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Aquifer parameters determination for large diameter wells using neural network approach

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Abstract

Use of artificial neural networks (ANNs) is becoming increasingly common in the analysis of groundwater hydrology and water resources problems. In this research, an ANN was developed and used to estimate aquifer parameter values, namely transmissivity and storage coefficient, from pumping test data for a large diameter well. The ANN was trained to map time–drawdown and well diameter data (input vector) to its corresponding transmissivity and storage coefficient values (output vector). Based upon a pre-specified range of aquifer parameters, the input vectors were generated from the analytical solution of Papadopulos and Copper for large diameter well in a homogeneous, isotropic, non-leaky confined aquifer. The ANN was trained with a fixed number of drawdown data points corresponding to a varying pre-specified range of aquifer parameters and time-series values. Once the network is trained to an acceptable level of accuracy, it produces an output of aquifer parameter values for any input vector. The results obtained with the ANN are in good agreement with published values. A significant advantage of the ANN approach is that it overcomes the problem of determining the storage coefficient, which when determined by traditional type curve matching method is of questionable reliability.

Keywords: Aquifer parameters; Artificial neural network; Large diameter well; Backpropagation algorithm; Well hydraulics; Training sets

1. Introduction

Almost all well hydraulics models are based on the assumption that the pumped well is a line source. This assumption proposed by Theis (1935) may not be valid if the well bore storage effects are significant. Effects of well bore storage may become important when the aquifer transmissivity and storage coefficient are small or the pumped well diameter is large. Papadopulos and Cooper (1967) developed an analytical solution and type curves in and around a large diameter well in a homogeneous and isotropic non-leaky confined aquifer by taking into consideration the water derived from storage within the well. Later, Moench (1985) presented mathematical models that combine the leaky aquifer theory of Hantush (1960) with the theory of flow to a large diameter well. In their analysis, Sakthivadivel and Rushton (1989) estimated aquifer parameters taking into account the dynamic seepage face. They analyzed both pumping and recovery test data. Herbert and Barker (1990) reanalyzed the same data using the Papadopulos and Cooper (1967) model. Results of these analyses are used for comparison with those obtained by the artificial neural network (ANN) approach.

The ANN technology is an alternate computational

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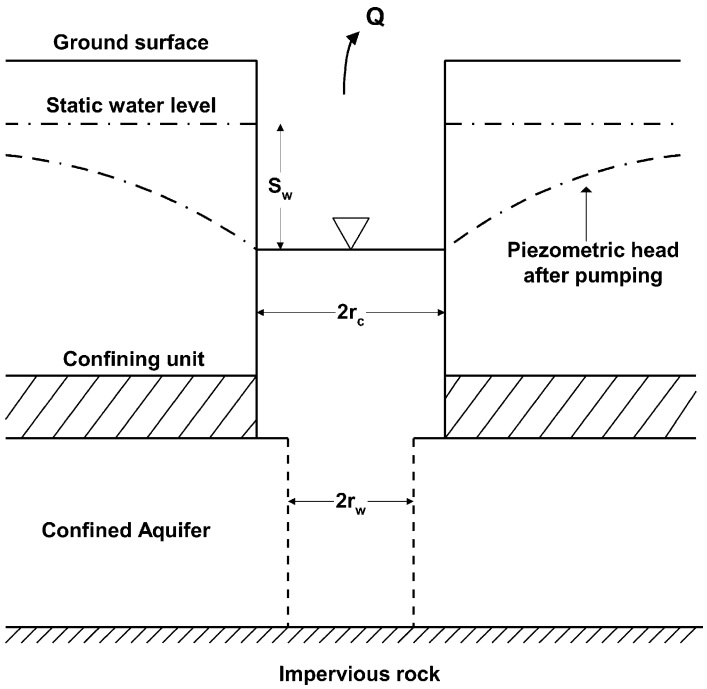

Fig. 1. Ideal large diameter well in a confined aquifer (after Papadopulos and Cooper (1967)).

approach based on theories of the massive interconnection and parallel processing architecture of biological systems. The main theme of ANN research focuses on modeling of the brain as a parallel computational device for various computational tasks that were performed poorly by traditional serial computers. ANNs have a number of interconnected processing elements (nodes) that usually operate in parallel and are configured in regular architectures. The collective behavior of ANN, like a human brain, demonstrates the ability to learn, recall, and generalize from training patterns or data.

The application of ANNs is becoming increasingly common in the analysis of groundwater hydrology and water resources problems (Hsu et al., 1995; Clair and Ehrman, 1996; Poff et al., 1996). The modern view of neural networks began in the 1940s with the work of McCulloch and Pitts (1943), who showed that networks of artificial neurons could compute any arithmetic or logical function. McCulloch and Pitts were followed by Hebb (1949), who proposed that classical conditioning is present because of the properties of individual neurons. He proposed a mechanism for learning in biological neurons. The first practical application of ANN came in the late 1950s, with the invention of the perceptron network and associated learning rule by Rosenblatt (1958, 1962) and Widrow and Hoff (1960) on the ADALINE. Minsky and Papert (1969) are often credited with the addition of one or two-layered perceptron networks.

