Rock Failure Mechanism / Механизмы разрушения горных пород

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Zonal type structures around the openings in highly stressed rock masses

Deformations and damages around workings and drill holes are usually zonal. They appear when strain in masses at great depth become recurrent and macro defects result in the areas, where principle compression tangential stresses take place causing a radial compression in the intermediate zones. The article presents a mathematical model of highly compressed rocks based on the principles of non-Euclidian mechanics and nonequilibrium thermodynamics as well as the methods of determining the parametres of the model. A satisfactory conformity between the results of the theoretical and experimental investigations has been obtained. The dependence of the principle structural parametres of the model on the affecting factors has been established as well.

Key words: mesostructures, rock failure, openings, zones, modelling.

Introduction

Failure conditions can take place in the boundary areas of the openings at high depths mining and drilling operations. In some cases the failure has a zonal character, where zones of tensile macrocracks alternate with relatively monolithic rock mass [1, 18] . Many attempts have been made to describe the zonal character of rock mass failure near openings based on classical mechanics [2, 16, 19]. But no one has been able to explain all the properties of zonal failure structures without the introduction of new assumptions in every new case.

Recently, a new gauge theory has been applied to solids to describe the whirl fields of plasticity in high energy conditions [6, 14, 15]. The main principle of gauge theory is incompatibility of the deformation in damaged solid. But it does not apply to the zonal failure phenomenon of rock masses near openings. In this paper we demonstrate an example of a description of the phenomenon by employing the gauge mathematical model. The description is based on the rock mass hierarchical block systems [10]. A rock sample in this conception is shown as the first level of a hierarchical system and the rock mass on the opening scale corresponds with the second one.

Mathematical model

Rock at a high depth is modeled by a faulted structure which is far from the state of thermodynamical equilibrium due to damage accumulation. Also it is subjected by compressive stresses at infinity. The boundary-value problem is formulated as determining the stress state of a weightless solid plane with damage. The stresses prescribed at infinity, model a gravity field. The plate contains a round hole that models an unsupported underground opening (Fig. 1). Due to the polar symmetry of the problem the equilibrium equations are as follows:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\varphi\varphi}\right) = 0, \sigma_{r\varphi} = 0, r_0 \le r < \infty, \tag{1}$$

where σ_{rr} is the normal radial stress, $\sigma_{\varphi\varphi}$ is the normal tangential stress, and $\sigma_{r\varphi}$ is the shear stress. At the boundary of the opening ($r = r_0$) and at infinity, the following stresses are applied:

$$\sigma_{rr} = 0 \text{ at } r = r_0; \sigma_{rr}, \sigma_{\varphi\varphi} \to \sigma_{\infty} \text{ at } r \to \infty, \tag{2}$$

where $\sigma = \gamma \times H$, γ is the rock density (N/m^3), and H is the opening depth (m).

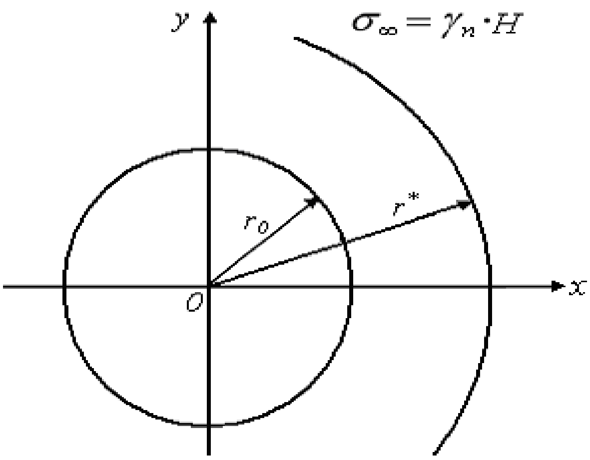

Fig. 1. Design diagram of an unlined opening task

The rock mass at a great depth is modeled by the material, where commonly the conditions of deformation compatibility ε_{ij} are not met:

$$R = \frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} - 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} \ne 0. \tag{3}$$

The damage parameter R is expressed by the equation [5]:

$$\Delta^2 R - \gamma^2 R = 0, \tag{4}$$

where Δ is the Laplace operator and γ is the model parameter.

