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Deriving historical equilibrium-line altitudes from a glacier length record by linear inverse modelling

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Abstract: Glaciers have fluctuated in historic times and the length fluctuations of many glaciers are known. From these glacier length records, a climate reconstruction described in terms of a reconstruction of the equilibrium-line altitude (ELA) or the mass-balance can be extracted. In order to derive a climate signal from numerous glacier length records, a model is needed that takes into account the main characteristics of a glacier, but uses little information about the glacier itself. Therefore, a simple analytical model was developed based on the assumption that the change in glacier length can be described by a linear response equation. Historical length observations, the climate sensitivity and the response time of a glacier were needed to calculate historical equilibrium-line altitudes. Both climate sensitivity and length response time were calculated from a perturbation analysis on the continuity equation. The model was tested on 17 European glacier length records. The results revealed that the ELA of most glaciers increased on average 54 m between AD 1920 and 1950. The results of the analytical model were compared to mass-balance reconstructions calculated with a numerical flowline model and derived from historical temperature and precipitation records. The findings lead us to believe that the analytical model could be very useful to gain information about the historical mass-balance rates and ELAs.

Key words: Glacier fluctuations, equilibrium-line altitude, mass-balance, inverse modelling, response time, climate sensitivity, glacier retreat, historical period.

Introduction

Studying glaciers is a useful method for understanding the historical climate. Glacier length records especially contain information on how the climate has changed. Therefore, extracting climatic information from glacier length records, which implies inverse modelling, contributes to the knowledge of the historical climate, particularly in regions with few meteorological records. Normally, the climate signal extracted from length records is represented as a mass-balance history or as a reconstruction of the equilibrium-line altitude (ELA).

Callendar (1950) was probably the first to attempt to extract a climate signal from the dimensions of a glacier. He presented a relation between the height of the firn-line and the glacier length, including the width at the glacier snout, the glacier width and slope at the firn-line altitude and a constant ratio between the accumulation and ablation area. Other simple methods to reconstruct the ELA are: (1) the median elevation of a glacier (Manley, 1959), which is the elevation midway between the glacier snout and the base of the headwall; (2) the THAR (toe-to-headwall altitude ratio), which is a fraction of the height range of the glacier (Meierding, 1982); (3) the ratio of the accumulation area to the total area (AAR) (Porter, 1975). Benn and Lehmkuhl (2000) discussed the applicability of these and other commonly used methods for different glacier types. However, none of the methods above considers the response time of a glacier. Haeberli and Hoelzle (1995) proposed a simple parameterization scheme taking into account the basic glaciological characteristics to determine the mean historical mass-balance of glaciers. Nye (1965) was the first to use a numerical method to infer the mass-balance history of a glacier from its length fluctuations. Oerlemans (1997), Wallinga and van de Wal (1998) and Mackintosh and Dugmore (2000) used a numerical flowline model and the procedure of dynamic calibration to derive a mass-balance history. The advantage of a numerical flowline model is that the geometry, the climate sensitivity and the response time of each particular glacier are taken into account. However, numerical flowline models need lots of information, which is not always available. Therefore, they cannot be applied to a large number of glacier records.

The aim of this research was to develop a simple analytical model that is applicable to many glacier length records to derive the mass-balance history of a glacier. The model should take into account the main characteristics of a glacier, including the response time. Nevertheless, it should only use a limited amount

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of information about the glacier. The model was tested on 17 European glaciers, the length records of which are shown in Figure 1. At the top of the graph, glaciers from the western Alps are shown, followed by glaciers from the eastern Alps, Scandinavia and Iceland. These 17 glaciers, which all retreated between 1850 and 1980, were selected because their length records are among the longest of all European glacier records. The mass-balance and ELA reconstructions calculated with the analytical model were compared with climate reconstructions derived from other methods. To apply the analytical model, the length response time and the climate sensitivity of each glacier need to be known. A method to calculate these parameters is described in this paper, and the obtained values were compared with values calculated from numerical models.

The analytical model

A glacier responds to a change in the mass-balance (and the ELA) by changing its length. The size of the length change depends on several factors, such as the geometry, the slope and the mass-balance profile of the glacier. The glaciers used in this study have relatively small length fluctuations compared to the total glacier length. For relatively small length fluctuations, it is assumed that the change in glacier length can be described by a linear response equation:

$$\frac{dL'(t)}{dt} = -\frac{cE'(t) + L'(t)}{t_{rL}} \qquad (1)$$

where $L'(t)$ is the glacier length with regard to a reference length (L_0) [m], t is time [a], c is the climate sensitivity [–], $E'(t)$ is the ELA with regard to a reference altitude (E_0) [m] and t_{rL} is the length response time of the glacier [a] (see also Figure 3). The concept of this analytical model was first put forward by Oerlemans (2001). The reference length, L_0, is calculated as the mean of a glacier length record. The climate sensitivity is a factor that relates a change in the glacier's steady-state length to a change in the ELA. A climate sensitivity of 20 implies a total glacier retreat of 20 m when the ELA has increased by 1 m. For large length fluctuations, the analytical model is not valid because the glacier geometry can change significantly when the glacier retreats or advances over a large distance. As a result, the response time and the climate sensitivity cannot be kept constant.

