

Estimation of earthquake magnitudes from epicentral intensities and other focal parameters in Central and Southern Europe

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SUMMARY

The publication of an earthquake catalogue by Kárník in 1996 (a continuation and revision of an earlier one (1969)) makes important data available covering one century of the seismic history of Central and Southern Europe. It allows us to study in detail empirical relations between the magnitude and other focal parameters. In this study well-known relations combining two or three focal parameters, $M = A + BI_0 + C \log(H)$, $M_s = D + EI_0 + F \log(H)$, $M_L = G + OI_0 + P \log(H)$, $M_L = Q + RM_s + S \log(H)$, are investigated (M , Kárník's magnitude; M_L , local magnitude; M_s , surface wave magnitude; I_0 , epicentral intensity; H , focal depth in kilometres). The data show a considerable scatter with respect to the relations above. The relations are considered useful, if the following significance criteria are fulfilled. (1) The data sets comprise a minimum of 20 entries. (2) The partial correlation between the two most important parameters is greater than 70 per cent. (3) The parameter of least importance still influences the correlation of the others by more than 5 per cent. The partial correlation coefficients help to decide whether the data are to be rejected as insufficient for the regression analysis or to determine the level beyond which it is useful to perform a regression analysis excluding the parameter of lowest importance. Two kinds of regression are carried out: (1) standard linear regression assumes that only M or M_L , respectively, are in error, while the remaining two parameters are error-free. (2) Orthogonal regression assumes that all three parameters have errors. This is the case for the data in the catalogue used here.

The orthogonal regression $M = -1.682 + 0.654I_0 + 1.868 \log(H)$, with a standard deviation of ± 0.284 , differs considerably from Kárník's empirical relation $M = 0.5I_0 + \log(H) + 0.35$ for shallow foci, but agrees well with the results of earlier studies by the authors for earthquakes in SE Europe. The data set M , I_0 , H (for $H < 50$ km) fails criterion (3). The orthogonal least-squares fit without $\log(H)$ has been found as follows: $M_s = 0.550I_0 + 1.260$, with a standard deviation of ± 0.412 . We observe systematic regional deviations from this relationship, which need further investigation. The correlation analysis shows that M_L and M_s are weakly linked with $\log(H)$, but the correlation between M_L and M_s is very high (93 per cent). Therefore, the orthogonal relation between M_L and M_s without the $\log(H)$ term was chosen: $M_L = 0.664 + 0.893M_s$, with a standard deviation of ± 0.163 . The correlations between M_L , I_0 and $\log(H)$ do not fulfil the significance criteria.

For the purpose of earthquake hazard analysis the orthogonal regression visualizes simultaneously the errors of all input data, i.e. δM_{Li} , δM_{si} and $\delta \log(H_i)$. Our new relationships result from orthogonal regression analysis using a large high-quality data set. They should be applicable in Central and Southern Europe unless there are regional relationships available that fit the data better.

Key words: Europe, magnitude–intensity relations, orthogonal regression.

Additional suggested key words: correlation coefficient, earthquake catalogues, local magnitude, surface wave magnitude.

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1 MOTIVATION

Richter (1935) introduced the magnitude M_L as a measure of the strength, power or effect of an earthquake. He had in mind mainly the derivation of a simple parameter, which can be determined quickly and easily in seismological observatory practice and which meets the needs for public announcements.

Yet, this concept becomes complicated as many definitions of magnitude are in use, which rely on simple measurements of seismic amplitudes as well. The local magnitude M_L mentioned above is based on the measurement of the maximum amplitude of a standard Wood–Anderson seismograph. The surface wave magnitude M_s is derived from the amplitude of surface waves with a period of 20 s. Similar definitions are used for the body wave magnitudes m_b , m_B or Kárník's magnitude M . In general, M_L , M_s , M , m_b or m_B give different values for the same earthquake, but they are related by empirical relations. This is caused by the use of amplitudes of different wave types at different frequency ranges on different components from seismograms recorded at different stations, applying different calibration functions. Sometimes, the magnitude of an earthquake is given together with some kind of error, usually the standard deviation of individual station magnitudes.

The greatest European earthquakes occurred before the installation of seismographs in 1900; we have narrative sources of them since approximately 1000 AD. Thus, their magnitudes cannot be measured but only estimated from macroseismic intensity data (e.g. Ambraseys 1985; Albarello *et al.* 1995; Scotti *et al.* 1999; López Casado *et al.* 2000). All kinds of possible uncertainties in the historical messages have to be taken into account. Many of them do not fit into the conception of error analysis generally used in seismology. It is necessary to calibrate the magnitudes of these historical earthquakes using comparable events for which we have both reliable instrumental and high-quality macroseismic data.

In these cases we may ask how reliable such a magnitude estimate is and whether a principal minimum limit of error exists that never can be passed below. This question is not only academic. It touches the responsibility of seismologists to public authorities and is of high importance when the seismic hazard at a site is investigated.

The author's opinion is, that beyond an estimation of the magnitude of the design earthquake, an estimate should also be provided on how reliable this estimation is. Errors of the design earthquake magnitude should be considered in any seismic hazard analysis.

2 AIM OF THIS STUDY

The aim of this study is to gain an indirect estimate of the magnitude from macroseismic data and other focal parameters by using empirical relations. In contrast to earlier studies we want to apply both the conventional 1-D regression and the orthogonal regression as a least-squares fit approximation. The comparison will shed light on the differences between the different types of error information as well as the practical importance of the orthogonal errors.

3 MAGNITUDE ESTIMATIONS

We want to find out whether or not the following relations between magnitudes and macroseismic parameters can be established for a given earthquake region:

$$M = A + BI_0 + C \log(H) \quad (1)$$

$$M_s = D + EI_0 + F \log(H), \quad (2)$$

$$M_L = G + OI_0 + P \log(H), \quad (3)$$

$$M_L = Q + RM_s + S \log(H), \quad (4)$$

with best-fitting coefficients $A, B, C, D, E, F, G, O, P, Q, R$ and S . This path is promising, if the data $M_L, M_s, I_0, \log(H)$ and I_0 correlate significantly.

If not enough events in the earthquake region are available, well-known empirical relations between M_s and other focal parameters can be used. For instance, if catalogue entries of I_0 and H only are available the formulae suggested by Kárník (1969, in the following referred to as KA69),

$$M = 0.35 + 0.5I_0 + \log(H), \quad (5)$$

where $M \cong M_s$ for $H < 60$ km and $M \cong m_B$ for $H \geq 60$ km with medium-period body wave magnitude m_B as defined by Gutenberg (1945) may be used. If M_L is available only, then

$$M_L = 0.71M_s + 1.46 \quad (\text{Ambraseys \& Bommer 1990}), \quad (6)$$

may be used. Note that coefficients in formulae (5) and (6) may differ for different seismic regions. So we have to prove whether the applied formulae are the appropriate ones.

4 THE DATA BASE

We used entries from Kárník's earthquake catalogue. He published the European Earthquake catalogue 1901–1955 (KA69) and prepared its continuation up to 1990. After his death the work was completed by K. Klíma, who compiled Kárník's material and published it in the form of a catalogue for the period 1901–1990 Kárník (1996, in the following referred to as KA96). It contains rather brief explanations of the tables only. Therefore, with regard to the methodology, we refer to the many explanations in KA69, while we use the data of KA96.

