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The azimuthal dependence of surface wave polarization in a slightly anisotropic medium

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SUMMARY

Compact analytical formulae are derived for particle motions of surface waves in a weakly anisotropic, flat-layered medium. Theory only incorporates coupling between fundamental mode Rayleigh and Love waves but comparison against numerical results show that a good match is achieved for anisotropy up to approximately 10 per cent. Among various types of particle motions for Rayleigh and Love waves, it is proposed that Love wave vertical polarization, defined by the ratio of the vertical to the transverse displacement, may be a good, cleanly measurable quantity. Derived formulae show that vertical polarization contains 2θ and 4θ azimuthal dependence, similar to the well-known phase velocity variations. Formulae show depth kernels explicitly and can be used for the inversion of anisotropic parameters. The measurement of polarization as a function of azimuth then provides constraints on anisotropy under a seismic station. The derived formulae are a natural extension of Smith & Dahlen's results on phase velocity to particle motions.

Key words: anisotropy, particle motion, surface wave.

1 INTRODUCTION

Understanding anisotropy in the crust and mantle is of potential importance because anisotropy is related to geodynamically important physical quantity in the Earth such a strain field at depth. In the past, the main source of information for anisotropy came from (1) *Pn* azimuthal anisotropy (e.g. Hess 1964; Francis 1969; Raitt *et al.* 1969), (2) *P*-wave particle motion (e.g. Schulte-Pelkum *et al.* 2001), (3) *S*-wave splittings (e.g. Vinnik *et al.* 1989; Silver & Chan 1991; Silver 1996), (4) surface wave azimuthal anisotropy (e.g. Forsyth 1975; Tanimoto & Anderson 1985; Montagner & Nataf 1986; Tanimoto 1986; Nishimura & Forsyth 1989; Montagner & Tanimoto 1991) and (5) surface wave particle motion (e.g. Park & Yu 1993; Pettersen & Maupin 2002). In addition, there are many important theoretical contributions for the anisotropic effects for body waves, in general (e.g. Jech & Pšenčík 1989; Chevrot & van der Hilst 2003). All of these approaches provide important information but have limitations due to the nature of specific seismic waves used to recover anisotropy. Our understanding of anisotropy is therefore based on complimentary information from all types of measurements and will probably continue to be in a similar state in the near future. It is then desirable to explore new methods of measurements which will bring new types of constraints on the anisotropy of the interior. This paper is an attempt for such an endeavour.

In this paper, we examine surface wave polarization as a new source of information for anisotropy. Our proposed approach is similar to Park & Yu (1993), the fifth item (5) in the above list, in that we examine surface wave polarization but it differs in that, while Park and Yu examined teleseismic data for tectonically interesting paths, we seek to retrieve azimuthal variations of particle motion at a single station and to obtain constraints on anisotropic structure under this seismic station. Specifically, we take a look at surface wave particle motions and see how they change with azimuth of arrival. Such an approach requires data from all azimuth and was not viable before at most stations. With the maturity of global seismic networks, however, we believe it is becoming a viable approach.

This study basically extends the analysis by Smith & Dahlen (1973), who demonstrated azimuthal dependence of surface wave velocities for the first time. In the next section, we derive formulae for surface wave particle motions in an anisotropic medium from which it is clear that there are many anisotropic effects in surface wave particle motions which can potentially be explored. Among those many effects, we propose that Love wave vertical polarization may be a good quantity to cleanly measure anisotropic effects in data because, in principle, effects from lateral heterogeneity are separable. We then derive the formulae for Love wave vertical polarizations. Because the method is essentially a perturbation theory, the derived formulae will be tested against direct numerical integration results in the following section in order to verify its validity. In the final section, we discuss an interesting qualitative aspect of particle motions where clockwise versus counter-clockwise particle motions switch with azimuth. Such a change may be observed in a robust manner.

