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P–S converted wave: conversion point and zero-offset energy in anisotropic media

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Abstract

Here, we analyze two aspects related to converted-wave seismology: the conversion point problem and the zero-offset energy phenomenon. We show the anisotropy effect on the conversion point position and evaluate the error introduced in data processing due to neglecting anisotropy. We demonstrate that the location of the conversion point depends strongly on elastic properties of the media. The exact position of the conversion point can move relatively to the isotropic conversion point and can be located either inside or outside the vertical plane. Also, we discuss and demonstrate the possibility of recording PS zero-offset energy in horizontally layered media and how to predict some of anisotropy properties from field messurements.

Keywords: Converted wave; Conversion point; Zero-offset energy; Anisotropy

1. Introduction

Where do P–S conversions occur? This is an important question in converted wave seismology (CWS). Assuming that P–S conversion occurs predominantly in reflectors (Yuan et al., 1998; Zhu et al., 1999), the determination of the conversion point offset is very important to carry out seismic processing of converted wave. The final product, obtained after handling the data, depends on the theoretical assumptions used in the determination of these points; mainly because, rather than CMP binning used in pure-mode processing, common-conversion-point (CCP) binning techniques are usually applied instead.

The strong dependence on physical assumptions concerning rock velocities in locating offsets for conversion points (CP) has been discussed in the last few years (Tessmer and Behle, 1988; Thomsen, 1999 and others). In general, the search of CP is done under the assumption of isotropic media. In the past few years, several publications have dealt with offset determination for converted waves (C–W) in anisotropic media. Although these works are limited to transversely isotropic media with vertical axis of symmetry (VTI), they constitute an important advance in CWS (Yuan and Li, 2001).

The relevance of anisotropic effects related to C–W becomes very important when physical and processing parameters are estimated from real data. The recovery of these parameters is impossible or very imprecise when extremely simplified assumptions are made. Recently, Yang and Lawton (2002) using Thomsen's

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approach quantified the errors in the CP determination when VTI media are approximated by an isotropic one, while Artola et al. (2003) analyzed the errors in the CP determination when azimuthally anisotropic media are approached with isotropic approximation. The latter work showed that the exact CP is located outside the vertical plane (offline component of displacement of the exact CP). This offline displacement depends mainly on the anisotropic nature of the subsurface, the offset/depth ratio X/Z and source-receivers geometrical arrangement. This fact can significantly affect the spatial resolution of subsurface seismic images.

Another aspect in the C–WS context is whether the PS zero-offset energy (ZOE) is null or not. The simple theory for horizontal isotropic reflections predicts null P–S reflectivity for vertical incidence. However, non-zero P–S zero-offset energy (ZOE) can be frequently observed.

Due to popularization of VTI, HTI and orthorhombic (no rotation around any horizontal axis) symmetry in applied seismology, several reflectivity studies adopted these symmetries. All of these symmetries provide a zero PS-ZOE. However, if we consider non-vertical or non-horizontal anisotropy axes, it is possible to justify non-zero ZOE.

In the upper crust, fractures are considered the main cause of the azimuthally anisotropic features of seismic data. Although it is often believed that fractures in the earth have predominantly vertical direction, there is some geological and geophysical data indicating that certain mechanisms, such a strong oblique stresses, might cause the presence of dipping fractures in the subsurface (Grechka and Tsvankin, 2001). When dipping fractures or cracks are affecting any isotropic background, they generate a special type of reflectivity response. In this context, non-zero PS reflectivity for normal incidence can be expected for horizontal and plane reflectors. In fact, the frequent observation of P-S ZOE can be predicted by the theory. This fact can be easily observed by analyzing approximated formulations for reflection coefficients in arbitrary anisotropic media.