We have to discuss KA69 first, as it provides the basic conception of a unified magnitude M . It contains entries such as focal coordinates, origin times, and several magnitude and epicentral or maximum intensity [MSK] entries. In this study we will call Kárník's magnitude M as defined by Kárník himself: '...the magnitudes based on surface waves (M_{LH}) were taken as representative for shallow earthquakes and those based on body waves $M_B = m$ for intermediate and deep earthquakes, respectively. . . ' (KA69, p. 41).

28 per cent of the magnitudes M in KA69 have been derived from non-instrumental observations. Therefore, the M data are basically inhomogeneous. Nevertheless, they provide the basis of one important result of KA69, the empirical equation of Kárník's magnitude given in eq. (5), which is recommended for earthquakes in Europe. The equation is based on data from approximately 1300 earthquakes with known M, I_0 and H that occurred between 1901 and 1955 in Europe, the Mediterranean and Balkan countries. Eq. (5) is frequently quoted and applied later for estimations of earthquake magnitudes, if instrumental records are not available (Franke & Gutdeutsch 1974; Meidow 1995 and many others).

The concept of Kárník's magnitude can be of great practical help if eq. (5) can be proven as a reliable prediction formulae for M_s or m_B if only macroseismic data I_0 and depth values H are available. The extended KA96 provides much more earthquake data and, thus, allows us to test the validity of eq. (5).

We regard KA96 as an important data base, useful for related studies. However, after having worked with both versions, KA69 and KA96, we have to make the following critical comment. In KA96 a magnitude 'M', slightly different from Kárník's original definition of M (KA69, p. 41), is used. In columns 45 and 46 of the computer file of KA96 M is defined as the 'surface wave magnitude M_s (LR or

Sg waves), with the average value determined in a uniform way: a standard for all events of $h < 60 \text{ km}$. However, there are still some M entries with $h \geq 60 \text{ km}$ in this catalogue, sometimes different, sometimes equal to m_B with the reference to Kárník's unpublished manuscript KVM 1968 (see KA96, p. 18). The given assignment poses questions to investigators who want to use or to prove the empirical formulae (5).

5 METHODS

5.1 Significance criteria for earthquake data distributions

The question of whether the scatter of data justifies relations with a 3-D distribution according to eqs (1), (2), (3) or (4), can only be answered if we define an allowable uncertainty of the data base used in predicting M , M_s or M_L , respectively. In other words, we have to find quantitative *significance criteria*. These criteria have to be formulated in such a way that the above question can be answered with either 'yes' or 'no'. As an example, we may call eq. (1) as being 'not valid' if the basic data set M, I_0, H does not agree with these criteria. Then eq. (1) has to be rejected. In this case, either another simpler model has to be used, or, if no simpler model can be found, the problem has to remain unsolved. Naturally, the significance criteria depend on the practical aim of the investigation and, thus, on the personal decision of the investigator. It might be expected that the rms error of the derived magnitude is an appropriate measure of the significance of an empirical relation. Yet, it will be shown in the next section, that this choice is questionable here. The large scatter and the inhomogeneous origin of data in time and space cause complicated error distributions. Under these circumstances it may be difficult to interpret the mean square error. Therefore, we use the following significance criteria:

(1) The simplest way to improve the basis is to increase the number of data. With a lack of better assumptions we decide that the number of earthquakes of the data set must be greater than 20.

(2) The second criterion is defined as the tolerable minimum partial correlation coefficient between focal parameters in eqs (1)–(4). Let us regard the relation (1) between $x = I_0, y = \log(H)$ and $z = M$ as an example. Following the notation of Schönwiese (2000, p. 182ff.) we write the 3-D partial correlation coefficient

$$r_{xz \cdot y} = \frac{r_{xz} - r_{xy}r_{yz}}{\sqrt{(1 - r_{xy}^2)(1 - r_{yz}^2)}},$$

where r_{xy}, r_{xz} and r_{yz} are the respective 2-D correlation coefficients. $r_{xz \cdot y}$ represents the correlation between M and I_0 , when the influence of $\log(H)$ is eliminated. $r_{xz \cdot y} \gg r_{xz}$ represents the case where the influence of $\log(H)$ masks the correlation of M and I_0 . The respective statements hold for $r_{xy \cdot z}$ and $r_{yz \cdot x}$. Thus, the partial correlation coefficient provides the necessary information concerning the degree of independent correlation between two parameters if a third parameter plays a role. With a lack of a better assumption we decide that the partial correlation coefficient between the most important parameters must be larger than 70 per cent.

(3) The parameter with the lowest influence diminishes the correlation of the others by more than 5 per cent, i.e. in the present example $|\frac{r_{xz \cdot y} - r_{xz}}{r_{xz \cdot y}}| > 0.05$.

In the case where all criteria are satisfied a prediction eq. (1) is established by regression methods.

If the significance criteria (1) and (2) are satisfied, but criterion (3) is not, then a simpler regression model excluding the parameter of lowest correlation has to be used.

If significance criteria (1) and (2) fail then the task has to remain unsolved (see Fig. 1).

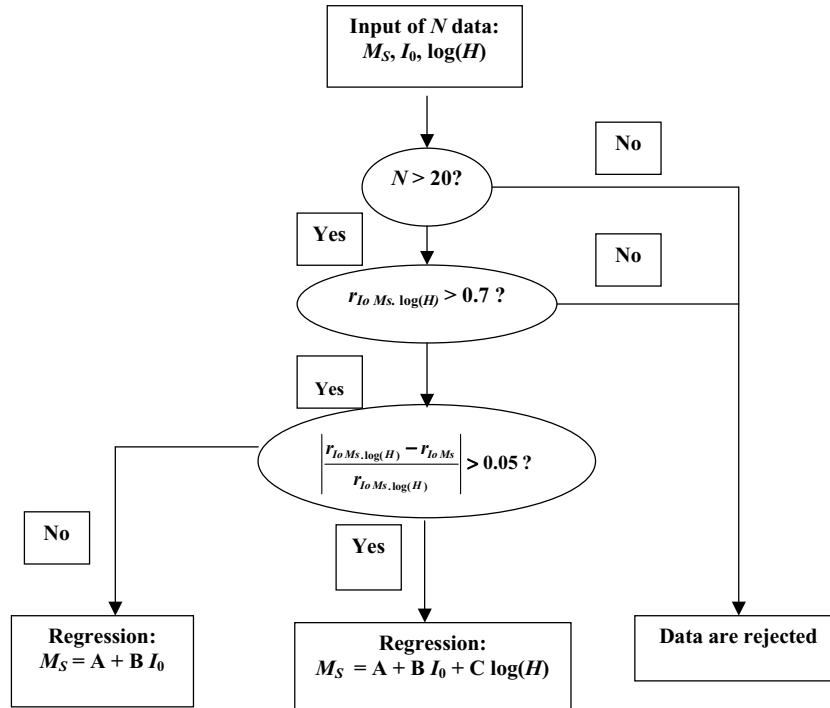


Figure 1. Flow chart of significance criteria and the choice of the equations.

5.2 Regression analysis

The next step consists in the determination of an empirical relation by regression:

$$M = A + BI_0 + C \log(H). \quad (1)$$

All input parameters, M_i , I_{0i} and H_i are in error. Nevertheless, most investigators apply a least-squares fitting approximation that regards M as subject to error, but $\log(H)$ and I_0 as error-free. Under this incisive presumption a best-fitting approximation of M is carried out. The sum of squared errors v_i^2 of observed M_i is minimized by variation of A , B and C according to

$$\sum v_i^2 = \sum [M_i - A - BI_{0i} - C \log(H_i)]^2 = \min. \quad (7)$$

We will call M when determined from eq. (1) with A , B and C according to standard regression (7) as $M^{(7)}$.