The second key development of the 1980s was the backpropagation algorithm for training multi-layer perceptron networks, which was discovered independently by several different researchers. The most influential publication of the backpropagation algorithm was by Rumelhart et al. (1986). This algorithm was the answer to the criticisms Minsky and Papert had made in the 1960s. Addition of a middle (hidden) layer to a multi-layer perceptron network, together with a clear explanation of the backpropagation learning algorithm, overcame many of the limitations of the one or two-layered perceptron ANNs. Since 1986, the variety of ANNs has rapidly expanded. Recently, Aziz and Wong (1992) have used the ANN approach for aquifer parameter determination. They utilized Theis (1935) and Hantuch and Jacob (1955)

solutions for confined and leaky-confined aquifer conditions to derive input–output patterns based on a pre-determined range of values.

In this research, an ANN was developed to simulate the relationship between an input vector consisting of time–drawdown and well diameter data to an output vector consisting of its corresponding aquifer transmissivity and storativity values. The output of the ANN was then verified with the existing analytical solution, and compared with published values. Three examples are presented and discussed for illustration.

2. Drawdown in large diameter well: Papadopulos and Cooper solution

Papadopulos and Cooper (1967) developed an analytical solution and type curves in and around a large diameter well in a homogeneous and isotropic non-leaky confined aquifer. They took into consideration the water derived from storage within the well, and assumed a horizontal aquifer with a constant thickness, and a constant discharge for a fully penetrating well.

The governing second order partial differential equation is:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad r \geq r_w \tag{1}$$

where s is the drawdown in the aquifer at a distance r at time t, S the storage coefficient of the aquifer, T the transmissivity, and r_w is the effective radius of well screen. The geometry of large diameter well in a non-leaky confined aquifer is shown schematically in Fig. 1.

The initial conditions are:

$$s(r,0) = 0 \qquad r \geq r_w \tag{2}$$

$$s_w(0) = 0 \tag{3}$$

and the boundary conditions are:

$$s_w(r_w,t) = s_w(t) \tag{4}$$

$$s(\infty,t) = 0 \tag{5}$$

$$2\pi r_w T\frac{\partial s(r_w,t)}{\partial t} - \pi r_c^2\frac{\partial s_w(t)}{\partial t} = -Q \qquad t \geq 0 \tag{6}$$

where $s_w(t)$ is the drawdown in the well at time t and r_c is the radius of the well casing in the interval over which the water level declines.

With the initial and boundary conditions stated above, Eq. (1) was solved using the Laplace transform method, and the following solution was obtained (Papadopulos and Cooper, 1967; Papadopulos, 1967; Reed, 1980):

$$s(r,t) = \frac{Q}{4\pi T}F(u,\alpha,\rho) \tag{7}$$

where

$$F(u,\alpha,\rho) = \frac{8\alpha}{\pi}\int_0^\infty \frac{C(\beta)}{D(\beta)\beta^2}\partial\beta \tag{8}$$

and

$$C(\beta) = \left[1 - \exp\left(-\beta^2\frac{\rho^2}{4u}\right)\right][J_0(\beta\rho)A(\beta) - Y_0(\beta\rho)B(\beta)] \tag{9}$$

where

$$A(\beta) = \beta Y_0(\beta) - 2\alpha Y_1(\beta)$$

$$B(\beta) = \beta J_0(\beta) - 2\alpha J_1(\beta)^2 \tag{10}$$

$$D(\beta) = [A(\beta)]^2 + [B(\beta)]^2$$

$$u = \frac{r^2 S}{4Tt} \qquad \alpha = \frac{r_w^2 S}{r_c^2} \qquad \rho = \frac{r}{r_w} \tag{11}$$

J_0 (and Y_0), and Y_1 represent zero-order and first-order Bessel functions of the first and second kind, respectively.

The drawdown inside the pumped well is obtained at $r = r_w$ and expressed as:

$$s_w(t) = \frac{Q}{4\pi T}F(u_w,\alpha) \tag{12}$$

where

$$F(u_w,\alpha) = F(u,\alpha,1) \tag{13}$$

and

$$u_w = \frac{r_w^2 S}{4Tt}. \tag{14}$$

Values of $F(u,\alpha,\rho)$ are computed by numerical integration of Eq. (8). Papadopulos and Cooper (1967) generated a family of type curves of $s_w/(Q/4\pi T)$ versus $1/u_w$ with one curve for each α.

Aquifer parameters are determined by fitting observed drawdown data to one of the families of type curves and finding a match point. The method is analogous to the Theis type curve method, except it involves more than a single type curve. Details of the procedure can be found in Theis (1935) and Papadopulos and Cooper (1967).