The inverse of the linear response equation can be used to calculate the historic ELAs:

$$E'(t) = -\frac{1}{c}\left(L' + t_{rL}\frac{dL'}{dt}\right) \qquad (2)$$

If $E'(t)$ is multiplied by the mass-balance gradient at the equilibrium line altitude (β_E), this equation can be used to calculate a mass-balance reconstruction ($B'(t)$). Methods to estimate the length response time and the climate sensitivity are given in the next section. First, how the time derivative of a length record can be calculated is described.

The glacier length records shown in Figure 1 are not smooth records. Linear interpolation between the observed data points

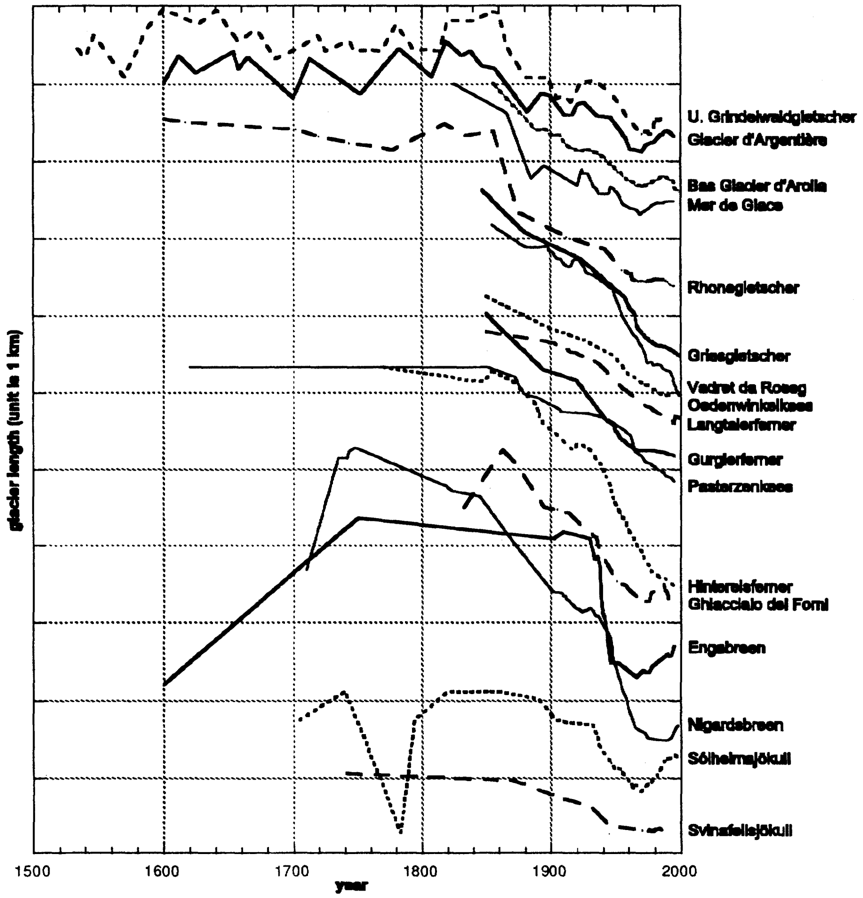

Figure 1 Glacier-length fluctuations of 17 European glaciers.

is applied, assuming that the glaciers have not fluctuated substantially during the periods between these points. It is likely that they were not larger during the period in between the data points, otherwise moraines would have been deposited. However, glaciers could have been smaller during these periods. Taking the time derivative from these length records is not a straightforward exercise, because the derivative will be discontinuous at the data points. To obtain smooth time derivatives, we tried to fit polynomials to the length records and to calculate the time derivative of these polynomials. However, we found that not every length record can be represented well by a polynomial fit of a certain degree. Then we used Fourier functions to describe the length records and calculate the time derivatives. The advantage of Fourier series is that this method allows separation of timescales. Nevertheless, most length records only show a retreat in length and little fluctuations. The results revealed that these length records especially are difficult to describe by Fourier decompositions.

We concluded that a better solution would be to apply a Gaussian filter to the length records and to calculate the time derivatives with central differences. A Gaussian filter is a weighted average, and the weight for each year (w_i) is defined by a Gaussian function. The filtered length record $L'(t)$ can be calculated with:

$$L'(t) = \frac{\sum_{-N}^{N} w_i \Lambda'(t+i)}{\sum_{-N}^{N} w_i} \tag{3}$$

where $\Lambda'(t)$ is the original length record, as shown in Figure 1. The Gaussian function is defined as:

$$w_i = e^{-i^2/\tau^2} \tag{4}$$

where τ is the timescale. The filtered length records were calculated using a timescale of 10 years. Accordingly, N was taken as 15 years. Figure 2 gives an example of the filtered length record and its time derivative of Hintereisferner. The time derivative does not show any discontinuities at the data points.

