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GEOPHYSICS

Evidence of Nonlinearity of the Chandler Wobble in the Earth's Pole Motion

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It is well known that rotational motions with annual and Chandler periods dominate in the dynamics of the location of the Earth's poles. The nature of the Chandler wobble is not clear. It is most frequently attributed to elasticity and differential rotation of different internal spheres of the Earth [1, 2]. Recently, it was shown [3] that the Chandler-type motions are possible in a model of solid Earth if this model is nonlinear with an annual external influence (the deviation of the equatorial section of the Earth's shape from the pure circular form is taken into account).

Using the wavelet analysis of the time series of the North Pole coordinates, we have distinguished for the first time doubled and tripled Chandlerian oscillations with variable amplitudes. In addition, we have discovered some fingerprints of oscillations with periods close to the free Eulerian nutation of the Earth and its superharmonic 1 : 2. The existence of such oscillations can only be explained within the framework of the nonlinear approach.

We used the initial data of the North Pole coordinates during the January 5, 1962–July 18, 2004 period, which are more exact than the data of the previous years. The time discreteness of these data (5 days) is more detailed compared to the time series of the previous observations. In order to omit the trend, we applied a 6-yr-running averaging to the time series and subtracted the averaged series from the initial ones.

The trajectory of the averaged component of the North Pole motion (NPM) on an (x, y) plane is shown in Fig. 1 (below). Since the positive direction of the *X*-axis corresponds to the southward displacement of the pole along the Greenwich meridian and the positive direction of the *Y*-axis corresponds to the displacement along 90° W, the figure suggests a general displacement (trend) of the pole toward Greenland. Thirty loops superimposed over the trend during the 36-yr-long period of the time-average series yields the mean period of the loop approximately equal to 1.2 yr (~14 months or 430 days), which coincides well with the known estimate of the Chandler period [2]. It is natural to suppose that the loops are actually related the Chandler wobble of the poles.

Figure 1 (upper plot) shows a pure rotational component of the NPM. The rotation is counterclockwise (in the W–E direction). The amplitude of the rotational component varies in time. Its time evolution is shown in Fig. 2 (upper plot). The graph in Fig. 2 (lower plot) shows the pattern of time evolution of this amplitude (abscissa) and the pattern over time scales (ordinate, logarithmic scale with base = 2). The plots are based on complex transformation of the time series of the amplitude of the rotational component using the Morlet function of wavelet transform (WT). The frequency characteristic of the Morlet function was taken so that the wavelet scale of the maximum response of the WT to any harmonic oscillation in the transformed series coincided exactly with the period of this harmonic. To our knowledge, the WT was used for the first time to study the motion of the Earth's poles. However, this method has been used to study variations in the velocity of the Earth's rotation in [4, 5]. The wavelet pattern in Fig. 2 is limited by the maximum scale equal to $1280\sqrt{2}$ days, starting from which the wavelet amplitude (WA) becomes extremely large and reflects the prevalence of the ~6-yr-period oscillation, which is clearly seen in the initial WT-free time series. In the range of wavelet scales smaller than $1280\sqrt{2}$ days and up to ~80 days (i.e., from 3–4 yr to approximately one season), the WA is sufficiently large as compared to the scales smaller than 80 days.

The energy spectrum (periodogram) of the transformed series (Fig. 2, right) confirms the significant amplitudes of these interannual and seasonal oscilla-