2 FORMULATION OF THE PROBLEM

2.1 Definitions

We consider propagating, infinitesimal surface waves in a perfectly elastic, weakly but generally anisotropic, non-gravitating half-space, the properties of which are a function of depth alone. We let x, y and z be the Cartesian coordinates, with the $\hat{\mathbf{x}}$ axis pointing east, the $\hat{\mathbf{y}}$ axis pointing north, and the $\hat{\mathbf{z}}$ axis pointing upward (Fig. 1); the surface $z = H$ is a free surface where three stress components disappear $\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$. For direct numerical integration in a later section, we assume existence of an isotropic half-space at bottom where we can start the integration with analytical solutions for an isotropic half-space (e.g. Takeuchi & Saito 1972). We define the top of this half-space as $z = 0$ which can be made as deep as we want. We let $\mathbf{k} = (k_x, k_y)$ be the horizontal wave vector of a surface wave in this medium.

We let $\mathbf{u}(\mathbf{r}, t)$ be the infinitesimal elastic displacement field in the medium. Defining ω to be the angular frequency of surface waves, we assume that $\mathbf{u}$ is of the form

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{s}(z) \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)], \tag{1}$$

where $\mathbf{s}(z)$ is the vertical eigenfunction, decaying at large depth, and $\mathbf{r} = (x, y)$.

We define the Cartesian components of the strain tensor ϵ_{ij} by

$$\epsilon_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j), \tag{2}$$

where i and j vary among x, y and z. Stress σ_{ij} and strain relations are given by

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \tag{3}$$

In the following analysis, we make use of the stationarity of the Lagrangian density defined by

$$\mathcal{L}(u_i^*, u_i) \equiv \omega^2 \int_{-\infty}^{H} \rho u_i^* u_i \, dz - \int_{-\infty}^{H} C_{ijkl}\epsilon_{ij}^*\epsilon_{kl} \, dz, \tag{4}$$

where repeated indices assume summations over x, y and z.

2.2 Ansatz

We assume that the medium is only weakly anisotropic and the eigenfunctions can be written as a combination of Rayleigh and Love wave eigenfunctions in the reference, isotropic medium. The validity of this assumption will be verified by comparing the results against the direct integration results in the next section.

Let the eigenfunctions of the reference isotropic medium be $\mathbf{u}_{\mathrm{R}}$ and $\mathbf{u}_{\mathrm{L}}$ where the subscripts refer to Rayleigh waves and Love waves, respectively. We assume that the eigenfunctions of the anisotropic medium can be written as

$$\mathbf{u} = a_{\mathrm{L}}\mathbf{u}_{\mathrm{L}} + a_{\mathrm{R}}\mathbf{u}_{\mathrm{R}}, \tag{5}$$

where a_{L} and a_{R} are the coefficients to be determined from the stationarity of the Lagrangian.

The form of $\mathbf{u}_{\mathrm{L}}$ and $\mathbf{u}_{\mathrm{R}}$ can be taken as

$$\mathbf{u}_{\mathrm{L}} = [-\beta W(z), \alpha W(z), 0]\mathrm{e}^{ik(\alpha x + \beta y) - i\omega t} \tag{6}$$

and

$$\mathbf{u}_{\mathrm{R}} = [\alpha V(z), \beta V(z), i U(z)]\mathrm{e}^{ik(\alpha x + \beta y) - i\omega t}, \tag{7}$$

where $\alpha = \cos\theta$ and $\beta = \sin\theta$ and θ is the direction of the wave vector measured from the x-axis (Fig. 1). In eqs (6) and (7), $W(z)$ is the Love wave eigenfunction, normalized by

$$\int_{-\infty}^{H} \rho W^2 \, dz = 1 \tag{8}$$

and $U(z)$ and $V(z)$ are Rayleigh wave eigenfunctions normalized by

$$\int_{-\infty}^{H} \rho(U^2 + V^2) \, dz = 1. \tag{9}$$

We substitute eq. (5) in eq. (4) and seek the stationarity state of the Lagrangian by using the relations $\partial\mathcal{L}/\partial a_{\mathrm{L}} = 0$ and $\partial\mathcal{L}/\partial a_{\mathrm{R}} = 0$. The problem is then reduced to an eigenvalue–eigenvector matrix problem given by