Calculation of the length response time and the climate sensitivity

The length response time and the climate sensitivity of a glacier can be determined by a numerical flowline model, as done previously for Hintereisferner (Greuell, 1992), Pasterzenkees (Zuo and Oerlemans, 1997), Unterer Grindelwaldgletscher (Schmeits and Oerlemans, 1997), Rhonegletscher (Wallinga and van de Wal, 1998), Nigardsbreen (Oerlemans, 1997), Glacier d'Argentière (Huybrechts *et al.*, 1989) and Sólheimajökull (Mackintosh, 2000). However, we prefer a simpler method based on a perturbation analysis on the continuity equation (Oerlemans, 2001). After Reynolds decomposition and neglecting higher-order terms of the continuity equation for a glacier volume, a perturbation equation is obtained:

$$\frac{dV'}{dt} = \overline{H_0}\frac{dA'}{dt} + A_0\frac{d\overline{H'}}{dt} \tag{5}$$

where V' is the glacier volume with regard to the reference volume [m^3], $\overline{H'}$ is the mean glacier thickness with regard to the reference thickness ($\overline{H_0}$), [m] and A' is the glacier area with regard to the reference area (A_0) [m^2]. The reference situation of the glacier is thus defined by L_0, A_0 and $\overline{H_0}$ (Figure 3). According to equation (5), a change in glacier volume is directly coupled to a change in the area and the thickness of a glacier.

It is assumed that a change in the glacier area is only related to a glacier length fluctuation. The corresponding volume change is then calculated by multiplying the length change by the width and the thickness of the glacier tongue. The width of the glacier tongue is assumed to be constant. If we also assume that the change in glacier thickness is proportional to the change in glacier length, the volume change of a glacier can be written as:

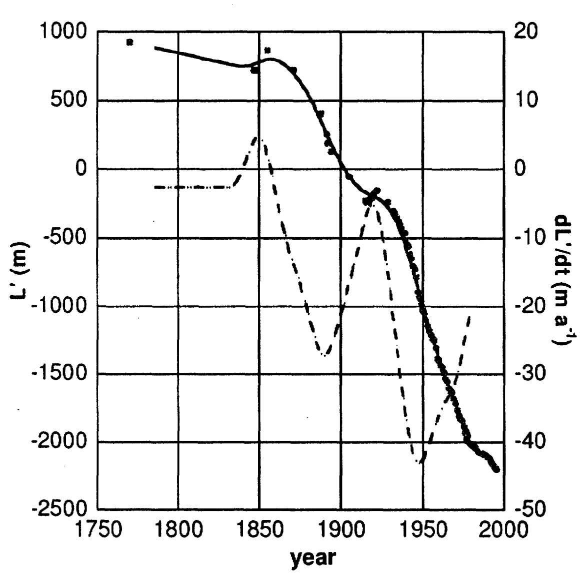

Figure 2 The length record of Hintereisferner filtered with the Gaussian filter and the time derivative of the length record (dashed line). The black dots indicate the individual data points.

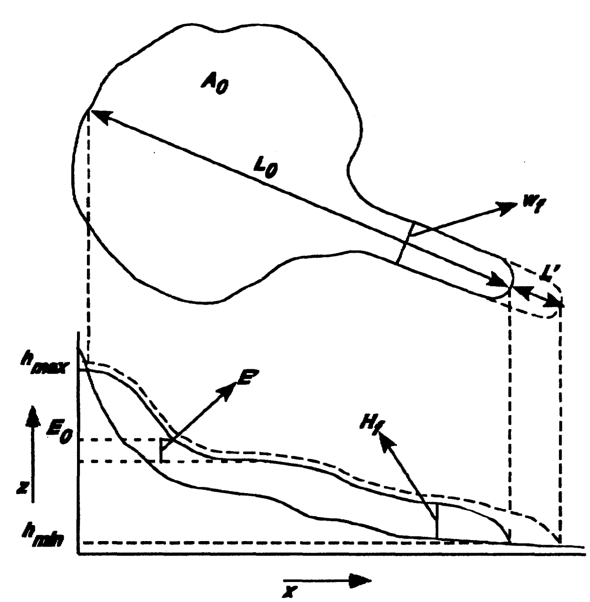

Figure 3 Schematic outline of a glacier, showing some of the parameters of the analytical model.

$$\frac{dV'}{dt} = (w_f H_f + \eta A_0)\frac{dL'}{dt} \tag{6}$$

where η is a constant that relates the mean glacier thickness to the glacier length, w_f is the characteristic width [m] and H_f the characteristic thickness of the glacier tongue [m] (Figure 3).