3. ANN approach

ANNs can be visualized as a set of interconnected nodes arranged in layers. The input layer contains one node for each of the input variables. In multi-layer network, the output of one layer constitutes the input to the next layer. Note that ANN may have more than one hidden layer. For example, in the ANN architecture shown in Fig. 2, the nodes of one hidden layer interconnect with each node in the input and output layers. Outputs of neurons are used as input to other neurons in the network. A numeric weight is associated with each of the inter-node connections.

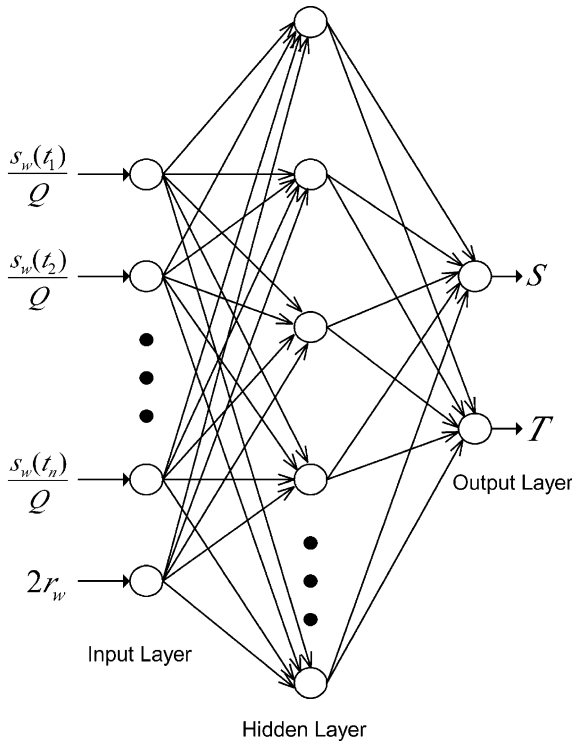

Fig. 2. Designed ANN used in the aquifer parameter determination.

The nodes themselves process the values entering the node to produce an output value.

Training of the ANN consists of calculating the output vector from the input vector, comparing the ANN calculated outputs with the actual outputs, and then adjusting the weights and the bias in the transfer function for each neuron so as to minimize the difference between ANN-output and actual values. Each input–output pattern in the training set is presented to the ANN many times until the difference between ANN-output and actual values becomes insignificant. The mean square sum of differences between target and ANN-output value serves as a measure of the goodness of fit.

The main advantage of an ANN is that unlike traditional analytical methods, it does not depend upon a graphical solution that requires curve matching, a common source of error when estimating aquifer parameter values. In addition, because an ANN does not depend solely upon the physical parameters used in the analytical approach, it can be designed with many different architectures to achieve optimal performance. That is the inputs and outputs of ANN do not necessarily have to be identical to the input and output parameters used in the adopted analytical solution. Thus the ANN can be designed to use all or partial input–output parameters to achieve an optimal performance. However, a disadvantage is that architecture parameter values are not known a priori to training, potentially requiring a lot of trial and error to achieve satisfactory results. If an ANN is poorly designed and/or trained, it produces output values with errors that are unacceptably high, which in some cases may even be physically infeasible.

The performance of ANN is sensitive to its physical architecture, such as the number of input nodes, hidden layer nodes, and output nodes. The appropriate architecture of ANN is highly problem-dependent. The type of ANN used in this study is a multi-layer feed-forward perceptron trained with the use of backpropagation learning algorithm (Rumelhart et al., 1986). The backpropagation algorithm makes use of robust optimization techniques to solve an unconstrained non-linear optimization. The mechanism of the learning algorithm can be summarized as follows:

(1) Connection weights are first randomly

initialized within some small positive and negative random values.

(2) Input–output patterns are selected randomly from the training sets and presented to the ANN.

(3) Actual network output is calculated for the current input.

(4) Mean square error (MSE) as a measure of performance is calculated as:

$$\mathrm{MSE} = \frac{1}{N} \sum_{x_i \in U} \sum_{j=1}^{n} [a_j(x_i) - b_j(x_i)] \tag{15}$$

where $a_j(x_i)$ is the actual value of the jth output neuron for input vector x_i, $b_j(x_i)$ is the ANN output of the jth neuron in the output layer, U is the set of input vectors, n is the number of neurons in the output layer, and N is the total number of training patterns.

(5) Connection weights are adjusted to minimize the MSE according to the rule

$$\mathbf{w} \leftarrow \mathbf{w} - \mu \nabla E(\mathbf{w})$$

where $\mathbf{w}$ is the vector of network weights, $E(\mathbf{w})$ is MSE with respect to $\mathbf{w}$, and μ is the learning rate. This rule states that the present weight is altered by an amount that is proportional to the negative derivative of the error with respect to that weight. The learning rate determines the rate at which the weight should be modified after each iteration.