$$\begin{pmatrix} A & E \\ E & B \end{pmatrix} \begin{pmatrix} a_{\mathrm{L}} \\ a_{\mathrm{R}} \end{pmatrix} = \omega^2 \begin{pmatrix} a_{\mathrm{L}} \\ a_{\mathrm{R}} \end{pmatrix}, \tag{10}$$

where A, B and E consist of azimuthally dependent terms up to $4\,\theta$. We use the notations

$$A = \int_{-\infty}^{H} dz\,(L_0 + L_1 \cos 2\theta + L_2 \sin 2\theta + L_3 \cos 4\theta + L_4 \sin 4\theta), \tag{11}$$

$$B = \int_{-\infty}^{H} dz\,(R_0 + R_1 \cos 2\theta + R_2 \sin 2\theta + R_3 \cos 4\theta + R_4 \sin 4\theta), \tag{12}$$

$$E = \int_{-\infty}^{H} dz\,(M_1 \cos 2\theta + M_2 \sin 2\theta + M_3 \cos 4\theta + M_4 \sin 4\theta), \tag{13}$$

where the isotropic term in E identically disappears ($M_0 = 0$). Formulae for L_0–L_4, R_0–R_4 and M_1–M_4 are given in the Appendix. Formulae for L_0–L_4 and R_0–R_4 are given in our notation, but are equivalent to those of Smith & Dahlen (1973). Those for M_1–M_4 are our contribution.

For a given wavenumber k, Love wave phase velocity is generally larger than that of Rayleigh wave phase velocity. Therefore, in the remainder of this paper, we assume $A > B$.

2.3 Eigenvalues

Eigenvalues for the matrix problem (10) are given by

$$\omega^2 = \frac{A + B \pm \sqrt{(A - B)^2 + E^2}}{2} \tag{14}$$

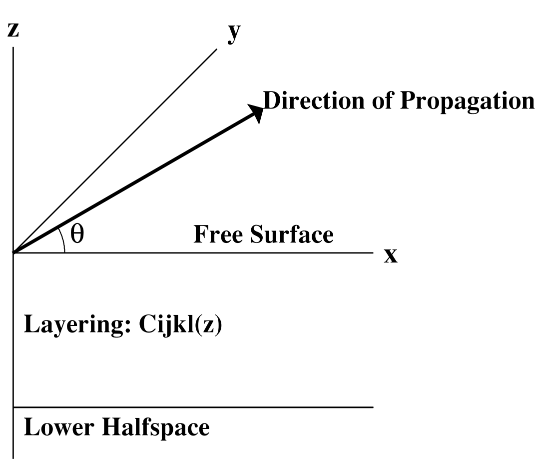

Figure 1. Definitions of the coordinate system. Azimuth of the wave vector (θ) is measured from the x-axis.

but simplifies further to the following form in the limit of weak anisotropy under $|A - B| \gg E$:

$$\omega^2 = A + \frac{E^2}{A - B}, \quad B - \frac{E^2}{A - B}. \tag{15}$$

The terms with E are clearly second order due to the above assumption $|A - B| \gg E$. Thus, to first order, the two eigenvalues are $\omega^2 = A$ and B. E vanishes from the eigenvalue formulae and makes our results equivalent to those of Smith & Dahlen (1973).

However, first-order corrections to the eigenvectors contain terms proportional to E. A normalized eigenvector (up to first order) associated with the eigenvalue $A + E^2/(A - B)$ is given by

$$(a_{\mathrm{L}}, a_{\mathrm{R}}) = \left(1, \frac{E}{A - B}\right) \tag{16}$$

and that associated with $B - E^2/(A - B)$ is

$$(a_{\mathrm{L}}, a_{\mathrm{R}}) = \left(-\frac{E}{A - B}, 1\right). \tag{17}$$

We will refer to the former as the quasi-Love wave because particle motions are predominantly those of Love waves and the latter as the quasi-Rayleigh waves for the same reason.