Changes in the glacier volume are caused by changes in the mass-balance, described by:

$$\frac{dV'}{dt} = A_0 B' + A_0 \beta \overline{H}' + w_f B_f L' \tag{7}$$

The first term on the right-hand side is the volume change caused by a perturbation of the mass-balance rate, B' [m]. B' can be expressed in a change in the ELA (E') by dividing B' by the mass-balance gradient at the equilibrium line altitude (β_E). The second term is a volume change resulting from the feedback between the mass-balance and the surface elevation of the glacier. β is the average mass-balance gradient over the glacier and $\overline{H}'$ can be described as $\eta L'$. The last term represents a volume change due to a change in glacier length, where B_f is the melt rate at the glacier terminus [m]. Combining and rewriting equation (6) and (7) yields:

$$\frac{dL'}{dt} = \frac{\beta_E A_0}{\eta A_0 + w_f H_f} E' + \frac{\eta \beta A_0 + w_f B_f}{\eta A_0 + w_f H_f} L' \tag{8}$$

Comparing this equation with equation (1), the length response time and the climate sensitivity can be derived:

$$t_{rL} = -\frac{\eta A_0 + w_f H_f}{\eta \beta A_0 + w_f B_f} \tag{9}$$

$$c = \frac{\beta_E A_0}{\eta \beta A_0 + w_f B_f} \tag{10}$$

With these equations, the climate sensitivity and the length response time of each glacier were calculated. It should be noted that equation (9), the expression for the length response time, also holds for the volume response time, because volume and length changes are coupled (equation (6)). Normally, the volume response time is smaller than the length response time, because the glacier volume is more directly affected by changes in the mass-balance (Oerlemans, 1997). Therefore, length response times calculated with equation (9) are expected to be smaller than the real length response times.

If the mass-balance/surface-elevation feedback is not taken into account (i.e., $\eta = 0$), the length response time (equation (9)) corresponds to the volume timescale derived by Jóhannesson *et al.* (1989). The volume timescale of Jóhannesson is always smaller than the length response time calculated with equation (9). Futhermore, the climate sensitivity decreases if the mass-balance/surface-elevation feedback is not taken into account. If we also assume that the glacier's width is constant along the glacier, the climate sensitivity corresponds to the often-used expression reported in Paterson (1994):

$$\frac{dL'}{dB'} = \frac{L_0}{B_f} \tag{11}$$

However, most glaciers do not have a uniform width. Normally, the glacier tongue is narrower than the mean glacier width. In that case, equation (11) underestimates the climate sensitivity.

To calculate the length response time and the climate sensitivity, some information about the glacier is needed. First of all, the thickness of the glacier snout (H_f) must be estimated. If we assume a constant driving stress, the ice thickness at any point on the glacier can be calculated from the surface slope (Paterson, 1994). The surface slope multiplied by the glacier thickness is then a constant, and the thickness of the glacier snout follows:

$$H_f = \frac{\overline{H} \cdot s}{s_f} \tag{12}$$

where s_f is the surface slope at the glacier snout, estimated from a topographical map, and s is the average surface slope of the glacier. s was derived from the maximum altitude, the minimum altitude and the length of the glacier obtained from Haeberli *et al.* (1998). $\overline{H}$ is the mean glacier thickness, which was calculated from an expression proposed by Oerlemans (2001):

$$\overline{H} = \left(\frac{\mu L}{1 + \nu s}\right)^{\frac{1}{2}} \tag{13}$$

where L is the glacier length [m] and μ and ν are constants (~9 m and ~30 respectively) determined by a numerical model (Oerlemans, 2001). Taking the derivative of equation (13) yields an expression for η:

$$\eta = \frac{d\overline{H}}{dL} = \frac{1}{2}\left(\frac{\mu}{(1 + \nu s)L}\right)^{\frac{1}{2}} \tag{14}$$

A typical value for η is 0.006. Values for the glacier area (A_0 were also obtained from Haeberli *et al.* (1998). w_f was estimated from a topographical map and β_E and β were calculated from mass-balance measurements. For β_E the mass-balance gradient of the ablation area was used. Mass-balance measurements were not available for all glaciers, and, if absent, the mean of the mass-balance gradients of nearby glaciers was used.

Length response time and climate sensitivity results

Table 1 shows the length response times and the climate sensitivities for the 17 European glaciers calculated with the analytical model. For some glaciers, the climate sensitivity, the length response time and the volume response time calculated with numerical models are given. The analytical length response times were expected to be smaller than the numerical length response times, as was explained in the previous section, because the analytical length response time is in fact a volume response time. However, the results indicate that the analytical length response times correspond better to the numerical length response times than to the numerical volume response times. Hintereisferner is an exception because the numerical length response time of Hintereisferner (94 a) is much larger than the analytical length response time (62 a).

The differences between the numerical and analytical values can be associated with topographical effects. For instance, the climate sensitivity increases if the glacier tongue reaches a point where the valley is narrow or the bed slope small. A numerical model takes into account these topographical features, whereas the analytical method assumes a constant bed slope and a constant width of the glacier tongue.