2. PS conversion point in anisotropic media

The PS-mode behavior in anisotropic media is rather complex. Jílek (2001) has shown some of the inherent complexities of the propagation of PS waves. In general, for non-isotropic and non-VTI media, it can be demonstrated that the conversion point is located outside the vertical plane, even in homogeneous media, since the Snell's law cannot be satisfied in that vertical plane (Fig. 1). The distance between the conversion point and the vertical plane depends mainly on the anisotropic nature of the subsurface, the offset/depth ratio X/Z and source-receivers geometrical arrangement.

Often, the determination of the conversion point is tackled by considering the rock formation as isotropic. This simplified approach may lead to errors due to neglecting anisotropy. Even for the simplest anisotropic case, as VTI media (Fig. 2), the discrepancy between the true position of the conversion point, X_c, and the approximated one, X_c^{ISO}, can be in the order of hundreds of meters, even for an offset/depth ratio, (X/Z), less than 2. On the other hand, in some special VTI cases, the position of the conversion point practically coincides with that in the isotropic case, even when ε and $\delta \neq 0$. This means that besides the dependence on (X/Z) and on vertical Vp/Vs ratio, the discrepancy, or consonance, between conversion points of isotropic and anisotropic media depends on the values assumed by the anisotropic parameters ε and δ. Usually, in more general cases of azimuthal anisotropy, as in the case of orthorhombic symmetry, the conversion point is no longer located in the vertical plane defined by source/receivers, i.e., the

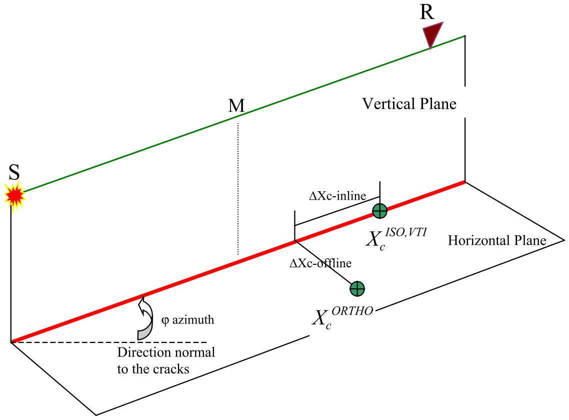

Fig. 1. Conversion point in isotropic and azimuthally anisotropic media.

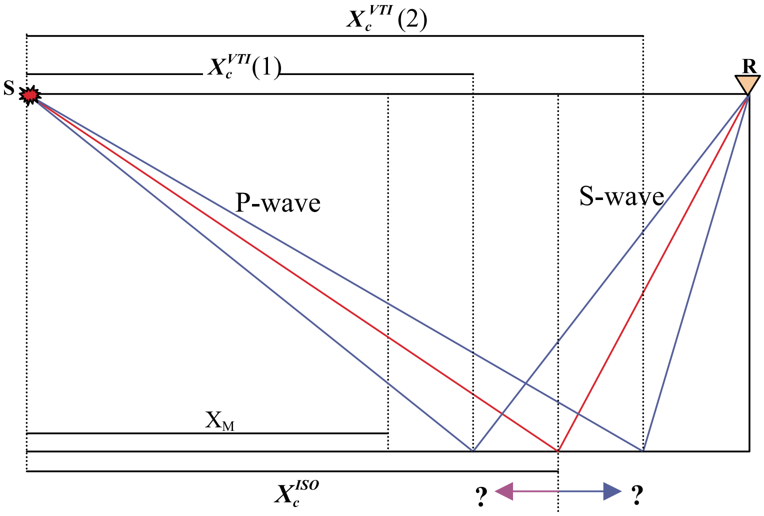

Fig. 2. Conversion point in isotropic and VTI media.

discrepancy of the conversion point has inline and offline components.

A VTI model was built starting from an isotropic model using Thomsen (1986) method. An orthorhombic model was built from VTI model introducing vertical cracks in the medium. The compliance tensor associated to this orthorhombic model can be determined with the aid of the *linear slip* crack model, according to Schoenberg and Helbig (1997) and Schoenberg and Douma (1988) or with *penny-shaped* crack model according to Hudson (1981).