As the problem is plane- and axi-symmetrical, Equation (4) in polar coordinates reads:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 R = \gamma^2 R. \tag{5}$$

The solution of Equation (5) decreasing at $r \to \infty$ is:

$$R(r) = aJ_0\left(\sqrt{\gamma}r\right) + bN_0\left(\sqrt{\gamma}r\right) + cK_0\left(\sqrt{\gamma}r\right), \tag{6}$$

where J_0, N_0, K_0 are the Bessel, Neumann, and MacDonald functions of zero order, respectively.

Non-classical boundary conditions and task solution

At the opening boundary the rock mass undergoes considerable destruction; therefore the damage parameter R should not equal zero. Assuming that all zones of rock destruction are equivalent and are of the same origin, we introduce the extremum of the function condition in the boundary $R(r)$ and the following zones of destruction. Therefore boundary conditions for the function $R(r)$ are:

$$R'(r)\big|_{r=r_0} = 0,\ R'(r)\big|_{r=r^*} = 0, \tag{7}$$

where r^* is determined experimentally.

The equation for the first invariant of stresses $\sigma = \sigma_{zz} + \sigma_{rr} + \sigma_{\varphi\varphi}$ is:

$$\Delta\sigma = \frac{E}{2(1-\nu)}R, \sigma \to 2(1+\nu)\sigma_{\infty}, r \to \infty, \tag{8}$$

with the determined function R, where E is the modulus of elasticity and ν is the Poisson ratio.

The solution of (8) gives the equations for the stress components:

$$\begin{aligned}
\sigma_{rr} &= \sigma_{\infty}\left(1 - \frac{r_0^2}{r^2}\right) - \frac{E}{2\left(1-\nu^2\right)\gamma^{3/2}} \cdot \frac{1}{r} \times \\
&\times\left[aJ_1\left(\sqrt{\gamma}r\right) + bN_1\left(\sqrt{\gamma}r\right) + cK_1\left(\sqrt{\gamma}r\right)\right]; \\
\sigma_{\varphi\varphi} &= \sigma_{\infty}\left(1 + \frac{r_0^2}{r^2}\right) - \frac{E}{2\left(1-\nu^2\right)\gamma} \times \\
&\times\left[aJ_0\left(\sqrt{\gamma}\cdot r\right) + bN_0\left(\sqrt{\gamma}\cdot r\right) - cK_0\left(\sqrt{\gamma}\cdot r\right)\right] + \\
&+ \frac{E}{2\left(1-\nu^2\right)\gamma^{3/2}} \cdot \frac{1}{r}\left[aJ_1\left(\sqrt{\gamma}\cdot r\right) + bN_1\left(\sqrt{\gamma}\cdot r\right) + cK_1\left(\sqrt{\gamma}\cdot r\right)\right],
\end{aligned} \tag{9}$$

where r is the distance from the centre of the opening to the selected point in the rock mass.

Stress calculation and criteria of failure

Solution (9) implies that stresses around the opening are spatially oscillating (Fig. 2).

The zones of failure appear in the areas where the conditions of cracking under compression are met:

$$K_I = 2\left(l/\pi\right)^{1/2} \cdot \left(\gamma_1 \cdot \sigma_1^0 - \gamma_3 \cdot \sigma_3^0\right) \geq K_{Ic}, \tag{10}$$

where l is the half-length of fracture faults of the rock mass and is assumed to be equal to the minimum half-length of a tensile macrocrack which is unstable in stress conditions (m); σ_1^0, σ_3^0 are the maximum and minimum of major stresses, respectively (MPa); γ_1, γ_3 are empirical factors; K_I is the coefficient of stress intensity (MPa × $m^{1/2}$); and K_{Ic} is the fracture toughness of rock material (MPa × $m^{1/2}$).

