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Great explosive eruptions on Kamchatka during the last 10,000 years: Self-similar irregularity of the output of volcanic products

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Temporal irregularity of the output of volcanic material is studied for the sequence of large (V≥0.5 km^3, N=29) explosive eruptions on Kamchatka during the last 10,000 years. Informally, volcanic productivity looks episodic, and dates of eruptions cluster. To investigate the probable self-similar clustering behavior of eruption times we determine correlation dimension D_c. For intervals between events 800 - 10000 yr, D_c ≈1(no self-similar clustering). However, for shorter delays, D_c = 0.71, and the significance level for the hypothesis D_c <1 is 2.5%. For the temporal structure of the output of volcanic products (i.e. for the sequence of variable-weight points), a self-similar "episodic" behavior holds over the entire range of delays 100-10000 yr, with D_c=0.67 (D_c <1 at 3.4% significance). This behavior is produced partly by the mentioned common clustering of event dates, and partly by another specific property of the event sequence, that we call "order clustering". This kind of clustering is a property of a time-ordered list of eruptions, and is manifested as the tendency of the largest eruptions (as opposed to smaller ones) to be close neighbors in this list. Another statistical technique, of "rescaled range" (R/S), confirms these results. Similar but weaker-expressed behavior was also found for two other data sets: historical Kamchatka eruptions and acid layers in Greenland ice column. The episodic multiscaled mode of the output of volcanic material may be a characteristic property of a sequence of eruptions in an island arc, with important consequences for climate forcing by volcanic aerosol, and volcanic hazard. **Index Terms:** 3250 Mathematical Geophysics: Fractals and multifractals; 8414 Volcanology: Eruption mechanisms; 8434 Volcanology: Magma migration. **Keywords:** explosive eruption, Kamchatka, volcanic episodes, fractal, temporal clustering, significance.

Introduction

The mode of evolution of volcanic process is a major question important both for general understanding of volcanic phenomena and also for applications regarding the impact of volcanoes on climate and human habitat. Over the geological time scale, the temporal character of volcanism has been clearly demonstrated to be non-uniform, *episodic* for such processes as ocean ridge volcanism, hot spot volcanism, explosive volcanism in island arcs (Sigurdsson, 2000), and trap (areal basalt) volcanism (Makarenko 1982). With respect to island arcs, this episodic style was established, on the basis of the study of volcanic ash layers in ocean-bottom boreholes, both for particular island arcs and on a global scale (Kennet et al. 1977; Rea and Scheidegger 1979: Cambray and Cadet, 1996; Prueher and Rea, 2001). However, these studies analyzed episodicity only in qualitative terms; no formal description for the episodic temporal structure of these volcanic processes was proposed. A description of such a kind was recently suggested by Pelletier (1999) who studied mostly the spatial and spatio-temporal structure of intrusions over geological time. He found that the actual episodicity in formation of intrusions is far from being completely "wild": when treated as random objects, intrusions are distributed in time (moreover, in space-time) in a statistically self-similar (fractal) manner. This mode of behavior means, first, that episodes or bursts of various possible durations or temporal scales are present within data (a multi-scaled behavior); therefore there is no preferred duration of an episode or a burst, and no preferred repeat period between bursts. Secondly, the behavior is qualitatively (and quantitatively) similar for the time segments of different duration (say, 10 kyr or 10 Myr). The results of Pelletier confirm the general idea of an episodic behavior, and suggest that the episodicity of volcanic process may be generally (or typically) self-similar. For historic time scales, indications of a statistically self-similar or fractal behavior was found by DuBois and Cheminee (1991) for eruptions of basaltic volcanoes; and by Godano and Civetta (1996) for Etna; similar style was deduced by Chouet and Shaw (1991) for the burst-like behavior of a developing eruption.

On the other side, many studies (e.g. Wickman, 1966; Ho et al., 1991; Jones et al., 1999) either assume or try to prove that eruptions of a particular volcanic center or of an area behave in another way: purely randomly (as a Poisson process) or with periodic tendency, and not as episodes. Note that in terms of the number of events, episodes must reveal itself as clusters of representing points on a time axis. We are interested here in the properties of entire volcanic areas, rather than of individual centers. In this respect, we must mention the study of De la Cruz-Reina (1991) who analyzed the global summary eruption data for the last five centuries. He found that the distribution of numbers of events within decade periods visually reminds the Poisson distribution, thus supposedly supporting the Poisson process idea. However, his most characteristic data (their Fig 1, cases VEI≥4 and VEI=4) show very clearly that both unusually overpopulated decades and unusually underpopulated decades are more abundant than would be expected for the Poisson case. It is just this kind of behavior that can be expected when events come in clusters rather than in a purely random ("Poissonian") manner. Of course, this deviation may be partly related to the underreporting of data for first 3 centuries of the study period. However a visual inspection of the data set used by De la Cruz-Reina reveals evident temporal clusters or episodes, e.g. for the following time periods: 1760-1800, 1810-1819 (with Tambora), 1870-1889(with Krakatoa) , and 1900-1919 (with Katmai). Thus, this data set actually supports the idea of an episodic behavior, rather than contradicts it.

The episodes of explosive arc volcanism can have a profound impact on society, directly or through their potential for global environmental change. However, the episodic style of activity is more or less established at present only for long time scales, between 2 and 100 Myr. The study of shorter scales, comparable to a human time scale, are evidently of high importance. In this line, we plan to analyze in the present study the expression of an episodic/clustered behavior in the catalog of large Holocene explosive eruptions of Kamchatka Peninsula for the last 10000 yrs. The catalog is assumedly complete for events with the volume of products $V \geq 0.7\text{-}1$ km^3 and is unique in terms of reliability, area and time coverage. It should be noted that for Kamchatka, as well as for other subduction-related volcanic zones, the volcanism is predominantly explosive (Melekestsev 1980, Sigurdsson 2000). When analyzing the preliminary version of the catalog, Braitseva et al. (1995) noticed two prominent episodes of enhanced production rate of volcanic material; on a time axis they are seen as dense groups or clusters. These two bursts of activity were first-class catastrophes that converted a large fraction of the territory of Kamchatka into a desert and, as witnessed by archeological data, pressed the native population out, to the north, for many centuries. The question arises: was the appearance of these two bursts a matter of a pure chance, or a manifestation of some intrinsic property of volcanic process. The publications cited above suggest that the second explanation is more probable, and the present study confirms this guess.

Limited but relatively high-quality data for Kamchatka Holocene eruptions permit us to study formally the long-term temporal structure of the output of explosive volcanic products over a broad volcanic area. In the following we study statistical properties of event distribution over time axis (when all events are treated equally, as if they have similar, equal weights) , and also the properties of the product output proper (when event sizes are properly respected during the data analysis). As an useful preliminary step, we also analyze the size distribution of events.

With our limited data volume, we need special efforts to prove the reality of our conclusions. For this reason, to demonstrate the presence of the self-similar/fractal behavior, we apply in parallel two very different statistical techniques: "correlation integral technique", and "rescaled range analysis". Formally, the coincidence of conclusions derived by two techniques does not prove much, because the results are predictably correlated; but in the real life, two successful tests still make the result more convincing.

After we have shown the presence of self-similar clustering (that is, episodicity) in Holocene Kamchatka data, we wish to understand whether the result is specific for this data set or more general. Hence we analyze two more catalogs: one containing historic Kamchatka events, and another that consists of acid peaks in a column of Greenland ice. The results mostly support the idea that the episodic/clustering behavior is typical.

The general approach

Three different ways to produce irregularity of volcanic output. Definition of irregularity

Eruptions of volcanoes are usually episodes of activity divided by quiet periods. Thus the history of eruptions, and of explosive eruptions in particular, may be represented approximately as the set of points on the time axis whose positions represent some characteristic moment of each eruption, like its the paroxysmal stage. For prehistoric eruptions whose dates are determined by, say, radiocarbon method, this representation is quite natural, and we will follow this simplified approach.

Among statistical properties of the history, that is, catalog, of eruptions, one can study the following aspects in a separate manner: (1) statistical distribution of sizes of eruptions (ignoring their times or order); (2) temporal structure of eruption times (ignoring sizes); and at last (3) temporal structure of volcanic output proper. The third aspect is somewhat complicated because it partly incorporates the first two. As we found in the progress of the present investigation, a specific object of study can also be isolated, independent of the first two, namely (3a) the properties of the time-ordered list of sizes. Why all these properties deserve any study at all? First, we merely wish to describe the actual behavior of a natural volcanic process; second, we can shed some light on the problem of irregularity/episodicity of volcanic output, which has more general geological importance. All three mentioned aspects of the eruption catalog are important in this respect.

Now let us consider the phenomenon of irregularity. The output of volcanic products from the Earth's interior to its surface is evidently irregular: it takes place as short bursts named eruptions, these bursts are highly variable in size, and the time intervals between them look random. These facts regarding the short-term irregular behavior are trivial. One might expect however that when sufficiently long intervals of time are considered, this irregularity smoothes out, and such a notion as the rate of accumulation of volcanic material can be meaningfully introduced. Indeed, one of the most important geological effects of eruptions is just this accumulation, and its rates are common numerical parameters used in discussing paleovolcanism. Empirically, of course, there is no problem: you merely divide the product volume by the duration it took to accumulate. However, does this number have any clear mathematical meaning? For example, can one *prove* that there is any *intrinsic*, significant difference between two volcanic areas if one of them has, for a given time interval, five times larger empirical production rate than another? Note that we are very cautious in specifying a volcano "extinct," just because we intuitively qualify its zero production rate during the last, say, 3000 years as not very significant! Therefore, the phenomenon of irregularity deserves attention.

We shall briefly discuss now how the three listed aspects of the eruption catalog are related to irregularity. We start from the hypothetic case when the effect of temporal structure is absent, and the rate of eruptions (in events per unit time) is constant. Then theory (Mandelbrot 1982) says that the variation among sizes may have rather different effect on the observed long-term average output rate. One characteristic case is when the eruptions are of variable, random, even unlimited, size but extremely large eruptions are very rare. Formally, the statistical distribution of eruptions sizes must have finite variance (this holds, e.g., for the normal, log-normal and exponential distribution laws). Then the long-term or smoothed behavior of the process is nearly uniform, with well-defined mean rate of the volcanic product output. In another (and realistic) case, eruptions sizes vary «wildly»: the size distribution lacks variance and even mean value; this case is often called «heavy-tailed» distribution. We shall see soon that the size-frequency law for eruptions is near to the power law with the exponent about 0.75. In this case, mathematical expectation or mean for the sum of volumes does not exist, and we have no hope to find any meaningful estimate of mean output rate. Formally, both (mean and mean rate) are infinite. On the empirical side, this will mean that observed empirical average output rates will have the following unpleasant properties: (1) they oscillate wildly; (2) estimates over different intervals will not match one another; and (3) they will, on the average, depend on the size of the observation interval (the longer is the interval, the greater is the empirical rate). This kind of long-term behavior is not related to discreteness of the process, and may hold even for continuous processes. We believe that the term «irregularity» should be reserved for this specific kind of behavior, and we will further follow this restricted use.

