Mathematical Geology, Vol. 38, No. 6, August 2006 (© 2006)
DOI: 10.1007/s11004-006-9044-x

Hypsometric Curves as a Tool for Paleosurface Mapping[1]

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A procedural paleosurface mapping tool, using hypsometric curves and digital elevation models, was developed and applied to three hydrographic basins that erode common areas in the coastal ranges of Brazil, in southeastern South America. The method consists of identifying areas favorable for the occurrence of paleosurfaces and their corresponding surficial formations as correlated to specific erosion events. Attributes from hypsometric curves, and logarithmic functions fitted to proper curve segments, are combined with common morphometric properties of paleosurfaces given a set of criteria. Zones satisfying the criteria are mapped as pertaining to specific paleosurfaces. Field survey confirms the agreement between predicted and observed occurrence and absence of paleosurfaces.

KEY WORDS: Surficial formations, relief modeling, hydrographic basin, Paraná, Serra do Mar.

INTRODUCTION

Paleosurfaces can be recognized in geologic and geomorphic records at regional scales. They represent time intervals long enough for distinctive correlated features to develop, and must be distinguished among themselves by their descriptive attributes (Widdowson, 1997). Current recognition and classification limitations make paleosurfaces difficult to map, be they exposed, covered, buried, or exhumed. A semiautomatic quantitative procedural paleosurface mapping method is therefore desirable. The search was driven toward the statement and validation criteria to identify areas where the occurrence of paleosurfaces and their corresponding surficial formations are expected. These formations are correlated to specific climatic and transport conditions during erosional stages, when erosion and transport were limited by low gradient, stable base level, progressive leveling by weathering and chemical removal. The easy availability of digital elevation

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data and processing capacity stimulate the use of hypsometric curves and digital elevation to represent relief forms and models as basic mapping tools.

In hypsometric curves, ordinate values represent altitude, expressed as elevation above sea level, and abscissa values represent cumulative frequency or areas under given contour intervals around watersheds or summit points. Being a summation of area intervals per altitude, the region under a hypsometric curve therefore represents the amount of rock between a river outlet and the erosion surface. Strahler (1952a) refers to it as the *hypsometric integral*. The shape of a hypsometric curve changes from concave-convex to concave as the basin reaches the equilibrium (mature) stage. Concave curves denote planations.

Although a very sensitive attribute of basins with different slopes, a hypsometric curve may be strongly influenced by diverse geometry of the drainage network (Willgoose and Hancock, 1998). This sensitivity turns hypsometric curves into a recognized tool for calibrating and comparing landscape evolution models (Hancock and Willgoose, 2002), for comparing catchments that differ in extent and steepness, for evaluating the geomorphic maturity of catchments and landforms (Strahler, 1952b, 1956); and for identifying connections among erosion processes, catchments geometry and network form (Willgoose and Hancock, 1998). Research was then addressed to demonstration that hypsometric curves can be used to identify paleosurfaces in multi-history landscapes by manipulating their attributes.

REGIONAL SETTING AND DATA

The method used here was applied to three neighboring hydrographic basins in southern South America (State of Paraná, Brazil; Figure 1): the Nhundiaquara, Capivari, and Alto Iguaçu basins. The basins erode common range areas in the Serra do Mar, bear Cenozoic superficial formations but have very different base levels. The Alto Iguaçu basin drainage flows inward on the continent to the Rio de la Plata basin, with very low altimetric width, watersheds with flat zones around 950 m and large alluvial plains around 850 m of altitude (Figure 2a). The Nhundiaquara basin drainage flows directly to the Atlantic Ocean, with alluvial plains near sea level, steep escarpments and flat shoulder-like steps around about 950 m (Figure 2b). The Capivari basin drainage discharges in the Atlantic Ribeira basin; it has an alluvial plain around 600 m, flat watershed around 900 m and flat shoulders around 1200 m. The region surveyed is under subtropical conditions, with mean annual temperatures between 18°C and 20°C and mean annual rainfall of about 1,500 mm. Climate is *Cfb* in the Köppen classification.

Digital terrain elevation data from Shuttle Radar Topography Mission (SRTM, 2003) and field-documented geomorphic features, paleosols and correlative deposits were used to define paleosurfaces and identify landscapes whose advanced erosion cycles are governed by their base levels and limited by transport capacity.

Figure 1. Geographical location of the study area.

GEOLOGICAL SETTING

The Alto Iguaçu hydrographic basin developed on terrains of Paleoproterozoic gneisses, Neoproterozoic granites, and Cenozoic sedimentary rocks. Capivari hydrographic basin terrains present a variety of geological types including Neoproterozoic phyllite, quartzite, carbonate, gneiss, granite, and parts of the Neogene Guabirotuba Formation. Nhundiaquara basin terrains include Proterozoic gneiss and granite, and also Quaternary sediments. Brittle and ductile structures in the three basin areas include faults and folds mainly striking northeast. Cretaceous NE-SW diabase dikes cut older structures (Figure 3, modified from Biondi, Cava, and Soares, 1989).