As a criterion function the following dependence is taken:

$$K(r) = K_I / K_{Ic}. \qquad (11)$$

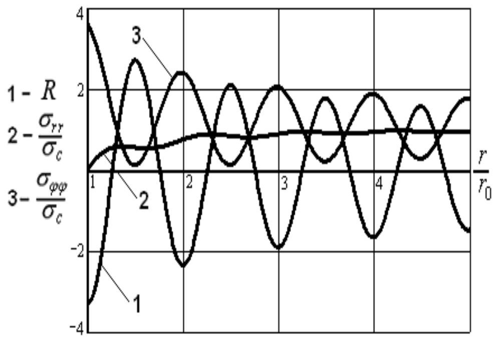

Fig. 2. Oscillating character of the stresses and R – functions in the rock mass surrounding the underground opening

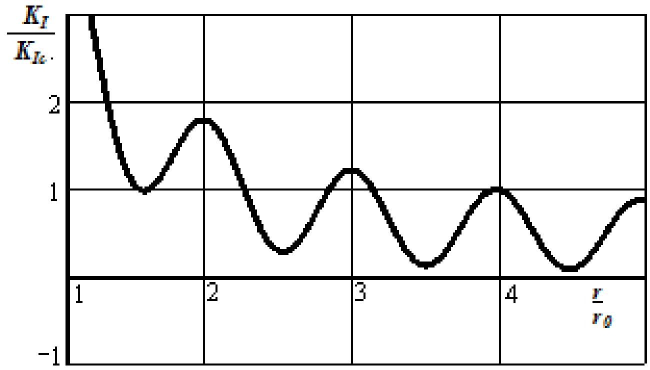

Fig. 3. Oscillating character of the criterion function

At K_I / K_{Ic} < 1 there is no failure around the opening; at K_I / K_{Ic} ≥ 1 the fracture starts to appear. The criterion function, as well as the stress components and damage parameter R, have an oscillating character (Fig. 3).

Model parameters

The research on rock mass zonal failure was carried out for an unlined opening. In order to run the calculations, algorithms and programs were developed that included formulas for calculating the damage function R(r), stresses, and criterion function $K_r(r)$. On the basis of the developed programs the experiment was carried out with three types of parameters being analysed.

The first type included parameters of model γ, c, which are determined from the experiment taking into account the all-around compression at a high depth. The second type includes parameters that characterize the mechanical properties of rock mass: E, the modulus of elasticity (MPa); ν, the Poisson ratio; σ_c, the uniaxial compression strength (MPa); and also the value of gravitational stresses in the rock mass, σ_∞ ($\gamma_r H$, where γ_r – is rock density, H – depth). The third type includes parameters in Formula (10) which characterize the cracking structure of the rock mass: the half-length of rock fracture faults l and the fracture toughness of rock material K_{Ic}, and also the coefficients γ_1, γ_3 (below he dependence γ_3 / γ_1 is used).

Parameter γ can be determined using the procedure of statistical analysis of the natural research results of the zonal failure process in deposits in Russia (the Far Eastern part, Siberia, Donbass) and China. A linear character of dependence between the relative distance from the opening contour to the middle point of the first failure zone and uniaxial strength of the rock was determined:

$$r^*/r_0 = 0.0083\sigma_c + 0.748, \qquad (12)$$

where r^* is the distance from the opening contour to the middle point of the first failure zone which has been found from experimental data.

The relationship between parameters γ and r^* / r_0 is linear too but avoid the long description the determination of it number can be achieved according with Table 1.

According to recent research the rock mass can be shown as a hierarchical block medium [3–8]. When the physical character of failure on the neighboring hierarchical levels is the same, the macrodefect size of the lower level can be determined as a mesodefect of the corresponding higher level [3, 5]. So this low is reflected in the conservation shear-tensile character of the rock failure on the neighboring levels of samples and mass in conditions of high stress [8].

