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Quick and accurate *Q* parameterization in viscoelastic wave modeling

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ABSTRACT

We introduce a procedure for including the attenuation factor Q in a consistent manner in seismic modeling and show 3D examples. The Q fitting over a chosen frequency band involves two algorithms: The first creates starting values of relaxation times, and the second does nonlinear inversion using the results of the first as initial values. The resulting Q function gives a good approximation to a constant Q over the chosen frequency band. The algorithm is combined with a finite-difference (F-D) code that includes topographies in 3D seismic media. The velocity-stress formulation for viscoelastic wave modeling is used with an arbitrary number of relaxation mechanisms to model a desired Q behavior. These equations are discretized by high-order F-Ds in the interior of the medium, and we gradually reduce the F-D order to two at the stress-free surface, where we implement our free-surface boundary conditions. The seismic F-D algorithm is applied to a marine seismic experiment, with and without viscoelasticity, to emphasize the importance of including physical attenuation and dispersion in seismic modeling. Their inclusion, even for marine surveys, is clearly important for lossy ocean bottoms. Our procedure for more accurate modeling of physical dispersion and attenuation may increase future motivation to include viscoelasticity in seismic inversion.

INTRODUCTION

Since publication of Day and Minster's (1984) Padé approximant method for numerical, full-viscoelastic wave modeling, more accurate techniques for the displacement-stress formulations have been developed (Carcione et al., 1988a; 1988b; Carcione, 1993). These formulations transform the time convolutions involved in the viscoelastic constitutive relationships to first-order partial differential equations by introducing memory variables. This procedure was extended to the velocity-stress formulation by Robertsson et al. (1994). For this formulation, Blanch et al. (1995) developed a quick and accurate (for low-loss media in particular) procedure for modeling a constant Q behavior (Q is the attenuation factor). Hestholm (1999) used the Blanch et al. (1995) procedure together with velocity-stress curved-grid equations to model variable free-surface topography. Efforts have been made in the area of estimating the necessary time-relaxation variables for given Q functions (Asvadurov et al., 2004; Carcione et al., 2002), and Moczo and Kristek (2005) gave an overview and relationship between rheologic models and recent procedures to model viscoelasticity.

We present an improved Q-parameterization scheme for viscoelastic modeling by combining two previously known methods. The method of Blanch et al. (1995) is used to produce input values to the nonlinear optimization Nelder-Mead algorithm for improved parameter estimation. The governing viscoelastic wave equations in this work are the velocity-stress formulation with an arbitrary number of relaxation mechanisms. These equations are the Cartesian equations in a curved grid transformed to a computational, rectangular grid (Hestholm, 2002, equations 1–18). Boundary conditions (Hestholm and Ruud, 2002) in the velocity-stress formulation are implemented at the top of the numerical grid, and absorbing boundaries (Cerjan et al., 1985) from the remaining boundaries. Numerical simulation of a marine seismic experiment is then presented to illustrate how important the inclusion of viscoelasticity may be, even in a marine environment.

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VISCOELASTIC Q PARAMETERIZATION

The viscoelastic velocity-stress wave equations used in this work are given in equations 1–18 of Hestholm (2002), using L arbitrary relaxation mechanisms in the medium. They are the momentum conservation equations, Hooke's law, and the memory-variable equations for a medium bounded above by a topography function and are not given here because of space considerations. Our boundary conditions for free-surface topography (Hestholm and Ruud, 2002; Hestholm, 2003) are implemented at the top of the grid. For numerical discretization of the interior wave equations, staggered eighth-order finite differences (F-Ds) (Kindelan et al., 1990) are gradually reduced to staggered sixth-, fourth-, and second-order F-Ds (Fornberg, 1988) when approaching the free surface. Along all remaining boundaries, the method of Cerjan et al. (1985) is employed for exponential wavefield damping.