Now imagine the unrealistic case when eruption sizes are distributed with finite variance, or even that all eruptions are similar in size. Does this guarantee us against irregularity? The answer is no. There exists another, independent source of irregularity, and it is the episodic/clustering style of the temporal structure of the process. In the simplified case of equal-sized events, we can meet irregularity (i.e. instability of long-term averages) of the second kind, related to the self-similar, multi-scale clustering of event dates already discussed in Introduction (not any clustering will have such consequences).

The two listed possible causes of the irregular self-similar behavior are now well-known (Mandelbrot 1982 and many later publications). In the progress of analysis of Kamchatka Holocene data we revealed one more source of irregularity, that may occur only for a sequence of variable-weight events. Irregularity may be caused by selective, specific grouping of large-volume events. Note that we emphasize here the *size-dependent differences* of a clustering behavior, as different from the already discussed common clustering, which is insensitive to event size. To exclude the contribution of common clustering, it is convenient to reject completely any information on accurate event dates, and keep only the temporal *order* of events of different sizes. Thus we consider now a time-ordered list of eruptions with dates omitted, so that the common clustering, that is, the increased amount of relatively short time intervals between events, cannot manifest itself at all. In such a list, however, a large-size event may have an unusually high probability to have large-sized neighbors, and this tendency can also result in irregularity. We shall use the term «order clustering» for this specific property of a variable-weight point distribution. We reiterate that the sequence of event dates may be clustered, "completely random" (Poissonian) or even periodic: «order clustering» may manifest itself in any case, and it can result in a specific kind of irregularity. Note that order clustering can appear even with common, «light-tailed» size distributions, and is therefore able to generate irregularity even when both its other causes (heavy-tailed size distribution and clustering of dates) are absent. In reality, however, common and order clustering are combined, and special efforts are needed to separate effects of each.

This combined manner of manifestation of common and order clustering makes it difficult to perceive clearly the difference between the two concepts. A reader might resort to the following common-life example. Imagine a long old-style highway crossing cities, towns, villages and open country. Biggest buildings form very tight clusters (city centers); multi-storey houses (in cities and towns) also cluster, but less tightly; and one-storey buildings are most spread (over cities, towns and villages) but still qualify as clustered when looked at against an open country. Now consider houses as unit points specified by their position in miles from the beginning of the road. These points will evidently form clusters (called cities, towns, etc). Oppositely, consider a plain list of volumes of houses ordered along the road, with actual distance information omitted. In this list, large volumes will be close neighbors much more often than in a similar list with a random order, thus showing order clustering. (To realize a list with random order, imagine the initial distance-ordered list to be written on cards that are afterwards shuffled.) Note that in this example, expressed clusters of both kinds are concentrated at similar positions, namely, cities. In other words, the clustering behavior of the two described kinds is "correlated"; this is an important property of the hypothetic situation. One can imagine (or simulate) other modes of non-purely-random behavior (like negative order clustering: regular villages and isolated big buildings in an open country).

Generally, one can treat a distribution of product volumes over an interval as an example (or, rather, manifestation) of a generalized point density function called "random measure". When such measures show multi-scaled mode of behavior (like houses in our example) they are called multifractals. In a number of studies, multifractal properties of geophysical point distributions were analyzed (e.g. Hirabayashi a.o.1992). This approach might be particularly interesting in our case, with "variable-weight" points. Unfortunately, with our small data set we are unable to determine reliably the set of fractal dimension parameters that consistently define a multifractal; below we limit ourselves by only one multifractal parameter, correlation dimension. Both common and order clustering may contribute to multifractal behavior, separately or jointly. However, when we look at our point process from the multifractal viewpoint, we are concerned with resulting mass concentration over each scale, and less interested in what way this concentration has been formed. On the other side, from the volcanological viewpoint, the two modes of clustering are logically different, and it is interesting to study them separately. Therefore, below we will apply statistical tests that measure both the joint effect of the two temporal properties, and also each effect individually. To clarify matters it should be noted that, when events with a power-law size distribution come without a multi-scaled temporal structure (e.g. periodically or as a Poisson process), volume distribution over time axis is *not fractal;* still, it is *self-similar*.

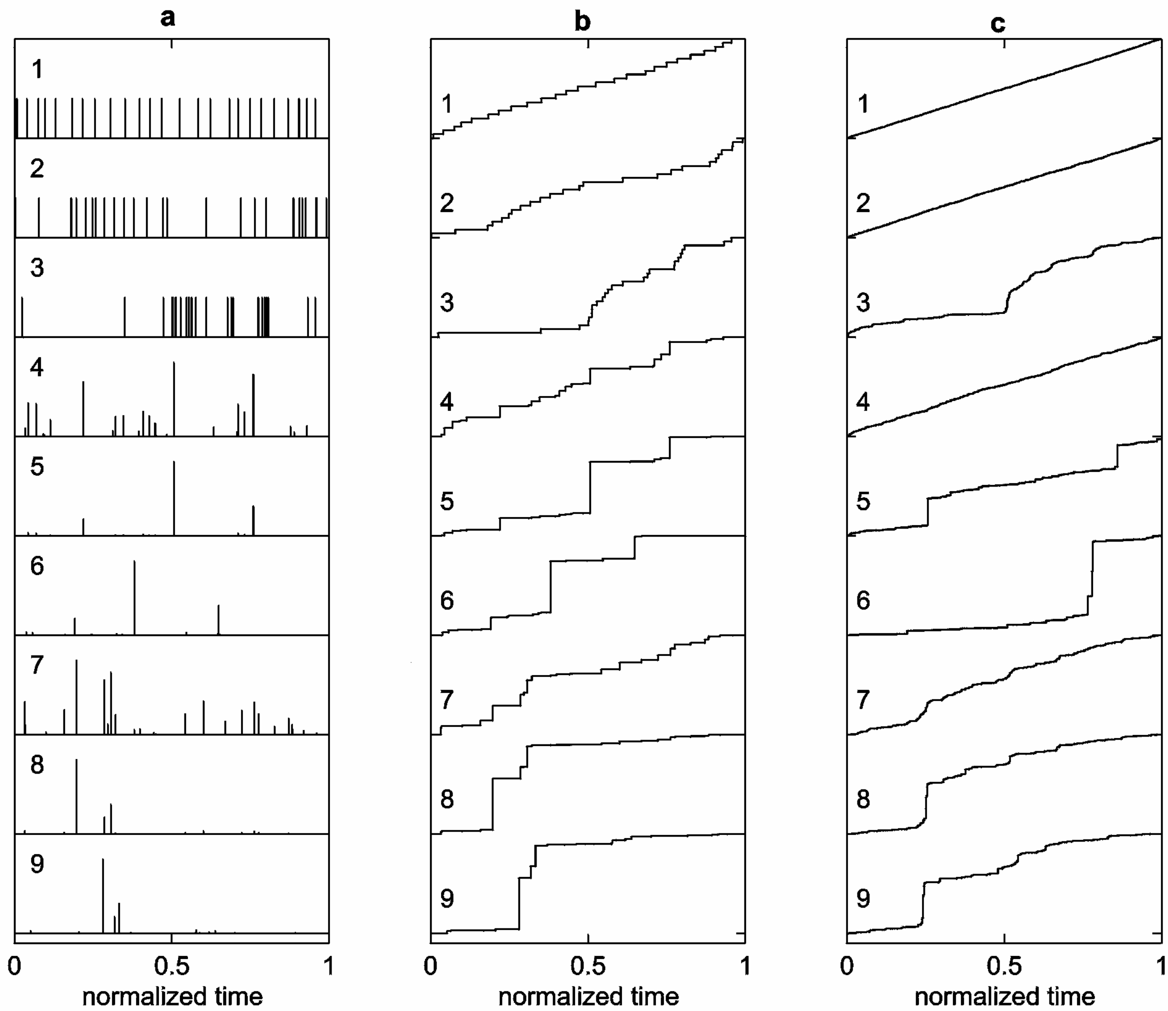

Figure 1. Examples of the regular and irregular temporal behavior of event sequences. a - short sequences (*N*=25), individual events are shown by bars; b - same sequences depicted as cumulative volume vs time, individual events are seen as steps of the stairs; c - long sequences (*N*=250-2000), again given as cumulative volume, most individual events are unresolved in this view. Three causes of irregularity, or instablity of long-term average output rate are illustrated, isolated or in combinations: heavy-tailed, in particular power-law volume distribution (HT), multiscale (self-similar) clustering of event dates (common clustering, CC) and self-similar «order clustering» among larger events (OC). Cases 1-3: equal-volume sequences. 1-almost periodic process, 2 - Poisson process, 3 - clustered process (CC factor only). Cases 4-9: variable-volume sequences. 4 - dates Poissonian, volumes distributed exponentially; 5 - dates Poissonian, volumes distributed by the power law (*b*=0.75) (HT factor only); 6 - like 5, but dates clustered (HT+CC); 7 - dates Poissonian, «order clustering» is present, volumes distributed exponentially (OC only); 8 - like 7, but volumes distributed by power-law (HT+OC); 9 - dates clustered, «order clustering» is also present, and volumes are distributed by power-law (CC+OC+HT). A persistent irregular behavior is seen for Cases 3 and 5-9, to be compared to a regular behavior in Cases 1,2 and 4. In section b of the plot, irregularity seen for Cases 2 and 4 is only apparent, caused by discreteness of the process; for large *N* (section c) it disappears

Size distribution of eruptions

Statistical distribution of eruption sizes was studied by some authors, and preliminary discussion of their results is useful. For the whole world, De la Cruz-Reina (1991) noted that the size-number relationship is near to a power law; similar law (for all but the greatest eruptions) is given in a graphical form by Simkin and Siebert (1994). That is, for a given period and territory, the number of events whose volume of products V is between $V/10^{0.5}=0.316V$ and $10^{0.5}V=3.16V$ is

$$n(V)=\text{const } V^{-b} \quad (1)$$

or

$$\log_{10} n(V)=a - b \log_{10}V. \quad (2)$$

For the value of b parameter (or, simply, b-value), De la Cruz-Reina gives b=0.789; (to be accurate, he uses the VEI parameter instead of $\log_{10}V$). On the basis of Fig. 10 of (Simkin and Siebert 1994)) we found the estimate b=0.75 over the volume range 10^6-10^{11} m^3 ; we believe this estimate to be more accurate. For the greatest eruptions (V>10^{11} m^3), Equation 1 overestimates the observed value showing a tendency to «saturation».

For an ideal power law (1), one can replace the analysis of distribution of "differential" numbers $n(V)$ in logarithmically similar bins by the analysis of the (non-normalized) cumulative distribution law $N(V)$, defined as the number of events with volume $V'>V$. Then for a data set that follow the $n(V)$ relationship (1) one can easily obtain:

$$\log_{10}N(V)=a'-b \log_{10}V \qquad (3)$$

with the *same* parameter b as in (1) and (2). (The obligatory coincidence of exponents may be not evident; it implies from the following argument: define $n'(V)$=d $N(V)$/d $\log_{10} V$; when $n(V)\propto V^{-b}$, one may assume $n'(V)$ d $\log_{10}V \propto V^{-b}$ d $\log_{10}V$, then $N(V) = \int n'(V)\, d \log_{10}V \propto \int V^{-b}\, d \log_{10}V \propto \int V^{-b-1} dV \propto V^{-b}$.) With limited data volumes, Equation 3 may be more convenient for analysis as compared to (2).