Table 1

Meanings of the parameter γ of the model

Middle part of the 1st zone, $r^* = 1 - r / r_0$	$r_0 = 1.75$	$r_0 = 2.0$	$r_0 = 2.5$	$r_0 = 3.0$
0.7	26.486	20.279	12.978	9.012
0.8	20.313	15.552	9.953	6.912
0.9	16.080	12.311	7.879	5.471
1.0	13.050	9.991	6.394	4.440

The algorithm for the determination of the mathematical model's parameters consists in the next steps:

After the rock sample strength reaches the limit of strength σ_c, the limit of residual strength σ_c^{res}, Young modulus E, and Poisson ratio ν are determined. Then by using F$_o$rmula (12) the emplacement of the $_f^{irs}$t failure zone's middle point r^* / r_0 can be found. Then after the substitution of these data in the Table 1, the first correction parameter of t$_h$e mathematical model γ can be determined.

The maximum diameter r of the rock sample minerals d_{max} and maximum mesocrack disclosure $h^* \approx d_{max}$ are determined and after that the minimum half-length of the tensile mesocrack $l_{mezo} \approx (2,5 - 5) \times d_{max}$, is calculated. The critical half-length of the tensile macrocrack in the sample is determined by the formula

$$l_* = \frac{h_* \times E}{4\left(1-\nu^2\right)\gamma_1 \times \sigma_c}$$

and the stress intensity factor of the rock mass is calculated by the formula

$$K_I^{mass} = \gamma_1 \sigma_c^{rec} \sqrt{\pi l_{meso}^{mass}}.$$

Then, the stress intensity factors of both the rock sample and the rock mass are written in the equation of equality and the parameter of the model "C" is determined as a result of the calculation. For this purpose the equation of equality is applied:

$$K_I^{mass} = 2\gamma_1 \sigma_c^{rec} \sqrt{l_{meso}^{mass} / \pi} =$$
$$= K_I = \sqrt{l/\pi}\left(\gamma_1 \sigma_{\varphi\varphi} - \gamma_3 \sigma_{rr}\right), \quad (13)$$

where the "C" parameter is used in the right-hand part of Formula (13) (see (9)).

Programs for the determination of the last destruction zone (for lined and unlined openings: $\hat{A}_1$ and B_2 respectively) and programs for calculation of the radial length of fracture zones (for lined and unlined openings: C_l and C_2 respectively) were developed. Program charts with a brief description were also developed. The patterns of the changes in the zonal structure of rock mass failure depending on various factors were obtained.

The main parameters of the zone structure were identified: the number of zones of failure, the location of the furthest fracture zone from the opening boundary (the last zone of failure); the creation of relative critical stresses of failure zones; and also the value of the radial length of failure zones.

Results of research

As a result of the modelling experiment on the basis of the adopted mathematical model we determined that the parameters of the zonal structure depend slightly on the values of the elastic modulus of rocks E and the Poisson ratio ν. This conclusion corresponds to the data obtained during laboratory studies (when the elasticity varies in 10 times, the critical stresses of zone creation change by 2–5% on average). The research on fracture zones was carried out for rather solid rock (σ_c = 150 MPa) and for weak rock (σ_c = 15 MPa). In order to run the calculations, algorithms and programs were developed that included formulas for calculating the expressions of defectiveness R(r), stress, and the criterion function K(r).

The results of the solid rock case study are demonstrated by the application of the developed method to the problem of zonal failure in the Nikolaevskij ore mine (Dalnegorsk, Russia). The forecasted depth of development of the cracking zone is shown in Table 2.

Table 2

Forecasted depth of development of zonal failure in the Nikolaievskij ore mine

Number of failure zones	I	II	III	IV
Relative critical stress of zone formation	1.3	2.3	2.9	3.3
Depth of zone appearance (m)	520	920	1160	1320

The amplitude parameter “C” is dependent on the modulus of deformation and the Poisson ratio of the rock mass (Fig. 4).