Southern Brazil remained under subtropical semiarid to humid conditions from the breakup of the Gondwana supercontinent through the Cenozoic. Vegetation cover fluctuated between savanna, grassland and tropical forest. Remnants of three extensive erosion paleosurfaces have been described in the region (Figure 4). The first, probably a very restricted Paleogene summit planation is found between 1,200 and 1,400 m in the mountains of Serra do Mar (Freitas, 1951; Bigarella, 1954) and has been considered correlated to the South America Planation of King (1956). In the western coast, it presents typical laterite crusts (Serra das Almas; Soares and others, 2002). The two other paleosurfaces are more preserved and have been described from the First Paraná Plateau. The older, probably Neogene Alto Iguaçu surface was described in Almeida (1952; after Fuck and others, 1969).

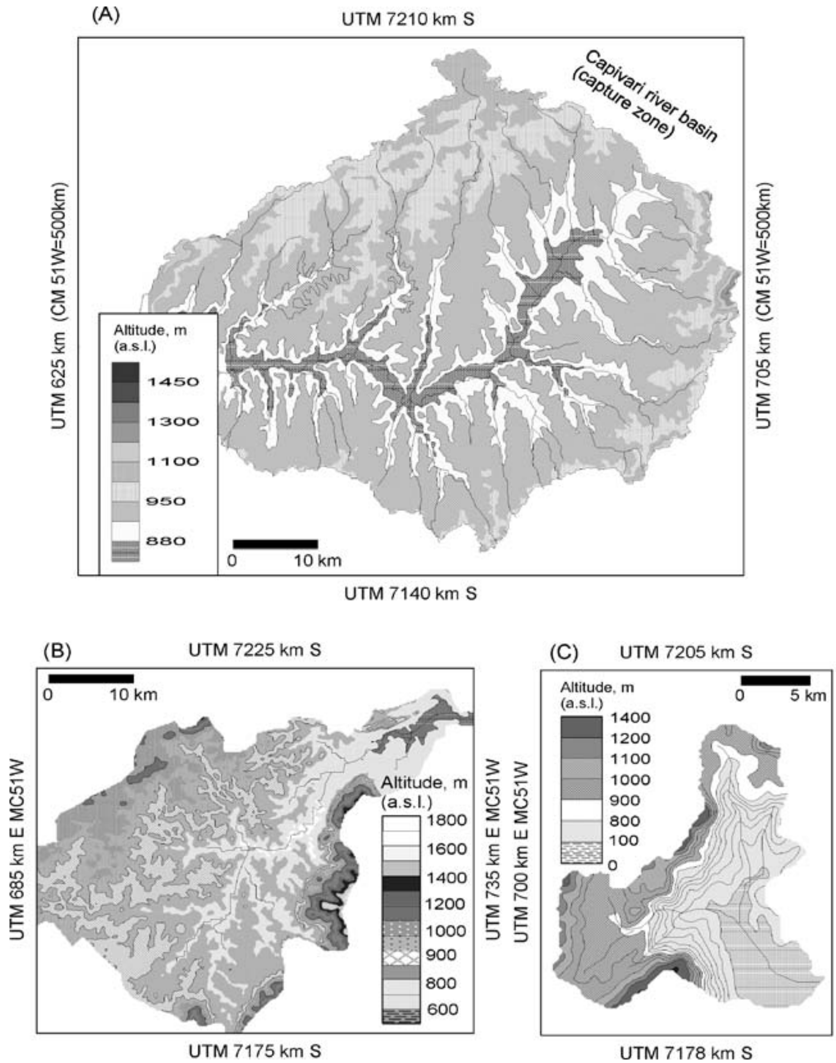


Figure 2. Relief contours in meters above sea level of the hydrographic basins used as example.

It is supposed to be contemporaneous with the immature Neogene sediments of the Guabirotuba Formation, which means that the surface is buried by no more than 100 m of sediments. The youngest, Curitiba paleosurface, corresponds to an erosion surface that sculptured sedimentary rocks of the homonymous basin and parts of the exposed basement. This paleosurface, above 900 m, is surrounded by high watersheds that remained unplanned (Figure 2). Its correlative deposits

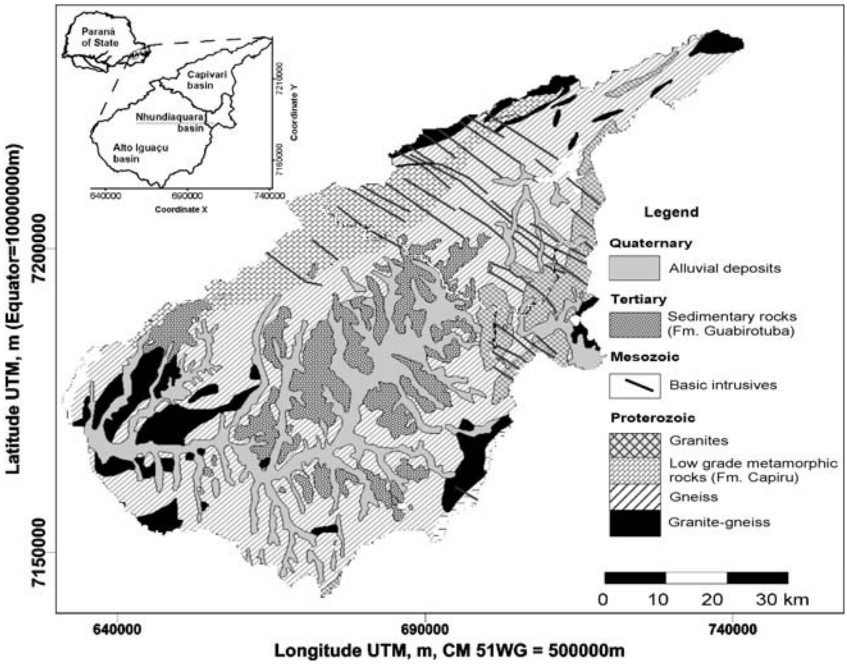


Figure 3. Simplified geological map.