To check whether the observed data follow a power law is an important aim for preliminary data analysis. Power laws are related to self-similarity, and they are ubiquitous as size distributions for natural phenomena (they work for earthquakes, faults, intrusions, gold and oil fields, islands, lakes, meteoroids, rivers, mud flows, avalanches and much more). Therefore, to find one more example of such a distribution may be not a great attainment. However, it is interesting and useful to know the specific b-value of our data set. Deviations of our data from a power law may say us important things about specific properties of the phenomenon in question, or indicate some deficiencies of the data set used.

Temporal structure of eruption dates

Possible structures for the distribution of eruption dates over the time axis also deserve some preliminary discussion. Although any observed event list is unique, one can view it as an example, or "realization", of a random point process, like telephone calls or Geiger counter clicks. A common approach in practice is to compare the properties of the observed event distribution with those of some reference, model random event flux. The standard reference event flux, called Poisson point process, takes place when points on a line (i.e., dates on the time axis) are "purely random". Denote λ the mean event rate (point density), measured in events per second or per year. For the Poisson process, the event rate λ is a constant and does not depend on the position of a time subsegment Δt where we determine it, in at least two aspects: (1) it does not change with (absolute) time and (2) it is insensitive to positions of other points. (More formally, see e.g. Cox & Lewis (1966), let Δp be the probability to find a point on a small subsegment Δt of the large segment $(0,T)$ of the time axis. For Poisson process, Δp is merely proportional to Δt : $\Delta p=\lambda\Delta t$.) If a real event flux deviates from this reference model, both assumptions (1) and (2) may fail. When (1) is invalid, this merely means that the event rate varies in time ($\Delta p=\lambda(t)\Delta t$ with $\lambda(t)\neq$const). When (2) fails, the probability of an event depends on the position of other events, making events interdependent. Typically, they either repulse one another, resulting in a periodic tendency, or attract one another, resulting in a tendency to group together, that is, to form clusters or "episodes". All these modes of behavior may have certain volcanological meaning. Observing a plain Poisson flux suggests the uniformity of the underlying process and independence of events (the radioactive decay is the well-known example). Time-dependent rate indicates that events are independent but their cause varies in time (like telephone calls over 24 hours). Repulsing of events may be indicative, e.g., of a limited-size reservoir that needs time to be refilled. At last, the clustering style may indicate the mutual support, (or contagion, or positive feedback) among events. A rather general class of a clustering behavior can be represented by introducing the "intensity" function $r(d)$ that determines the probability $\Delta p = r(d)\Delta t$ for an event B to occur on a small time segment Δt that is situated at a time interval $d = |t_B - t_A|$ before or after a given event A. The Poisson process is specified by $r(d)=\lambda$=const. We speak of a clustering tendency when $r(d)$ is a decreasing function of d. Nonformally, the nearer a potential neighbor is located, the larger are the chances that it is actually present.

Among random point processes with clustering, those that attracted much attention recently are so named fractal or statistically self-similar (or "self-affine"). In this case, the clustering tendency is manifested in the manner that is «uniform» for all time scales. In particular, there is no such thing as the characteristic, typical duration of a cluster. The appropriate $r(d)$ function is the inverse power law decay of intensity:

$$r(d) \propto d^{-a} \qquad (4)$$

This model, with a single numerical parameter a, is sometimes quite adequate for description of natural clustered point processes. However, applying this concept to observational data one must take into account that the self-similar object is only a mathematical concept. For any natural object, one can observe the self-similar behavior only within a limited range of scales, between the "lower fractal limit" d_1 and "upper fractal limit" d_2 (these limits may be «natural» or arise from observational constraints). Thus, one must be prepared to find the self-similar behavior only over a limited range of inter-event delays; in other words, in a limited frequency band. Such an object is called "band-limited fractal." The actual values of d_1 and d_2 represent an important part of data description

Examples

We now illustrate the described concepts using simulated event sequences that show various modes of irregular temporal behavior. Fig. 1a shows the temporal structure as points over the time axis, whereas Fig. 1b shows the same examples as integrals (cumulative sums) along the time axis. All examples in Fig. 1ab correspond to the fixed small number of events N=25. Fig. 1c is similar to 1b, but with much larger N=250-2000, so that an asymptotic behavior may be clearly seen. As explained above, we specify the behavior as "irregular" when the fluctuations of the average output rate are so intense that they do not flatten out with increasing number of events. The presence or absence of irregularity is well seen on long-term cumulative plots. Cases 1, 2 and 3 show sequences of identical (constant-size) events, that are distributed in time in the following modes: (1) almost periodically, (2) as a Poisson process («purely random»), and (3) with self-similar clustering, respectively. One can naturally see no irregularity for the periodic case, for small or large event number. Some irregularity may be suspected for the Poisson case, but it is only apparent, related to the discrete structure of the process; it essentially disappears as the number of events increases. Therefore, for large N, the behavior of periodic and Poisson processes is quite similar. For this reason, we will not give more examples for the periodic case. For the clustered case, a clear irregular behavior is seen, and it evidently persists even when the number of events is large.

The further examples show events with non-equal, random weights. In Case 4 (the reference case) we combine the Poisson temporal behavior with the exponential distribution of volumes. This size distribution has finite variance, and the dispersion of volume values, though formally unlimited, is not sufficiently "wild" to produce an irregular behavior, as seen on the example graph. In Case 5, instead of the exponential law, we use realistic heavy-tailed power-law type of distribution (b=1). Now irregularity is evidently present, both in short and long term, but it is not related to any specific temporal structure: the only cause of irregularity is the effect of the variable event size. In Case 6, we changed the situation of Case 5 in additional respect: we added clustering of event dates (like in Case 3). One can note (only at large N) that irregularity somewhat increased, as could be expected. On another line, in Case 7 we modified the Case 4 (Poissonian dates, exponential law for volumes) introducing order clustering. Here one can see that even with a «light-tailed» law, and without any clustering of dates, irregularity is still present, generated by order clustering alone. Replacing here the exponential law with the power-law in Case 8, one observes further increase of irregularity. At last, in Case 9, all three factors: 'heavy-tailed' size distribution, and two types of clustering, are combined.

Procedures of data analysis

We now describe the data analysis technique we employ. The particular questions that we wish to answer are: (1) is the sequence of event dates clustered; and if yes, does it show self-similar properties; (2) does the sequence of «variable-weight» points show the self-similar behavior in general, and if yes, can the contributions from order clustering and common clustering to such a behavior be separated.

In the analysis of the clustering behavior of event dates, the common first step is to compare them to a Poisson sequence (Cox & Lewis (1966). If no significant deviations from the Poissonian behavior can be observed, this will mean either that the events occur independently and at a constant rate, or that the volume of our data set is too small to draw any conclusive judgment. In such a problem, there is a traditional statistical approach, that of analysis of inter-event intervals. For the Poisson process, these intervals are known to be distributed exponentially, so for the probability of a particular interval d' to be below some d we have:

$$\mathrm{Prob}(d'<d)= P(d)=1 - \exp(-d/d_0) \qquad (5)$$

where $d_0 = 1/\lambda$ is the mean inter-event interval. The d_0 parameter is estimated as T/N where T is the duration of the analyzed period and N is the number of observed events. $P(d)$= Prob($d'<d$) is called cumulative distribution function (CDF). Often it is more convenient to work with the complementary cumulative distribution function (CCDF) $P_C(d)$= Prob($d'>d$) = 1-$P(d)$. In the already discussed case of eruption sizes, it is $P_C(V)$ that is the power law; similarly, in the case under study, it is $P_C(d)$ that is exponential.

As the clustering property was already informally noticed with our data, it is reasonable to apply a model that can reproduce it. Hence we will use the Weibull distribution of intervals:

$$\mathrm{Prob}(d'<d)=P(d)=1 - \exp(-(d/ d_0)^{\gamma}) \qquad (6)$$

This distribution has two parameters d_0 (again related to the event rate) and γ, which is of key interest. A set of points on a segment, with intervals between points distributed according to the Weibull law is either clustered when γ<1 (small intervals are more often than in the Poisson case) , or has the tendency to periodicity when γ >1 ("repulsing": small intervals are less often than in the Poisson case). At γ =1, the Weibull law reduces to the exponential one (5). The choice of the Weibull law to represent clustering is dictated mainly by the considerations of convenience; however, we expect that our conclusions will no more that marginally depend on this specific choice.

The Weibull model for intervals is primitive and cannot represent the clustering tendency in an adequate way. The reason is that a real physical factor, that assumedly makes dates to cluster, cannot be bounded by the ends of a single inter-event interval; whereas in the Weibull model for intervals, the clustering tendency exists only for a pair of adjacent events, and not between, say, side members of a triplet. This is an intrinsic deficiency of any model that is based on statistical distribution of interval durations only (the «renewal process model»).

The positive result of the test based on the Weibull law would mean that deeper analysis makes sense, based on the Equation 4. To do this, we can analyze all possible delays between events, not only those between adjacent ones, and look for the effects of genuine self-similar clustering. By forming a histogram of all observed d values, the Equation 4 may be almost immediately used in data analysis that yields the value of the exponent a. However, the estimates obtained in this way often show large scatter. Numerically more stable technique may be based on the integral $C(d) = \int_0^d r(u)du$ called correlation integral (Hirabayashi et al. 1992). Thus we calculate all $N_P = N(N\text{-}1)/2$ inter-event delays d_{ij}, sort them in increasing order and then find the normalized cumulative sum

$$C(d)=(1/N_P)\, N(d_{ij} <d) \qquad (7)$$

This definition assumes that data are "ideal": the number of pairs $N_P \to \infty$, and data are collected over an infinite segment. Our data are however from a bounded time segment; to compensate related distortions, the actually used $C(d)$ values were additionally normalized so as to guarantee that the relationship $C(d) \propto d^1$ (see explanation below) will be obtained for data generated by a Poisson process on a bounded segment. (The normalization we use originates in the standard "triangular" normalization procedure used when estimating correlation function of a random process observed within a limited time window.)