Reconstructions of the equilibrium-line altitude

Figure 4 shows the length record of Glacier d'Argentière filtered with the Gaussian filter and the reconstructed ELAs calculated

Table 1 Climate sensitivities and length response times calculated by the analytical method (ana) and by numerical models (num) and volume response times calculated by numerical models for several European glaciers; the numerical values are taken from literature referenced in this article

Glacier	Climate sensitivity		Length response time [a]		Volume response time (num) [a]
	ana	num	ana	num	
Glacier d'Argentière	25	35	33	27–45	
Mer de Glace	57		56		
Bas Glacier d'Arolla	16		48		
Rhonegletscher	31	32	53	58	36
Griesgletscher	17		61		
U. Grindelwaldgletscher	64	49	52	34–45	
Vadret da Roseg	29		52		
Gurglerferner	30		41		
Oedenwinkelkees	9		37		
Langtalerferner	27		59		
Pasterzenkees	32	32	62	70–137	34–50
Hintereisferner	25	31	62	79–109	56–78
Ghiacciaio dei Forni	62		76		
Engabreen	39		53		
Nigardsbreen	84	63	64	63–73	38–47
Sólheimajökull	53	26	87	65–73	58–72
Svinafellsjökull	20		60		

with the analytical model. The figure clearly indicates that changes in the ELA precede changes in the glacier length.

The reconstructed equilibrium line altitudes of the 17 European glaciers are shown in Figure 5. The ELAs of most glaciers show a similar pattern: after 1850 the ELAs increase, indicating a warmer period or less snowfall. At the beginning of the twentieth century, there is a small decrease in most ELAs. After that, the ELAs increase until around 1950 and then decrease slightly. The ELA reconstructions of Unterer Grindelwaldgletscher and Glacier d'Argentière are among the longest records and show very similar fluctuations. However, the amplitudes of the fluctuations differ.

The ELA reconstruction of Rhonegletscher shows a large increase between 1850 and 1870 compared to the other reconstructions. This strong increase is probably an artifact of the analytical model, which uses a constant climate sensitivity and a constant length response time. Between 1850 and 1870, the terminus of Rhonegletscher rested on a small sloping surface, much smaller than the mean slope, implying that the glacier length was actually very sensitive to a change in the ELA. If a larger climate sensitivity was applied to Rhonegletscher, a smaller increase in the ELA would have been calculated.

Hintereisferner reveals two very steep increases in the ELA after 1850, interrupted by significantly lower ELAs at the beginning of the twentieth century. The total increase in the ELA of Hintereisferner is rather large compared to other ELA reconstructions of glaciers in the Alps. The analytical model is probably less valid for Hintereisferner, because the relative decrease in length over the total period of Hintereisferner is large: 30%. The other glaciers in the Alps retreated on average 18%.

Sólheimajökull's high ELAs around 1760 and low ELAs just before 1800 are necessary to obtain its enormous retreat before and growth after 1780. Svinafellsjökull, however, does not show low ELAs before 1800. Still, the large ELA fluctuations of Sólheimajökull fit with documented changes in the Icelandic climate and sea-ice extent (Ogilvie, 1992): during the 1780s, sea ice remained unusually close to Iceland, and during some years it was still present along the south coast of Iceland in August. Besides, the large ELA fluctuations are confirmed by ELA reconstructions of Mackintosh and Dugmore (2000) that were calculated for Sólheimajökull with a numerical flowline model.

The ELA reconstructions of glaciers located in the same area exhibit resemblances. Therefore, means of the ELA reconstructions of the Icelandic and the Scandinavian glaciers and the glaciers of the western and eastern Alps were calculated (Figure 6). The eastern part of the Alps was separated from the western part by the 9°E meridian. Means were calculated over the period 1872–1974 because ELA reconstructions of all glaciers were performed for this period. The mean ELAs show an increase after 1915. This is 45 m for the Icelandic glaciers, 59 m for the Scandinavian glaciers, 47 m for the western Alps and 64 m for the eastern Alps. Apparently, air temperatures and snowfall have changed more in the eastern Alps than in the western Alps. The ELAs in northern Europe reached a maximum before 1950 and then decreased, unlike the ELAs of the Alps, which were at maximum after 1950 and show a smaller decrease after that. Furthermore, it is striking that the ELAs in the western Alps were also at a minimum before 1900, unlike the other ELA reconstructions.

Comparison of the results with other records

It would be ideal to compare the ELA reconstructions with long records of ELA or mass-balance observations. However, these

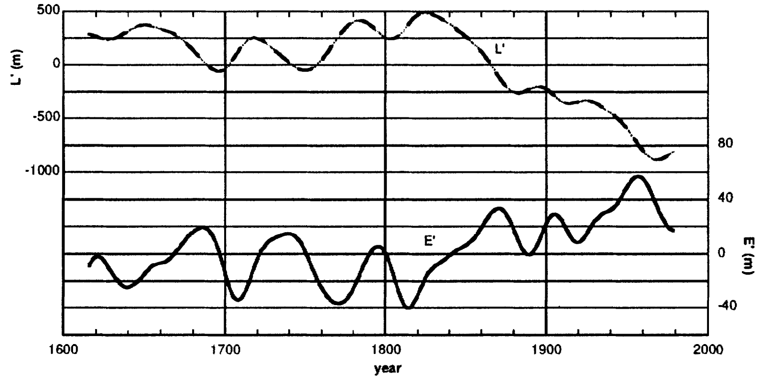

Figure 4 The length record of Glacier d'Argentière filtered with the Gaussian filter (dotted line) and the ELA reconstruction calculated with the analytical model (solid line).