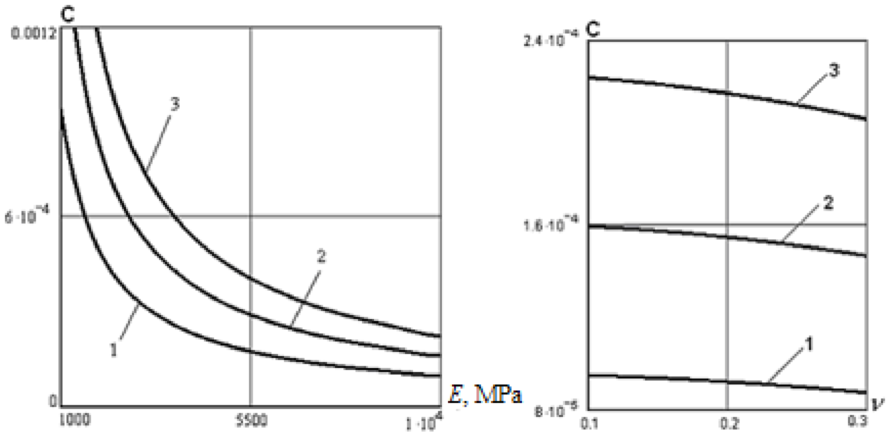

Fig. 4. Relationships of the model amplitude parameter with the deformation modulus E and Poisson ratio ν under conditions of different values of rock mass failure strength: 1 – K_{Ic} = 1.5MPa × $m^{1/2}$ × d; 2 – K_{Ic} = 2.0MPa × $m^{1/2}$ × d; 3 – K_{Ic} = 2.5MPa × $m^{1/2}$ × d

The precision of the correlation between theory and experiment has been estimated by comparing the results of in situ measurement of the radial displacements near the openings at high depth (Nikolaevskij ore mine) with the model calculation results (Fig. 5). It was determined that the difference between the forecasted and measured data was no more than 50% in the four radius field around the opening.

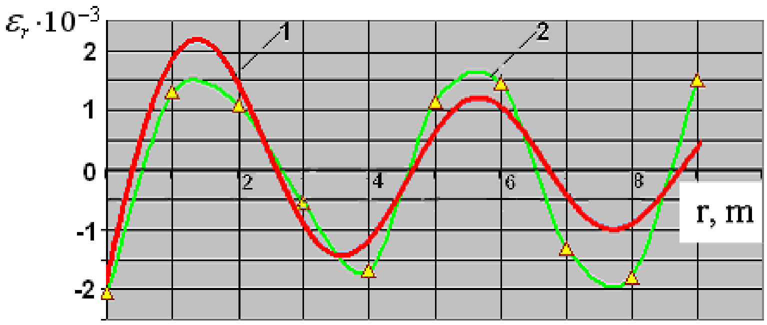

Fig. 5. Comparison between theoretical (1) and experimental (2) data of radial deformations

The comparison of the results of the analytical and experimental studies of the weak rock also shows their good convergence (Table 3). The research on zonal failure of the rock mass was carried out for an unlined opening.

Table 3

Comparison of the theoretical and experimental [9] results (unlined opening)

Parameter	Method	Elements of zonal failure structure			
		1st zone	2nd zone	3rd zone	4th zone
1. Location of the furthest zone boundary, r / r_0	Experiment	1.03	2.23	3.40	4.54
	Theory	1.28	2.17	3.09	3.97
	Deviation, %	24.3	–2.7	–9.1	–12.6
2. Relative critical zone stresses, σ / σ_c^{res}	Experiment	1.1	2.2	2.7	–
	Theory	0.95	2.1	3.1	–
	Deviation, %	–13.9	–4.5	14.8	–

It was determined that the basic factor that influences the parameters of zonal failure structure is the value of stresses that act within the rock mass (opening depth). With the increase of stresses, the number of failure zones increases and their radial length increases until it reaches neighboring zones. The closer the zone is located to the opening boundary, the faster the process. The boundary of the last zone of failure moves further into the rock mass (Fig. 6, left).