For fractal data, C(d) must follow the power law:

…

Table 1. The largest explosive eruptions in Kamchatka during the last 10 000 years

Average ^{14}C age, yr. B.P. [$]	Calendar age, yr.[@]	Accepted age, yr.	Volume of tephra, km^3 [$]	Source volcano (tephra code)
-	AD 1964	+1964	0.6-0.8	Shiveluch (SH_{1964})
-	AD 1956	+1956	1.2-1.3	Bezymianny (B_{1956})
-	AD 1907	+1907	1.5-2	Ksudach, Shtyubel cone ($KSht_3$)
-	AD 1854	+1854	~1	Shiveluch (SH_{1854})
265±18	AD 1641 (1652) 1663	+1652	≥1	Shiveluch (SH_1)
965±16	AD 1021 (1034) 1157	+1034	≥2	Shiveluch (SH_2)
1090±31	AD 889 (978) 1015	+978	0.8-1	Ksudach, Shtyubel cone ($KSht_1$)
1450	AD 628	+628	≥1	Shiveluch (SH_{1450})
1404±27	AD 614 (653) 670	+653	≥2	Shiveluch (SH_3)
1478±18	AD 550 (606) 638	+606	9-10	Opala, Baranii Amphitheater crater (OP)
1806±16	AD 147 (236) 317	+236	18-19	Ksudach (KS_1)
2506±31	BC 791 (762, 624, 598) 428	-624	1-1.5	Khodutkinsky crater (KHD)
2553±46	BC 807 (780) 524	-780	≥1	Shiveluch (SH_5)
2800	BC 922	-922	≥1	Shiveluch (SH_{2800})
3512±18	BC 1886 (1873, 1839, 1812, 1807, 1781) 1748	-1812	≥4	Avachinsky (AV_1)
4020 ± 49	ÂÑ 2850(2558, 2530, 2497)2409	-2530	≥0.6	Avachinsky (AV_2)
4105±31	BC 2866 (2616) 2504	-2616	≥1.5	Shiveluch (SHdv)
4482±31	BC 3339 (3261, 3244, 3101) 3034	-3244	≥1.1	Avachinsky (AV_3)
4606±58	BC 3509 (3359) 3103	-3359	1.2-1.4	Iliinsky (ZLT)
4628±90	BC 3634 (3366) 3046	-3366	0.9-1	Chasha crater (OPtr)
5489±27	BC 4360 (4342)4262	-4342	≥1.34	Avachinsky (AV_4)
6007±38	BC 4956 (4907) 4797	-4907	7-8	Ksudach (KS_2)
6284±23	BC 5268 (5251) 5147	-5251	0.5-1	Ksudach (KS_3)
6957±30	BC 5926 (5769) 5711	-5769	12-13	Khangar (KHG)
7151±51	BC 6110 (5977) 5875	-5977	≥8-10	Avachinsky (I AV2)
7531±37	BC 6423 (6377) 6225	-6377	4-5	Kizimen (KZ)
7666±19	BC 6530 (6459) 6422	-6459	140-170	Kurile Lake-Iliinskaya caldera (KO)
7889±67	BC 7008 (6642) 6495	-6642	13-16	Karymskaya caldera (KRM)
8826±40	BC 8005 (7922) 7702	-7922	1.5-2	Ksudach (KS_4)

[$]Radiocarbon ages are averaged and calibrated to calendar ages according to the technique by Stuiver and Reimer, 1993. The data are from Braitseva et al., 1997a,b, 1998; Dirksen and Ponomareva, 1998; Pevzner et al., 1998; Ponomareva et al., 1998; and Volynets et al., 1999.
[@] The 'Calendar age' column is organized as: a(bb)c where a - c is the interval estimate, and each b is a single individual date estimate.
[$]The volume values used in calculations were obtained from this column by: (1) taking the middles of interval estimates and (2) treating ' ≥' signs as 'equal' signs.

$$C(d) \propto d^{D_c} \qquad (8)$$

or

$$\log_{10} C(d) = \text{const} + D_c \log_{10} d \qquad (8a)$$

where $D_c = 1 - a$ is, by definition, the so named "correlation fractal dimension" of a point set on a line. The Poisson process is the limiting, non-fractal case when $r(d)$=const, $C(d) \propto d$, $D_c = 1$ and $a=0$.

When we treat our eruption catalog as a sequence of variable-weight points ("a marked point process"), we need more advanced analytical means. One useful approach is the well-known «rescaled range» or *R/S* ratio technique that yields the Hurst exponent value *H* (Mandelbrot 1982, Feder 1988). The *R/S* ratio procedure that we use departs from mass distribution function $m_d(t)$ over a segment, formed from known dates and volumes. To calculate *H* value, we divide the initial segment into a number of subsegments of the size k_i=1, 2/3, 1/2, 1/3, 1/4... of the total length. For each k_i and for each subsegment (t_1,t_2), we calculate cumulative mass function $m(t) = \int_{t_1}^{t} m_d(u)du$ and then determine auxiliary linear function $m_{tr}(t)=(t-t_1)(m(t_2)-m(t_1))/(t_2-t_1)$. Then we find the difference $m_s(t)=m(t)-m_{tr}(t)$ and obtain the value of «range» $R=\max(m_s(t))-\min(m_s(t))$. Then the S^2 value is estimated as the average variance of $m_d(t)$ over the same segment. Finally, scaling *R* by *S* we obtain an individual «rescaled range» *R/S* value. For all subintervals with the same k_i, *R/S* values are averaged, to give the empirical function $R/S(k_i)$. The value of *H* is found fitting *R/S* data by a straight line in a log-log scale:

$$\log(R/S) = \text{const} + H \log(k_i) \qquad (9)$$

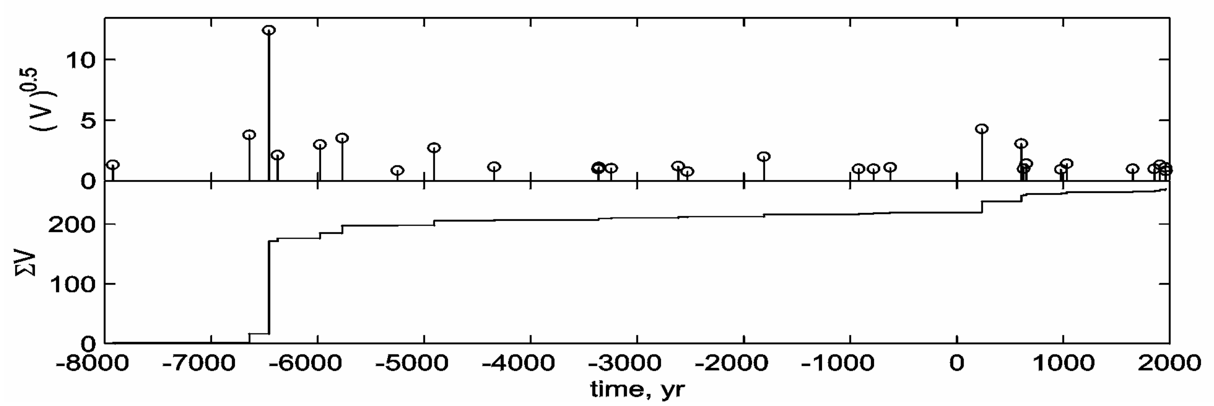

Figure 2. Time evolution of Kamchatka Holocene explosive eruptions. top - the sequence of individual volumes, the height of a stem is proportional to $V^{1/2}$ for visual clarity; bottom – the same data as a cumulative plot (total volume accumulated up to a given date).

For the reference non-fractal case, estimated H must be near 0.5; larger values of H (in case of acceptable fit) indicate multiscaled variations of fractal kind.

It should be mentioned that with our discrete kind of data, many shorter subsegments arise with zero mass (no one event). Over such a segment, $m_d(t)$ equals to zero, R=0, S=0, R/S is meaningless and must be discarded, and this distorts the results of the R/S technique. For this reason, we used the R/S technique only in a limited relative subsegment size range k_i from 1/4 to 1 (this empirical choice is based on numerical experiments with samples of N=30). This means that in our case, the self-similar behavior can be studied by this method for long intervals only.

Such problems do not occur with another technique to analyze variable-weight point distribution on a segment, which is merely a generalized version of the above-described correlation integral technique. To determine the modified correlation dimension we replace in (7) cumulative number function $N(d)$ by cumulative weight function $W(d)$. The contribution into $W(d)$ from an event pair (i, j) with weights V_i and V_j equals to their product $w(d_{ij})=V_iV_j$. The versions of $C(.)$ function and D_c parameter modified in this way we denote $C_v(d)$ and D_{cv}. To estimate $C_v(d)$ function we calculate the cumulative sum:

$$C_v(d) = W(d)/W(\infty) = \sum_{d_{ij}<d} w(d_{ij}) \Bigg/ \sum_{\text{all } d_{ij}} w(d_{ij}) \qquad (10)$$

and then determine D_{cv} by fitting a power law to it.

To reveal the presence of the common and/or order clustering behavior in a particular data set, we use the following scheme. As explained above, the deviation of H from 1/2 or D_{cv} from unity (when significant) suggests self-similar irregularity of the process. This deviation may be caused by the following factors: (1) dates of (all) events tend to be clustered, or (2) largest events come in groups. In a study aimed at showing the presence of each factor, another factor is an interfering one, and we would like to suppress its effect. To implement this idea, we can compare H estimates for real data with those for specifically modified data. First, we can «turn off» the order clustering factor keeping common clustering unaffected. With this end in view, we can randomly shuffle the actual values of volumes among events, preserving all dates, and then compare the resulting H estimates with the initial one. If order clustering existed in initial data, new estimates must

Table 2. The largest historical explosive eruptions on Kamchatka during the period AD 1735 - 1993*

Year / month of eruption	$\log_{10}$ (tephra volume, m^3)	Volcano
1985	7	Bezymianny
1984	7	Bezymianny
1980/04	7	Bezymianny
1980/06	7	Gorely
1979	7	Bezymianny
1977/08	7	Kliuchevskoi
1977/03	7	Bezymianny
1975	8	Tolbachik cones
1965	7	Kliuchevskoi
1964	8	Shiveluch
1963	7	Karymsky
1956	9	Bezymianny
1945	8	Avachinsky
1941	7	Tolbachik
1938	7	Avachinsky
1937	7	Kliuchevskoi
1934	7	Karymsky
1930	7	Gorely
1929	7	Gorely
1926	8	Avachinsky
1925	7	Kliuchevskoi
1923	8	Zheltovsky
1911	7	Kliuchevskoi
1907	9	Ksudach
1901	7	Iliinsky
1898	7	Mutnovsky
1890	7	Kliuchevskoi
1883	7	Kliuchevskoi
1878	7	Kliuchevskoi
1854	8	Shiveluch
1848	7	Mutnovsky
1832	7	Gorely
1829	8	Kliuchevskoi
1828	7	Gorely
1827	8	Avachinsky

1779	7	Avachinsky
1776	8	Opala
1762	7	Kliuchevskoi
1740	7	Tolbachik cones
1737	7	Avachinsky

* The data are from Simkin and Siebert (1994), refined according to Melekestsev et al. (1990, 1994), Vlodavets (1957), Fedotov and Masurenkov (1991).