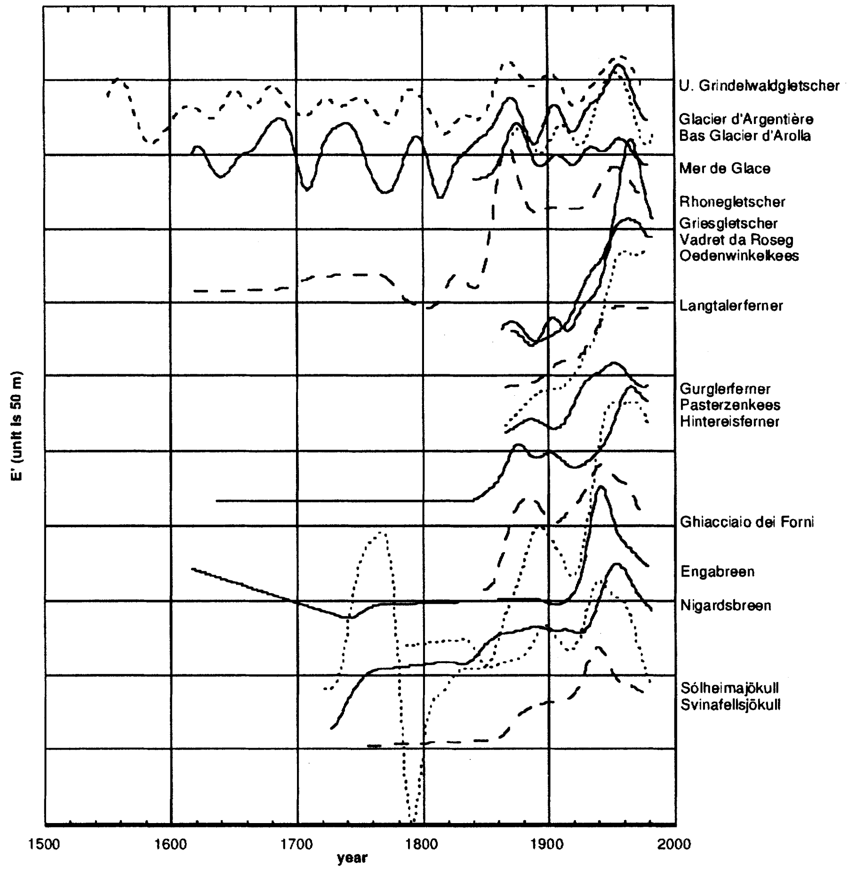

Figure 5 Reconstructed ELA records of 17 European glaciers calculated with the analytical model.

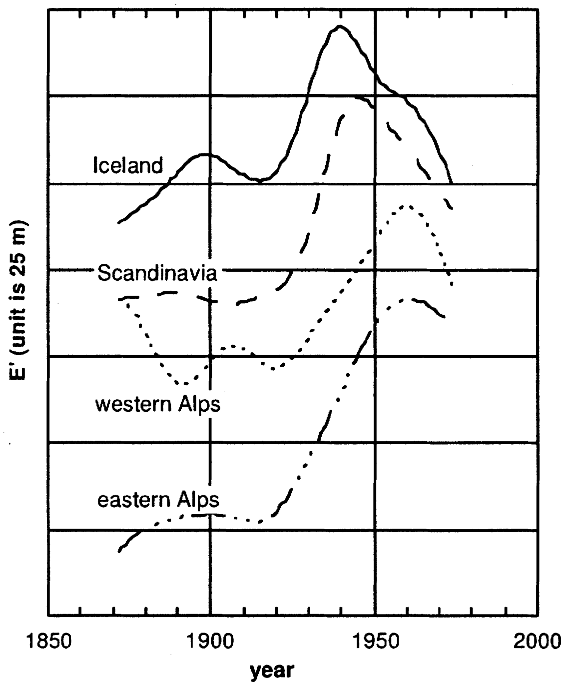

Figure 6 Means of the reconstructed ELA records of Iceland, Scandinavia, the western Alps and the eastern Alps.

long observation records do not exist. Therefore, the results of the analytical model were compared with different types of data: results from a simple method and from a numerical model and from historical temperature and precipitation records.

Haeberli and Hoelzle (1995) calculated the average change in the mass-balance over 1850–1970 from 13 length records of glaciers in the Alps. They assumed that the glaciers were stationary at the beginning and at the end of this period. Subsequently, they supposed that one full dynamic response to a step change in the mass-balance would explain the glaciers' retreat over this period. They then calculated a mean step change in the mass-balance of -0.5 ± 0.1 m w.e a^{-1} and an average mass loss of 0.2–0.3 m w.e. a^{-1} during 1850–1970. Mass-balance changes over the same period were also calculated for the glaciers in the Alps with the analytical model. The average of the mass-balance changes resulted in -0.3 ± 0.1 m w.e a^{-1}, which is smaller compared to the step change in mass-balance calculated by Haeberli and Hoelzle. The difference between the results of the two methods could be explained by the different approaches. First, the analytical model takes into account the response time and does not assume steady-state situations before 1850 and after 1970. Second, Haeberli and Hoelzle use equation (11) to determine the climate sensitivity of a glacier, which underestimates the climate sensitivity.