include very immature sediments such as channel-fill deposits; typical *in situ* or allochthonous paleolatosol formations with fine ferruginous crusts (Salamuni, Salamuni, and Ebert, 1999) are superposed to either correlative deposits or correlative erosion surfaces (Soares and others, 2002).

Topography and geomorphic features are different for the three basins: the common features are planed summit points, shoulders around mountain massifs and plane-top watersheds. Those features can be seen everywhere, but are scattered

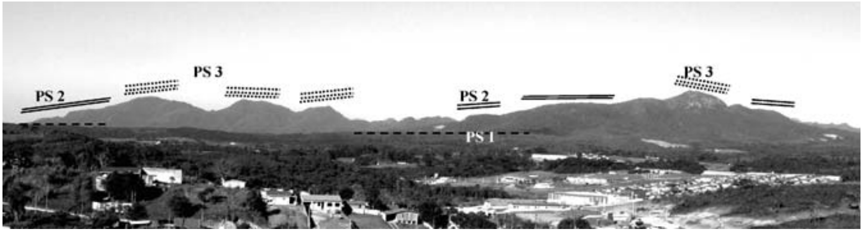


Figure 4. Panoramic views of geomorphic features considered correlatives with Curitiba (PS 1) Alto Iguaçu (PS 2) and South American (PS 3) paleosurfaces.

and isolated remnants (Figure 4). Some remnants have been mapped only locally and classified because of the difficulties to find criteria to discriminate and correlate the paleosurfaces.

METHODOLOGICAL DEVELOPMENT

Paleosurface remnants are large areas with small altitude changes that appear as flat segments in hypsometric curves. Surface formations covering such areas denote uplift of ancient base levels. Each step or terrace in a hypsometric curve is then assumed to represent a remnant segment of a hydrographic basin that approached its base level. The relief gradient in a hydrologic basin dominated by a fluvial erosion base level is directly proportional to the distance from the river mouth and proportional to the altitude, or inversely proportional to the distance from the heads, and can be modeled by a logarithmic function.

Altimetric data (SRTM, 2003) were used for analysis and construction of histograms and hypsometric curves. The points $P_{(x,y,z)}$ are geo-referenced in coordinates X and Y, have a Z elevation above sea level (altitude H) and represent an unit grid-area sampled (180 m × 180 m). The histograms for the several classes of H were constructed and the cumulative frequency was converted to area (A) between the highest point and the H level. A hypsometric curve represents the cumulative frequency of points with altitude H within class intervals, from the highest to the lowest point. In order to compare different basins, the H and A values are converted to fractions of their respective maximum values to give comparable hypsometric curves.

The experimental hypsometric curves were segmented between minimum (H_{omin}) and maximum (H_{omax}) observed values of altitude in histograms; the minimum (H_{omin}) is taken from the modal lower class boundary, because this modal class represents the larger area of altitude interval; this class therefore represents the lower mean slope interval and as a consequence, the closest preserved level to the early (eroded) base level of the hydrographic basin. The operation is taken in order not to consider altimetric data related to erosion events governed by successive base levels.

A logarithmic function was fitted to each segment by regression analysis using the least-squares method:

$$H = a - b \ln(A),$$

where H is the minimum altitude to which area A is circumscribed; a and b are the estimated parameters. The a is the maximum predicted altitude, and b is a coefficient that is proportional to the average declivity in the altitude interval. This allows us to calculate the minimum paleosurface altitude. Consider A to be the total basin area; other parameters calculated were the vertical separation from the current base level, and the mean slope as a function of the altitude

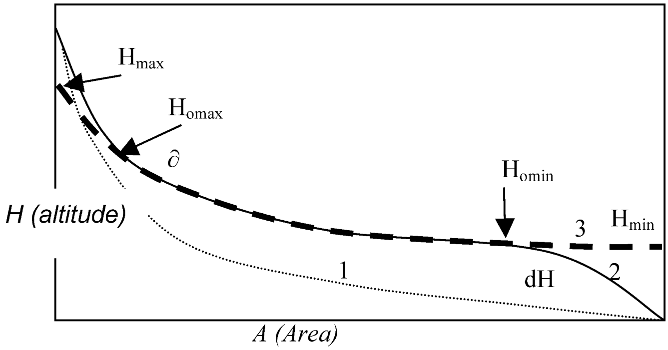


Figure 5. Hypsometric curve attributes considered for paleosurface mapping: observed curve (2), ideal curve (3), lowest topographic position of local base level alluvial deposits (1), plotted as vertical distance (dH) function of H; expected slope (∂) as a function of H, minimum and maximum observed and estimated H values (H_{omin}, H_{min}, H_{omax}, H_{max}).

to be calculated (Figure 5). The minimum coefficient of determination (squared Pearson r) accepted for all fitting operations was 0.99.