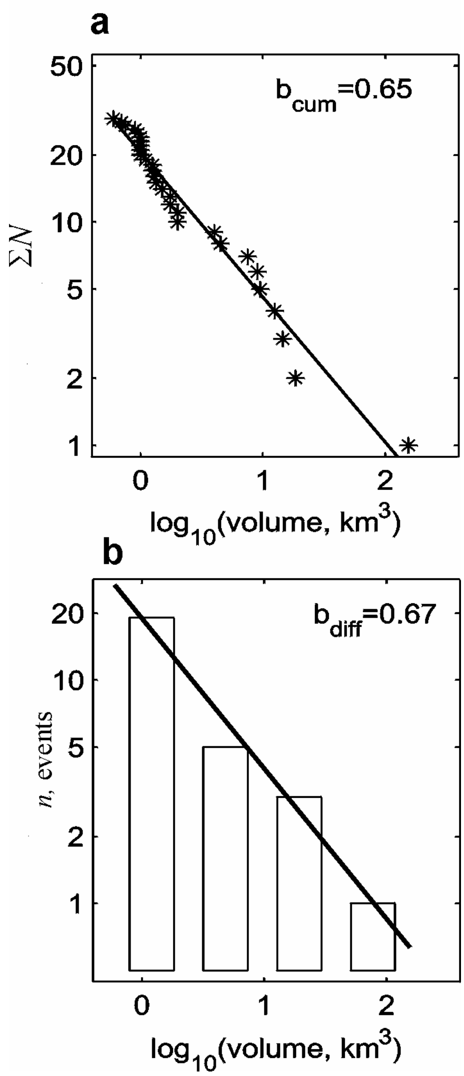

Figure 3. Frequency-size distribution of Kamchatka Holocene explosive eruptions. a - cumulative *N(V)* plot (the number of events with volume *V′* above *V*, versus *V*); b - same data as a histogram. Straight lines are power-law fits.

be significantly lower (nearer to 1/2) than the initial one. We wish to determine the significance level for the hypothesis «shuffling produces *H* values that, on the average, are below the initial one». With this aim, we can repeat random shuffling many times and find the empirical distribution of *H* values for the modified data. Comparing the value of *H* obtained from real data with this distribution immediately gives us the level of significance.

Completely similar procedure, only with approaching unity from below instead of 1/2 from above, may be performed for the D_{cv} parameter. In the same manner we can study separately the common clustering property. In this case we keep the ordered list of volumes intact, but «turn off» the clustering of dates by replacing real dates by random Poissonian dates. Again, if common clustering exists in initial, unperturbed, data, new *H* estimate must be, on the average, below the unperturbed one. (Or, new D_{cv} estimate must be larger.). At last, we can «turn off» both factors of the temporal structure, and preserve only the pure effect of volume variability that must not produce any fractal-like behavior alone.

The data set under study

Kamchatka Peninsula hosts more than 25 active volcanoes and hundreds of monogenetic vents which form a 700-km long volcanic belt from Shiveluch (56.65° N 161.36° E) in the north to Kambalny (51.30° N, 156.87° E) in the south. Volcanism in Kamchatka is produced by subduction of the Pacific plate at (modern) rate of 9-10 cm/yr (Minster and Jordan, 1978; Geist and Scholl, 1994); it is mainly explosive in character (Melekestsev, 1980; Fedotov and Masurenkov, 1991). In Table 1 we give the catalog of the largest explosive eruptions of the Kamchatka volcanoes during the last 10,000 years. It is based on two comprehensive published lists (Braitseva et al., 1997 a,b) augmented by new data: (Braitseva et al.,1998; Dirksen and Ponomareva, 1998; Pevzner et al., 1998; Ponomareva et al., 1998; Volynets et al., 1999).

The eruptions have been identified and documented during many years of tephrochronological studies. Their ages have been estimated by radiocarbon dating of associated organic material (Braitseva et al., 1993). Radiocarbon ages were averaged and calibrated to calendar ages according to the technique by Stuiver and Reimer (1993). The catalog is assumedly complete for the explosive eruptions with the volumes of products of more than 0.8-1 km^3. The temporal structure of the catalog can be seen on Fig. 2.

In Table 2 we also present a catalog of historical explosive eruptions on Kamchatka, with tephra volumes of 0.01 km^3 or more, for 1735-1993. The catalog is based on (Simkin and Siebert, 1994), refined according to Melekestsev et al. (1990, 1994), Vlodavets (1957) and Fedotov and Masurenkov (1991).

The size distribution of eruptions

The distribution of volumes of products of eruptions listed in Table 1 are depicted in Fig. 3 both in cumulative and histogram forms. The shape of the cumulative plot suggests a power law hypothesis that predicts a straight line in log-log scale. The quality of linear fit is quite acceptable. On the right side (largest events), the point that represents the singular Kurile Lake caldera eruption that produced about 50% of all products fits the line quite good. On the opposite side, no significant deviation from the straight line is seen near the threshold value of about 0.5 km^3; this fact suggests a high degree of completeness of the data set. With only 29 points at hand, the histogram plot is much less stable; but it also does not contradict the linear-shape assumption. The estimate of *b*-value was obtained by the linear fit of the cumulative plot and the histogram, yielding, respectively, *b*= 0.65 and 0.67, with estimated error of about 20%. The estimated *b*-value of 0.65 for Holocene Kamchatka data set (as well as *b*=0.60 for the historical data set, see below) seem to be somewhat lower than the value of 0.75 for global data derived above from Simkin and Siebert (1994); however, this difference is not established reliably.

One can ask what may be the effect of possible errors of estimated volumes on the derived *b* estimate. Because of the cumulative character of the plot in Fig.3, the possible effect of random, non-systematic errors is in fact minor. It is easy to show that even twofold random errors ($\sigma(\log V)$=0.3), if added to true volume data, would change the true *b* estimate only marginally.

Table 3. Data analysis for the Holocene Kamchatka data set (data of Table 1, *N*=29)

	γ^*	$D_c{}^{\&}$ (100-10000 yr)	$D_c{}^{\&}$ (100-800 yr)	$D_{cv}{}^{\$}$ (100-10000 yr)	$H^{@}$
H_0 value	1	1	1	1	1/2
Observed value	0.73	0.98	0.71	0.67	0.58
"Po" value[1]	1.05	1.00	1.00	1.12	0.48
sigma[1]	0.26	0.07	0.14	0.23	0.06
significance[1]	7%	44%×	2.5%	3.7%	3.2%
"Sh" value[2]				1.08	0.48
sigma[2]				0.23	0.05
signif[2]				4.8%	3.1%
"Po&Sh" value[3]				1.11	0.48
sigma[3]				0.23	0.06
signif[3]				3.4%	3.4%

[*] Parameter of the Weibull law.
[&] Correlation dimension for the given range of interval duration *d*.
[$] "Variable-weight" correlation dimension.
[@] Hurst's exponent.
[1] Mean and standard deviation for modified data obtained by changing actual dates to Poissonian sequence, and the significance level for the violation of the null hypothesis of Poissonian sequence.
[2] Same, with the event dates undisturbed and the sequence of actual volume values randomized by shuffling. The null hypothesis here is that large-volume events do not "attract" one another.
[3] Same, both dates and volume sequences are randomized. The null hypothesis is the lack of statistical self-similarity.
Complete lack of significance (*Q*>20%) is indicated by (×) sign.

The temporal structure of Holocene volcanic explosive activity on Kamchatka

The clustering of event dates

A first look at the event distribution over time (Fig. 2) gives the impression that events come in tight clusters, and that this tendency is particularly expressed for larger events. We will begin the study of the temporal structure of eruptive activity with analysis of the sequence of eruption dates, ignoring at first the differences of event sizes. First we check the presence of the clustering by fitting the Weibull law to intervals between events, then analyze the same tendency using correlation integral. Further, we include eruption volumes into consideration, and look for the self-similar properties of the product output proper. All numerical results related to these analyses are given in Table 3. The reader should address to this and subsequent tables for a systematic presentation of our analyses; only selected results are given in the graphical form

To show the presence of clustering we fit the observed distribution of inter-event time intervals by the Weibull law and test whether its parameter γ is significantly below unity. A convenient approach in the problem of comparison of observed data to a particular distribution is the use of probability paper, i.e. to represent the observed CDF $P(d)$ using specific scales on the *P* and *d* axes that are non-uniformly stretched in such a manner that the theoretically expected dependence is represented by a straight line. Fig. 4a represents the data on the Weibull law probability paper. The slope of data points immediately gives the estimate γ=0.73. The value γ=0.73 is considerably below unity, and seems to indicate remarkable clustering. The approximating straight line fit (that corresponds to the «ideal» CDF) is acceptable, with some deviations at tails of the distribution. One outlier value at lower *d* indicates probable limited resolution of the data over time axis, further assumed with some tolerance to be 100yr.

An important question is the significance of the observed deviation of the interval distribution from our null hypothesis of the Poisson case (with γ=1). To determine it we directly modeled 5000 realization of Poisson process with 29 points and estimated the γ parameter by the same procedure as the one used with the observed data set. In 345 out of 5000 cases, the estimated γ was below 0.733. Therefore, the significance level *Q* for the hypothesis that the observed distribution deviates from the Poisson model is approximately 0.0690, or below 7%. This value is not impressively small, meaning that our result might be obtained without any clustering in one case out of 14; but we consider the result as acceptable for a very limited data volume that contains only 28 intervals; therefore we believe that the clustering tendency is a real one.

Then we calculated C(*d*) function and estimated D_c for data. The result is seen on Fig. 4b. For the entire range *d*=100-10000 yr, the formal estimate of the slope of the plot in the log-log scale gives the estimate D_c=0.98. However, this estimate is not very meaningful: one can see that C(*d*) dependence does not remind a single straight line; rather, we have two linear branches. The first branch, for longer delays (*d*=800-10000 yr), has $D_c \approx 1$. For the second branch, with short delays (100-800 yr) we find (Fig. 4c) D_c = 0.71.