The weight update equation may be written as

$$w^m = w^{m-1} - \mu \frac{\partial E^m}{\partial w^{m-1}} + \xi(w^{m-1} - w^{m-2}) \tag{16}$$

where m is the updating index and ξ is the momentum factor.

The momentum factor helps the network to learn fast, since part of previous weight change is applied to the current weight change.

(6) Steps (2)–(5) are repeated for each pair of input–output vector in the training set, until no significant change in the MSE is detected for the system.

After training is completed, the final connection weights are kept fixed, and new input patterns are presented to the network to produce the corresponding output consistent with the internal representation of the input/output mapping. For a more detailed discussion of the learning algorithm, the interested reader is referred to Werbos (1974), Parker (1985), Le Cun (1985), and Rumelhart et al. (1986).

4. Application and discussion

In this research, a three-layered ANN was designed to process an input vector consisting of both time–drawdown data $s_w(t)$ and the effective well radius r_w in order to generate an output vector consisting of aquifer parameter values for both transmissivity and storativity. The components of the designed ANN are shown in Fig. 2. Training and testing vector sets are generated using Eq. (13). Each training vector set contains number of neurons equal to the number of time–drawdown data points plus the well diameter in its input layer, and two neurons in the output layer.

The training sets are used to develop the appropriate ANN architecture and weight connections for the specific application. Following ANN development, its ability to accurately map the input to the output is verified using the testing sets. The testing sets consist of input and output vectors not used during training. This ensures that the ANN has effectively learned to generalize the problem, and has not simply 'memorized' the training sets, a problem associated with 'over-training'.

To generate training sets via Eq. (13), a range of aquifer parameter values must be specified and then combinations of these values are used during the generation process. The question that might arise here is what range of aquifer parameter values should be selected for training? In fact, the network can be trained with any given range of input values. However, the performance of the network during the testing phase is largely determined by its training phase, because after training, the connection weights are kept fixed during the testing phase. This means that selection of an appropriate range of input values for generating training sets is important for developing an ANN capable of accurately estimating property values over the expected range.

In accordance with the above, it is suggested that the network be trained in two steps, herein referred to as the macro-scale and micro-scale training steps. The macro-scale training is the training of the network when a very wide or very narrow range of T and S values are selected randomly. The reason for this

selection stems from a lack of information regarding the actual values, which is usually the case for most newly investigated aquifers. Unfortunately both ranges are highly questionable and may affect the learning and generalization process of the network. It is assumed that the best performance of ANN occurs when the values of the testing pattern fall within a narrow range of the training pattern. This implies that the selection of wide range of values far from the real ones may cause inefficient generalization of the network. On the other hand, selection of narrow range of input values may have two possible outcomes: (1) if the real test values fall within the selected range, perfect training is expected, and (2) if it falls outside the selected range, inefficient learning and generalization might occur.

The micro-scale training of the network occurs when the range of values of T and S are selected appropriately. Appropriate selection of these values occurs when some information about T and S values is available. This information could be in the form of rough quantitative or qualitative estimates of T and S values. In any case, the range of training values can be managed and refined to achieve optimal ANN performance. This suggests that prior knowledge of aquifer parameters should be used as valuable pieces of information in the ANN approach.

In this work, both macro- and micro-training scales are used. For both training scales, drawdown data are normalized $\{s_w(t)/Q\}$ with a well discharge of 1 m^3/day. In the macro-scale training step, a total of 4557 training patterns of normalized drawdown data were generated using 26 input drawdown data points corresponding to 26 time-series data. Wide ranges of aquifer parameter values were selected as: (1) T values ranging from 50 to 650 m^2/day with a step size of 30, (2) S values ranging from 0.00005 to 0.01505 with a step size of 0.0005, and (3) $2r_w$ values ranging from 0.1 to 3.1 m with a step size of 0.5 m.

Aquifer response to pumping depends largely on its properties. For example, certain combinations of T and S values produce a complete and fast aquifer response, while other values produce slow aquifer response. In this study, the 26 time-series data points are selected to generate the corresponding drawdowns from a range of time values spanning from as early as 0.1 min. to as late as 18 h. The time-series data values vary within the specified time span depending on the combination of T and S values. The question here is how to test field experimental data that have a number of drawdown data points different from the number of input data points of the designed and trained ANN. In fact two cases could occur here. The first case occurs when the number of drawdown data inputs is less than that required by the trained ANN. Actually examples 2 and 3 below are typical examples of this case. Both examples have a number of drawdown data less than those of the trained ANN (26 data points).