2.1. VTI model

The starting model consists of a single 1-km thick isotropic homogeneous layer: Vp = 3 km/s, Vs = 1.5 km/s and $\rho = 2$ g/cm^3. Making Vp and Vs as Vp$_0$ and Vs$_0$, vertical velocities in a VTI model, one can determine a set of stiffness tensors for several combinations of ε, δ and γ (Thomsen's parameters). The ray tracing code ANRAY 4.20 (Pšenčík, 1998) was used to determine the conversion points. The source and receivers arrangement is composed by 40 inline receivers equally spaced by 50 m, resulting in a minimum offset of 50 m and in a maximum offset of 2 km. For this special case of symmetry, the conversion point is located in the vertical plane and the particle displacement of the converted wave P-SV is polarized in the plane of incidence. As a result, the discrepancy between X_c^{VTI} and X_c^{ISO} is a distance measured in the vertical (inline) plane. Hence this discrepancy can be calculated subtracting the inline coordinate of X_c^{ISO} from the inline coordinate of X_c^{VTI} ($\Delta X_c = X_c^{\mathrm{VTI}} - X_c^{\mathrm{ISO}}$).

The discrepancy between X_c^{VTI} and X_c^{ISO} for several values of ε and δ is plotted in Fig. 3a–d. From the figures, it can be seen that the discrepancy depends on values associated to the Thomsen parameters as well as on the ratio offset/depth. The value and direction of the conversion point displacement depend on the relation between ε and δ. For a fixed value of ε, a variation in the value of δ can result in X_c^{VTI} closer to the source or closer to the receiver, when compared to X_c^{ISO}. Also, certain combinations of values of ε and δ result in X_c^{VTI} very close to X_c^{ISO}. It can be seen in Fig. 3a and b that for the pairs $(\varepsilon, \delta) \rightarrow (0.1, 0.075)$; $(0.15, 0.12)$, X_c^{VTI} is very close to X_c^{ISO}. It might be that the good results obtained in converted-wave processing, assuming isotropy, may be associated to this fact.

2.2. Orthorhombic model

In order to evaluate the effect of anisotropy in conversion point location for more realistic cases, a new model was built introducing in the VTI model, a vertical crack system with crack density $\eta = 0.1$. Due