2.4 Particle motion

The eigenfunction of the quasi-Love wave is obtained from (5) and (16) as

$$\mathbf{u} = \left(-\beta W + \frac{E}{A - B}\alpha V, \alpha W + \frac{E}{A - B}\beta V, i\frac{E}{A - B}U\right). \tag{18}$$

The surface particle motion for Love waves in an isotropic medium is linear in the transverse direction. However, the form of eq. (18) indicates that the particle motion in an anisotropic medium is no longer linear; it is clearly elliptical because of a small vertical component with a phase lag denoted by the imaginary factor i. Also within the horizontal plane, the particle motion is no longer strictly transverse; because of this, the particle motion in the horizontal plane is no longer perpendicular to the direction of wave vector, although it should be close to 90° under weak anisotropy (Fig. 2).

The other eigenfunction is that of the quasi-Rayleigh wave and is given by

$$\mathbf{u} = \left(-\frac{E}{A - B}\beta W + \alpha V, \frac{E}{A - B}\alpha W + \beta V, iU\right). \tag{19}$$

The surface particle motion for Rayleigh waves in an anisotropic medium is (already) an ellipse and is polarized within the plane defined by the radial and the vertical directions. In an anisotropic medium, the particle motion remains as an ellipse, but the plane of polarization deviates from the radial direction by a small angle (Fig. 2) due to the introduction of small transverse component in eq. (19).

2.5 Love wave vertical polarization

Examinations of the formulae for particle motions in eqs (18) and (19) (Fig. 2) suggest that some anisotropic effects may be hard to observe due to other complicating factors in the real Earth. For example, deviation of the quasi-Rayleigh wave particle motion from the radial–vertical plane is difficult to measure because the same effect can be created by lateral refraction of surface waves (e.g. Laske 1995). In such a case, it would be impossible to distinguish this lateral refraction effect from the anisotropic effect.

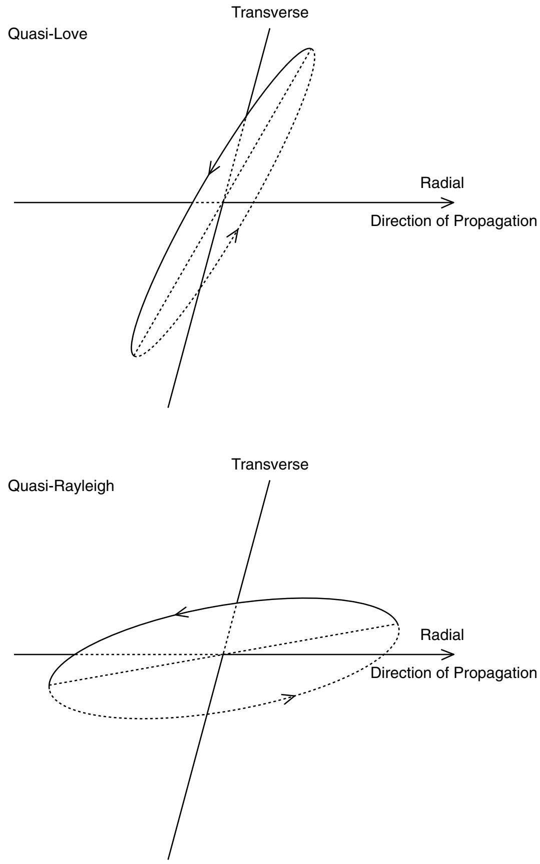

Figure 2. Love wave surface particle motion becomes elliptical instead of linear in an anisotropic medium because of the small vertical component. Also the plane of polarization is no longer transverse. The plane of the Rayleigh wave particle motion also tilts with respect to the plane defined by radial and vertical directions.

