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Technical Note

Tidal fluctuations in a leaky confined aquifer: localised effects of an overlying phreatic aquifer

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Abstract

Damping of tidal head fluctuations in a leaky confined coastal aquifer is enhanced by leakage into an overlying phreatic aquifer. We show that the phreatic aquifer is, however, resistant to the leakage flow and in particular, a deep phreatic aquifer can reduce the leakage effects significantly. An analytical solution, based on a vertical flow model for the phreatic aquifer, is derived for quantifying the role of this upper free water body in tidal propagation in the lower semi-confined aquifer.

Keywords: Semi-confined aquifer; Phreatic aquifer; Tidal fluctuations; Leakage

1. Introduction

Tidal fluctuations in a leaky confined coastal aquifer are damped significantly due to leakage into an overlying phreatic aquifer. Jiao and Tang (1999) presented an analytical solution to a simple model describing this phenomenon. Their model assumes that the phreatic aquifer is static at all times. This assumption was based on observations that tidal fluctuations in the phreatic aquifer usually diminish over a short distance from the coastline because of high damping, i.e. $A(x) = A_0 \exp(-\kappa_d x)$, where $A(x)$ is the amplitude of the head fluctuations at distance x from the shore, A_0 is the amplitude at the shore and κ_d is the damping rate. The damping rates for tidal fluctuations in a phreatic aquifer can be 10–20 times higher than those for tidal fluctuations in a confined aquifer. Later, Li et al. (2001) and Jeng et al. (2002) relaxed the assumption of a static phreatic aquifer and examined dynamic effects of the overlying aquifer on tidal fluctuations in the lower confined aquifer. They showed that these two aquifers interact with each other in response to tides.

Depending on the local geology, a coastal phreatic aquifer can have very low connectivity to the sea, e.g. with a barrier, such as a seawall, of very low hydraulic conductivity at the shore. In this case, tides will not propagate directly in the phreatic aquifer. However, there remains the question of whether we can treat the phreatic aquifer as a static water body. Clearly, the phreatic surface will respond to local leakage from the semi-confined aquifer, which in turn affects the

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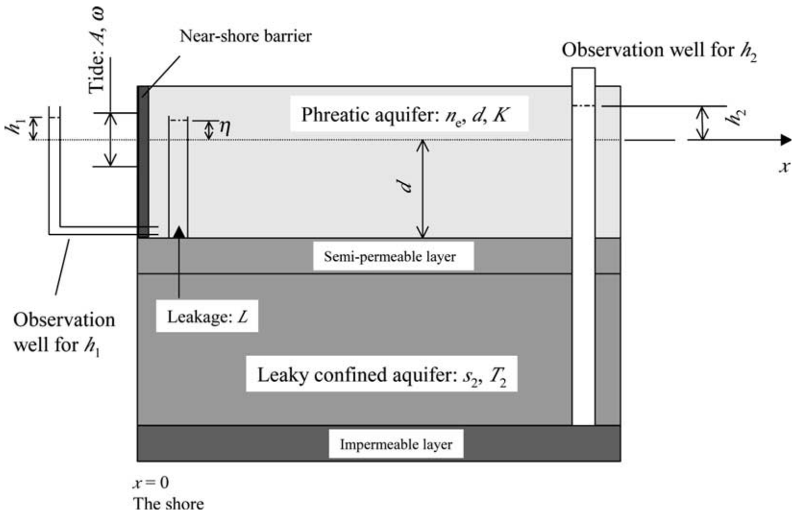

Fig. 1. Schematic diagram of a leaky confined aquifer with an overlying phreatic aquifer.

leakage flow and tidal fluctuations in the leaky confined aquifer. In this paper, we examine these effects based on a vertical flow model.

2. Problem set up

As shown in Fig. 1, while tides propagate in the confined aquifer, the overlying phreatic aquifer is not affected directly by the oceanic oscillations because of a near-shore barrier. However, the connection between the two aquifers through leakage will transmit the tidal signals to the upper layer. The governing equation of tidal fluctuations in the confined aquifer is (Jiao and Tang, 1999),

$$s_2 \frac{\partial h_2}{\partial t} = T_2 \frac{\partial^2 h_2}{\partial x^2} + L(h_1 - h_2), \tag{1}$$

where h_2 is the head in the confined aquifer (the datum is set at the mean sea level); s_2, and T_2 are the storativity and transmissivities of the confined aquifer, respectively; and L is the specific leakage of the semi-permeable layer. Observe that h_1 is the head at the lower boundary of the phreatic aquifer (i.e. the upper boundary of the semi-permeable layer) and has been assumed by Jiao and Tang (1999) to be constant. However, the leakage, which has been included in Eq. (1), must also influence h_1 to satisfy mass conservation. The semi-permeable layer shown in Fig. 1 is assumed to be very thin, having negligible storage. We shall quantify the effect of leakage across the semi-permeable layer using a vertical flow model for the phreatic aquifer, assuming that the horizontal flow there is much less than the vertical flow. This assumption will be justified with the solution obtained later. As a result of the leakage, the water table will fluctuate according to,