Other Attributes

Slope. Very low slope values are typical of paleosurface remnants, and they may be estimated as a function of the altitude in the basin geometry models. Such estimations are most difficult to accomplish because the slope depends on the basin geometry and early geometry may only be approximated by present form. It is known that the mean slope increases progressively upstream and high values are found in shoulder remnants near mountains. The conversion of the surrounding area to distance from summit points was done in a simple way: It has been assumed that N summit points lie in segments around the basin (Figure 6). In order to have no space problems, six points, for example, were considered to be equally distributed, remembering that the goal is to define the averaged slope for a given interval of altitude. Contour lines of H_i are substituted for by concentric circles with radius R_i. The area A_i of the basin is equivalent to double the area of the circle. Considering the symmetry, a half area of a hydrographic basin bears all classes of altitude with a mean distance R from the summits. The average slope at a mean distance $R_{(H)}$, where the set of points with altitude H exists, may then be calculated with the first derivative of a hypsometric curve, substituting A by $(2\pi R^2)$.

$$H = a - b\ \ln(2\pi R^2)$$

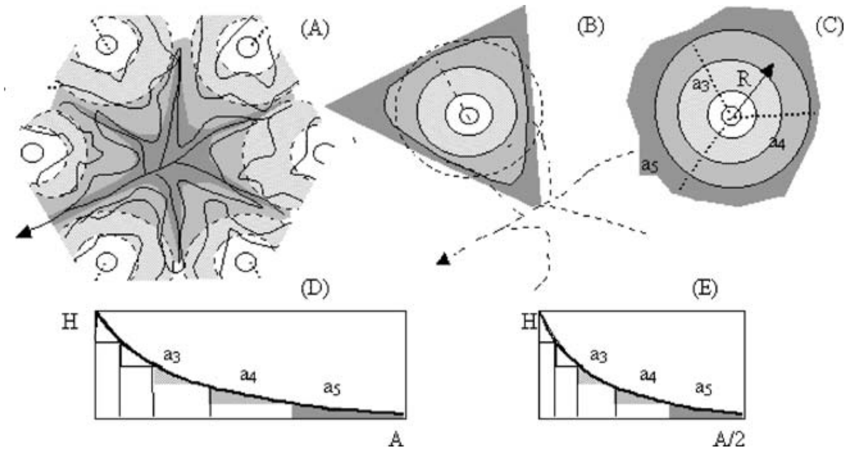


Figure 6. Schematic illustration of the reasoning associated to the substitution of area A, which envelopes elevations higher than H, by concentric circles around remnant summit points, in order to estimate mean slope: a1, 2, ..., 5, are the half area between contours values of H. (A) – Simulated hydrographic basin with drainage net, contour curves (lines) and hypsometric zones regularized as circles (shaded). (B) and (C) – half area of figure (A) rearranged and grouped as concentric regular curves; (D) and (E)— Hypsometric curves for (A) and (C) showing that $H = f(A/2)$ may be substituted by $H = f(R)$.

Taking the derivative and going back to the relation of H and A, we have:

$$\partial = \frac{2b}{\sqrt{\left(\frac{1}{2\pi}\right) \cdot \left[\exp\left(\frac{H-a}{-b}\right)\right]}}$$

Using this equation, the expected values for slope δ as a function of H were estimated for each point, $P_{(X,Y)}$ in the worksheet, for comparison with georeferenced slope values obtained from digital terrain model.

Convex Relief Features. In paleosurface remnants, dissection is absent or negligible and convex reliefs dominate, with low profile curvatures (Dalrymple, Blong, and Conacher, 1968). This implies negative or null slope curvature (C $\leq$ 0) in the digital terrain model.

Convex Watersheds. paleosurface remnants are preserved in watershed remnants; so the plan curvature of the digital terrain model is expected to be positive, with curvature radius in the watershed. Incision zones will have negative curvature, with radius in the valley.

Elevation Variability. For the same reason, the local elevation variance (V) is minimum within paleosurface remnants. Shallow hydraulic incisions cause small lateral change in contour lines, which results in low and dominantly convex downward curvatures ($L \leq 0$). Outside the remnants of paleosurfaces, local valley incision is more pronounced, giving higher values of topographic unlevelling. This space variable was surveyed as the difference between maximum and minimum altitude values obtained within appropriate searching windows.

Paleosurface Prediction

The morphometric parameters considered for paleosurface prediction were then established: (1) maximum calculated elevation for a fitted segment; (2) minimum fitted segment elevation; (3) altimetric distance to the nearest neighbor base level; (4) maximum expected slope as a function of altitude (with some tolerance); (5) profile curvature minus tolerance >0; and (6) plan curvature minus positive tolerance >0. Combining data as spatial variables, these morphometric parameters were used to discriminate paleosurface areas. They were discretized into 1 or 0 according to whether they satisfied the criteria or not, a tolerance value being considered, and recombined by the product of variable value (using the *and* operator). As a result, the value 1 represents the expected presence of the paleosurface, whereas 0 represents its expected absence. Large sets of contiguous points were marked as paleosurface presence. Field surveying was carried out to validate the results. Surficial formations, alluvium or colluvium deposits, and paleosols were the diagnostic criteria used in field work to identify paleosurfaces.