On the other hand, we propose that Love wave vertical polarization may be a relatively clean parameter to constrain anisotropy. We define the Love wave vertical polarization by the ratio of the vertical to the transverse displacements. From (18) this is given by (to first order)

$$\epsilon = \frac{E}{A - B}\frac{U(H)}{W(H)}, \tag{20}$$

where $U(H)$ and $W(H)$ are surface values of eigenfunctions. This can be rewritten, to first order, in the form

$$\epsilon = E_1 \cos 2\theta + E_2 \sin 2\theta + E_3 \cos 4\theta + E_4 \sin 4\theta, \tag{21}$$

where

$$E_i = \frac{U(H)}{W(H)} \frac{\int_{-\infty}^{H} M_i \, dz}{\int_{-\infty}^{H} (R_0 - L_0) \, dz} \tag{22}$$

for $i = 1$–4. Here, we approximated $A - B$ by their first terms R_0 and L_0.

Non-zero values for ϵ occurs even in an isotropic medium if heterogeneity near the station is strong. However, such an effect should be isotropic and does not contain azimuthal dependence. On the other hand, anisotropic effects should show azimuthal dependence of 2θ or 4θ as eq. (21) indicates. Therefore, we claim that, by measuring the azimuthal dependence of this quantity, we should be able to measure effects of anisotropy cleanly. There may be some practical observational difficulty in avoiding interference from body wave signals, but as long as 2θ or 4θ dependence are detected, their interpretation will be obtained relatively straightforwardly by using the formula (21).

3 COMPARISON WITH DIRECT INTEGRATION

The formulae derived in the previous section are basically based on a perturbation theory and thus the basis functions, used to describe the solution, are somewhat restricted (fundamental mode Rayleigh and Love wave eigenfunctions). Therefore, depending upon the degree of anisotropy, its validity may be suspect. In order to check this point, we made comparison of the numerical results of the formula (21) against the results obtained by direct numerical integration; in the latter case, we set up a full 6×6 anisotropic system of elastodynamic equations of motions consisting of six first-order differential equations and integrated this system starting from three independent solutions in the lowermost half-space. This type of approach is a natural extension of Gilbert & Backus (1966) who worked on separable *P-SV*- and *SH*-type problems and has been used ever since. Matrix approach for stacked homogenous layers was derived for the same problem by Crampin (1975), although our preference is for vertical integration scheme because it allows one to treat continuously varying anisotropy in the *z* direction.

We used a simple one-layer over a half-space model. The upper layer has a thickness of 30 km with density of 3000 (kg m^{-3}). It has transversely isotropic symmetry with the symmetry axis pointing in the *x*-direction. In the first example (Fig. 3a), *P*-wave velocity along the *x*-axis is 6.765 (km s^{-1}) (hereafter $\alpha_{\rm V}$) and *P*-wave velocity in the perpendicular plane ($\alpha_{\rm H}$) is 6.435 (km s^{-1}). *S*-wave velocity along the *x*-axis ($\beta_{\rm V}$) is 3.9975 (km s^{-1}) and *S*-wave velocity in the perpendicular plane ($\beta_{\rm H}$) is 3.8025 (km s^{-1}). Both *P*-wave velocity and *S*-wave velocity have 5 per cent anisotropy. In the lower half-space, we assumed an isotropic medium with density 3300 (kg m^{-3}), *P*-wave velocity 8.0 (km s^{-1}) and *S*-wave velocity 4.6 (km s^{-1}). In the second example, we used an isotropic *P*-wave velocity and 10 per cent anisotropy for *S*-wave velocity in the upper layer.
Direct integration of the 6×6 system was performed by starting integrations from three independent solutions in the isotropic lowermost half-space. For a given wavenumber k and an azimuth θ, three independent solutions can be written analytically (e.g. Takeuchi & Saito 1972). Integrations were performed by the fourth-order Runge–Kutta method and eigenfrequencies were determined by matching the free-surface boundary conditions at $z = H$.