To overcome this problem, an interpolation procedure is proposed and implemented in this paper. The procedure is based on interpolating a number of drawdown data points (complementary data set) that, when combined with the existing real data, constitute the equivalent number of drawdown data points used to train the ANN. For example, if the number of the existing experimental data is 20, then six data points are needed for interpolation. To minimize uncertainty and alteration of existing data, interpolated data points are evenly distributed over all time spans and each point is interpolated between two adjacent and close existing real data points. The second case occurs when the number of available drawdown data points is more than that of the trained ANN (not implemented here). However, it is suggested in this case that the input vector be selected from the available data so that the number of drawdown data points is equivalent to the number used to train the ANN. In addition, the selected data set must span all times (early to late) of the available data. Note that in both cases the time-series data are not required in training the designed ANN but their corresponding drawdowns are.

In the micro-scale training step, the wide range of aquifer parameter values are narrowed based upon rough estimates of the real values. The rough estimates of T and S values are obtained by fitting the time–drawdown curve under question to a family of time–drawdown curves generated from the analytical solution (not presented). The fitted type curve corresponds to a certain α value. Estimates of T and S are then obtained from the analytical solution. Adding upper and lower margins to obtained T and S values constitute the final range of values used in generating a micro-scale training set. In this study, an order of magnitude was added and subtracted from the upper and lower estimated S values, respectively, forming

Table 1
ANN influential parameters used during training

Parameter	Value
Learning rate	0.8
Momentum	0.6
Scaling interval	[−0.8, +0.8]
Convergence criteria	0.001
Learning count	30,000
Random seed	257
Activation function	Sigmoid
Number of input nodes	27
Number of output nodes	2
Number of nodes in the hidden layer	35
MSE	0.0733
Correlation coefficient (R)	0.933

the upper and lower extreme values. For T, 50% of the maximum and minimum values were added and subtracted to derive the upper and lower extreme values, respectively.

It is not known a priori how many hidden layers or hidden nodes are needed for optimal ANN performance. The number of hidden layers or nodes and network parameters, must be determined by the modeler, often on a trial and error basis. An increase in the hidden layer nodes increases the number of weights in the network, which in turn increases the ability of the ANN to learn the underlying complex relationship between the input and output patterns. This increase in learning is problem-dependent. In this research, the best ANN performance (i.e. the minimum MSE) was achieved using one hidden layer containing {1.3× (input neurons + output neurons)} hidden nodes. The influential parameters used during macro-scale training are given in Table 1. These parameters are commonly specified in most NN that use backpropagation algorithm.

Scaling interval is one of the most critical parameters particularly when sigmoid or hyperbolic tangent activation functions are used. These functions respond nearly in linear fashion to summations between about −2 and +2. If a large input value (e.g. 1000) is used, then the derivative of the sigmoid is close to zero. Since the derivative is a multiplier in the weight update equation, learning stops for neurons with such large summation values. Therefore, it is important to select scaling intervals within the specified range.

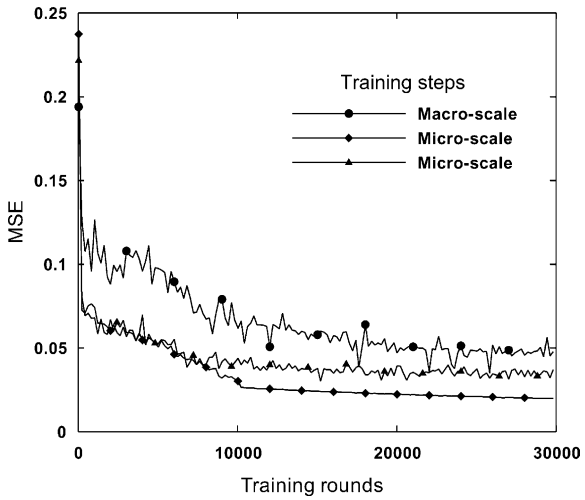

Fig. 3. Development of MSE during macro-scale training of the designed ANN for example 1 (•) and micro-scale training of example 2 (▲) and example 3 (◆).

During the training process, the MSE is computed each time a training pattern is presented to the ANN (learning counts). The evolution of the MSE during macro-scale training corresponding to a learning count of 30,000, which is the total number of times an input/output vector is presented to the ANN, is depicted in Fig. 3. The rapid decrease in MSE at the beginning of training is followed by a subtle decay until no or nearly non-noticeable change occurs. This behavior is typical of most ANN learning, and is attributable to influential parameters used during initialization and/or training, including random seed, learning rate, random initialization of connection weights, and random selection of patterns for training. As a measure of correlation between ANN-output and actual outputs, the Pearson moment correlation coefficient (R) is calculated (Table 1). Note that a perfect agreement between actual and ANN-output occurs when the computed R approaches unity.