The method (7) is incorrect for two reasons: As mentioned above, the $\text{rms}^{(7)}$ of standard regression (7) communicates an incorrect understanding of the true error as it ignores the errors of $\log(H)$ and I_0 . Additionally, eq. (1) is not reversible. It does not hold for calculation of I_0 from M and $\log(H)$ or calculation of $\log(H)$ from I_0 and M after respective conversion of eq. (1). In this study the non-reversibility is indicated by the arrow ' \leftarrow ' instead of the equality symbol '=' as follows:

$$M^{(7)} \leftarrow A + BI_0 + C \log(H). \quad (8)$$

Naturally, the irreversibility of the standard regression eq. (7) holds for data sets with more than two input parameters as well. Yet, most of the earlier publications use standard regression and disregard its fundamental disadvantage.

The presumption that all 'input parameters' M_i , I_{0i} , $\log(H_i)$ are associated with errors is more realistic. The orthogonal regression takes this into account. It minimizes the orthogonal distance h_i of the i th data point M_i , I_{0i} , $\log(H_i)$ from the 'plane' in the M , I_0 , $\log(H)$ 'space'. In its simplest form it ignores different weighting factors of the input data. Bormann & Khalturin (1975) and Bormann (2000) emphasize a great advantage of the orthogonal regression: it provides a reversible regression equation. This means that eq. (1) with A , B and C determined by orthogonal regression may be used for the calculation not only of M but also, after conversion of the equation, of $\log(H)$ or I_0 as well.

The orthogonal error h_i is found by the 3-D HESSE normal equation as follows:

$$P = n_M M + n_{I_0} I_0 + n_{\log(H)} \log(H), \quad (9)$$

$$h_i = P - n_M M_i - n_{I_0} I_{0i} - n_{\log(H)} \log(H)_i, \quad (10)$$

where P is distance of the 'plane' $P = \text{constant}$ from the origin, $(n_M, n_{I_0}, n_{\log(H)})$ is the normal 'vector' of length 1 of the 'plane' with

$$n_M^2 + n_{I_0}^2 + n_{\log(H)}^2 = 1 \quad (11)$$

M_i , I_{0i} , $\log(H_i)$ is the input data $i = 1, \dots, N$, h_i is the normal distance of the data point M_i , I_{0i} , $\log(H_i)$ to the plane P .

Lagrange's method turns out to be very successful in finding the extrema of a function with one side condition, in our case:

$$\sum h_i^2 = \sum [P - n_M M_i - n_{I_0} I_{0i} - n_{\log(H)} \log(H)_i]^2 - \lambda (n_M^2 + n_{I_0}^2 + n_{\log(H)}^2 - 1) = \min. \quad (12)$$

λ is the Lagrange parameter, which takes the side condition into account. Eq. (12) has to be minimized by variation of P , n_M , n_{I_0} and $n_{\log(H)}$. The method is well known in seismic signal analysis

(Robinson & Treitel 1980, and others). Its application to the present task is described by Gutdeutsch *et al.* (2000a). $\text{rms}^{(12)} = \sigma$ refers to h_i and differs slightly from $\text{rms}^{(7)}$. In many cases we find $\text{rms}^{(12)} < \text{rms}^{(7)}$. This does not mean that the errors of M , I_0 and $\log(H)$ are smaller than that of the standard regression. This observation explains our decision to use the correlation instead of the rms as significance criterion. We can visualize this effect by the *equivalent errors* $\delta M^{(12)}$, $\delta I_0^{(12)}$ and $\delta \log(H)^{(12)}$. The symbol ' δ ' indicates that it is not identical to $\text{rms}^{(12)}$. For example, $\delta M^{(12)} = \sigma/n_M$ is the error of M if $\delta \log(H)^{(12)} = 0$ and $\delta I_0^{(12)} = 0$. Therefore, we regard $\delta M^{(12)}$ as an important informative measure of the error of M derived from the orthogonal regression σ .

Shifting the coordinate system I_0 , $\log(H)$, M to the centre of gravity of the data set

$$I_0^{\text{mean}} = \frac{\sum_{i=1}^N I_{0,i}}{N}$$

$$\log(H)^{\text{mean}} = \frac{\sum_{i=1}^N \log(H)_i}{N}$$

$$M^{\text{mean}} = \frac{\sum_{i=1}^N M_i}{N}$$

removes A in eq. (1) and P in eq. (10), respectively. Both, standard regression (7) and the orthogonal regression (12) form 'planes' that cross the centre of gravity of the data set. From this the important conclusion follows that the standard regression (7) and the orthogonal regression (12) coincide exactly at the centre of gravity of the data and agree well in its neighbourhood.

In this study we carry out both, the standard regression (7) and the orthogonal regression (12).

6 RESULTS

6.1 Discussion of M , I_0 , H data

KA96 includes $N = 3362$ earthquakes with M , I_0 , H entries and $1 \leq H < 300$ km. In view of the restriction discussed in chapter 4 we used Kárník's original definition of $M = M_S$ (columns 45 and 46 in computer file KA96 for $H < 60$ km) and $M = m_B$, respectively, m_b (column 40 and 41 in computer file KA96 for $H \geq 60$ km). Some entries have been obtained by macroseismic (i.e. semi-quantitative) methods. We expect that the significance of the instrumental magnitude exceeds that of the macroseismic magnitude. The significance probably increases with the number of stations N_{stat} providing magnitude estimates. Table 1 shows a comparison of correlation coefficients of four subsets of H , I_0 , M data sets.

The effect of the preselection is visualized in Figs 2(a)–(d) (data distribution Table 1, line 3). From Table 1 we conclude that:

- the preselection of data with high I_0 quality has reduced the number of samples down to 66 per cent, but increases the partial correlation coefficients of M and I_0 by 30 per cent;
- the negative correlation of $r_{I_0 \log(H)-M}$ and the correlation $r_{M I_0 \log(H)}$ is enhanced when doubtful I_0 are excluded;
- the correlations $r_{M \log(H)-I_0}$ and $r_{I_0 \log(H)-M}$ appear as rather independent of the data quality, which increases from line 1 to line 4. A possible explanation could be that the quality of H determinations of the data sets is hardly improved by the selections according Table 1.

First, we attempted to retrieve Kárník's eq. (5) using data material from the same time span 1901–1955 that he had used. The results were not satisfying. In an earlier publication, a geographical window was found that covers Central Europe and Italy where a

Table 1. Partial correlation coefficients of four different M , I_0 , H data sets from KA96 with focal depths in the range $1 \leq H < 300$ km. According to Kárník's definition, $M = M_s$ for $1 \leq H \leq 60$ km and $M = m_B$ for $60 < H \leq 300$ km. The criterion 'doubtful I_0 ' is provided in KA96.