RESULTS

The current base level is different for each basin, but all hypsometric curves show a significant correlative flat or terrace with similar slope and altitude in their middle parts. Such flats or terraces are indicative of paleosurfaces or relief-leveling related to ancient base levels. The very high base level of the Alto Iguaçu basin is sustained by the erosion barrier that marks the transition between the Paraná basin and its basement. The Ribeira River represents the base level for the Capivari basin. For the Nhundiaquara basin, the Atlantic Ocean is its base level. The hypsometric curves are presented in Figure 7. Steps or flats are easily seen around an altitude of 950 m above sea level.

The hypsometric curves of the basins in oriental Serra do Mar (Alto Iguaçu and Capivari) resemble each other in shape, but differ in valley incision, which is deep in the north towards the Atlantic (Capivari), and smooth in the western inner part of the continent (Alto Iguaçu). Active tectonic features prevail in the central parts of the Atlantic exoreic basins, being represented in the hypsometric curves as anomalously extended linear steps or abrupt slopes, especially in the Nhundiaquara basin.

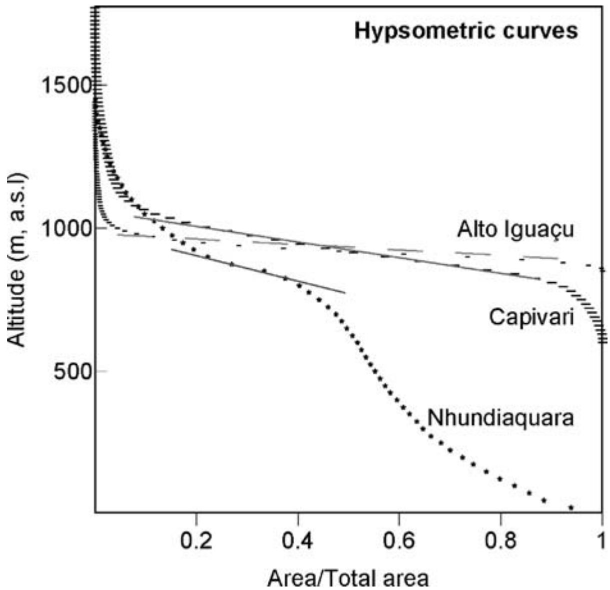


Figure 7. Hypsometric curve for Nhundiaquara, Capivari, and Alto Iguaçu hydrographic basins, with observed altitude values (H) and cumulative area (A), re-scaled to 1: area above altitude H divided by total area.

Modal values from histograms (Figure 8) indicate the altitudes of maximum preserved areas of the paleosurface and the altitudes for preservation and curve segmentation. In the Alto Iguaçu basin, two modal values occur around 850 m and 1,050 m. They correspond to flats that are unnoticeable from simple hypsometric curve inspection. In the two other basins, the second modal value occurs only in the upper set of points. These modal points are considered as the lower elevation points with maximum paleosurface preservation: two paleosurfaces are identifiable in the hypsometric curves.

The three basin curves show two flats that are indicated by the distribution of observed $H \times A$ points. In order to fit logarithmic curves to the point sets in these flats, two concave upward segments were delimited by the lower and upper elevation points of each segment (Table 1; Figure 9). Fitting curves for the two set of points (Figure 10) for the three steps and three basins provided the a and b parameters, with goodness of fit $r^2 > 0.99$. The fitted curve slope and curvature are larger for paleosurface 1 in Nhundiaquara basin, possibly because its modeled portion lies closer to the water heads than in the other two basins. Alternative interpretations for the larger curvature are tectonic tilting or higher rates of base level lowering.