The designed ANN was then tested and applied to three different examples, the results of which are presented below. Example 1 measures the performance of the designed network, and is used to check the validity of the methodology. In examples 2 and 3, the ANN approach was applied using reported data obtained from literature. The same ANN architecture was used in the three examples; however the number

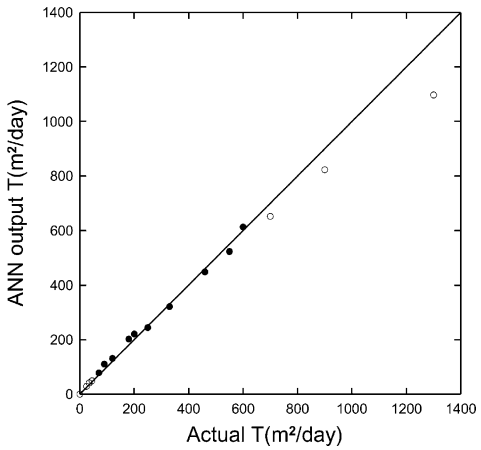

Fig. 4. Actual and ANN output for selected values of transmissivity within trained range (filled circles) and out of trained range (empty circles).

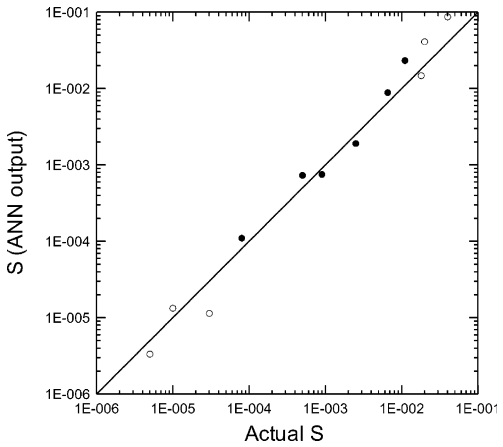

Fig. 5. Actual and ANN output for selected values of storativity within trained range (filled circles) and out of trained range (empty circles).

of training sets varied, depending upon the range of aquifer parameter values used in the micro-scale training step as explained above.

4.1. Example 1: measure of performance

Testing data are selected and used in this example to assess the generalization performance of the designed network. If an ANN is capable of generalizing well, its fitted function accurately estimates output values outside of the generated training data. Average generalization performance (AGP), a percentage measure of ANN capability for correctly predicting outputs, was computed with the following formula

$$\mathrm{AGP} = \frac{1}{\mathrm{NTP}} \sum_{i=1}^{\mathrm{NTP}} 100 - \mathrm{ABS}\left(\frac{\mathrm{av}_i - \mathrm{ov}_i}{\mathrm{av}_i}\right) \qquad (17)$$

where av_i and ov_i are the actual and the ANN output values of the ith tested pattern respectively and NTP is the number of tested patterns. This measure was used to quantify ANN performance with two data sets not used during training.

The first test set contained data within the range of values used in ANN training. A total of 75 new input patterns selected randomly from the specified training range were presented to the trained ANN. No micro-scale training is performed here, since the input patterns are selected over the specified range of aquifer parameter values. Due to similarities in output, Figs. 4 and 5 show a scatter plot for only 10 of the 75 estimated and actual T and S values. Note that points falling on the 45-degree line indicate a perfect match between actual and estimated values. The AGP of the 75 tested patterns achieved for this test set was 95.5%. This high performance indicates that the ANN is well trained and is capable of generalizing.

The second test set consisted of data selected from outside the training range. Combinations of six pairs of T and S values were selected from below and above the lower and upper training limits. Their corresponding normalized drawdown data were presented to the trained ANN as a test set. Values estimated by the ANN for this test set are depicted as empty circles in Figs. 4 and 5 for T and S, respectively. As shown, T values below the lower training limit are on average overestimated by the ANN by 14%, and those above the upper training limit are on average underestimated by 10%. Regardless of their statistical significance, the obtained percentages add some confidence in using the designed ANN for predicting around the upper and lower training limits. Note that because of plotting small values over a large range (Fig. 4), estimated T values below the lower training limit were not on average estimated more accurately than T values above the upper training limit. Similarly, S values estimated by the ANN are on average underestimated when below the lower training limit and

Table 2
Reported time–drawdown data of examples 2 and 3

Example 2 (Wikramaratna, 1985)		Example 3 (Sakthivadivel and Rushton, 1989)	
Time (min)	Drawdown (m)	Time (min)	Drawdown (m)
0.167	0.34	11.5	0.13
0.5	0.84	21.6	0.24
1	1.36	**24.8**	**0.32**
1.25	**1.54**	28.1	0.36
1.5	1.71	**34.6**	**0.44**
2	1.97	41.0	0.48
3	2.32	49.7	0.55
2.9	**2.51**	**55.1**	**0.60**
5	2.69	60.5	0.64
10	3.11	**63.0**	**0.68**
15	3.31	65.5	0.71
17.5	**3.8**	**72.4**	**0.76**
20	3.44	79.2	0.79
30	3.63	**85.7**	**0.85**
40	3.75	92.2	0.88
45.8	**3.8**	**103.0**	**1.00**
50	3.85	113.8	1.10
60	3.92	**121.0**	**1.22**
90	4.09	128.2	1.30
104.2	**4.15**	158.4	1.18
120	4.21	**162.0**	**1.27**
150	4.3	165.6	1.31
180	4.37	**175.7**	**1.36**
213.8	**4.43**	185.8	1.39
240	4.49	**193.7**	**1.41**
300	4.58	201.6	1.44