Line number	Specification	Number of samples	$r_{M \log(H)-I_0}$	$r_{I_0 \log(H)-M}$	$r_{M I_0-\log(H)}$
1	All data M , I_0 , H	3362	0.54	-0.47	0.60
2	Specification as line 1, but doubtful I_0 excluded	2206	0.66	-0.60	0.79
3	Specification as line 2 but $N_{\text{stat}} \geq 4$	527	0.51	-0.56	0.78
4	Specification as line 2 but $N_{\text{stat}} \geq 6$	327	0.53	-0.62	0.81

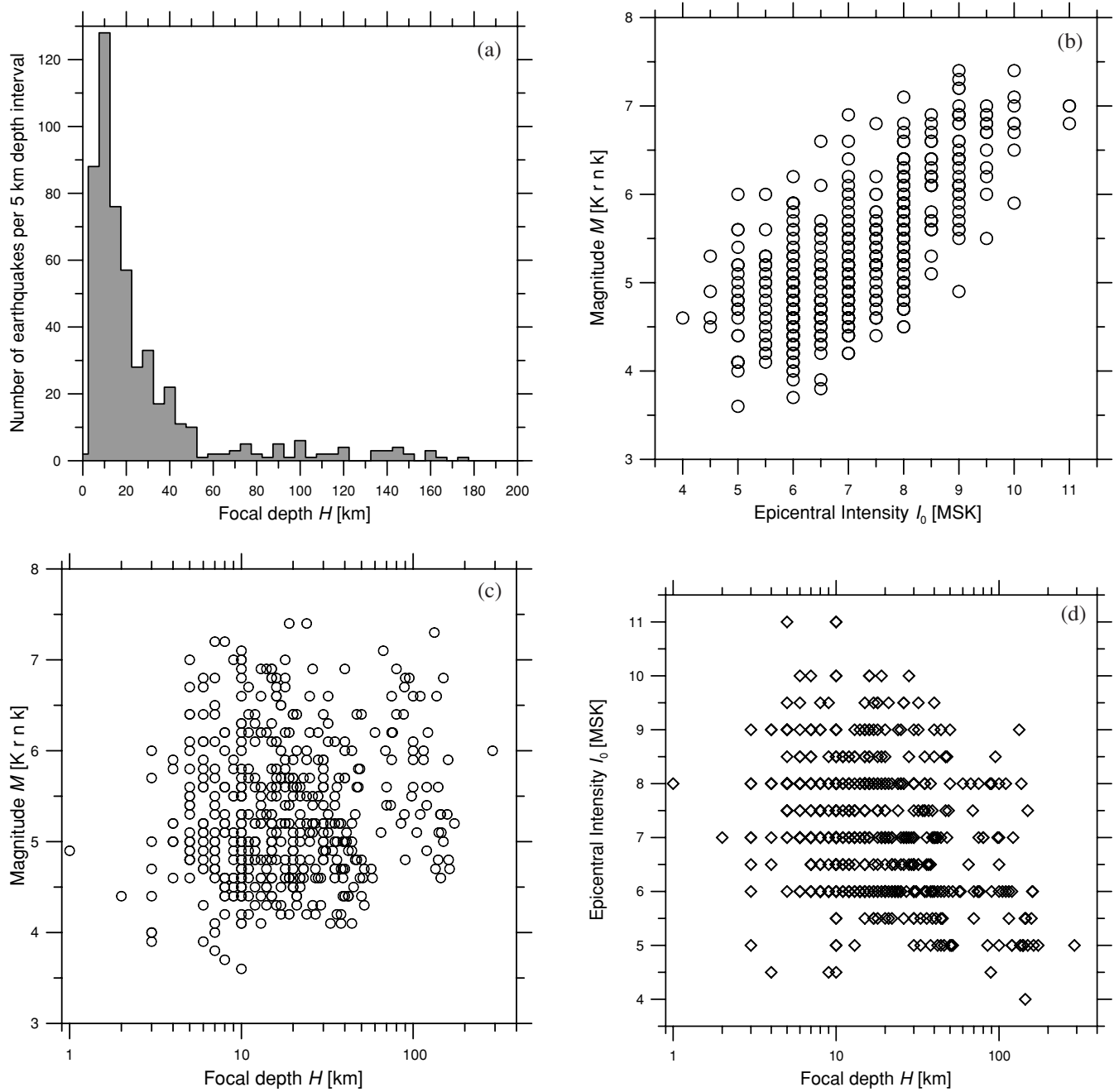


Figure 2. (a) Histogram showing the number of events per focal depth interval = 5 km, $N = 527$ events with $N_{\text{stat}} \geq 4$, $1 \leq H < 300$ km, events with doubtful I_0 excluded (data set see Table 2). The difference in the structure of the data distribution of H below and above 60 km can be explained by Kárník's method of M determination. In this figure one data point with $H = 290$ km has been omitted. (b) $M-I_0$ [MSK] distribution, data see Fig. 2(a) and Table 2. (c) $M-\log(H)$ distribution, data see Fig. 2(a) and Table 2. (d) I_0 [MSK]- $\log(H)$ distribution, data see Fig. 2(a) and Table 2.

Table 2. Correlation coefficients of a data set with doubtful I_0 excluded and $N_{\text{stat}} \geq 4$ (line 3 in Table 1) from KA96 with focal depths in the range $1 \leq H < 300$ km.

$r_{I_0 \log(H)}$	$= -0.32$	$r_{I_0 H} = -0.32$
$r_{I_0 \log(H)M}$	$= -0.56$	$r_{I_0 H \cdot M} = -0.57$
$r_{I_0 M}$	$= 0.70$	
$r_{I_0 M \cdot \log(H)}$	$= 0.78$	$r_{I_0 M \cdot H} = 0.79$
$r_{\log(H)M}$	$= 0.12$	$r_{HM} = 0.13$
$r_{\log(H)M \cdot I_0}$	$= 0.51$	$r_{HM \cdot I_0} = 0.52$

better agreement with eq. (5) exists (Gutdeutsch *et al.* 2000b). The authors conclude that the data set KA69 confirms eq. (5) by the standard regression eq. (7) with a maximum error of 0.1 magnitude units. Table 1 shows, that the unselected data set does not satisfy the significance criteria. Hence, we use the preselected data set with doubtful I_0 excluded and $N_{\text{stat}} \geq 4$ (line 3 in Table 1). Its frequency distribution (Fig. 2a) shows the effect of Kárník's definition of M very clearly. The frequency of shocks with $H \geq 60$ km follows a law that is different from that at $H < 60$ km (see Fig. 2a). The correlation coefficients are presented in Table 2. In Table 2 as well as in Tables 4–6 (see Sections 6.2 and 6.3), we added the respective correlation coefficients with H as parameter in order to compare the correlations of I_0 or the magnitude with $\log(H)$ and H to test the presumption of eqs (1)–(4). Obviously the differences between the correlations with $\log(H)$ and H are small. The investigation of H , instead of $\log(H)$, or a combination of both will be the subject of another study.

Figs 2(b)–(d) visualize the correlation. The high correlation coefficient $r_{I_0 M \cdot \log(H)} > r_{I_0 M}$, $|[r_{\log(H)M \cdot I_0} - r_{\log(H)M}] / r_{\log(H)M \cdot I_0}| > 0.05$ and $N > 20$ satisfy the significance criteria. Regression formulae have been established as follows:

$$M^{(7)} \leftarrow 0.785 + 0.505 I_0 + 0.737 \log(H),$$

$$\Delta M^{(7)} = \pm 0.468 \quad (\Delta M^{(\text{Kárník})} = \pm 0.501), \quad (13)$$

$$I_0^{(7)} \leftarrow 2.306 + 1.210 M - 1.275 \log(H),$$

$$\Delta I_0^{(7)} = \pm 0.725, \quad (14)$$

$$\log(H)^{(7)} \leftarrow 1.185 + 0.347 M - 0.250 I_0,$$

$$\Delta \log(H)^{(7)} = \pm 0.321, \quad (15)$$

$$M^{(12)} = -1.682 + 0.654 I_0 + 1.868 \log(H) \quad (16)$$

with $P = 0.7584$, $n_{I_0} = 0.2951$, $n_{\log(H)} = 0.8424$, $n_M = -0.4509$ and $\sigma = \pm 0.285$,

$$\delta M^{(12)} = \sigma / n_M = 0.631,$$

$$\delta I_0^{(12)} = \sigma / n_{I_0} = 0.965,$$

$$\delta \log(H)^{(12)} = \sigma / n_{\log(H)} = 0.338.$$

The range of validity of eq. (16) is approximately $4 \leq M \leq 7$ and $H = 300$ km (see Figs 2a and b).