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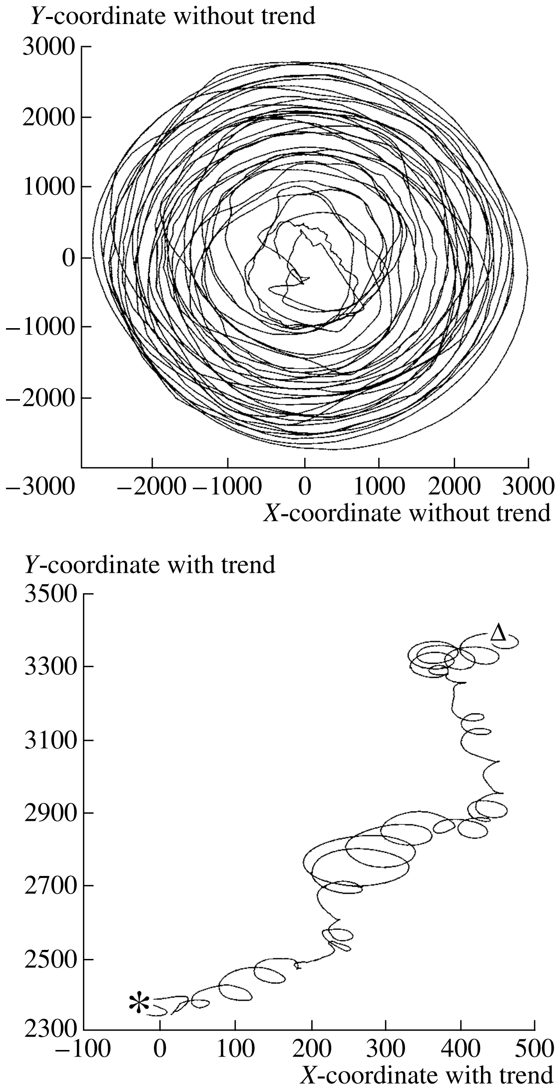

Fig. 1. The lower plot shows trajectories of running 6-yr mean coordinates of the Earth's North Pole with a general trend oriented to Greenland. The initial point of the trajectory is marked with an asterisk; the final point, with a triangle. The upper plot shows the trajectory of deviations of the North Pole coordinates, which represents the rotational component of the NPM from mean values for sequential 5-day intervals over the January 5, 1962–July 18, 2004 period.

tions. This spectrum range shows numerous peaks of spectral density with two significant peaks at periods near 1280 days and slightly greater than $640\sqrt{2}$ days. The exact values of their periods are equal to tripled (~1280 days) and doubled (860 days) Chandler periods, respectively. The spectral region adjacent to these peaks is filled with insignificant peaks, whose periods range from the doubled to single Chandler period. The single peak clearly seen at a period of ~320 days can be related to the Eulerian period of the Earth's free nutation (~10 months). Finally, one can see a slight increase in the spectral density at periods smaller than $160\sqrt{2}$ days. It is difficult to determine whether this increase (statistically negligible) coincides with a half-year period or with a superharmonic 1 : 2 of the free Eulerian nutation, or whether this is a manifestation of both factors.

Returning to the WA pattern in Fig. 2, one can see that the WA does not remain constant on any of the wavelet scales corresponding to one of the periods of spectral peaks mentioned above. The WA always varies in time; i.e., it is an amplitude-modulated oscillation. Moreover, the WA can also be a frequency-modulated parameter. For example, in the beginning of the time interval considered here (1962–1974), the WA is large on all wavelet scales greater than 640 days. In 1974–1992, the WAs of ~640 days were relatively small, while relatively large amplitudes were displaced to the scale of ~320 days. In the subsequent years, relatively large amplitudes were again displaced to the scale of 640 days or more, and they were small for the scale of ~320 days. However, these variations seem difficult for unambiguous interpretation.