The significance level Q for the hypothesis «D_c <1 for d=100-800 yr » was determined by the already described technique, by comparison with the results of 5000 realizations of the Poisson process, to yield Q=2.5%. Thus, the D_c technique indicates that the fractal-like clustering behavior of event dates exist, but only in a limited range of delays. For the range of smaller and less accurate delays 25-100 yr this result also generally keeps true, as one can see from Fig. 4c

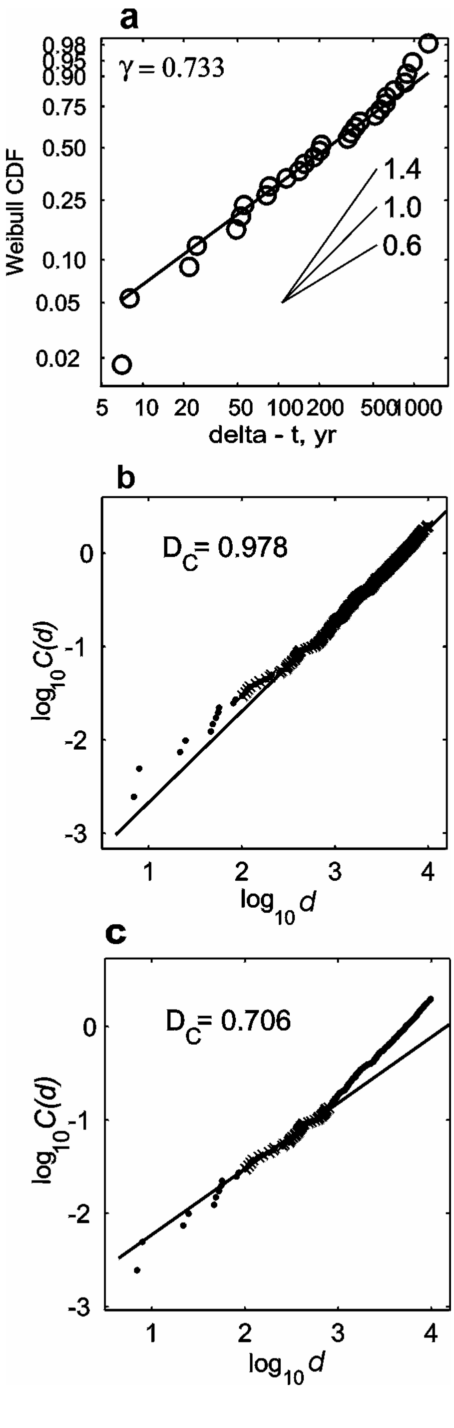

Figure 4. Analysis of the sequence of dates for Kamchatka Holocene eruptions (N=29). a - inter-event interval distribution on the Weibull probability paper; the value γ=0.73<1 of the Weibull law parameter indicates the clustering of event dates. b, c - determination of correlation dimension D_c for the same sequence. b - over the entire delay time range, when the contribution of longer delays is dominating, D_c =0.978≈1 indicates the absence of self-similar clustering over the entire range of delays. c - over the small delay range d<800 yr; the low value of D_c =0.706<1 indicates clear self-similar-like .clustering in this range.

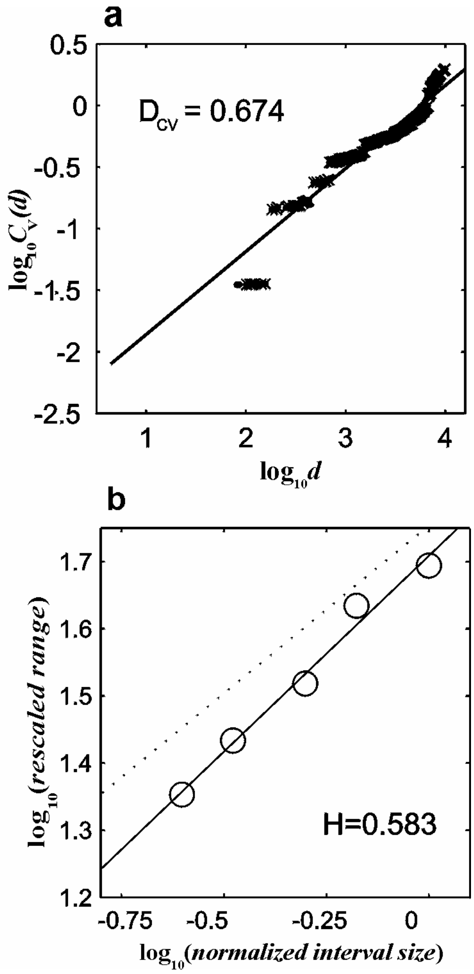

Figure 5. Analysis of the self-similar behavior of volcanic product output for Kamchatka Holocene data. a - estimation of correlation dimension D_{cv} , b - determination of the Hurst's exponent H over the interval duration range 2500-10000 yr (dotted line is a theoretical non-fractal reference case, H=½). The estimated values D_{cv}<1 and H>½ are the evidence of the self-similar behavior.

Self-similar properties of explosive eruption product output

We now pass to the analysis of eruptions as variable-weight events, starting from the correlation dimension D_{cv}. The estimated D_{cv} value is 0.67 (Fig. 5) which is much lower than unity, suggesting a pronounced self-similar behavior. To establish this fact statistically we again compare estimates of our parameters for real data with those for specifically modified data. First we turn off the clustering of dates replacing them by a Poissonian sequence, however preserving the ordered list of volumes («randomizing of dates»). With 5000 runs of randomizing dates, for the hypothesis «observed D_c value is below the one generated by any Poissonian sequence of dates» we obtain

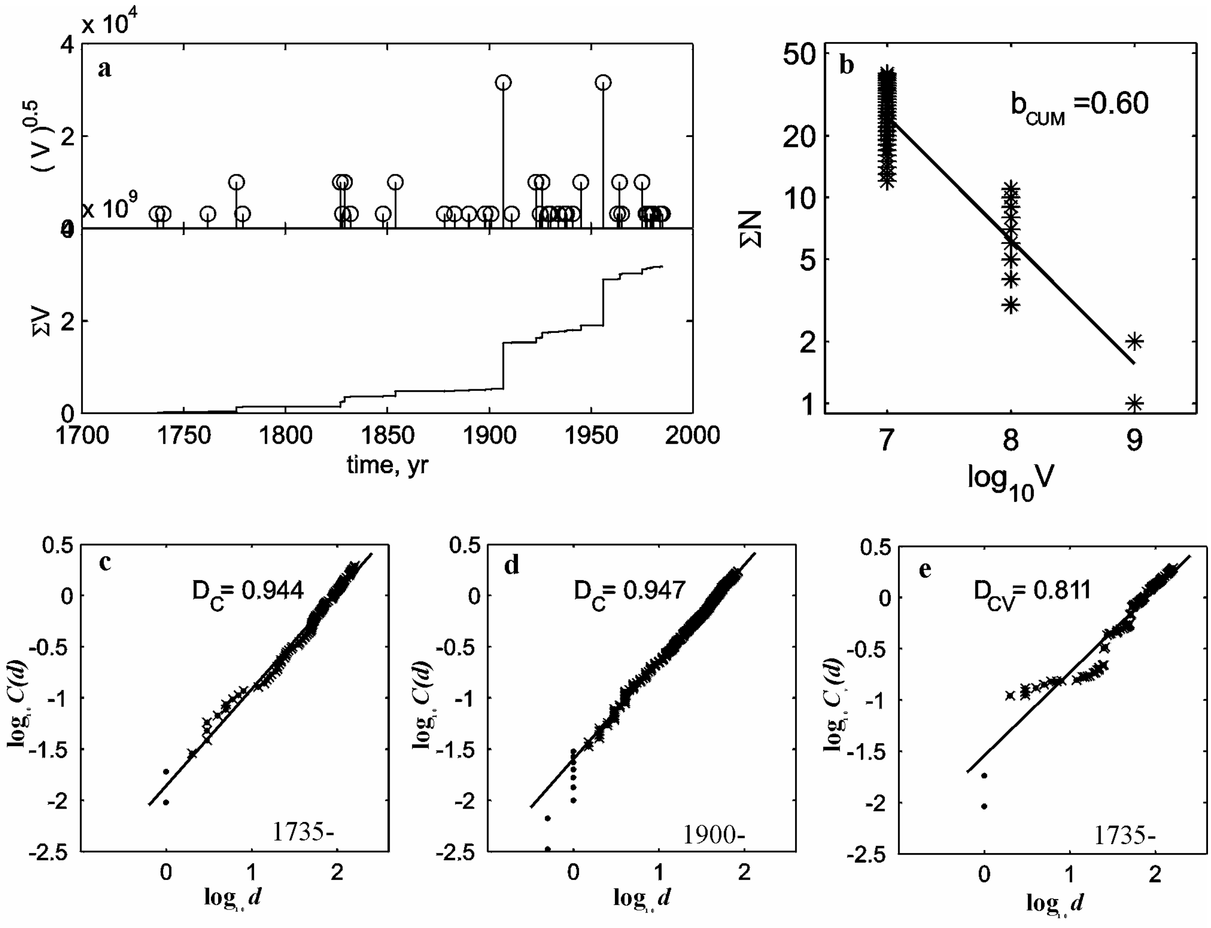

Figure 6. Analysis of the Kamchatka historical explosive eruption catalog. a - time evolution represented as an event sequence and cumulative volume similarly to Fig 2. Note the change of the event rate around 1900. b - the cumulative event size distribution; the line is the power-law fit. c, d - determination of correlation dimension D_c for the sequence of eruption dates, separately for two time periods. $D_c \approx 0.95<1$ suggests weak self-similar clustering of both date sequences. e - estimation of correlation dimension D_{cv} of volcanic product output for the earlier sub-period. The estimated D_{cv}=0.81<1 suggests the self-similar episodic behavior for 1735-1900.

Q=3.7% . Generally, a result of this kind is not unexpected because the presence of clustering has already been proven in other ways; but now it applies also to the 800-10000 yr range, where no clustering of event dates was seen before. (As was found by numerical experiments with variable number of events, the deviation of the simulated average D_{cv} (1.17) from the theoretical D_{cv} =1 is a typical artifact related to a small amount of data; we need not wary about this fact.)

In the opposite way, we can preserve the common clustering, but turn off order clustering, that is, keep dates intact but shuffle actual volume values randomly among events («randomizing event order»). Again with 5000 runs of randomizing event order, for the hypothesis «observed D_{cv} value is below the one generated by random shuffling of volumes» we obtained Q=4.8%. Thus, the «order clustering» phenomenon seems to be really present in our data. At last, we randomize both dates and event order, to obtain Q=3.4%. This check does not show anything unexpected.

An important property of our data is the difference between correlation dimension estimates for unit-weight events and variable-weight events, and in the latter case this dimension is smaller indicating more expressed fractal behavior. This fact probably reflects the above-discussed "correlation" (that is, mutual enhancement of effects) of common and order clustering.

Similar analysis was performed using the R/S technique. The estimated («observed») value of the Hurst's exponent H is 0.58 (see Fig. 5) which is somewhat above 0.5, suggesting a fractal behavior. To check this observation statistically we compared the observed H value with those obtained by randomizing dates or/and event order, with the same number of random tries as before. In the case of randomizing only dates we obtain Q=3.2%. Similarly, randomizing the event order yields Q=3.1%, and with both kinds of randomization we find Q=3.4%.

As mentioned, the results based on H values are related to longer time intervals (2500-10000 yr.). In this sense, the results regarding the clustering/fractal behavior that were obtained with γ and D_c estimates, on one side, and with H estimates, on the other side, are, to a certain degree, of a complementary character. Only the D_{cv} value characterizes the entire observed range of delays.