Eqs (13)–(15) are standard regression relations of $M^{(7)}$, $I_0^{(7)}$ and $\log(H)^{(7)}$ following eq. (7). $M^{(12)}$ is the result of the orthogonal regression according to eq. (12). The *non-least-squares* errors $\Delta M^{(\text{Kárník})}$ of Kárník's formulae (5) has been added in parentheses.

The one-sided relations (13)–(15) contradict each other considerably. This can be shown when we interpret them as equations and transform them into

Table 3. Comparison of magnitudes $M(I_0, \log(H))$ calculated using eqs (16), (19), (13) and (5) for $I_0 = I_0^{\text{mean}} = 7.16$ (mean value of the data set). Note that M estimates of the orthogonal regression (16), the mean of standard regression (19) and the standard regression (13) coincide at the centre of gravity of the data set $I_0^{\text{mean}} = 7.16$, $H^{\log \text{mean}} = 17.36$ km, $M^{\text{mean}} = 5.31$. M calculated from Kárník's formulae (5) deviates slightly from the centre of gravity.

H (km)	$M^{(12)}$ from orthogonal regression eq. (16)	$M^{(\text{ave})}$ from mean coefficients eq. (19)	$M^{(7)}$ from standard regression eq. (13)	M from Kárník's regression eq. (5)
1	3.00	3.38	4.26	3.93
2	3.56	3.85	4.62	4.23
5	4.31	4.47	4.91	4.63
10	4.87	4.94	5.13	4.93
15	5.20	5.22	5.26	5.10
17.16	5.31	5.31	5.31	5.16
20	5.43	5.41	5.36	5.23
30	5.76	5.69	5.49	5.41
50	6.18	6.03	5.65	5.63
100	6.74	6.50	5.87	5.93

$$M = -1.905 + 0.826 I_0^{(7)} + 1.053 \log(H), \quad (17)$$

$$M = -3.421 + 0.722 I_0 + 2.886 \log(H)^{(7)}. \quad (18)$$

The average values of the coefficients in eqs (13), (17) and (18) give rise to

$$M^{(\text{ave})} = -1.514 + 0.684 I_0 + 1.559 \log(H),$$

$$\Delta M^{(\text{ave})} = \pm 0.571. \quad (19)$$

The striking agreement between eqs (19) and (16) demonstrates that the average values of coefficients gained from the standard regression (7) provide a relation close to that of the orthogonal regression (12).

Note, that $M^{(7)}$ in eq. (13), $M^{(12)}$ in eq. (16) and $M^{(\text{ave})}$ in eq. (19) agree exactly at the centre of gravity $M^{\text{mean}} = 5.31$, $I_0^{\text{mean}} = 7.16$, $\log(H)^{\text{mean}} = 1.23$ with $H^{\log \text{mean}} = 17.14$ km = logarithmic mean value of H . This statement can be of some help as a general information with respect to the data set of KA96 (see Table 3).

Similar relations have been found for different regions, for instance by Franke & Gutdeutsch (1974) for East Alpine earthquakes.

Eq. (16) differs slightly from the comparable relation for earthquakes in SE Europe (Kaiser & Gutdeutsch 2001; Kaiser *et al.* 2001):

$$M^{(12)} = -1.62 + 0.65 I_0 + 1.90 \log(H),$$

$$\sigma = \pm 0.210, \quad (\text{Kaiser } et al. 2001). \quad (20)$$

Table 3 shows the distribution $M(H)$ for $I_0 = I_0^{\text{mean}} = 7.16$ for data 1901–1990.

Kárník's empirical relation (5) differs considerably from eq. (16), particularly for shallow focal depths. The most probable explanation for these differences is that the focal depths used by KA69 to derive his relationship are of 'low accuracy' (KA69, p. 28). We state that the use of only high-quality data as input in the regression analysis provides reliable relationships for the estimated magnitudes.

Eq. (16) is recommended according to its realistic presumption of input errors. This conclusion is supported by similar results found in different regions by the authors. However, the great range of the *equivalent error* $\delta M^{(12)} = \pm 0.631$ of the magnitude $M^{(12)}$ makes it comparable to the result of equation $M^{(7)}$. An agreement between

Table 4. Correlation coefficients of a data set with doubtful I_0 excluded, $N_{\text{stat}} \geq 4$, focal depths in the range $1 \leq H \leq 50$ km from KA96 (469 events).

$r_{I_0 \log(H)} = -0.18$	$r_{I_0 H} = -0.18$
$r_{I_0 \log(H) \cdot M_s} = -0.31$	$r_{I_0 H \cdot M_s} = -0.26$
$r_{I_0 M_s} = 0.78$	
$r_{I_0 M_s \cdot \log(H)} = 0.80$	$r_{I_0 M_s \cdot H} = 0.79$
$r_{\log(H) M_s} = 0.01$	$r_{H M_s} = -0.24$
$r_{\log(H) M_s \cdot I_0} = 0.25$	$r_{H M_s \cdot I_0} = 0.19$

both eqs (13) and (16) within ± 0.2 magnitude units is found for focal depths between 5 and 50 km. This example makes clear that the rms of the standard regression $\Delta M^{(7)} = \pm 0.468$ provides an incorrect understanding of the total error of M , which actually is much greater.

6.2 Relations of M_s , I_0 and H in KA96

Can the use of a smaller set of selected data with better quality increase the significance of relations between I_0 , M and $\log(H)$? Ambraseys (2001) points out that Kárník's magnitude M entries in KA69 are significantly corrected for depths > 50 km. This correction is based on uncertain assessments of focal depths. Additionally, they follow from the presumption that M_s is equivalent to the Gutenberg–Richter magnitude. Here, the authors conclude that Kárník's M_s estimates for subcrustal and intermediate focal depths are rather uncertain. We assume the same to be true for the KA96 data and thus we investigated a selected data set with focal depth $H \leq 50$ km (see Table 4).