The WT pattern shown in the lower left plot of Fig. 3 is easier for interpretation. It is known from the WT theory that the wavelet phase (WP) increase rate of complex WT is inversely proportional to the transformed harmonic period. If the transformed series includes a harmonic, whose amplitude is greater than that of the closest harmonics (a delta-shaped peak in the energy spectrum of the time series), then the WP increase rate for a scale smaller than the dominating harmonic period will be suppressed, and the velocities of harmonics with longer periods will be increased relative to the 'standard' case characterized by the absence of the dominating harmonics. In order to simplify the analysis, we can subtract the standard value from all WP values. In this case, if the dominating harmonic exists, then a pair of negative and positive WP values would appear in the WT phase pattern. The boundary between them would pass over the scale equal to the period of this harmonic. The bands of negative (positive) values would be located on the smaller (greater) scales, relative to the period of the dominating harmonic. Thus, the WT phase pattern can be used as an additional tool to distinguish the peaks in the energy spectrum. Since the difference between the actual and standard WP growth rates is accumulated in the course of time (if the spectral peak exists), the spectral peak can be very reliably identified, especially if it is supplemented by the spectrum of time-average WP values as functions of the scale. The position of the peaks is clearly determined in the spectrum on the basis of its transition across the zero point. It should be noted, however, that the frequency resolution of this method of spectral peak identification is low. If alternation is observed, i.e., the amplitude and/or frequency of the dominating harmonic varies in time (broad peak or a band of high spectral density in the energy spectrum), then the shape and temporal intensity of the above pair