Table 4. Data analysis for the historical Kamchatka data set (data of Table 2, N=40)*

	γ			D_c			D_{cv}			H		
	AB	A	B	AB	A	B	AB	A	B	AB	A	B
H_0 value	1	1	1	1	1	1	1	1	1	1/2	1/2	1/2
Obs. value	0.81	0.93	1.14	0.91	0.94	0.95	*1.25*	0.81	*1.76*	0.59	0.60	0.26
"Po" value	1.13	1.13	-	1.00	0.99	1.00	-	1.08	-	0.49	0.53	-
sigma	0.42	0.46	-	0.04	0.09	0.06	-	0.17	-	0.13	0.14	-
significance	11%	38%×	-	2%	32%×	18%	-	14%	-	27%×	34%×	-
"Sh" value							-	1.04	-	0.51	0.48	-
sigma							-	0.18	-	0.13	0.12	-
significance							-	17%	-	32%×	12%	-
"Po&Sh" value							-	1.10	-	0.50	0.54	-
sigma							-	0.22	-	0.14	0.14	-
significance							-	14%	-	37%×	37%×	-

*Results are given for the complete catalog for 1735-1993,, denoted AB, and for its two subcatalogs: for 1735-1900, N=15 (A) and for 1900-1993, N=25(B). In case of the absence of the sought effect, meaningless figures are replaced by a dash. Complete lack of significance (Q>20%) is indicated by (×) sign. For other notation see Table 3

Concluding remarks

Normally, such levels of significance as 2.5- 7% may be thought as not sufficiently low for the rejection of the null hypothesis. On the other side, the numerical values of parameters γ, D_c , D_{cv}, and H considerably deviate from their null-hypothesis values. Only much larger, unrealistic deviations of this kind might produce completely acceptable significance levels for a given volume of data. This means that our results, despite their moderate-to-low significance, could hardly be substantially better within the actual limitations regarding the amount of data. To obtain stronger results, data volume must be increased at least 2-3 times which is not expected in any near future.

The standard deviations of Table 3 (though estimated at the null hypothesis) may be viewed at as reasonable, (in all cases but H - conservative) estimates of the accuracy for the corresponding parameters. (However, one should not try to use these values for standard checks of significance: the simulated distributions are «lighter-tailed» than the normal law.)

We may conclude that we established with certain reliability the following facts regarding large explosive Holocene Kamchatka eruptions: (1) eruption dates are not completely random; instead, they are clustered, with the value of the Weibull parameter estimated as γ=0.73; (2) for the inter-event delay time range 25-800 yr, this clustering is fractal-like, with the estimate of correlation dimension D_c =0.71;(3) the output of volcanic products behaves fractal-like as manifested by the estimates of correlation dimension D_{cv} =0.67 (valid for the entire studied range of delays 25-10000 yr), and of the Hurst's exponent value H=0.58 (that describes the largest delays only); (4) common clustering of event dates contributes significantly to the fractal behavior; (5) in addition to common clustering, the new phenomenon of «order clustering» is revealed, and its presence is shown independently from common clustering.

Analysis of additional data sets

Historical catalog of Kamchatka

It was interesting to perform similar analysis with the list of large historical explosive eruptions of Kamchatka (Table 2). Fig 6a shows the time sequence of events. One can note that the event rate changes around 1900. Its value is about 1/11 yr. for 1735-1899, denoted A period, and 1/4 yr. for 1900-1993, denoted B period. The cause of this difference is unclear. On one side, it can merely reflect gaps in the data reporting for the earlier period. However, another cause is also possible: two catastrophic eruptions occurred during the later sub-period, and the event rate in their temporal vicinity could increase because of already mentioned clustering effects. The presence of the variable event rate present certain difficulties to our analysis. As for the volume distribution exponent, for the entire data set, denoted AB, we found it to be b_{cum}= 0.60 and b_{diff}=0.58 (with N=40). The cumulative plot (Fig. 6b) look like a staircase because initial data were already grouped in wide (one order of magnitude) volume bins; however, the general agreement with the power law assumption is evident. The estimates over sub-periods are b_{cum}= 0.63 (A) and b_{cum}= 0.51 (B). The value for the period A is near to our estimate for the Holocene data; this may indicate that gaps in data are in fact limited for this period. (Information on smaller eruptions has the largest chances to be lost; for this reason, the loss of data will be enhanced for the lower-volume data bin, resulting in too low b-value). This fact suggests that the rate variations over the entire period may well be genuine. As a final estimate of b we prefer the estimate for the entire (AB) period, equal to 0.60. Within its accuracy limits (about 15%) it does not contradict to our earlier estimate for the Holocene data set.

The Weibull parameter for the entire data set is γ=0.81, with significance of the hypothesis «γ<1» equal to Q=11% (Table 4). This result, however, may be misleading because this kind of analysis produces biased results in the situation of a variable event rate. Indeed, for subperiods we obtain: γ=0.94 (Q=38%) for A and γ=1.13>1 for B.

… **Table 5**. Data analysis for the Greenland ice core data set* (A≥26)

	γ	D_c	D_c (1-30yr)	D_{cv}	D_{cv}(1-30yr)	H
H_0 value	1	1	1	1	1	1/2
Obs. value	0.81 / 0.67	0.96 / 0.90	0.73 / 0.64	0.99 /0.94	0.71 / 0.60	0.62 / 0.62
"Po" value	1.03 / 1.04	1.00 / 0.99	0.99 / 0/97	-/ 1.01	1.03 / 1.08	0.52 / 0.52
sigma	0.16 / 0.21	0.028 / 0.04	0.15 / 0.22	-/ 0.06	0.22 / 0.37	0.15 / 0.15
significance	7% / 1.6%	10% / 3%	3% / 5%	-/ 17%	7% / 9%	23%× / 28%×
"Sh" value				-/ 0.91	0.75 / 0.67	0.63 / 0.60
sigma				-/ 0.95	0.11 / 0.16	0.10 / 0.14
significance				-/ 75%×	33%× /35%×	64%× /54%×
"Po&Sh" value				-/ 1.01	1.03 / 1.06	0.50 / 0.51
sigma				-/ 0.07	0.21 / 0.35	0.16 / 0.13
significance				-/ 16%	12% / 8%	21%× / 20%

*before the slash: period 0-1990AD, N=64; after the slash: period 0-1500AD, N=40. Complete lack of significance (Q>20%) is indicated by (×) sign. For other notation see Table 3 and 4.

With D_c parameter, we have no theoretical obstacles for analyzing data for the entire AB period. Thus we obtain the D_c estimate of 0.91, with impressive significance of the hypothesis «D_c<1»of Q=2%. For the variant of separate analysis of subperiods, we find: D_c = 0.94(Q=32%) for A and D_c = 0.95(Q=18%) for B. Therefore, the clustering behavior is established, but only on the condition that the joint analysis of the historical data set is justified.

The results for volcanic product output are even less decisive. Table 4 shows that for the period A, some reasonable indications of a self-similar behavior are present, with the noticeable Q value of 17% for the result of correlation integral analysis, when checked against the «order clustering» property (Fig. 6e). For *R/S* analysis, in the similar check we obtained Q=12%. For the period B, however, the temporal behavior does not show a self-similar episodicity at all. This is seemingly related to the fact that each of the two catastrophic eruptions (1907 Ksudach and 1956 Bezymyanny) represent a temporally isolated event on the scale of a few years, see Fig 6a. The results for the entire period are unstable: R/S ratio supports the fractal behavior (H=0.59) whereas the estimate of D_{cv} (=1.25) is against it. We may infer that Kamchatka historical data show some weak indications of a self-similar behavior but do not provide any independent convincing proof for it.

The list of presumed volcanic explosions of 0-1990 AD after Zielinski et al.

Zielinski et al.(1994) analyzed the record of explosive volcanic activity represented as H_2SO_4-enriched layers in the Greenland glacier ice column. Although these data cannot be considered as a calibrated event catalog of any fixed area (or hemisphere, or Earth), they represent a unique record of volcanic activity. We believe that the values of H_2SO_4 concentration parameter (which we denote A here) reflect true eruption sizes but in a heavily distorted manner because of many biasing factors such as the relative fraction of SO_2 in eruption products, distance, atmospheric circulation tendencies etc. However, when these factors can be believed to act in a manner independent from the properties thet we are studying here (time sequence and eruption size), their effect may be considered as that of random multipliers applied to the values of volume. Such random modulation can of course *destroy* any episodicity or clustering that is present in initial sequence, but it is incapable to *create* such clustering out of nothing. For this reason we considered meaningful and interesting to apply our approach to these data as well. We confined our analysis by the event list (Zielinski et al. 1994) covering the period 0 - 1990AD, with, initially, N=69 events (Fig 7, Table 5); this data set will be further referred as "Zielinski data" for brevity.

Fig 7b shows the distribution of A values. The appearance of the plot generally agrees to the power law if we assume that only the data above A=25 are complete. The b-value for event sizes represented by A parameter is b_{cum}=1.67, much larger than that for eruption volumes. If one assumes that eruptions that generated Zielinski data follow the power law of global eruption volumes, with b=0.75, and that the effects of all other relevant parameters can be represented by random multipliers that are independent of volume, then the parameter A behaves, approximately, as $(\text{volume})^{0.4}$, suggesting a fast decrease of the relative SO_2 content (at least in precipitation) with increased output volume.

To study the temporal structure of data we are highly interested in their uniformity. On the basis of the above discussion of A distribution we thus discarded five events with A≤25 from the subsequent analysis. Fig 7c shows the Weibull cumulative probability plot for time intervals; the γ estimate is 0.81 which means a considerable degree of clustering. The significance level for the hypothesis γ <1 is 7%.

Fig 8a shows the C(d) plot. Judging from Fig 7c, the smallest intervals seem to be lumped to 1 yr in the technology of Zielinski et al.. This will distort the C(d) plot at small d; to overcome the problem we cut off all event pairs data with d=1yr. For the entire plot, excluding 1-yr. intervals, D_c estimate is 0.96. This value is below unity with Q=10% . One can see however the limited range of lower slope between d=1.5 and d=30 yr. For this interval, D_c =0.73, with Q=3%.

Estimates related to the self-similar behavior of the "output" of the A parameter are as follows. Observed data over entire range of delays yield D_{cv} =0.99 (no self-similar behavior). For small delay range however, D_{cv} estimate is as low as 0.71 (see Fig. 8b); and Q=7% for the hypothesis

«D_{cv}>1» when we randomize dates. R/S ratio analysis gives H=0.62 (Fig. 8c); the hypothesis «H>0.5» has very poor Q=23% when we randomize dates. Randomizing event order does not change both D_{cv} and H significantly; hence, the «order clustering» does not manifest itself with these data.

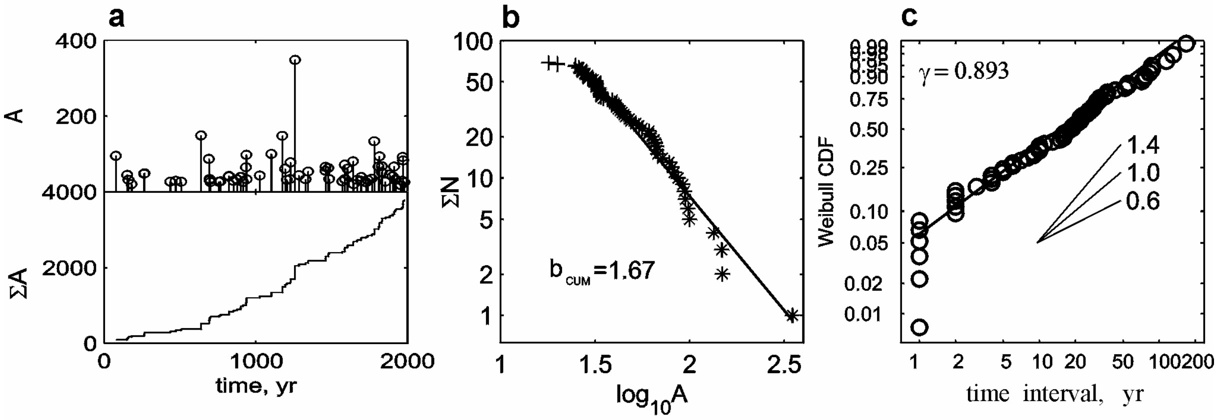

Figure 7. Analysis of Zielinski et al. (1994) data on eruption-generated H_2SO_4 concentration peaks in Greenland ice (denoted A) for 0-1990AD. a - time evolution represented as the sequence of individual peaks (top) and as a cumulative plot of A values (bottom). b - cumulative $N(A)$ distribution. Stars denote data and crosses denote rejected data, straight line is the linear fit to the power law. c - Weibull probability plot; linear fit with γ<1 indicates clustering.