$$n_e \frac{d\eta}{dt} = -L(h_1 - h_2), \tag{2}$$

where η is the elevation of the water table. Eq. (2) reflects the principal of mass balance in the local vertical column of the phreatic aquifer (Fig. 1). We also have the vertical flow equation, written as,

$$n_e \frac{d\eta}{dt} = K \frac{h_1 - \eta}{\eta + d}, \tag{3a}$$

where K and d are the hydraulic conductivity and thickness of the phreatic aquifer, respectively. In general, the amplitude of the oscillations is small relative to the aquifer depth, i.e. $\eta \ll d$, and hence Eq. (3a) can be simplified to (linearisation)

$$n_e \frac{d\eta}{dt} \approx K \frac{h_1 - \eta}{d}. \tag{3b}$$

Note that Eq. (3b) is similar to equation (18) of Nielsen and Perrochet (2000). Although the situation considered here is different from that of Nielsen and Perrochet's (2000) vertical column experiments, the

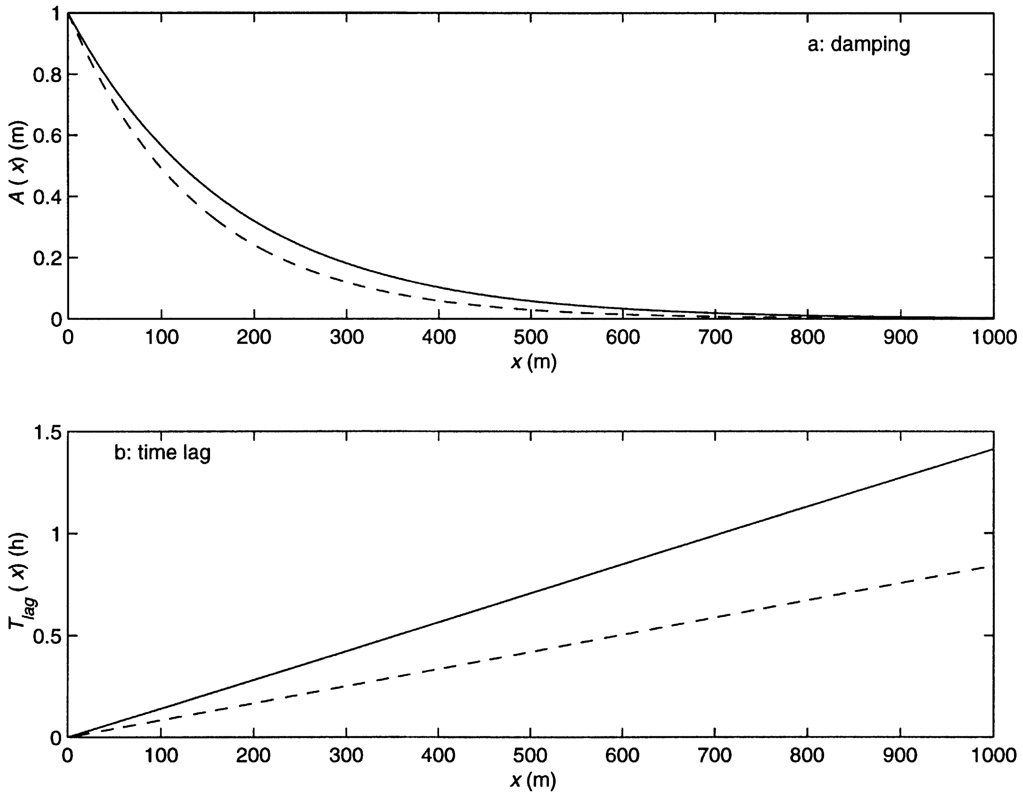

Fig. 2. Predicted amplitude damping (a) and time lag (b) of tidal water table fluctuations in the confined aquifer (h_2) by the present solution (solid lines) and that of Jiao and Tang (1999) (dashes).

vertical flow model applies to certain extent if the horizontal flow in the phreatic aquifer is small and negligible in the absence of direct tidal propagation from the sea (in the horizontal direction). The boundary conditions for the confined aquifer are

$$h_2(0,t) = A\cos(\omega t), \tag{4a}$$

$$\left.\frac{\partial h_2}{\partial x}\right|_{x=\infty} = 0, \tag{4b}$$

where A and ω are the tidal amplitude and frequency, respectively. Physically, Eqs. (4a) and (4b) describes a periodic boundary condition at the origin in an aquifer without a regional flow component. Only one tidal constituent is considered here. Since the problem is linear, solutions for multiple-constituents tides can be obtained easily from the solution given below using superposition.