Figure 7 shows how the analytical ELA reconstruction of Nigardsbreen compares to the ELA reconstruction calculated from a numerical model (Oerlemans, 1997). The variation in the ELA reconstructions is of the same size, but there is a phase difference between the two reconstructions. Although the time at which the ELAs start to decrease or increase is similar, the locations of the maxima and minima differ. This difference could be due to

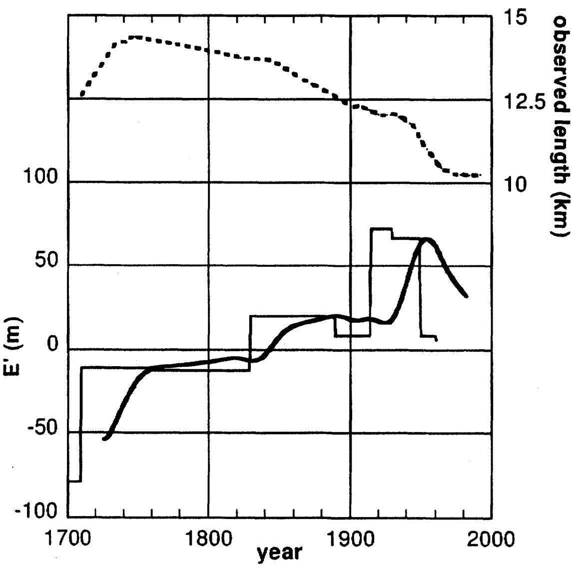

Figure 7 ELA reconstruction of Nigardsbreen calculated with the analytical model (thick line) and computed with a numerical model (Oerlemans, 1997) (thin line). The dotted line is the observed length record.

the numerical model, which uses a succession of step functions. Nine step functions were used to obtain the numerical ELA reconstruction as shown in Figure 7. Increasing the number of step functions will certainly influence the ELA reconstruction and probably lead to a reconstruction that is closer to the analytical reconstruction. Second, the analytical model is based on the assumption that the glacier length directly responds to a change in the ELA. However, from numerical studies it is clear that glacier volume does indeed respond immediately to a change in the ELA, but glacier length starts reacting somewhat later. Therefore, in the analytical model, a change in the ELA is closer followed by a length change. In the numerical model, it takes more time for the glacier length to respond to a change in the ELA. The analytical ELA reconstruction is accordingly shifted forward in time compared to the numerical reconstruction. Third, numerical models may respond too slowly to a mass-balance change due to the grid-point spacing, which was 100 m for Nigardsbreen.

Comparing a mass-balance reconstruction with climate records is difficult because changes in the mass-balance can be due to fluctuations in temperature, precipitation, sunshine duration or incoming radiation. A method to derive a mass-balance history from historical meteorological data is using seasonal sensitivity characteristics (Oerlemans and Reichert, 2000). A seasonal sensitivity characteristic is the dependence of the mass-balance on monthly anomalies in temperature and precipitation. Figure 8 shows the seasonal sensitivity characteristics of Griesgletscher and Rhonegletscher calculated by Oerlemans and Reichert (2000) from a mass-balance model of Oerlemans (1992). The figure illustrates that the mass-balance of Rhonegletscher is more sensitive to changes in precipitation than the mass-balance of Griesgletscher. Furthermore, a temperature increase in winter does not change the mass-balance of Griesgletscher and hardly influences the mass-balance of Rhonegletscher. Mass-balance reconstructions of these two glaciers in the Alps were calculated and compared with the reconstructions calculated with the analytical model. Therefore, long records of monthly precipitation and temperature anomalies were needed. The monthly precipitation of Beatenberg