Bold numbers: complementary data set.

overestimated when above the upper training limit. The maximum percentage discrepancy between actual and estimated S values was 217% and the minimum discrepancy was 38%. Unlike the type curve method, the difference between the actual and ANN estimated S values never exceeded an order of magnitude. In general, the ANN estimated T values more accurately than S values.

4.2. Example 2: analysis of Wikramaratna (1985) data

In this example, the pumping test data of drawdown in the well (s_w) versus time (t) for large diameter well are taken from Wikramaratna (1985). As reported earlier, the number of time–drawdown data points of this example is 20, and a complementary data set consisting of six points was added to form a valid input vector. For this particular example, the complementary data are obtained from the analytical solution. Table 2 lists time–drawdown data for this example, and the complementary data are presented in bold face.

One input vector consisting of normalized time–drawdown data and well diameter is presented to the trained network. Values of T, S, Q, r_w, and r_c are given as 86.4 m^2/day, 0.01, 432 m^3/day, 0.2, and 0.2, respectively. Assuming the Q, r_w, and r_c, are known, values of T and S are treated as idealized results from the pumping test. The MSE during micro-scale training of this network is shown in Fig. 3. The micro-scale training sets were obtained by narrowing the wide range of aquifer parameter values as explained above. Rough estimates of T and S (68.4 m^2/day and 0.0484, respectively) are obtained by the manual type curve matching procedure. The T and S values predicted by the trained network were 85.3 m^2/day and 0.00961, respectively. Table 3 shows results obtained by the ANN of this example as well as values of T and S obtained by the curve matching technique of Papadopulos and Cooper (1967). Results obtained by the network show very good agreement with the idealized T and S values, especially when micro-scale training is used. This indicates that the designed network is capable of learning and testing the non-linear behavior represented by the time-drawdowndrawdown data and accurately predicting the T and S values.

One of the advantages of the ANN approach is that it overcomes the questionable reliability in the estimation of S values obtained by traditional type curve matching methods. This is due to the fact that matching the time-drawdown data to the appropriate type curve match is highly subjective, and the shape of the type curve differs only slightly when α differs by an order of magnitude. The property of T is not as sensitive as S. This is made clear by Table 3, where three matching points corresponding to three different type-curves are presented, as well as the corresponding T and S values. As shown, the change in T values is not significant as compared to the change in S values, where the change is an order of magnitude. In short, T changes only slightly when moving from adjacent type curves, whereas S changes by a complete order of magnitude.

Table 3
Calculated (taken from literature) and predicted (ANN) aquifer parameters

Reported analysis		Calculated		Predicted (ANN)				
		T (m^2/day)	S	T (m^2/day)		S		
				Macro-scale training	Micro-scale training	Macro-scale training	Micro-scale training	
Wikramartatna analysis	Matching point 1	62.5	0.0846	61.8	85.3	0.0325	0.00961	
	Matching point 2	85.95	0.0119					
	Matching point 3	114.59	0.0011					
Sakthivadivel et al.		60	0.008	92	38.6	0.00685	0.00044	
Herbert et al.		32	0.0002					

4.3. Example 3: field data

The field data of this example are taken from the experiment conducted by Sakthivadivel and Rushton (1989). These data include time versus drawdown during pumping at a constant discharge of 67 m^3/day from a 1.88 m diameter well. Only 15 time–drawdown data were available (Table 2) as reported for this example. An input vector of time–drawdown data was prepared by adding a set of complementary data before presenting it to the trained network. Initial rough estimates of T and S that are used in generating a micro-scale training set were 53.7 m^2/day and 0.0013, respectively. The development of MSE corresponding to the micro-scale training step is shown in Fig. 3.

Aquifer parameters obtained by the ANN for both macro and micro-scale training steps are shown in Table 3. The table also shows the results obtained by Sakthivadivel and Rushton, as well as those by Herbert and Barker (1990) for the same aquifer. As expected, results obtained by the network trained in micro-scale are much better than those trained in macro-scale. The micro-scale trained network results closely agree with the published values of Herbert and Barker. Note that T and S values obtained by Sakthivadivel and Rushton are significantly different from those obtained by Herbert and Barker because the former analysis accounted for the dynamic seepage face. For more details, see Herbert and Barker (1990).