In order to evaluate the formula (21), we computed the eigenfrequencies and eigenfunctions of the reference isotropic medium (U, V and W). We simply took the average of $\alpha_{\rm V}$ and $\alpha_{\rm H}$ for *P*-wave velocity and the average of $\beta_{\rm V}$ and $\beta_{\rm H}$ for *S*-wave velocity in the upper layer and computed the eigenfrequencies and eigenfunctions of Rayleigh and Love waves.

Fig. 3(a) shows the comparison between the direct integration results (solid circles) and the results by the formula (21). In addition to Love wave polarization (bottom of Fig. 3a), excellent matches of azimuthal variations of phase velocities are shown in the top two panels. Clearly, the derived formula is valid up to 5 per cent anisotropy.

Fig. 3(b) shows the case where *S*-wave velocity had 10 per cent anisotropy. The match is still quite good, although we start to see some discrepancies between our theory and the direct numerical integration. If the anisotropy exceeds much more than 10 per cent, the validity of eq. (21) may thus be suspect.

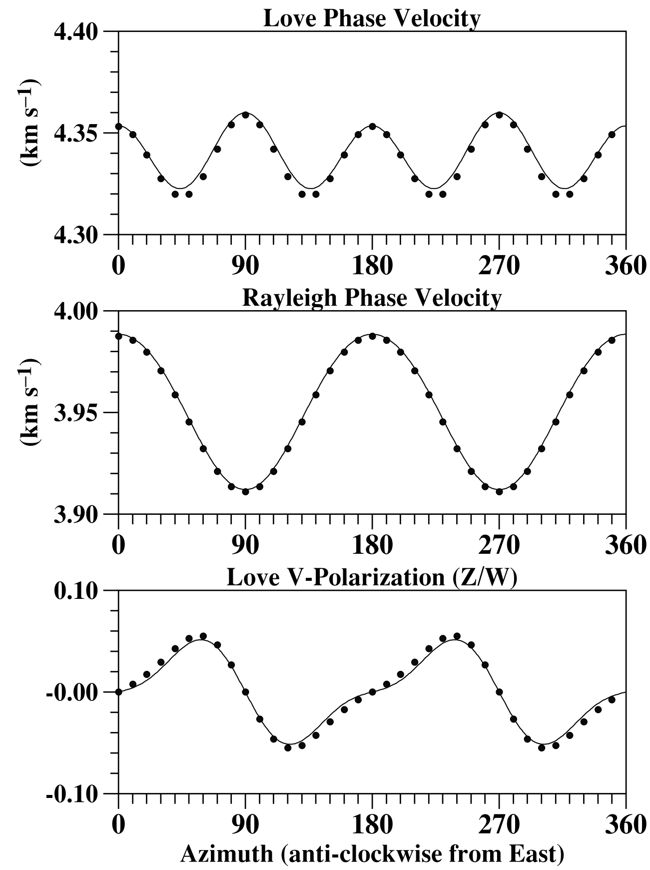

Figure 3. (a) Comparison between our analytical results and direct numerical integrations. The medium is transversely isotropic with the axis of symmetry pointing in the *x*-axis. Both *P*- and *S*-waves have anisotropic velocity differences of 5 per cent. Phase velocities of Love and Rayleigh waves are also shown in addition to Love wave vertical polarization data (bottom). (b) The same as (a) except for isotropic *P*-wave velocity and 10 per cent anisotropy for *S*-wave velocity.

