere in earth models with $\eta_{lm} < 5\times10^{21}$ Pa s as shown above [9]. Nakada [9] suggested that the difference may be attributed to viscoelastic relaxation of the lithosphere. However, results for E(100,10^{24},c), E(100,elastic,c) and EM(100) suggest that this is not a main cause. That is, the present-day PWS for EM(100) is nearly the same as that for E(100,10^{24},c), although there is no viscous relaxation of a lithosphere for $t<1$ Myr in $k_2^{T,H}(t)$ of EM(100) (see Fig. 1).

The relaxation time of the M1 mode is mainly sensitive to the lower mantle viscosity, and its value is longer than 3 Myr for earth models with $\eta_{lm} > 5\times10^{21}$ Pa s and much longer than the Quaternary ice age $\sim$1 Myr [16]. The predicted present-day PWS for $\eta_{lm} > 5\times10^{21}$ Pa s is, therefore, less sensitive to the M1 mode. On the other hand, excitation of the M1 mode is dependent on the magnitude of the density jump at 670 km depth (e.g., [7,16,17]), and $\Delta\rho_{670} = 388.57$ kg/m^3 for PREM [14]. The contribution of the M1 mode for earth models with a viscoelastic lithosphere is not easily estimated from the results in Fig. 1 because the Maxwell relaxation time of a viscoelastic lithosphere is similar to that of the M1 mode for $\eta_{lm} = 10^{21}$ Pa s. However, it is inferred from the results with an elastic lithosphere. For example, we consider the results for E(100,elastic,c). The M1 mode contribution to $k_2^{T,H}$, $\Delta k_2^{T,H}$(M1), corresponds to $k_2^{T,H}(t=10^5$ kyr)$-k_2^{T,H}(t=10^2$ kyr). On the other hand, the magnitude of k_f^T, the fluid limit of $k_2^{T,H}$, is similar in earth models with a viscoelastic lithosphere (Fig. 1). Its magnitude for an elastic lithosphere

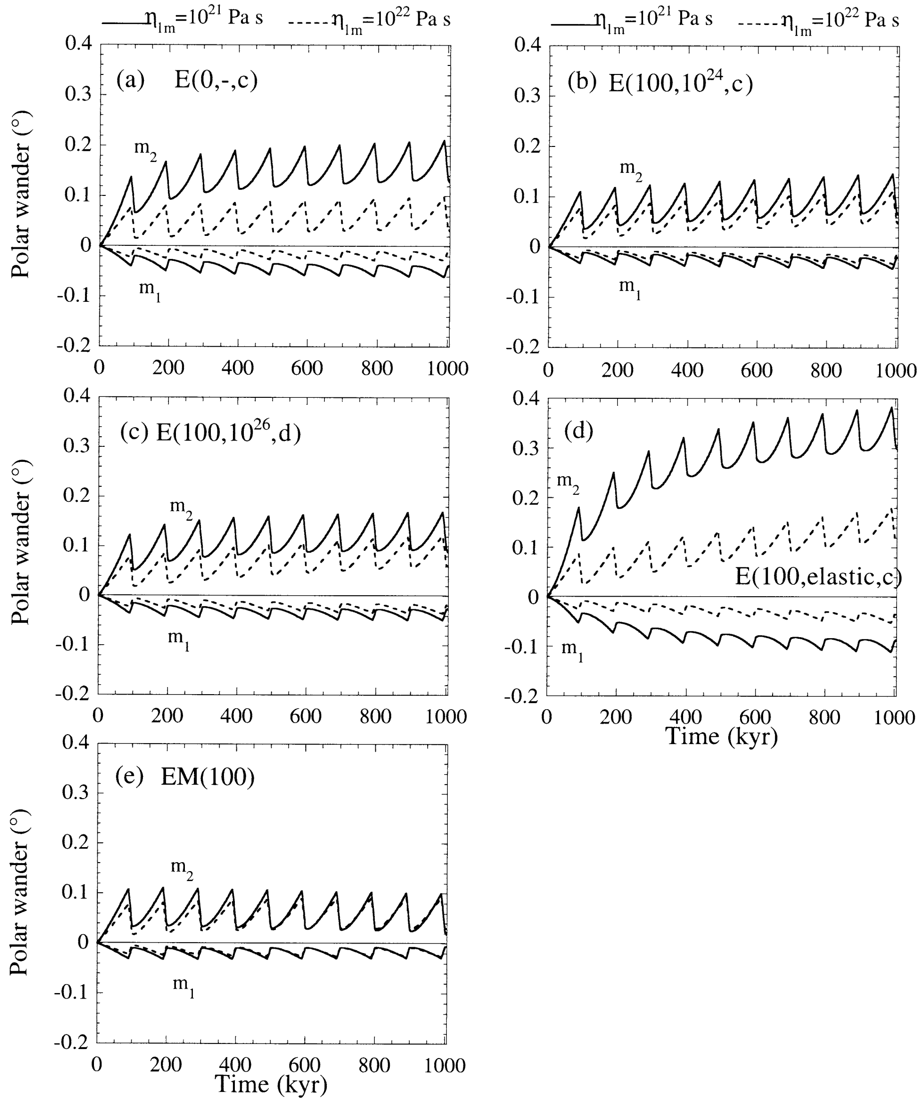

Fig. 3. Predicted $m_1(t)$ and $m_2(t)$ as a function of time for the seismological model PREM [14]. The upper mantle viscosity is 10^{21} Pa s, and the lithospheric thickness is 100 km. The solid and dashed lines correspond to the predictions for earth models with lower mantle viscosity (η_{lm}) of 10^{21} and 10^{22} Pa s, respectively. The notation for the model results is the same as in Fig. 2.

decreases significantly with increasing thickness. By considering these results, we discuss the role of the lithosphere and the M1 mode on the PWS.