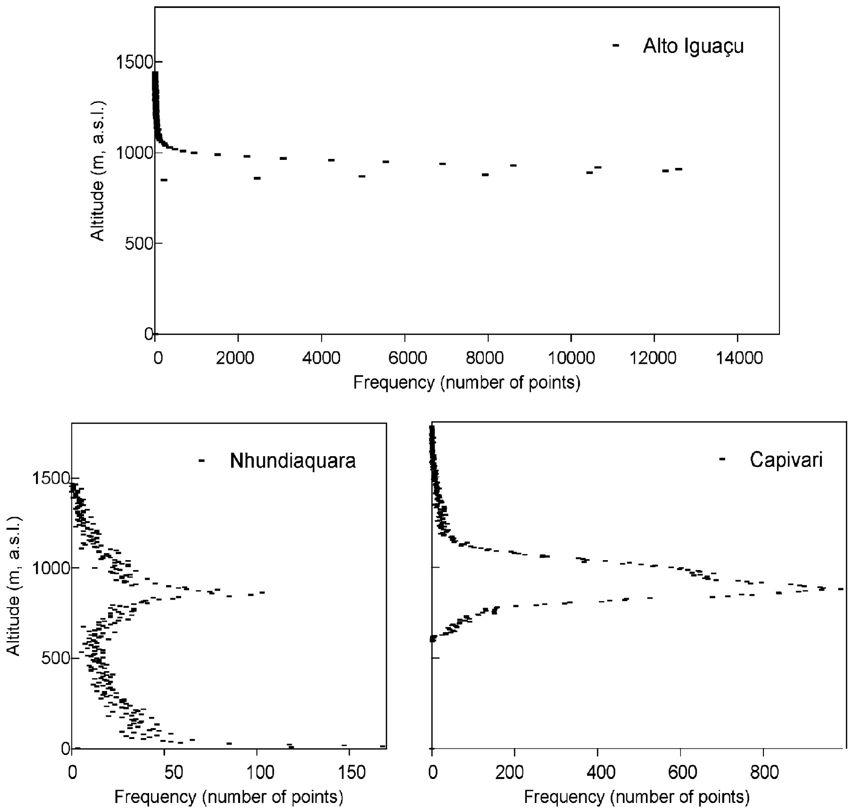


Figure 8. Simple frequency distributions for Alto Iguaçu, Capivari, and Nhundiaquara basins: number of points (pixels) for each class interval of altitude above sea level, before conversion to area.

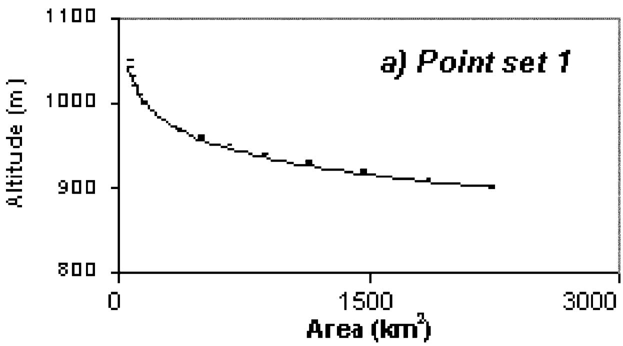
Both logarithmic curves fitted to the Alto Iguaçu basin data, when extended to the minimum estimated altitude, show that paleosurface 2 (defined by the higher set of points) projects itself below paleosurface 1 (defined by the lower set of points). This interesting finding indicates that the older paleosurface 2 was tilted and buried, as expected, by up to 100 m of Neogene alluvial deposits of the Guabirotuba Formation. Extrapolated paleosurface altitudes fall below 850 m along the Alto Iguaçu basin axis, marking the base of the overlying Guabirotuba formation (Figure 11).

Predictive maps for paleosurfaces 1 and 2 are presented. The map contours smoothed results. The smoothing was done by taking the moving average of 1 (present) or 0 (absent) values in a 3×3 matrix; the result is interpreted as the expected probability of points with the presence of paleosurface: when lower then

Table 1. Limits and Fitted Parameter Values for Logarithmic Models

Hydrographic basin	Provided altitude of point sets (m a.s.l.) Lower	Higher	$H = a + b \ln(A)$ Estimated parameters a	b	r^2
Paleosurface 1					
Nhundiaquara	840	1240	3791.4	−162.9	0.9965
Capivari	950	1150	2614.1	−83.0	0.9954
Alto Iguaçu	900	1050	1690.9	−36.6	0.9939
Paleosurface 2					
Nhundiaquara	1160	1350	3106.6	−119.9	0.9964
Capivari	1310	1515	3484.2	−127.8	0.9949
Alto Iguaçu	1070	1240	3006.9	−111.0	0.9994

Note. Lower and higher points refer to the high and low values of H for the boundaries of the selected segment or point set considered for paleosurface and used for regression analysis.



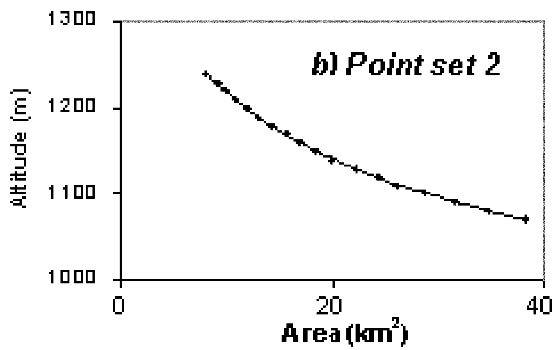


Figure 9. Two concave upward sets of points for Alto Iguaçu basin taken from hypsometric curve: (a) lower set corresponding to paleosurface 1; (b) upper set corresponding to paleosurface 2.