Discrepancies found in Fig. 3(b) may be partly related to restricted basis functions, namely fundamental-mode Rayleigh and Love waves with the same wavenumber only. There must be contributions from higher mode surface waves and fundamental modes with different (but adjacent) wavenumbers. It is certain that a better fit for the case in Fig. 3(b) can only be achieved by incorporating such wider range of basis functions. After all, even within the realm of first-order perturbation theory, eigenfunction perturbation is given, rigorously, by the infinite number of orthogonal eigenfunctions (e.g. Schiff 1968). We have only used the leading order term by as a guess (therefore called Ansatz) which matches exact (numerical) calculations reasonably well.

4 DISCUSSION: CLOCKWISE VERSUS COUNTER-CLOCKWISE PARTICLE MOTION

The main contribution of this paper is the derivation of analytical formulae for surface wave particle motions. We specifically proposed that Love wave particle motion may provide a new constraint on anisotropic structure under a particular seismic station and derived formulae for depth inversion. We believe formulae are useful practically because, with almost 15 yr long observation at some

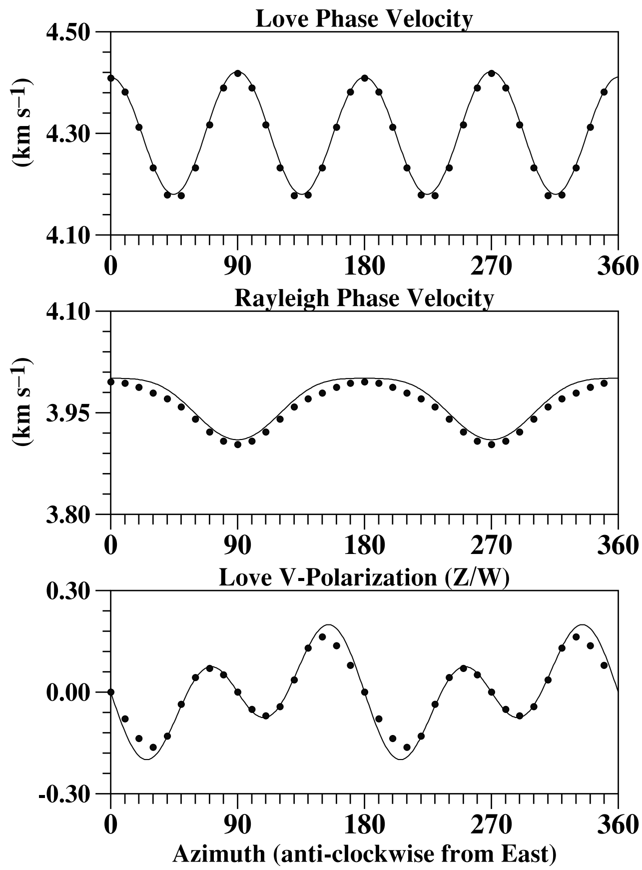

Figure 3. (*Continued.*)

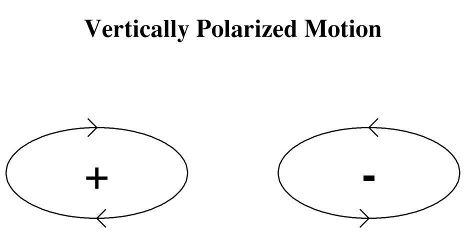

Figure 4. Switching of particle motions between clockwise and counter-clockwise motion must occur and may be easier to measure than the ratio of two axes in the ellipsoid defined by surface particle motion. The Love wave is assumed to propagate in and out of this plane.

broad-band stations, it is relatively easy to collect Love wave vertical polarization data at many stations. The key is to detect 2θ and 4θ azimuthal dependence in such measurements.

We suspect, however, that, since vertical displacement is small in Love wave signals, it may be hard to measure Love wave vertical polarization accurately and detect azimuthal dependence in this parameter. Under such circumstances, focusing on qualitative aspects of particle motions may be useful; our derived formula (21) implies that the direction of particle motions switch signs with azimuth. Depending on the sign of polarization, particle motion should be either clockwise (left in Fig. 4) or counter-clockwise (right in Fig. 4). In Fig. 4, Love wave is assumed to be propagating in and out of the plane. It is important to note that, since there is no isotropic term E_0 in eq. (21), this switching of particle motion must occur within 360°. Detection of such switching of particle motions (as a function of azimuth) may be easier than measuring the ratio of the vertical to horizontal amplitude accurately. This may be analogous to *P*-wave first motion analysis where sign of up-down motion provides critical information on nodal planes, while *P*-wave amplitudes are much harder to understand quantitatively.