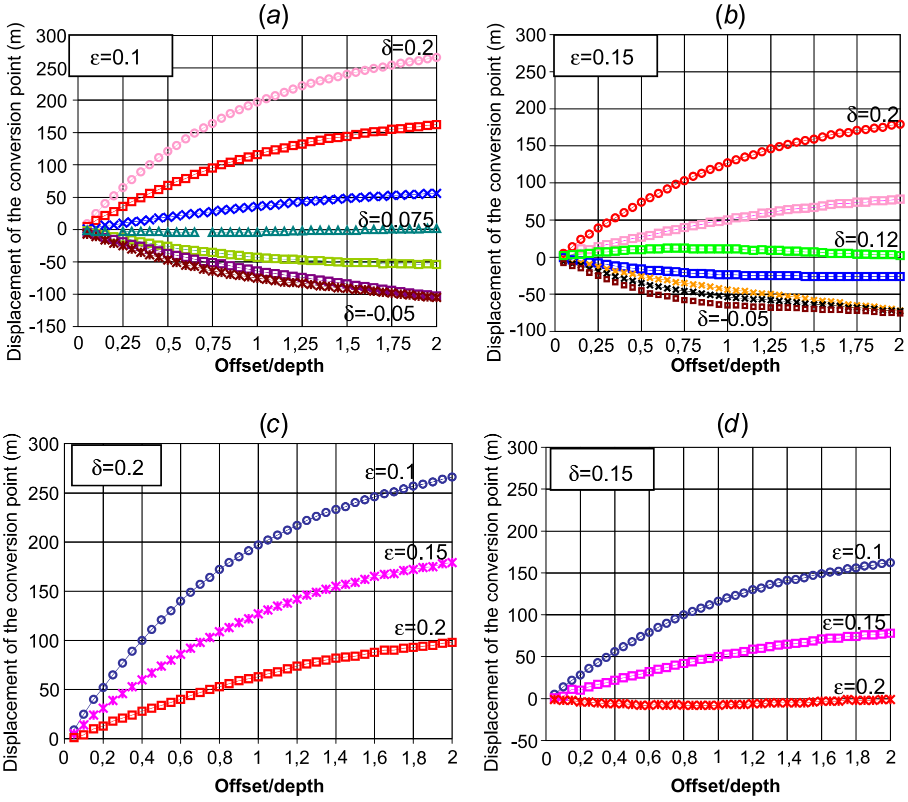

Fig. 3. VTI model. (a) and (b): Displacement of the conversion point in comparison to the isotropic case. Model with $\varepsilon = 0.1$, 0.15 fixed and δ variable. (c) and (d): $\delta = 0.2$, 0.15 fixed and ε variable.

to the presence of the fractures, the medium became orthorhombic (i.e., it is no longer azimuthally invariant). The polarization vectors of converted qS_1 and qS_2 waves are locally rotated by an angle Φ when compared to the correspondent converted SV and SH waves. In addition, because Snell's law cannot be satisfied in the vertical plane, the conversion point is located outside vertical plane. In consequence, X_c^{ORTHO} is displaced from X_c^{VTI} and X_c^{ISO} by inline and offline components. The offline component depends on the perturbation characteristics introduced to azimuthally invariant models. Fig. 4 shows the azimuthal variation of the conversion point coordinates associated to qS_1 and qS_2 for a fixed offset of 2 km and ratio offset/depth = 2. The conversion point coordinates for P–SV-mode in correspondent VTI and isotropic cases are also plotted. The inline and offline displacement components between X_c^{ORTHO} and X_c^{ISO} are shown in the Fig. 5. It can be seen that for certain

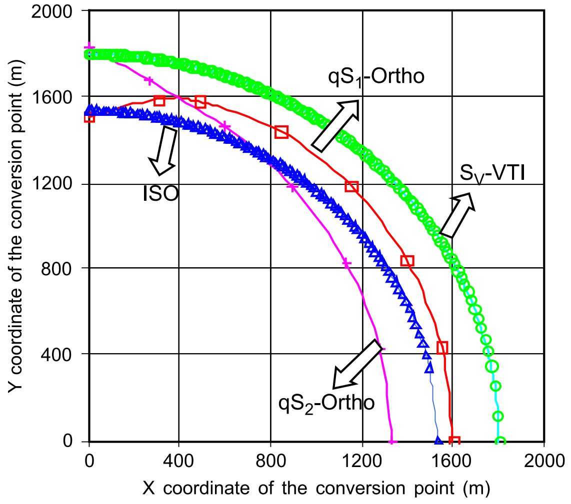

Fig. 4. Multiazimuthal conversion point for orthorhombic, VTI and isotropic cases.