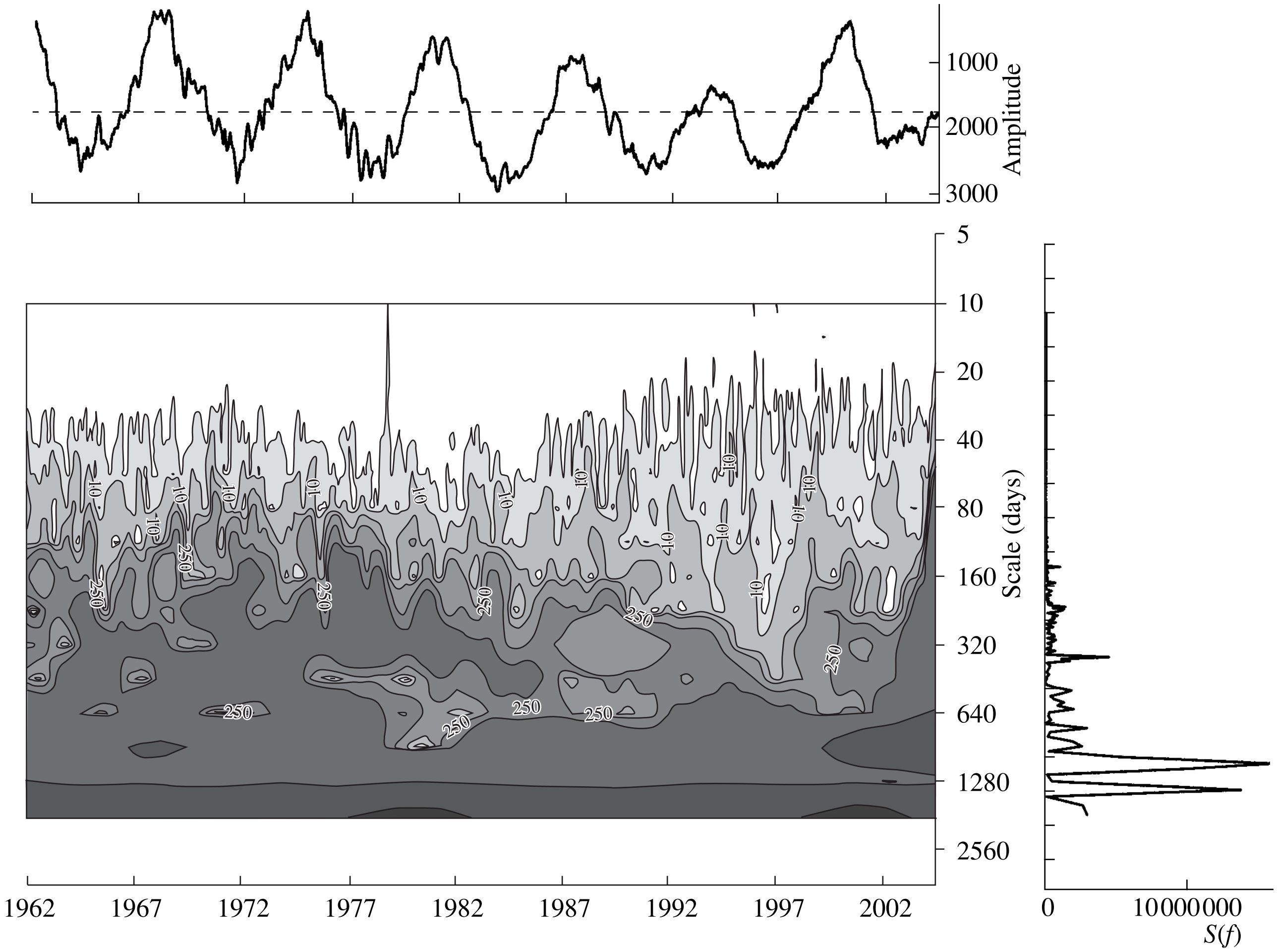

Fig. 2. The upper plot shows time series of the amplitude of the rotational component of the NPM. The lower left plot shows wavelet amplitudes of the WT of the time series shown above. Regions of large WA values are shown in dark color. The lower right plot shows energy spectrum (periodogram) of the time series shown above.

of regions changes and transitions across the zero point in the spectrum phase become vague.

The WP pattern clearly shows three pairs of such bands (Fig. 3). The first part is bounded by the wavelet scale of ~180 days. This boundary is clearly seen as a sharp transition across zero in the phase spectrum as a function of the wavelet scale (Fig. 3, right plot). Thus, the time series of the amplitudes of rotational motion of the pole includes the approximately half-year-long periodicity mentioned above during the consideration of the WA pattern and energy spectrum in Fig. 2. However, it is not possible to determine the exact period of this periodicity. It is most likely that both the half-year oscillation and superharmonic 1 : 2 of the free Eulerian nutation (approximately 5-month period) make a contribution to this period. The boundary between the next negative and positive bands and the corresponding transition from the negative to positive value in the spectrum of this phase falls within the scales of $320\sqrt{2}$ and 640 days. Although the transition across zero in the spectrum is less distinct than in the previous case, one can assume that the group of peaks of the doubled Chandler period corresponding to this boundary is also real. A clearer peak in the energy spectrum at the free Eulerian nutation of ~320 days in the phase pattern (Fig. 3) corresponds to the boundary between the positive (negative) WP values at smaller (larger) scales. The time-average value of the phase for this period is negative. This could be interpreted as evidence of the spectral density minimum at a period of 320 days. Actually, this may indicate that the total contribution of the peak with approximately a half-year period and the group of weaker peaks at the Chandler-type periods exceeds the contribution to the WP increase rate of a singular (but greater) spectral peak during the period of free Eulerian nutation. The third pair of the bands (negative on the smaller scales and positive on the greater scales) is bounded by the wavelet scale slightly smaller than 1280 days. It reflects the joint contribution of both (doubled and tripled) Chandler periods, whose peaks are quite prominent in the energy spectrum (Fig. 2) to the WP increase rate. Thus, there are no doubts in their reality.

Thus, we can conclude that the existence of various amplitude- and frequency-modulated oscillations with