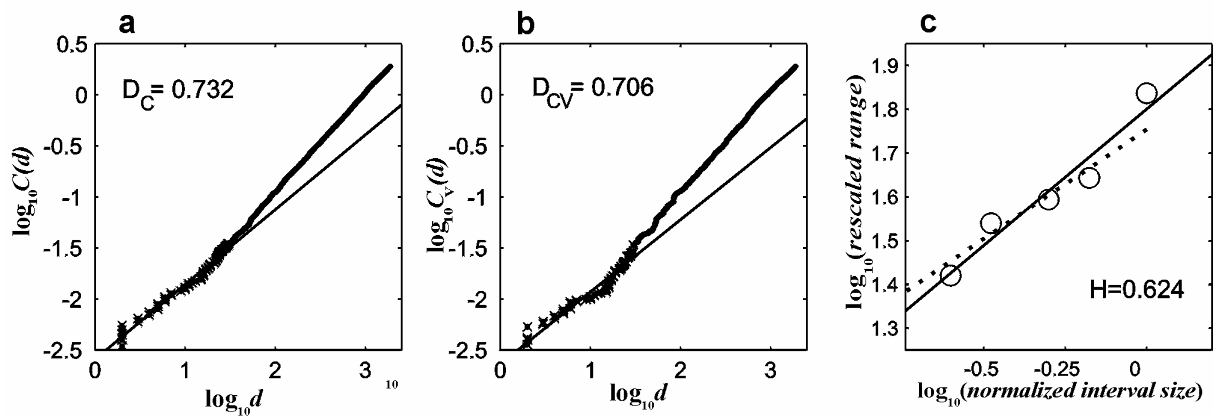

Figure 8. Analysis of self-similar properties of Zielinski et al. (1994) data. a- $C(d)$ plot for the sequence of dates; for small delays d<30yr, the estimated D_c <1 suggests the self-similar behavior within this range ofv delays. b - $C_v(d)$ plot, again suggesting the self-similar behavior of A "output" for small delays. c - rescaled range plot with estimated H=0.62>1/2 suggesting the self-similar behavior of A "output" for large intervals.

A remarkable feature of Zielinski data is the apparent temporal variation of clustering tendency. We found that if we exclude the data after 1500, the remaining 40 events show much more expressed clustering. Its parameters can be seen in Table 5 after slash in each cell of the table. Comparing to the entire data set, the values of γ, and D_c parameters decrease, and the value of H increases showing much more enhanced clustering of times. Order clustering is evidently absent. Visual inspection of stem plot on Fig 7a shows that all largest spikes are single, isolated events. This may mean that order clustering and common clustering need not appear in a correlated mode.

We performed similar analysis for the last five centuries of Zielinski data and found no manifestation of any kind of clustering. However, the entire data set, treated as a whole, still shows significant tendency to self-similar clustering. At this general level, this fact confirms that the episodic behavior is a common phenomenon and not a peculiarity of Kamchatka Holocene data.

Discussion

We have established above the statistically self-similar episodic tendency of volcanic product output from a volcanic region of Kamchatka (of the size of 700 km) over the range of time intervals from 100 to 10000 years. This observation is the main result of our study. In the Introduction we have already listed a number of examples of the episodic temporal behavior for time scales of 2-50 Myr. Our results significantly broadens the range of time scales where this kind of behavior was observed; they suggest that self-similar episodicity may represent a characteristic feature for island-arc volcanism for time scales from 100yr to 100 Myr

We wish to emphasize that in the present paper, our primary interest is the adequate *description* of the temporal structure. We believe that such things as the mechanism of volcanic phenomena in general and deep causes of an episodic behavior in particular can be discussed in detail only after establishing actual observational properties of the temporal (or, better, spatio-temporal) structure of the volcanic process. These properties may be complicated and even counter-intuitive, and definitely deserve attention per se. When understood at a phenomenological or descriptive level, these properties may supply a key evidence for the deduction of the mentioned mechanism. We do not intend to go so far, but will mention nevertheless a number of possibilities.

When looking for explanations for our observations, we must take into account an important aspect of the episodic behavior: the synchronous manner of observed variations over the territory. One well-know case of such a behavior is the phenomenon of synchronous eruptions of several volcanoes after a great earthquake (e.g. Darwin 1840). It gives us one possible explanation of the phenomenon, namely, the external excitation or acceleration of eruption processes by (transient) seismic wave or by jumps of static elastic deformation (their origin may be seismic or not). Both possibilities cannot be excluded; but this mechanism seems somewhat doubtful for cluster durations of tens or even hundreds of years.

Another possible synchronizing factor is the fluid pulse from the mantle. Blot and Grover (1967) and Blot (1973) have shown quite convincingly how, in several quite similar cases, some signals, that probably represent such pulses, propagate up along the New Hebrides Benioff zone and excite, in succession, a deep earthquake, an intermediate earthquake, and at last an eruption. Similar but somewhat less compelling correlations were found also for many other island arcs (Blot 1981). The assumed pulses may easily be imagined to have their effect over an area of the size of many hundred km along an island arc; and the durations of such pulses or of their groups may be imagined to vary in a wide range. Instead of ascending fluid, a portion of silicate liquid may propagate as well (or, more generally, the first may boost the second). In fact, most of the explosive eruptions in general and those under consideration (Table 1), in particular, are believed to have been triggered by injections of some new portions of ascending mafic melt into shallow silicic chambers (Sparks and Sigurdsson, 1977; Volynets, 1979; Volynets et al., 1999). So the observed clusters of larger explosive eruptions might have been caused by synchronous ascent of deeper mafic melts under the 700-km-long volcanic belt. Both with fluid or magma pulses, some specific mechanism akin to non-linear diffusion may operate.

A contradiction was noted (Gilluly 1973) between the uniform or moderately variable plate motion velocities and the highly episodic character of magmatic output. On the other side Rea and Scheidegger (1979) try to explain variations of volcanic output in island arcs just by the variations of relative velocity of plates. Of course, variations of plate velocities and/or of the amount of hydrated sediments that are dragged into the mantle by a subducted slab may really perturb the production rate of the mobilized silicate material that is being generated during subduction. However, this source of episodicity does not seem to be the principal one.

All mentioned possible explanations of the self-similar episodic behavior of explosive eruptions are based on some external forcing. Essentially, they merely shift the problem of the causes of self-similar temporal structure of volcanism to some other, even less explored phenomena. One can imagine, alternatively, that this behavior constitutes some intrinsic property of the volcanic process over a territory. Roughly speaking, explosive eruptions can represent cooperative phenomena, and a cluster of eruptions may be a sort of epidemic. Pelletier (1999) proposed some mechanisms, acting at "roots of volcanoes", that might result in this kind of behavior. Although his particular explanations may look speculative, they illustrate a mode of thinking that eventually may give us a real key to the problem.

We noted that as a result of a wildly irregular behavior, the output of volcanic products represents a difficult object in terms of data analysis: variations of empirical output rates must be huge, and must depend systematically on the duration of the interval of estimation. These rates are expected not to stabilize when this duration increases. On a finite Earth with a finite history duration, these phenomena of course must have an upper limit of scale in terms of both eruption/burst size and longest relevant time scale; but we are far from establishing these boundaries at present. The hint to existence of such boundaries is the tendency to saturation manifested in the size-frequency trend of Simkin and Siebert (1994, Fig. 10) for largest eruptions; this saturation is however a «soft» one and does not show any definite upper limit.

Three separate facts should be mentioned, each indicating that the episodic/clustering behavior is far from being a simple phenomenon. One feature is common for the Holocene Kamchatka data and for Zielinski data. In both cases, fractal dimension D_c seem to be lower at the shorter-delay part of the time scale in question: 25-800 yr. out of 25-10000 yr. in the first case, 1.5-30 yr. out of 1.5-2000 yr. in the second case. Technically, this means that our data, treated as point sets, are not strictly self-similar fractals. Their dimension depends on scale: the shorter the scale, the stronger is the expression of the self-similar behavior. In both cases, however, the volcanic output shows a generally fractal behavior over the entire range of scales in question. Another feature, common to historical Kamchatka data and Zielinski data, is the temporal variation of clustering tendency: initial data can be divided in two temporal segments with the different clustering behavior. At last, order clustering, well manifested in Holocene Kamchatka data, and weakly expressed in the first subperiod of historic Kamchatka data, is definitely absent in Zielinski data and in the second subperiod of Kamchatka historic data. In the last two cases, visual inspection even suggests that the largest events might prefer to appear in isolated manner ("negative order clustering"). Unfortunately, the data volumes are prohibitively too small to study these interesting tendencies in any detail..

Conclusions

For the Holocene Kamchatka explosive eruptions, the power-law frequency-size relationship is shown to be a reasonable description of data, and its parameter *b* is estimated, with a certain accuracy (*b*=0.65±20%).

For the same data set, the following results are demonstrated statistically (that is, the particular, clearly specified significance levels are calculated):

(1)The sequence of eruption times is not purely random; instead it shows clustering tendency. For a particular range of time scales (100-800 yr) this clustering is statistically self-similar (fractal); the numerical parameter “correlation dimension” (D_c), that defines the degree of clustering of eruption times, is estimated, with a definite accuracy (D_c=0.71±0.14).

(2)The sequence of eruption volumes (or the temporal evolution of the output of volcanic material) is fractally clustered (“episodic”) within the entire studied range of time scales of 100-10000 yr. This clustering is specified by the smaller value of correlation dimension (D_c=0.67±0.23); the clustering tendency is manifested also in the value of Hurst exponent H=0.58±0.06 for the time scales 2500-10000 yr.

The clustering of the sequence of eruption volumes is caused by two different effects: (1) the common clustering of eruption dates (the tendency known for other data sets) and (2) the newly established effect of “order clustering”, or specifically enhanced clustering of larger eruptions. These effects operate in a joint, correlated manner.

More or less clear manifestations of the self-similar clustering tendency are also found for two more event catalogs: historical eruptions of Kamchatka for 1735-1993, and events of the 0-1990 AD list of Zielinski et al (1994).

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