We first discuss the predictions for an elastic lithosphere. For models with $H = 200$ km, the relative strength $\Delta k_2^{T,H}(M1)/k_f^T$ is significantly larger than that for $H = 100$ km because the $\Delta k_2^{T,H}(M1)$ is similar in both models and k_f^T for $H = 200$ km is smaller than that for $H = 100$ km (Fig. 1). Thus, the contribution of the M1 mode for $H = 200$ km is significantly larger, resulting in an increase in the PWS for the $H = 200$ km model. The k_f^T value is similar in earth models with a viscoelastic lithosphere. Moreover, its value is significantly larger than that for an elastic lithosphere. That is, the magnitude of $\Delta k_2^{T,H}(M1)/k_f^T$ for a viscoelastic lithosphere is smaller than that for an elastic one. Therefore, in the case of a viscoelastic lithosphere, the M1 mode contributes insignificantly to the PWS even for earth models with $\eta_{\mathrm{lm}} < 5\times10^{21}$ Pa s, and the PWS is mainly sensitive to the lower mantle viscosity only. The present-day PWS for $\eta_{\mathrm{lm}} < 5\times10^{21}$ Pa s is, therefore, controlled by the relative strength of the M1 mode, $\Delta k_2^{\mathrm{T,H}}(\mathrm{M1})/k_{\mathrm{f}}^{\mathrm{T}}$, in which $k_{\mathrm{f}}^{\mathrm{T}}$ depends on an adopted rheology of the lithosphere. On the other hand, in earth models with $\eta_{\mathrm{lm}} > 5\times10^{21}$ Pa s, $\Delta k_2^{\mathrm{T,H}}(\mathrm{M1})/k_{\mathrm{f}}^{\mathrm{T}}$ is insignificant because the relaxation time of the M1 mode (>3 Myr) is significantly longer than the timescale of the Quaternary ice age, ~ 1 Myr. In this case, the present-day PWS is sensitive to the lower mantle viscosity only.

Observed present-day PWS is about 1°/Ma [18], and many causes for this have been discussed. For example, the PWS is sensitive to the present-day mass flux of the Antarctic and Greenland ice sheets (e.g., [8]), and to the mass distribution associated with mantle convection [19,20] and mountain building [21]. If, however, we assume that the observed PWS [18] is dominantly contributed by the Quaternary glacial cycles, then the rheological structure of the Earth's mantle inferred from the PWS depends on an adopted rheological model of the lithosphere, i.e., elastic or viscoelastic.

In order to estimate the rheological structure of the lithosphere, many geological and geophysical phenomena have been analyzed as reviewed by Watts [22]. The inferred lithospheric thickness and effective viscosity depend not only on lithospheric age but also on the details of load. For example, an effective elastic thickness of lithosphere will be dependent on the duration of external and/or internal loading as first indicated by Walcott [23]. Walcott [23] also indicated that estimates of the elastic thickness could be explained by a viscoelastic lithosphere with a typical relaxation time. Seismic estimates of lithospheric thickness (e.g., [24]), short-term elastic thickness, will give relatively large values, and can exceed 220 km beneath the stable shields [25]. The thickness of oceanic lithosphere increases with the square root of its age, and is approximately identical to the thickness of the thermal boundary layer defined by the 1100°C isotherm [26–28]. An elastic lithospheric thickness can also be evaluated from the sea-level variations due to the glacial rebound for a timescale of 10^3–10^4 years. For example, lithospheric thickness around the British Isles seems to be 50–70 km [29]. Estimates inferred from topographic and gravity measurements of the flexure of islands, trenches, and mountain belts with a characteristic loading timescale of greater than 10^6 years will usually lead to lower values of 10–30 km associated with a temperature of 300–450°C (e.g., [30–32]). On the other hand, studies using seismic and geoid data, and flow models suggest an effective lithospheric viscosity similar to the lower mantle viscosity (e.g., [33]). Thus, an effective lithospheric viscosity inferred from geological and geophysical phenomena seems to depend on the phenomenon examined. However, the magnitude of $k_{\mathrm{f}}^{\mathrm{T}}$ in Eq. 1 represents the k_2^{T} value in the fluid limit. Thus, it may be reasonable to adopt a viscoelastic lithosphere or an extremely high lithospheric viscosity of type EM in predicting the PWS due to the Quaternary glacial cycles.

Our study implies that the predicted PWS will not converge to the elastic case even if we adopt an earth model with a very high lithospheric viscosity. For example, consider an earth model with a viscoelastic lithosphere of $\eta_{\mathrm{lith}} = 10^{40}$ Pa s and 100 km thickness. The predicted PWS for this model must converge toward the elastic case. However, the prediction with $k_{\mathrm{f}}^{\mathrm{T}}$ defined by Eq. 5 (see also Eq. 1) will give the same value as that for EM(100). This may imply that the $k_{\mathrm{f}}^{\mathrm{T}}$ in Eq. 1 is intended to be the observed value (Mitrovica, personal communication, 2002). The $k_{\mathrm{f}}^{\mathrm{T}}$ for the PREM estimated from $3(C-A)G/R^5\Omega^2$ [10,11] is 0.9381, similar to those for earth models with a viscoelastic lithosphere (see Table 1). R is the radius of the Earth and G is the gravitational constant. In this case, as we expect on physical grounds, the only difference of polar wander between models will arise due to differences in the viscoelastic Love numbers for $t < 1$ Myr. Therefore the prediction for E(100,elastic,c) will provide the same prediction as EM(100), and PWS predictions for increasing lithospheric viscosity converge to the elastic case. Thus, we may have to use the observed $k_{\mathrm{f}}^{\mathrm{T}}$ in Eq. 1 because we are really concerned about perturbations to the observed state.

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