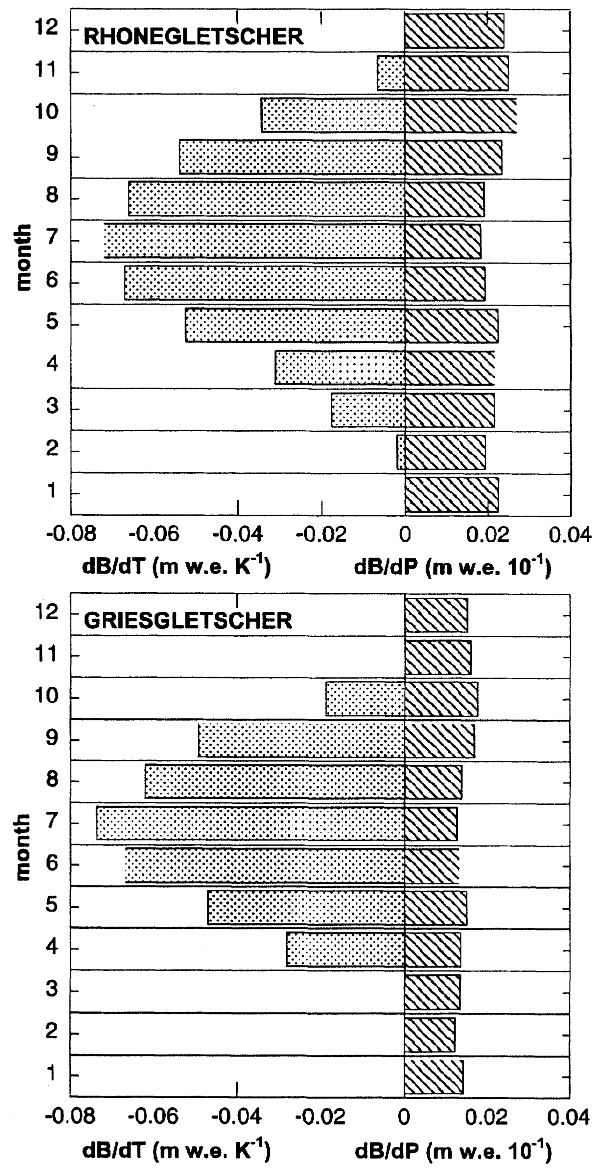

Figure 8 Seasonal sensitivity characteristics of Rhonegletscher and Griesgletscher calculated from a mass-balance model (Oerlemans and Reichert, 2000).

(Switzerland) and the monthly homogenized high-elevation (above 1500 m a.s.l.) temperature record of 46° N 8° E taken from Böhm *et al.* (2001) were used for both glaciers. The mass-balance reconstructions calculated from the seasonal sensitivity characteristics were then filtered with a Gaussian filter. The mass-balance reconstructions of Griesgletscher and Rhonegletscher and the mean mass-balance history of the glaciers of the western part of the Alps calculated with the analytical model are shown in Figure 9. The mass-balance reconstructions show similar fluctuations with amplitudes of similar magnitude. However, there is again a phase difference between the reconstructions. The analytical mass-balance reconstruction is shifted about 10 years forward in time, which is less than the phase difference between the analytical and the numerical model results.

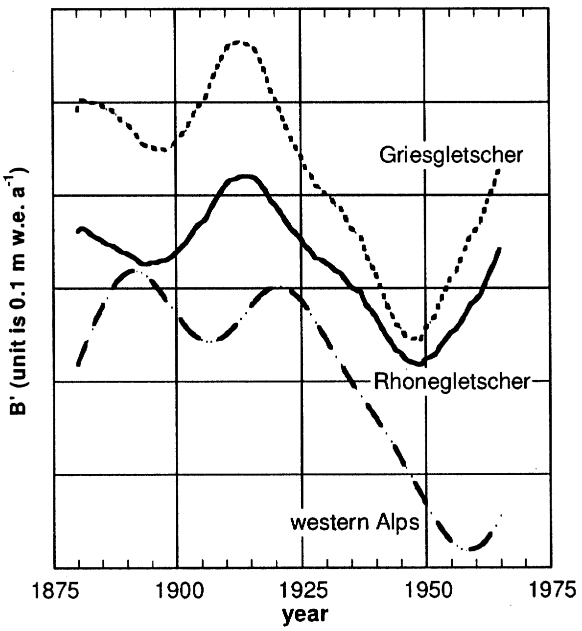

Figure 9 Mean mass-balance reconstruction of glaciers in the western Alps (dashed) and mass-balance reconstructions calculated with seasonal sensitivity characteristics of Griesgletscher (dotted) and Rhonegletscher (solid). A Gaussian filter is applied to the mass-balance reconstructions.

Conclusions

In this paper, an analytical method that calculates a mass-balance history or a reconstruction of the ELA from a glacier length record has been described. Application of the method to 17 European glaciers showed that the model is useful to derive a climate signal from a glacier length record. The calculated mass-balance fluctuations are in agreement with mass-balance reconstructions derived from numerical models and from temperature and precipitation records using seasonal sensitivity characteristics. The length response times and climate sensitivities calculated with the analytical model were in agreement with values calculated from numerical models.

An advantage of the model is that it takes into account the response time of the glacier, in contrast to other simple models, e.g., the AAR-method (Porter, 1975) or the method of Haeberli and Hoelzle (1995). Furthermore, it requires less information about a glacier than is needed for a numerical model.

However, the ELA reconstructions calculated with the analytical model were shifted forward in time by a decade compared to the numerical mass-balance reconstruction and the mass-balance reconstructions calculated from temperature and precipitation records. This is due to the model assumption that glacier length immediately responds to a change in the mass-balance. Another limitation of the model is that topography is not fully taken into account compared to a numerical model. In the analytical model, the glacier's geometry is represented in a schematic way. The topography is especially important when the slope of the bedrock and the valley width change along the glacier. Then, the climate sensitivity and the length response time depend considerably on the position of the terminus, which in turn will influence the mass-balance reconstruction. Therefore, the analytical model is not suitable for glaciers with strong variations in the slope and width of the glacier valley. Furthermore, the model is only valid for relatively small glacier length fluctuations.

These limitations have to be considered when applying this method to other glacier length records. The analytical model could be very useful to gain information about the historical climate of areas with limited climate records. It is planned to calculate mass-balance reconstructions with the analytical model of glaciers from different parts of the world.

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