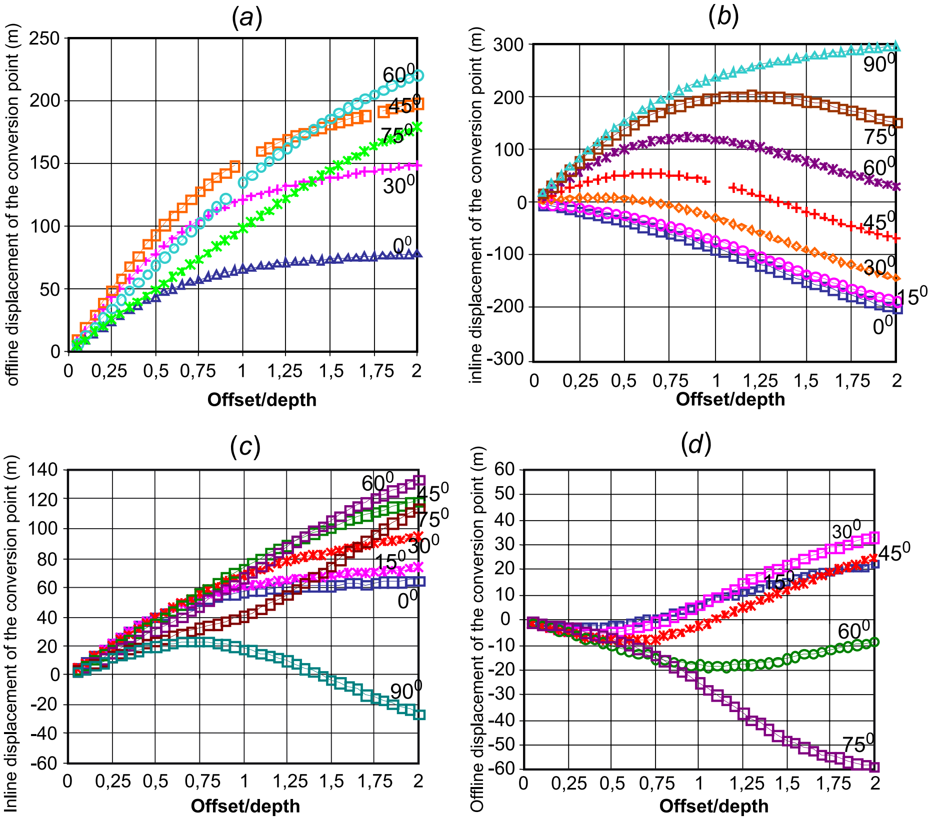

Fig. 5. Offline and inline displacements of the anisotropic conversion point for qS_2 and qS_1 components from isotropic case. The curves correspond to several values of azimuth with respect to the direction normal to the cracks.

values of azimuth, the offline component for qS_2 wave can be of the order of hundreds of meters.

3. Reflectivity for zero-offset in anisotropic media

In this section, we demonstrate that P–S converted wave energy for zero-offset in horizontally layered media scenarios is not necessarily null and depends on elastic properties of the subsurface.

Mueller (1991), among others, evidenced with field data the presence of significant energy for zero-offset/near-offset associated with the reflection of the converted wave and later on Thomsen (2002) pointed out its relevance.

One possible explanation of this observation is the presence of anisotropy and the fact that the reflectivity in anisotropic media is much more complex than in isotropic media. In isotropic media, the propagation directions and consequently the displacement vectors of the reflected and transmitted waves resulting from an oblique P wave incidence can be determined directly applying the Snell's law. Transmitted and reflected converted modes are generated to satisfy the displacement and stress continuity at the interface. As for the normal P wave incidence only displacement in the normal (z) direction is generated, there is no converted transmitted or reflected wave for such incidence. On the other hand, in some anisotropic media, the particle displacement vector of a qP wave propagating in the vertical (z) direction does not point out toward that direction. This occurs in media with oblique axis or planes of symmetry. So, in order to satisfy the boundary conditions at the interface, it is possible that converted transmitted and reflected waves are generated for normal P or qP wave incidence.

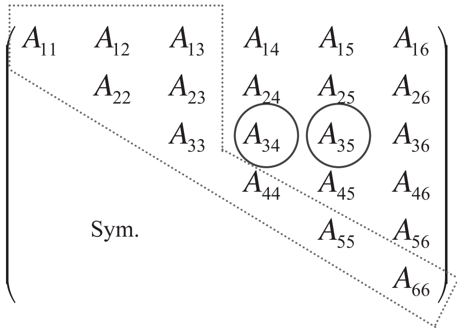

Fig. 6. Matrix of elastic coefficients.