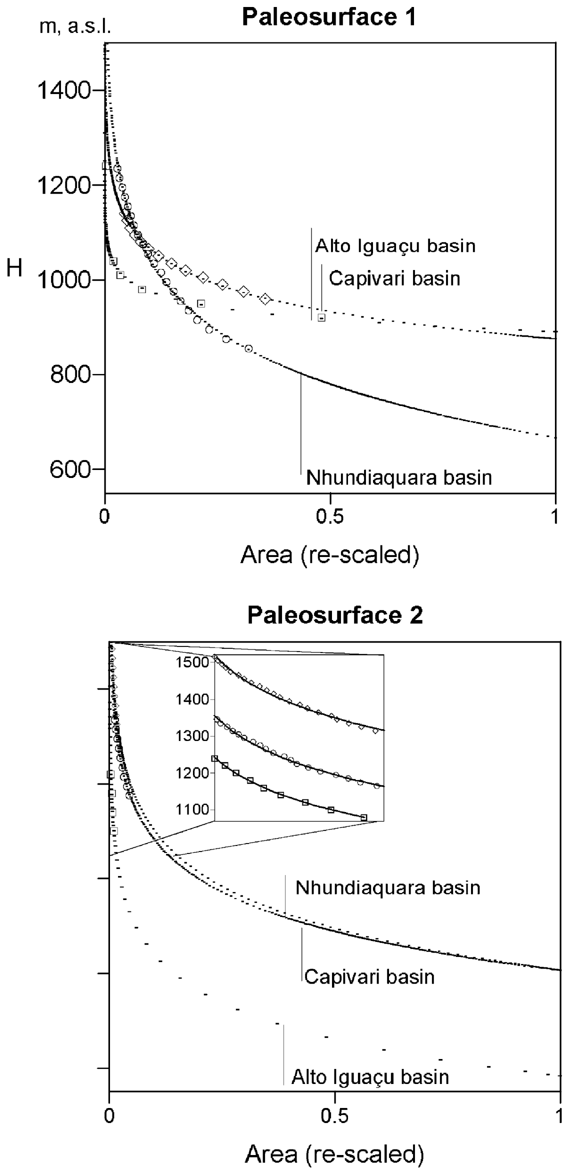


Figure 10. Hypsometric curves for the two modeled surfaces, 1 (A) and 2 (B), for the three basins. Estimated (dots) and observed values (Alto Iguaçu-squares, Capivari – circles and Nhundiaquara – diamonds). Insert in Figure B shows in detail re-scaled values for paleosurface 2.

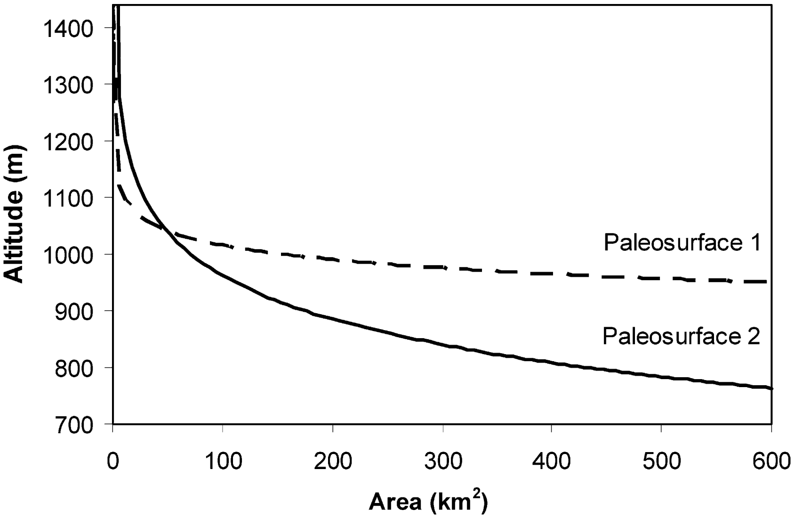


Figure 11. Comparison between two modeled paleosurfaces in Alto Iguaçu basin. Early paleosurface 2 is projected below paleosurface 1, suggesting that it is buried by Neogene Guabirotuba Formation, in the Curitiba sedimentary basin.

0.1, the paleosurface was considered absent; between 0.1 and 0.5, as possible (yes or no) and higher than 0.5 as predicted presence.

The extensive occurrence of paleosurface 1 remnants in the Alto Iguaçu basin reflects base level stability and slow erosion (Figure 12). Zones of expected occurrence of the older paleosurface 2 are near the mountains at the borders of the hydrographic basin (Figure 13). In the field, they appear as dissected shoulders dipping from the mountains toward the basins and bearing pebble pavements.

In Figure 12, zones of predicted occurrence of paleosurface 1 (Curitiba surface) fit very well with the field observations: occurrences of thick paleolatosol and ancient alluvial (or colluvial) deposits.

For operational reasons, the field work was done before the results and only in Alto Iguaçu basin. A comparison of the results is presented in Table 2.

The balance shows a good result, better when considering that uncertainty in the field arises mostly when low relief and surface formations are present but are not diagnostic of which surface it is part. The model identifies the paleosurface in 68% of field undefined cases; field undefined and present represent 93% of predicted cases of paleosurface 1.