The resulting correlation between I_0 and M_s is increased relative to the complete data set. It satisfies criterion (1) and (2). $r_{I_0 M \cdot \log(H)}$ is hardly influenced by $\log(H)$. Therefore, criterion (3) is not satisfied. Note that the correlation of $\log(H)$ with I_0 or M_s is worse relatively to the complete data set. This result has to be seen in relation to the frequency distribution of foci with depth H (see Fig. 3c). The exaggerated cumulation of foci at $H = 10, 20$ and 30 km might be explained by the decisions of the observers in view of the great uncertainty of many H entries. In such cases they are artefacts. They cause a considerable deviation from the true distribution and can worsen a good correlation between M , I_0 and $\log(H)$. The investigation of this effect shall be left to later studies. Because of the uncertainties in focal depth, we apply a least-squares fit without the $\log(H)$ term as follows:

$$\begin{aligned} M_s^{(7)} &\leftarrow 0.488 I_0 + 1.712, \\ \Delta M_s^{(7)} &= 0.464, \quad \Delta I_0^{(7)} = 0, \end{aligned} \quad (21)$$

$$\begin{aligned} I_0^{(7)} &\leftarrow 1.278 M_s + 0.541, \\ \Delta M_s^{(7)} &= 0, \quad \Delta I_0^{(7)} = 0.752, \end{aligned} \quad (22)$$

$$\begin{aligned} M_s^{(12)} &= 0.550 I_0 + 1.260, \\ \sigma &= 0.412, \quad \delta M_s^{(12)} = 0.470, \quad \delta I_0^{(12)} = 0.855. \end{aligned} \quad (23)$$

The range of validity of eq. (23) is approximately $4 \leq M_s \leq 7$, $H < 50$ km as visualized in Figs 3(b) and (c).

Schenk *et al.* (2000) use data of the Czech Republic, Poland, Slovakia areas. They carry out standard regression analysis both of $I_0(M_s)$ and $M_s(I_0)$ and recommend using the mean

$$\begin{aligned} M_s &= (0.6725 \pm 0.0818) I_0 [\text{MSK}] \\ &+ (0.3354 \pm 0.2704) \quad (\text{Schenk } et al. \text{ 2000}). \end{aligned} \quad (24)$$

Their result agrees well with the orthogonal regression, because the scatter of data is small enough. This can also be seen by the relation $\tan(2\varphi_{\text{orth}}) = 2 \tan(\varphi_{I_0}) / [1 - \tan(\varphi_{I_0}) \tan(\varphi_{M_s})]$, where φ_{orth} is the incline angle of the orthogonal regression. φ_{I_0} and φ_{M_s} are the respective incline angles of the best-fitting lines of standard regression with I_0 , respectively, M_s as input.

Albarelo *et al.* (1995) found a similar equation for earthquakes in Italy using ‘standard regression analysis’:

$$M_s = 1.35 + 0.50 I_{\text{max}} [\text{MCS}] \quad (\text{Albarelo } et al. \text{ 1995}). \quad (25)$$

Fig. 3(b) shows graphs of eqs (21)–(25). Obviously eq. (25) for Italian earthquakes predicts lower values of M_s for a given I_0 .

We recommend relationship (23) for application in Central and Southern Europe unless there is a regional relationship available that better fits the data. Note the large negative correlation between the coefficients A and B of $M_s = A + B I_0$ in eqs (23)–(25). The same tendency appears as a general feature of data sets of Italian earthquakes. On the basis of synthetic data Mucciarelli (1998) suspects there is a ‘pivotal phenomenon’. If this is the only reason, no hidden influence of an additional physical parameter exists. We conclude from our data that the influence of the focal depth H cannot be ruled out, but it appears to be not significant enough.

6.3 Relations M_L , I_0 and M_s in KA96

The data set 1901–1990 provides 269 earthquakes with M_L , I_0 , H entries and 99 earthquakes with M_L , M , H entries. The correlation is given in Tables 5 and 6. The correlation coefficient $r_{M_L I_0 \cdot \log(H)}$ is less than 70 per cent. Therefore, the M_L , I_0 , H -data do not satisfy significance criterion (2) and are rejected. $r_{M_s M_L \cdot \log(H)} = 0.933$ is nearly equal to $r_{M_s M_L} = 0.932$. According to significance criterion (3) we regard the influence of $\log(H)$ as negligible and establish a relation between M_L and M_s only:

$$\begin{aligned} M_L^{(7)} &\leftarrow 0.941 + 0.838 M_s, \\ \Delta M_L^{(7)} &= \pm 0.216, \quad \Delta M_s^{(7)} = 0, \end{aligned} \quad (26)$$

$$\begin{aligned} M_s^{(7)} &\leftarrow -0.304 + 1.035 M_L, \\ \Delta M_L^{(7)} &= 0, \quad \Delta M_s^{(7)} = \pm 0.240, \end{aligned} \quad (27)$$

$$\begin{aligned} M_L^{(12)} &= 0.664 + 0.893 M_s^{(12)}, \quad \sigma = \pm 0.163, \\ \delta M_s^{(12)} &= \pm 0.219, \quad \delta M_L^{(12)} = \pm 0.245 \end{aligned} \quad (28)$$

for $4 < M_L \leq 7$, $4 \leq M_s \leq 7$ (see Fig. 4), $M_L^{\text{mean}} = 5.18$, $M_s^{\text{mean}} = 5.05$, $\log(H)^{\text{mean}} = 1.16$, $H^{\log \text{mean}} = 11$ km. Fig. 4 shows the distribution M_L , M_s and the best-fitting regression lines. We conclude that the result shown in Fig. 4 is very stable and recommend eq. (28) for applications. Ambraseys & Bommer (1990) used 301 earthquakes with similar ranges of M_L and M_s and found by orthogonal regression (using our notation):

$$M_L^{(12)} = 0.71 M_s^{(12)} + 1.46, \quad (6)$$

where $\sigma = 0.21$ (Ambraseys & Bommer 1990).

Their data have been taken from a time window 1966–1989. They overlap our data in nine events only. In view of this fact the agreement is good.