Because of the increasing interest in the converted wave seismology, Zillmer et al. (1997), Vavryčuk (1999) and Jílek (2001) have proposed approximated formulations to evaluate such reflection coefficients which, in spite of their limitations, can be still applied to specific problems. From the analysis of these formulations, it can be shown that such reflectivity depends on the contrast of the stiffness components A_{34} and A_{35} of the two media. Using the general approach of Jílek (2001) for PS-wave reflection coefficients, it may be demonstrated that the P–S reflection coefficient for vertical incidence can be expressed as:

$$R_{c1} = K[(\Delta A_{35}\cos\psi + \Delta A_{34}\sin\psi)\cos\Phi - (\Delta A_{34}\cos\psi - \Delta A_{35}\sin\psi)\sin\Phi] \tag{1}$$

$$R_{c2} = K[(\Delta A_{35}\cos\psi + \Delta A_{34}\sin\psi)\sin\Phi + (\Delta A_{35}\cos\psi - \Delta A_{34}\sin\psi)\cos\Phi] \tag{2}$$

where K is a constant related to background isotropic media, Φ is the polarization angle, ψ is the azimuth of the incident plane and $\Delta A_{ij} = A_{ij}^{(2)} - A_{ij}^{(1)}$. The superscripts (1) and (2) stand for upper and lower medium, respectively.

From Eqs. (1) and (2), it can be noticed that for $\Delta A_{34} \neq 0$ and/or $\Delta A_{35} \neq 0$, R_{c1} and R_{c2} are non-zero and for $\Delta A_{34} = 0$ and $\Delta A_{35} = 0$, R_{c1} and R_{c2} are null. The consequence is that R_{ci} ($i = 1,2$) for zero-offset can be either zero or non-zero, depending on the contrast of the elastic parameters associated to the media.

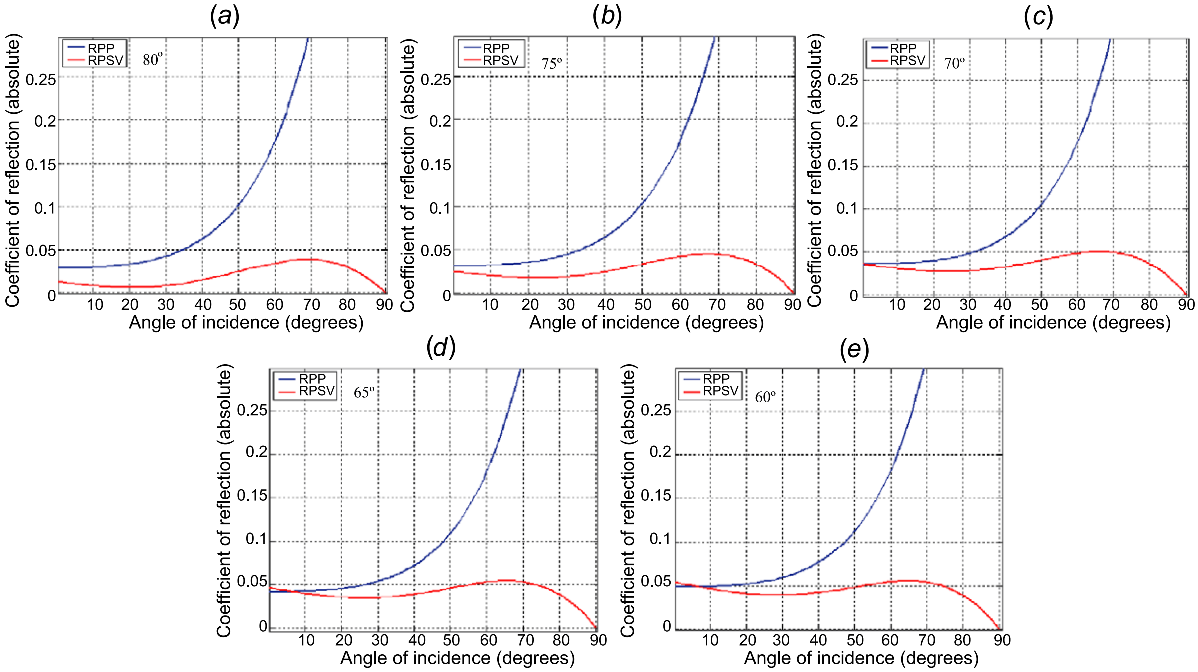

Fig. 7. P and PS reflection coefficients: Upper half-space is isotropic. Lower half-space is transversally isotropic media with inclined axis of symmetry. The anisotropy is induced by cracks with 80°, 75°, 70°, 65° and 60° of inclination. The interface is horizontal and plane.