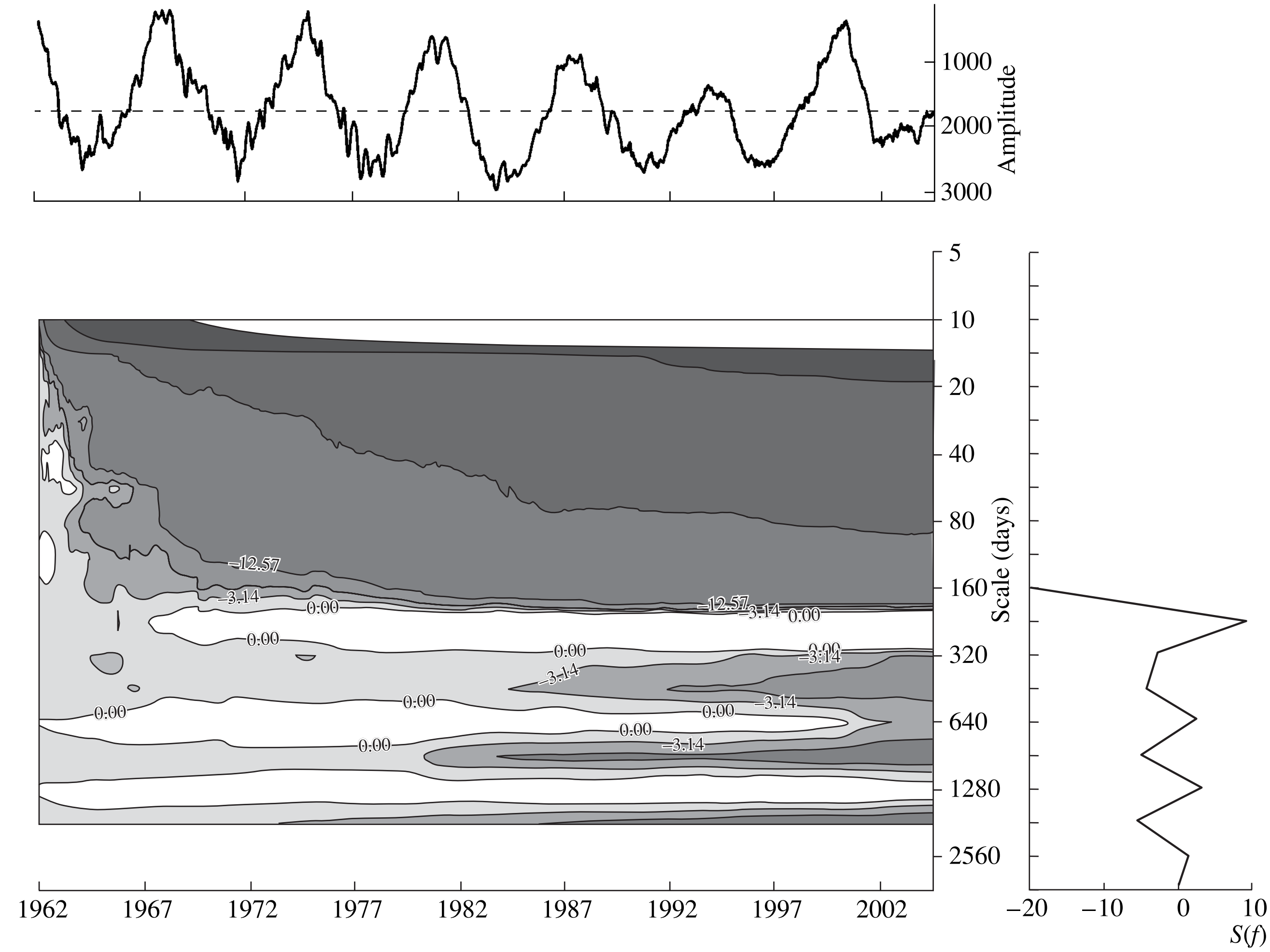

Fig. 3. The upper plot shows time series of the amplitude of the rotational component of the NPM. The lower left plot shows WT wavelet phases of the time series shown above. Regions of positive deviations of the phase from the standard WP values are shown in dark color. The lower right plot shows spectrum of the time-average WP values as function of the wavelet scale.

half, double, triple, and other periods is natural for nonlinear systems subject to periodical external forces. Therefore, we believe that the discovered phenomena of doubled and tripled Chandler periods and the half period of the free Eulerian nutation in the motion of poles can be explained within the framework of the nonlinear model of the Earth's rotation.

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