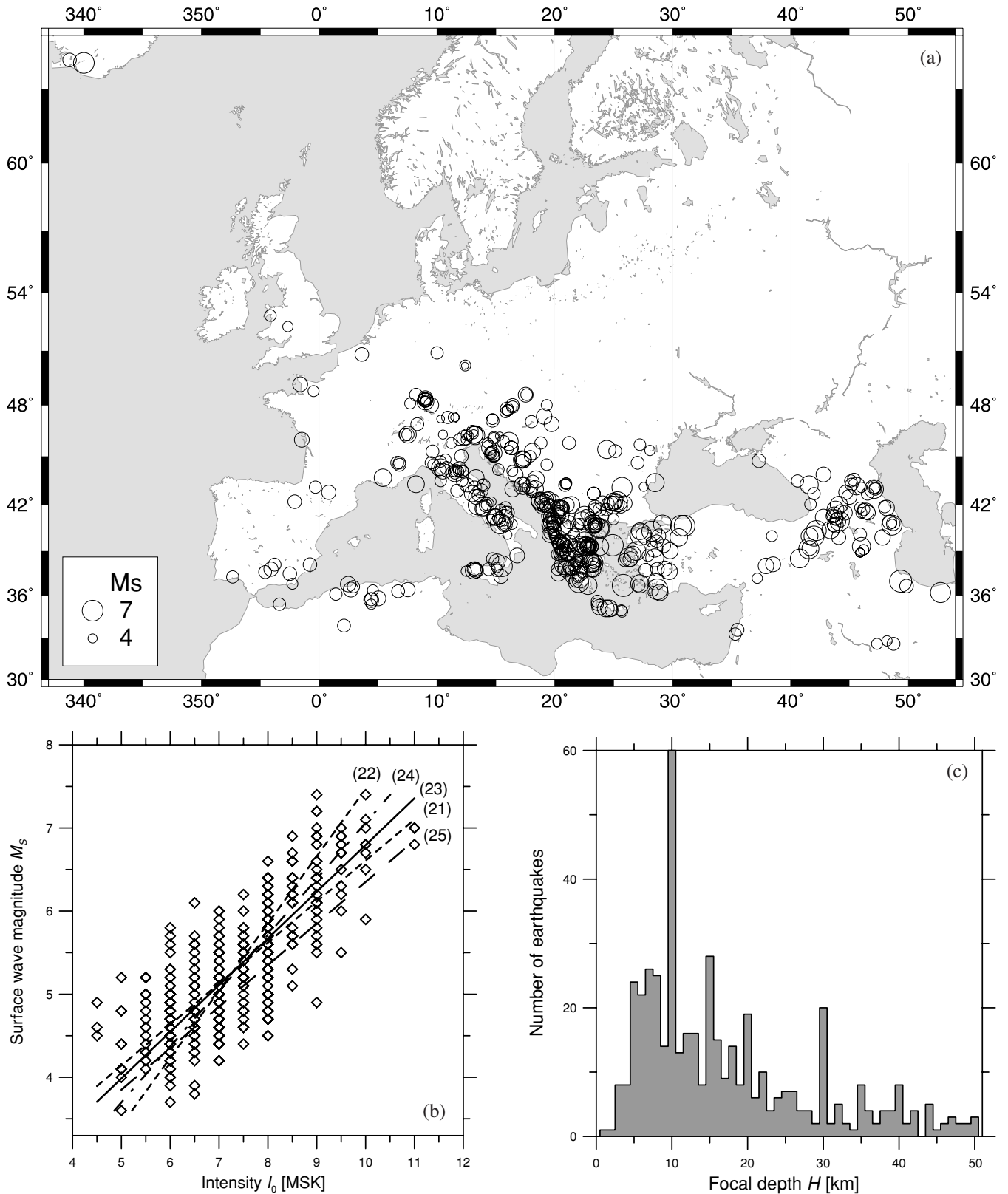


Figure 3. (a) Geographical distribution of epicentres of 469 earthquakes from KA96 used in the regression analysis, with M_s , I_0 , H entries, doubtful I_0 excluded, $N_{\text{stat}} \geq 4$, $1 \leq H \leq 50$ km. The diameter of circles is proportional to M_s . (b) Distribution $M_s(I_0)$, $N = 469$ events with $N_{\text{stat}} \geq 4$, $1 \leq H \leq 50$ km, events with doubtful I_0 excluded. The numbers in parentheses refer to the respective equation. Eq. (21), standard regression (7) M_s in error. Eq. (22), standard regression (7) I_0 in error. Eq. (23), orthogonal regression (12). Eq. (24), relation from Schenk *et al.* (2000). Eq. (25): relation of Albarello *et al.* (1995). (c) Histogram showing the number of events per focal depth interval = 1 km for $N = 469$ earthquakes from KA96 used in the regression analysis, with M_s , I_0 , H entries, doubtful I_0 excluded, $N_{\text{stat}} \geq 4$, $1 \leq H \leq 50$ km.

Table 5. Correlation coefficients of the data set with doubtful I_0 excluded, $N_{\text{stat}} \geq 4$, focal depths in the range $1 \leq H \leq 300$ km from KA96 (269 M_L , I_0 , H entries).

$r_{I_0 \log(H)} = -0.02$	$r_{I_0 H} = -0.06$
$r_{I_0 \log(H) \cdot M_L} = -0.34$	$r_{I_0 H \cdot M_L} = -0.31$
$r_{I_0 M_L} = 0.64$	
$r_{I_0 M_L \cdot \log(H)} = 0.69$	$r_{I_0 M_L \cdot H} = 0.69$
$r_{\log(H) M_L} = 0.35$	$r_{H M_L} = 0.28$
$r_{\log(H) M_L \cdot I_0} = 0.47$	$r_{H M_L \cdot I_0} = 0.42$

Table 6. Correlation coefficients of the data set with doubtful I_0 excluded, $N_{\text{stat}} \geq 4$, focal depths in the range $1 \leq H \leq 60$ km from KA96 (99 M_L , M_S , H entries).

$r_{M_S \log(H)} = -0.07$	$r_{M_S H} = -0.05$
$r_{M_S \log(H) \cdot M_L} = -0.14$	$r_{M_S H \cdot M_L} = -0.11$
$r_{M_S M_L} = 0.93$	
$r_{M_S M_L \cdot \log(H)} = 0.93$	$r_{M_S M_L \cdot H} = 0.93$
$r_{\log(H) M_L} = -0.02$	$r_{H M_L} = -0.02$
$r_{\log(H) M_L \cdot M_S} = 0.12$	$r_{H M_L \cdot M_S} = 0.09$

7 REGIONAL VARIATIONS IN THE RELATION $M_S - I_0$

In this section we briefly discuss apparent regional variations in the relation between M_S and I_0 . We also investigated regional distinctions in the relationship between M_S and M_L , which could be expected owing to different procedures used for the calculation of M_L , but did not observe any systematic pattern.

For each earthquake shown in Fig. 3(a) we calculated the difference between the instrumental M_S and $M_S^{(12)}$ calculated from I_0

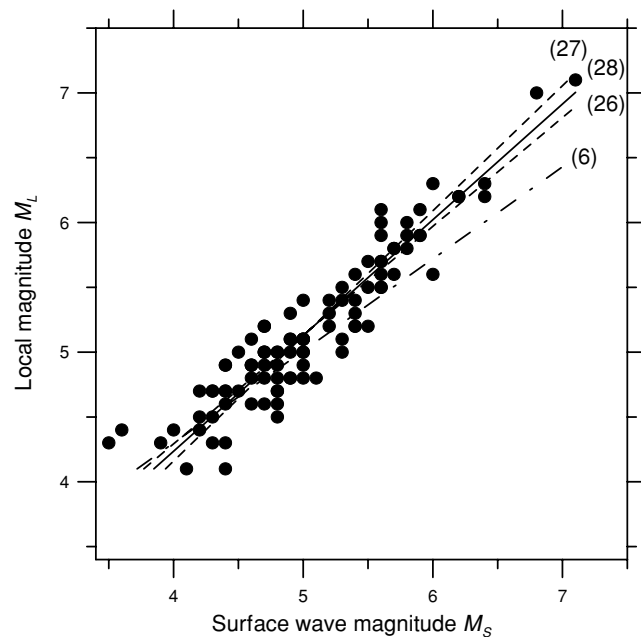


Figure 4. Distribution of 99 M_L , M_S entries of KA96 for $1 \leq H \leq 60$ km. Numbers in parentheses refer to the respective equation. Eq. (26), standard regression (7) assuming M_L is in error. Eq. (27), standard regression (7) assuming M_S is in error. Eq. (28): orthogonal regression (12). Eq. (6), relationship from Ambraseys & Bommer (1990).

using eq. (23) and display the results in the map of Fig. 5. The residual values $M_S - M_S^{(12)}$ fall in the range from -1.3 to $+1.3$ magnitude units (cf. Fig. 3b), which is equal to approximately 2.8 standard deviations. Fig. 5 clearly shows systematic regional variations with predominantly higher values (stars) for $M_S^{(12)}$ calculated from I_0 in Central Europe, the Alps, Italy, Algeria, and lower values (circles) especially in Greece, Bulgaria, Western Turkey, along the western coast of the Adriatic Sea, and in the Caucasus.

There are several possible explanations for the observed regional variations.

(1) Differences in the practice of intensity assignments and the use of different intensity scales (MKS, MCS); see the discussion of eqs (23)–(25).

(2) The consequence of a highly clustered data set to the parameters of the linear least-squares fit approximation (Pivot phenomena) as proposed by Mucciarelli (1998).