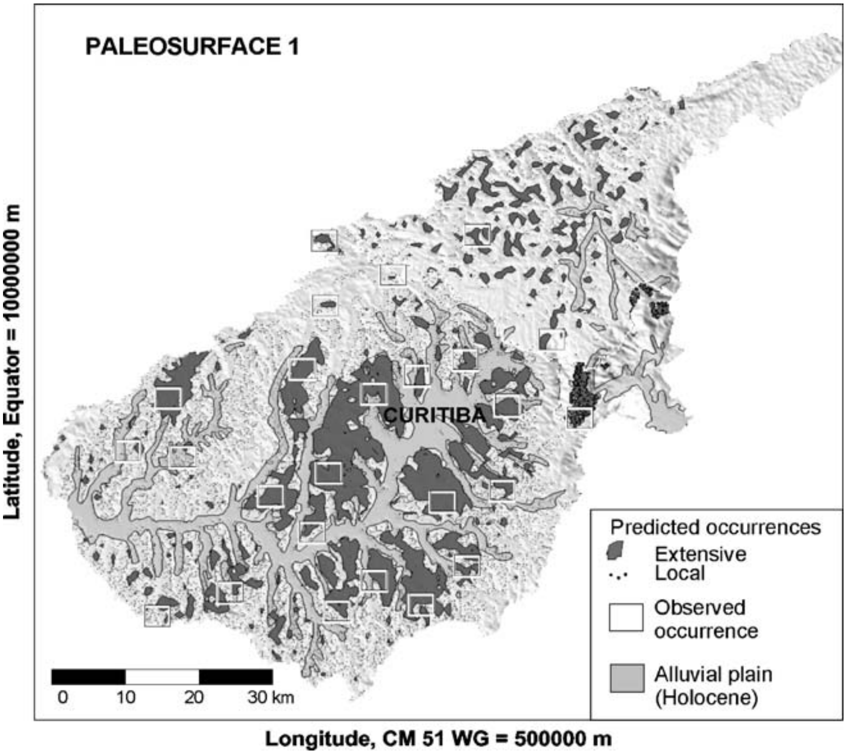


Figure 12. Predictive map for occurrence of paleosurface 1 (coordinates in meters).

CONCLUSION

Paleosurface remnants preserve relief attributes that may be followed in digital elevation models. Identification, classification and correlation of paleosurfaces in different hydrographic basins, at different scales and with varying degrees of dissection, are easily accomplished from hypsometric curves whose expected attributes might be taken as mapping criteria. Field observation on sampled points improves criteria and tolerance definition.

The method may be easily applied using available digital elevation data in a number of commercial software packages. Continental hinterland regions present extensive distribution of paleosurface formations that can be easily mapped, with significant implications in land use and mineral deposit formation.

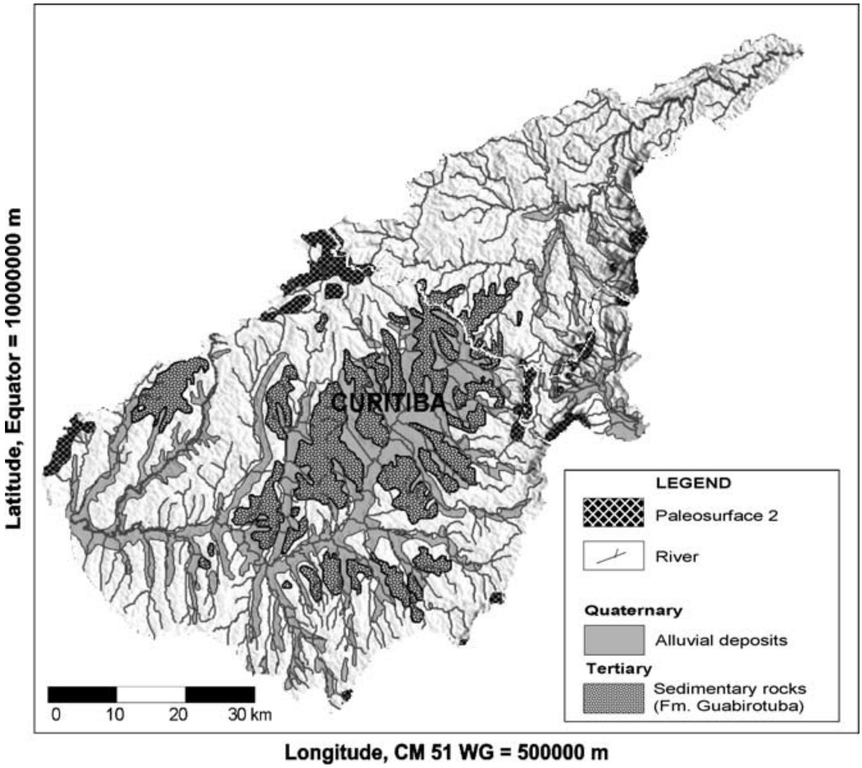


Figure 13. Predictive map for occurrence of paleosurface 2 (coordinates in meters); Guabirotuba Formation seems to occur over the paleosurface, as predicted by model and as have been hypothesized by some authors.

Table 2. Comparison of Results from Model and From Field Observations, Given as Fraction of Observed Number of Points for Paleosurface 1

	Field verification		
Model prediction	Present	Undefined	Absent
Present	0.54	0.68	0.09
Undefined	0.44	0.29	0.02
Absent	0.02	0.03	0.89
Total	1.00	1.00	1.00
Observed ($N = 111$)	39	28	44

ACKNOWLEDGMENTS

The authors thank the postgraduate program in geology at UFPR and colleagues for the many fruitful discussions, especially E. Salamuni and F.J.F. Ferreira; to the guest editor Hannes Thiergärtner and reviser for helpful suggestions that improved the understanding of the text; to CAPES, and IUGS – Geohost Program; SUDERHSA for the maps supplied; and to Priscila Kosop for English revision.

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