The parameters of the fracture structure of the rock mass also influence the character of zone failure. The radial length of the zones of failure decreases if the rock fracture toughness rises (Fig. 6, right). When the rock fracture toughness decreases, zones of failure appear at lower relative stresses and the distance of the last failure zone from the opening boundary increases.

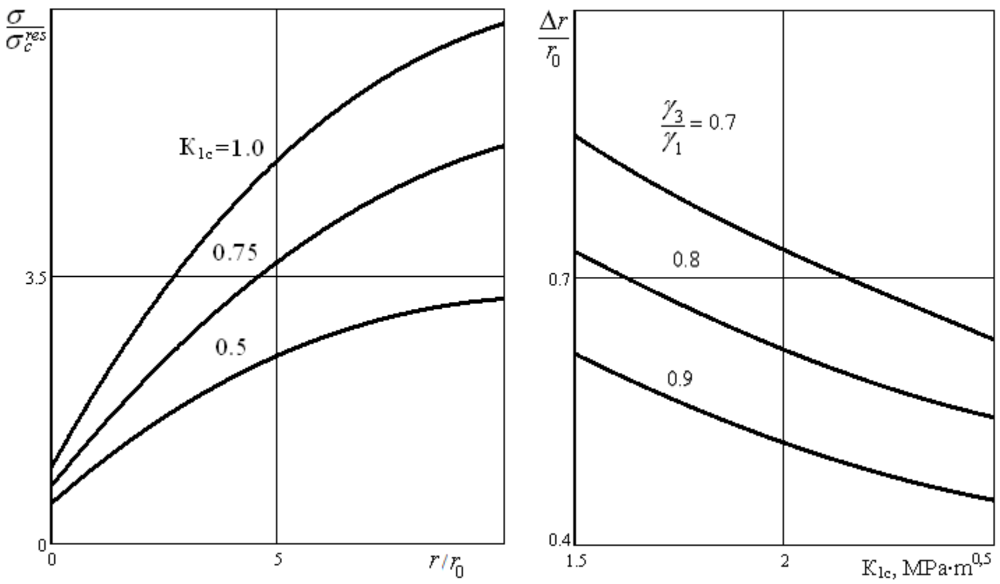

Fig. 6. Dependence of the position of the last failure zone on the relative stresses acting within the rock mass (left) and the radial length of the first zone of failure on the rock fracture toughness (right)

The activity of rock destruction also has a strong influence on the parameters of the zone failure structure. As the fracture faults length increases, the radial length of failure zones increases. This parameter decreases if the dependence γ_3 / γ_1 rises. Regularities determined for weak rock mentioned above are true for solid rock also.

Conclusion

The research conducted shows that as the depth of the opening rises the zonal character of rock failure becomes more expectable, which should be taken into consideration when designing a lining for such conditions.

A method for determining of the mathematical model parameters of zonal failure structure near to deep openings has been developed. A full quality and good quantitative correlation between theoretical forecasting and experimental research has been achieved.

Acknowledgments

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Механизмы разрушения горных пород / Rock Failure Mechanism

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Структуры зонального типа вокруг подземных выработок в сильно сжатых горных массивах

Деформирование и разрушения вокруг подземных выработок часто носят зональный характер. Механизм этого явления заключается в том, что напряжения в массиве в условиях больших глубин начинают приобретать периодический характер, а на участках действия максимальных сжимающих нормальных тангенциальных напряжений развиваются макродефекты, определяя радиальное сжатие промежуточных зон. Разработана математическая модель сильно сжатых горных пород, основывающаяся на принципах неевклидовой механики и неравновесной термодинамики. Разработаны методы определения параметров модели и получено удовлетворительное соответствие между результатами теоретических и экспериментальных исследований. Получены зависимости основных структурных параметров модели от влияющих факторов.

Ключевые слова: мезоструктуры, разрушение горных пород, выработки, зоны, моделирование.