(3) Systematic variations of the stress drop. High stress drop earthquakes radiate more high-frequency energy resulting in higher macroseismic intensities for a given M_S .

(4) Variations of average focal depth. However, as we showed in Section 6.2, the correlation between M_S and I_0 is practically independent of the focal depth for the present data set. So this explanation can probably be disregarded.

(5) Systematic differences in crustal attenuation and/or local site conditions.

Which and how these possible explanations contribute to the observed variations is important to understand and will be investigated in future work.

8 CONCLUSION AND OUTLOOK

In this study the development of empirical relations between the earthquake magnitude and macroseismic parameters is investigated.

(1) We presume a tolerable level of significance, from which on a least-squares fitting approximation of empirical relations between magnitudes and macroseismic data is useful. This tolerable level depends on the aim of the investigation and the personal decision of the investigator. The significance level is defined by three criteria taken from the correlation coefficients between the parameters used. These significance criteria in three-parametric cases such as M , I_0 and $\log(H)$ help to decide from which level of significance is it reasonable to perform a regression analysis excluding the parameter of lowest importance.

(2) The differences between the standard regression eq. (7) and the orthogonal regression eq. (12) are investigated. Empirical standard regression formulae are not equations in the mathematical sense but rather a one-sided attribution. This means that they do not have the property of reversibility as mathematical equations. In contrast to this feature the orthogonal regression can be treated as a mathematical equation. It is shown that standard and orthogonal regression apply different concepts of the least-squares error. The ‘equivalent error’, for instance δM_S , represents a measure, which can be interpreted from the viewpoint of standard regression analysis if the orthogonal regression has been carried out.

(3) The results of the orthogonal and the standard regression agree completely at the centre of gravity of the data set. With increasing distance to the centre of gravity the discrepancy between both increases.

The following conclusions follow from our experience with data from KA96.

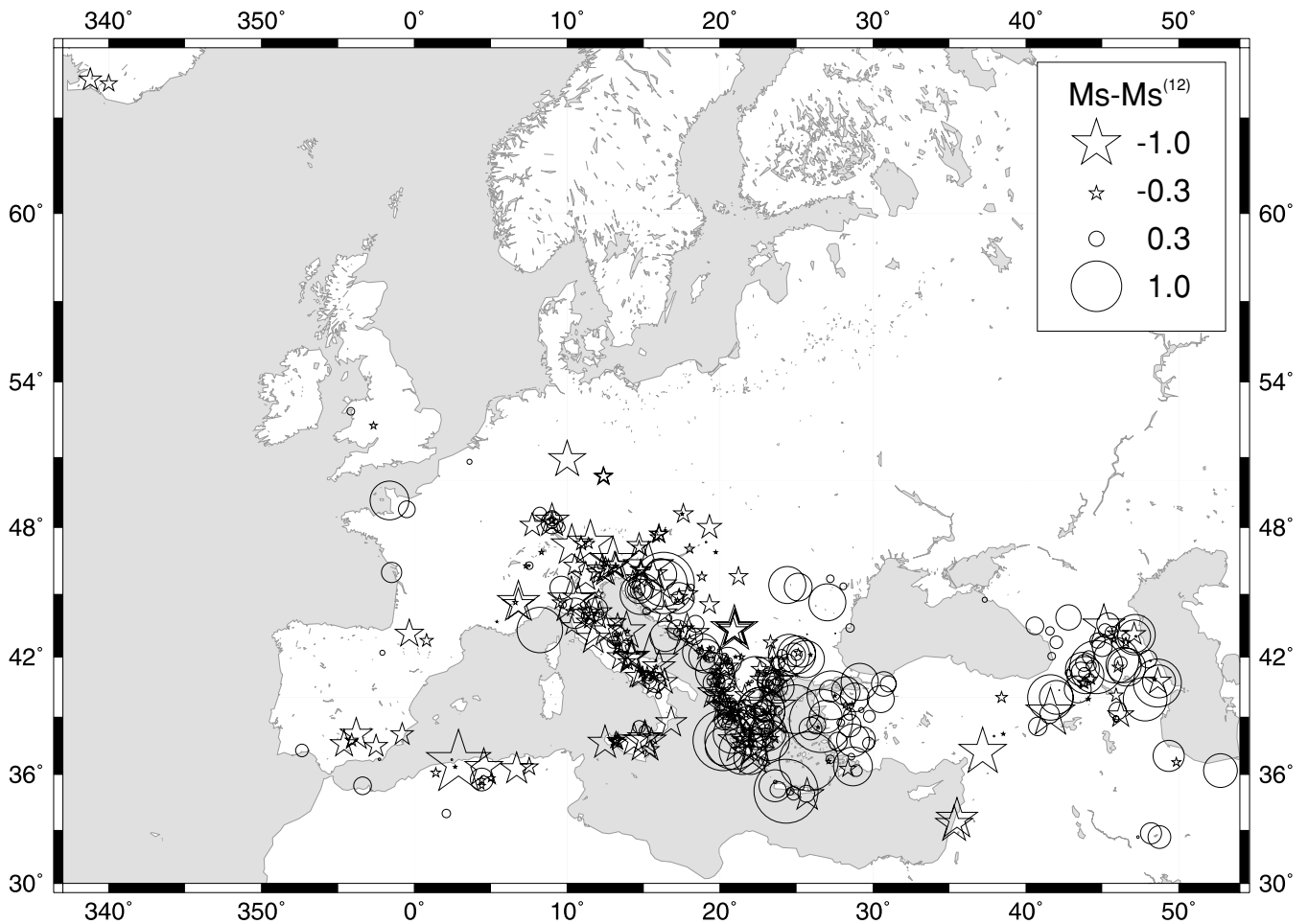


Figure 5. Regional differences in the relation between M_s and I_0 [MSK] for the 469 earthquakes shown in Fig. 3(a) used to derive eq. (23). Stars, M_s calculated from I_0 [MSK] using eq. (16) larger than the instrumental M_s , i.e. $M_s^{(12)} > M_s$. Circles, $M_s^{(12)} < M_s$. The size of the symbols is proportional to $|M_s - M_s^{(12)}|$.

(4) The mean values of the coefficients gained by the standard regression agree quite well with the coefficients of the orthogonal regression. This has been shown for both the 2-D and the 3-D orthogonal regression.

(5) The orthogonal least-squares fit in general yields a greater rms than the standard least-squares fit. The difference between both relations increases with decreasing correlation coefficients. One can regard this difference between both as a touchstone of the quality of the formulae used. For questions of earthquake hazard analysis the orthogonal regression is a considerable help as it visualizes simultaneously the errors of all input data.

(6) Eqs (16) and (23) are derived by orthogonal regression from a large high-quality data set. We recommend their application because they consider a realistic presumption of input errors and because they provide reversible equations. They should be applicable in Central and Southern Europe. However, considering the equivalent error $\delta M = 0.63$ and 0.47 , respectively, and obvious regional deviation variations, we deem it reasonable to derive regional relationships that better fit the data. Thus, they are also important for earthquake hazard assessments, especially in moderate or low seismicity domains (intraplate regions).

The magnitude is a helpful but imprecisely defined physical parameter. This fact explains the well known and lamentable lack of precision of magnitude values given in catalogues. Our study shows that a greater data set does not necessarily provide a better

basis for magnitude conversions. There is a limit of precision which the practitioner knows well. It corresponds to the order of 0.3–0.5 magnitude units.

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