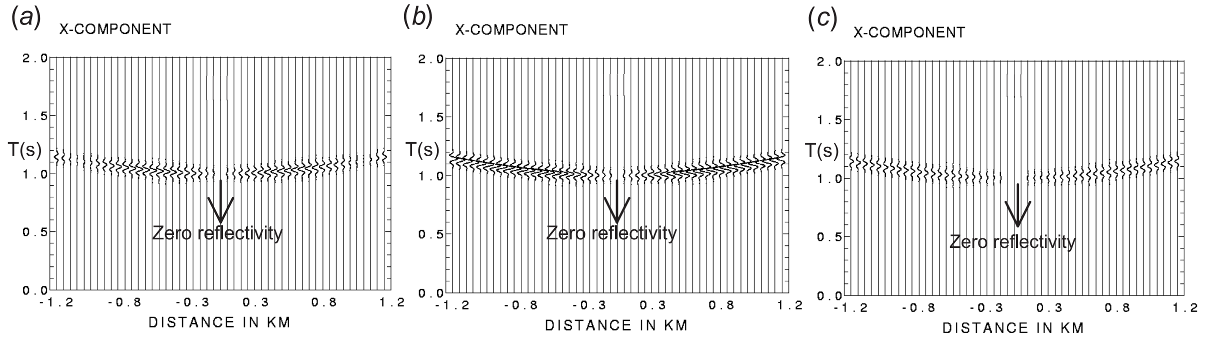

Fig. 8. Synthetic seismograms indicating null energy for PS mode at zero-offset. (a) ISO/HTI model. (b) ISO/ORTHO model. (c) ISO/VTI model.

3.1. In which cases is R_{ci} non-zero at zero-offset?

A general elastic parameter matrix is shown in Fig. 6. For isotropic media, the elements placed outside the region delimited by the red dashed line are always zero. They are also zero for VTI, ORTHO (orthorhombic) and HTI media, provided such media are referred to a coordinate system whose axes coincide with the principal axes of symmetry. It is also observed that the elements A_{34} and A_{35} of the elastic matrix are still zero for rotations around the vertical axis. It results that the vertically propagating P-wave in these media is purely longitudinal. As a result, R_{ci} for zero-offset is always zero.

For ITI (inclined transversally isotropy) and MONO (monoclinic symmetry, VTI + oblique fractures) media or for some other media of lower symmetry, A_{34} and/or A_{35} are non-zero elements. So, according to Eqs. (1) and (2), for those cases,