5. Summary and conclusion

In this paper, a new alternative approach to the curve matching method has been presented and demonstrated for aquifer parameter determination in large diameter wells. This approach is based on the application of ANN methodology, which has the capability of matching input to output. The designed network is trained to learn the underlying complex relationship between input and output patterns of normalized drawdown data generated from the analytical solution of Papadopulos and Cooper (1967) and its corresponding T and S values. The ANN was trained with a fixed number of input drawdown data points obtained from the analytical

solution for a pre-specified ranges of aquifer parameters values and time-series data.

The trained network is capable of producing aquifer parameter values for any given input pattern of normalized drawdown data and well diameter size. The values of aquifer parameters obtained using the ANN approach are in a good agreement with those obtained by other published results. Macro and micro-scale training introduced here shows a systematic way of network training. Prior knowledge about aquifer parameter values served as a valuable piece of information in the ANN approach. Because of its ability to mimic complex input–output relationships, the ANN methodology presented here proved to be a useful tool in the field of aquifer hydraulics. Both macro- and micro-scale training are recommended for optimal ANN design and performance in future work.

References

Aziz, A.R.A., Wong, K.V., 1992. A neural-network approach to the determination of aquifer parameters. Ground Water 30 (2), 164–166.

Clair, T.A., Ehrman, J.M., 1996. Variation in discharge, dissolved organic carbon and nitrogen export from terrestrial basins with changes in climate: a neural network approach. Limnol. Oceanogr. 41, 921–927.

Hantush, M.S., 1960. Modification of the theory of leaky aquifers. Journal of Geophysical Research 65 (11), 1627–1629.

Hantush, M.S., Jacob, C.E., 1955. Nonsteady radial flow in an infinite leaky aquifer. Am. Geophys. Union Trans. 36 (1), 95–100.

Hebb, D.O., 1949. The Organization of Behavior, Wiley, New York.

Herbert, R., Barker, J.A., 1990. Numerical analysis of large diameter wells with a seepage face—comment. J. Hydrol. 113 (1–4), 371–375.

Hsu, K.L., Gupta, H.V., Sorooshian, S., 1995. Artificial neural network modeling of the rainfall–runoff process. Water Resour. Res. 31, 2517–2530.

Le Cun, Y., 1985. Une procedure d'apprentissage pour resru a seuil assymetrique (a learning procedure for asymmetric threshold network). Proc. Cognit. 85, 599–604.

McCulloch, W., Pitts, W., 1943. A logical calculus of the ideas immanent in nervous activity. Bull. Math. Biophys. 5, 115–133.

Minsky, M., Papert, S., 1969. Perceptrons, MIT Press, Cambridge, MA.

Moench, A.F., 1985. Transient flow, to a large-diameter well in a confined aquifer with storativity semiconfining layers. Water Resour. Res. 21 (8), 1121–1131.

Papadopulos, I.S., 1967. Drawdown distribution around a large-diameter well. National Symposium on Ground-Water Hydrology, San Francisco, CA November, 157–168.

Papadopulos, I.S., Cooper, H.H. Jr., 1967. Drawdown in a well of large diameter. Water Resour. Res. 3 (1), 241–244.

Parker, D.B., 1985. Learning-logic, Technical Report TR-47. Center for Computational Research. Economic And Management Science, Massachusetts Institute Of Technology.

Poff, N.L., Tokar, S., Johnson, P., 1996. Stream hydrological and ecological responses to climate change assessed with an artificial neural network. Limnol. Oceanogr. 41, 857–863.

Reed, J.D., 1980. Type curves for selected problems of flow to wells in confined aquifers. US Geological Survey, Book 3, vol. 106. US Government Printing Office, Washington, DC, Chapter B3, pp. 4–5.

Rosenblatt, F., 1958. The perceptron: a probabilistic model for information storage and organization in the brain. Psychol. Rev. 65, 386–408.

Rosenblatt, F., 1962. Principle of Neuro-Dynamics: Perceptrons and the Theory of Brain, Mechanisms, Spartan, Washington, DC.

Rumelhart, D.E., Hinton, G.R., Williams, R.J., 1986. Learning internal representations by error propagation. In: Rumelhart, D.E., McClelland, J.L. (Eds.), Parallel Distributed Processing, vol. 1. MIT Press, Cambridge, MA.

Sakthivadivel, R., Rushton, K.R., 1989. Numerical analysis of large diameter wells with a seepage face. J. Hydrol. 113 (1–4), 43–55.

Theis, C.V., 1935. The relationship between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. Am. Geophys. Union Trans. 16, 519–524.

Werbos, P., 1974. Beyond regression: new tools for prediction and analysis in the behavioral sciences. PhD Dissertation, Harvard University, Boston, MA.

Widrow, B., Hoff, M.E., 1960. Adaptive switching circuits. IRS WESCON Conv. Rec. 4, 96–104.

Wikramaratna, R.S., 1985. A new type curve method for the analysis of pumping test in large-diameter wells. Water Resour. Res. 21 (2), 261–264.