3. Analytical solution

The solution of h_2 takes the following form,

$$h_2 = A\exp(-\kappa_d x)\cos(\omega t - \kappa_w x), \tag{5}$$

where κ_d (m^{-1}) is the damping rate and κ_w (m^{-1}) is the wave number for phase shift. For convenience, we rewrite Eq. (5) as,

$$h_2 = \mathrm{Re}[A\exp(i\omega t - \kappa x)], \tag{6a}$$

with the complex wave number defined as

$$\kappa = \kappa_d + i\kappa_w. \tag{6b}$$

At any distance x from the shore, since the only forcing oscillation is $h_2(x,t)$, h_1 must be related to h_2 and

$$h_1 = \mathrm{Re}[cA\exp(i\omega t - \kappa x)], \tag{7}$$

where c is a complex number, accounting for the damping and phase lag of h_1 with respect to h_2. The solution to Eq. (3b) is (Nielsen and Perrochet, 2000),

$$\eta = Re\left[\frac{cA\exp(\mathrm{i}\omega t - \kappa x)}{1 + \mathrm{i}\dfrac{n_\mathrm{e}\omega d}{K}}\right]. \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (2) gives,

$$c = \frac{L}{L + \dfrac{\mathrm{i}n_\mathrm{e}\omega K}{K + \mathrm{i}n_\mathrm{e}\omega d}}. \tag{9}$$

Based on Eqs. (1), (6a) and (7), κ is determined as

$$\kappa = \sqrt{\frac{\mathrm{i}\omega s_2 + (1-c)L}{T_2}}. \tag{10}$$

From Eq. (10), the damping rate κ_d and the wave number κ_w can be calculated. Compared with equation (A6) of Jiao and Tang (1999), the present solution, Eq. (10), includes an additional term, c, that accounts the effects of the moving water table on h_2.

4. Dynamic effects of the phreatic surface (water table) on tidal fluctuations in the leaky confined aquifer

As stated above, dynamic effects of the phreatic surface are contained in the parameter, c. The larger the magnitude of c, the greater these effects. As an example, we calculated c to be $0.33 - 0.02i$ for $L = 0.1\ \mathrm{d}^{-1}$, $n_\mathrm{e} = 0.3$, $d = 5$ m, $\omega = 6.28$ Rad/d (diurnal tides) and $K = 1$ m/d. Although the values of these parameters can vary over wide ranges, the values used here are realistic. Clearly, under such conditions, the dynamic effects will be important.

We compare the predictions of damping and time lags by the present solution and that of Jiao and Tang (1999) in Fig. 2. The results clearly show that the latter solution over-predicts the damping but under-estimates the time lag. In the calculation, $A = 1$ m, $s_2 = 0.001$, $T_2 = 2000\ \mathrm{m}^2/\mathrm{d}$ and other parameter values were the same as above.

Based on this example, one can also show that the maximum vertical head gradient in the phreatic aquifer is about six times larger than maximum horizontal gradient calculated based on h_1. This indicates that the assumption of neglecting horizontal flows in the vertical flow model is valid to certain extent. The ratio of the magnitude of the vertical flow to that of the horizontal flow can be estimated to be $|(L(c-1))/K\kappa|$. For confined aquifers, κ is small and hence the vertical flow can be the dominant process given that the leakage effect is important (i.e. large L).

From Eq. (9), one can derive the following condition, under which dynamic effects of the phreatic surface are important,

$$\frac{K}{dL} \ll 1. \tag{11}$$

This condition implies that a relatively deep phreatic aquifer ($d \gg K/L$) needs to be considered in studying the leakage effects on tidal propagation in the leaky confined aquifer. As the thickness of the phreatic aquifer increases, the magnitude of the leakage term in Eq. (10) is reduced. In the limit of $dL/K \to \infty$, c approaches unity, resulting the disappearance of the leakage term. This result, however, should be taken cautiously since the vertical flow assumption is no longer valid for c close to 1. Physically, a large phreatic aquifer thickness represents a significant resistance to the leakage flow and thus has the capacity of reducing the excessive damping effects of leakage on tidal fluctuations in the confined aquifer.

5. Conclusions

We have derived an analytical solution, based on a vertical flow model, for tidal head fluctuations in a leaky confined aquifer, including effects of the overlying phreatic aquifer that has negligible direct connectivity to the sea. These effects are demonstrated to be important under conditions of deep phreatic aquifers. The results show that under certain conditions, neglecting the horizontal flow component in the phreatic aquifer is reasonable, since the aquifer is not affected directly by the tides (i.e. no horizontal transmission).

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