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APPENDIX: FORMULAE FOR M_i, L_i AND R_i

Expressions for the coupling (M_i) are the main contribution of this paper. Expressions for the Love wave azimuthal terms L_i and the Rayleigh wave azimuthal terms R_i ($i = 0 - 4$) given below in our notations but are equivalent to Smith & Dahlen (1973). The same formulae (R_i and L_i) were also published in Montagner & Nataf (1986) and Tanimoto (1986; appendix A). Primes indicate differentiation with respect to z

$$M_0 = 0$$

$$M_1 = \tfrac{1}{2}(C_{xxxx} + C_{yyyy})k^2 W V - C_{zzxy} k U' W + C_{xzyz} W'(V' + kU)$$

$$M_2 = \tfrac{1}{4}(-C_{xxxx} + C_{yyyy})k^2 V W + \tfrac{1}{2}(C_{xxzz} - C_{yyzz})kU'W + \tfrac{1}{2}(-C_{xzyz} + C_{yzyz})(V' + kU)W'$$

$$M_3 = \tfrac{1}{2}(C_{xxxy} - C_{yyxy})k^2 V W$$

$$M_4 = \left[-\tfrac{1}{8}(C_{xxxx} + C_{yyyy}) + \tfrac{1}{4}C_{xxyy} + \tfrac{1}{2}C_{xyxy}\right] k^2 V W$$

$$L_0 = \left[\tfrac{1}{8}(C_{xxxx} + C_{yyyy}) - \tfrac{1}{4}C_{xxyy} + \tfrac{1}{2}C_{xyxy}\right] k^2 W^2 + \tfrac{1}{2}(C_{xzxz} + C_{yzyz})W'^2$$

$$L_1 = \tfrac{1}{2}(-C_{xzxz} + C_{yzyz})W'^2$$

$$L_2 = -C_{xzyz} W'^2$$

$$L_3 = \left[-\tfrac{1}{8}(C_{xxxx} + C_{yyyy}) + \tfrac{1}{4}C_{xxyy} + \tfrac{1}{2}C_{xyxy}\right] k^2 W^2$$

$$L_4 = \tfrac{1}{2}(-C_{xxxy} + C_{yyxy})k^2 W^2$$

$$R_0 = \left[\tfrac{3}{8}(C_{xxxx} + C_{yyyy}) + \tfrac{1}{4}C_{xxyy} + \tfrac{1}{2}C_{xyxy}\right] k^2 V^2 - (C_{xxzz} + C_{yyzz})kU'V + C_{zzzz}U'^2 + \tfrac{1}{2}(C_{xzxz} + C_{yzyz})(V' + kU)^2$$

$$R_1 = \tfrac{1}{2}(C_{xxxx} - C_{yyyy})k^2 V^2 + (-C_{xxzz} + C_{yyzz})kU'V + \tfrac{1}{2}(C_{xzxz} - C_{yzyz})(V' + kU)^2$$

$$R_2 = (C_{xxxx} + C_{yyyy})k^2 V^2 - 2C_{zzxy}kU'V + \tfrac{1}{2}(C_{xzxz} - C_{yzyz})(V' + kU)^2$$

$$R_3 = \left[\tfrac{1}{8}(C_{xxxx} + C_{yyyy}) - \tfrac{1}{4}C_{xxyy} - \tfrac{1}{2}C_{xyxy}\right] k^2 V^2$$

$$R_4 = \tfrac{1}{2}(C_{xxxy} - C_{yyxy})k^2 V^2.$$