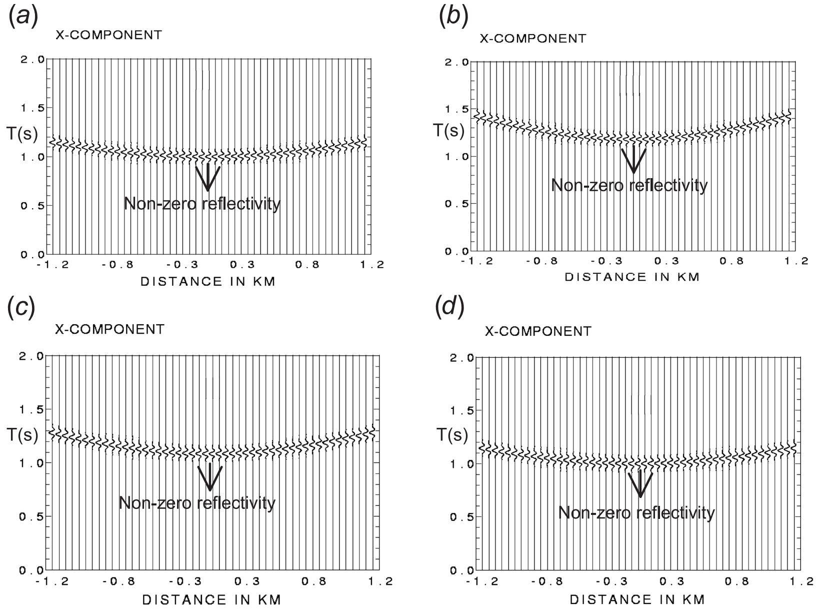

Fig. 9. Synthetic seismograms indicating non-zero energy for PS mode at zero-offset. (a) ISO/ITI model. (b) ORTHO/ITI model. (c) HTI/MONO model, and (d) ISO/MONO.

R_{ci} for zero-offset can be non-zero. Notice that this is still valid even when one of the two media is isotropic.

4. Numeric results using the exact formulation

The ideas discussed above were confirmed by numeric results obtained from exact formulation (Fig. 7). The models consist of an isotopic half-space into which incidence and reflection take place over ITI half-spaces with several values of inclination of symmetry axes.

The ITI half-space was built perturbing the isotropic background with oblique cracks. Each figure represents the PP and PS reflection coefficient as a function of incidence angle for cases where cracks with 80°, 75°, 70°, 65° and 60° of inclination are affecting an isotropic medium. One can observe that PS reflection coefficient values are not zero for normal incidence, and this coefficient value is becoming higher when diminishes the inclination of the cracks. It was also observed that PS reflection coefficient with the same or even higher values than PP mode for vertical incidence.

4.1. Synthetic seismograms

Figs. 8a–c and 9a–d shows synthetic seismograms obtained with the use of the ANRAY 4.2 ray-tracing package for seven different models. Table 1 summarizes the model configurations and the scenarios for zero and non-zero zero-offset converted wave reflectivity. From this table it can be pointed out that it may occur even when one of the two media, the incident one for instance, is isotropic.

Table 1
Configuration of the models

Figure	Upper half-space	Lower half-space	Zero-offset energy
(a)	ISO	HTI	no
(b)	ISO	ORTHO	no
(c)	ISO	VTI	no
(d)	ISO	ITI	yes
(e)	ORTHO	ITI	yes
(f)	HTI	MONO	yes
(g)	ISO	MONO	yes

5. Conclusions

Considering non-VTI anisotropic models, the conversion point is not placed in the vertical plane defined by the inline arrange of source–receivers. Conversion points determined for VTI models have displacements in the vertical plane. For orthorhombic models, conversion points have displacements composed by inline and offline components compared to the isotropic and VTI cases. It is important to note that there are combinations of Thomsen's parameters ε, δ, in VTI cases, for which X_c^{VTI} is very close to X_c^{ISO}. For more realistic cases (e.g., orthorhombic models), the offline component of the displacement in the location of conversion points depends mainly on the anisotropic properties of the subsurface, the offset/depth ratio X/Z and source–receivers geometrical configuration. The mislocation of the conversion points due to neglecting anisotropy can seriously affect the spatial resolution of subsurface seismic images when simplified assumptions are made.

Considering anisotropy, it is possible to predict PS non-zero reflectivity for zero-offset in horizontally layered media. An approximated formulation was used to give a better understanding of elastic parameters that control the PS conversion phenomenon. The conclusions drawn are confirmed by calculating synthetic seismograms using the ANRAY packed. We conclude that the contrasts of the elastic parameters, A_{34} and A_{35} of the two media, are the dominant factors that determine the PS zero-offset energy. As a consequence, if the incident and the transmitted media are isotropic, VTI, HTI or ORTO, $R_{ci}=0$ for zero-offset. For cases where at least one of the two media is ITI, MONO or lower symmetry, R_{ci} is in general non-zero.

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