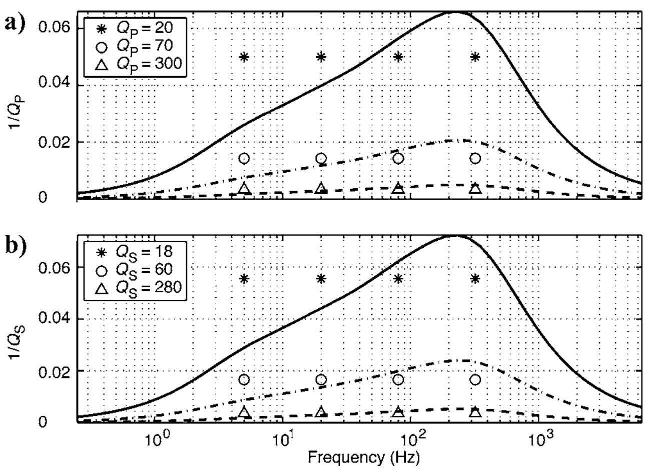

Figure 1. Modeled (a) Q_P and (b) Q_S versus frequency in three layers of a geologic model using only the Blanch et al. (1995) τ method. The symbols are the relaxation frequencies for which the $1/Q$ values are fitted: Stars (heavy curve) signify the lower half-space, circles (dash-dotted curve) the intermediate layer, and triangles (dashed curve) the upper layer.

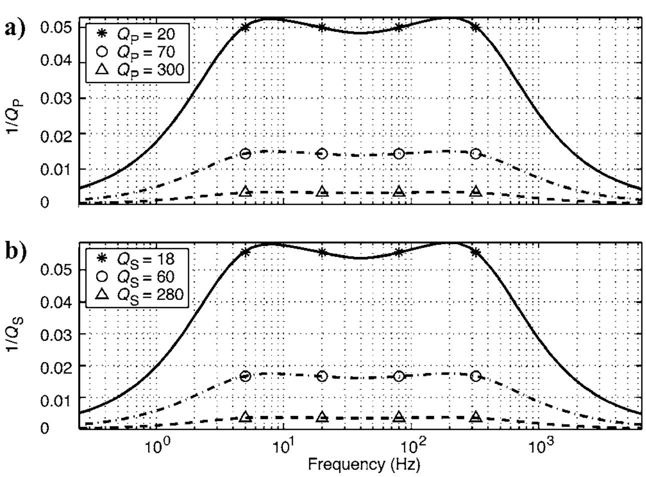

Figure 2. Modeled (a) Q_P and (b) Q_S versus frequency in the three layers of the model of Figure 1 using relaxation times from the Blanch et al. (1995) τ method input to the nonlinear Matlab inversion routine *fminsearch*. The symbols are the relaxation frequencies for which the $1/Q$ values are fitted: Stars (heavy curve) signify the lower half-space, circles (dash-dotted curve) the intermediate layer, and triangles (dashed curve) the upper layer.

To make modeling of Q sufficiently accurate, we use $L = 4$ relaxation mechanisms. The constant Q values are assumed different in each domain of a geologic medium. The τ method of Blanch et al. (1995) uses a least-squares procedure to estimate stress- and strain-relaxation times for viscoelastic wave modeling, given a desired constant Q as a function of frequency. The method is known to give good results, except for geologic media that have low Q values; in such media the estimated relaxation times commonly exhibit a monotonically decreasing Q with frequency. We found that using the Blanch et al. (1995) τ method by itself gave slightly less than satisfactory results for the approximation of a constant Q for chosen relaxation frequencies of 5, 20, 80, and 320 Hz (Figure 1). However, the τ method provided excellent starting values of stress- and strain-relaxation times for input to the nonlinear optimization Matlab routine *fminsearch*. This routine is an implementation of a direct-search simplex Nelder-Mead algorithm (Nelder and Mead, 1965; Lagarias et al., 1998); see Appendix A. This procedure gave values of stress- and strain-relaxation times that fitted constant Q values for P- and S-waves (Q_P and Q_S) very well over the range covered by the chosen relaxation frequencies (Figure 2). Figures 1 and 2 illustrate the resulting fit for the Q function to a desired constant Q value in each of three layers in a geologic medium, where the fit is exact for the selected relaxation frequencies. The square of the difference between the desired constant Q value in each layer and our fitted Q function was used as the cost function in the optimization function call. This cost function can be written as

$$\phi(\boldsymbol{\tau}_\epsilon, \boldsymbol{\tau}_\sigma) = \sum_{\omega_s} [Q_{\text{const}} - Q(\omega_s, \boldsymbol{\tau}_\epsilon, \boldsymbol{\tau}_\sigma)]^2, \qquad (1)$$

where ω_s represents the selected frequencies for exact fitting of the Q function, Q_{const} is the constant desired Q value, and $Q(\omega_s, \boldsymbol{\tau}_\epsilon, \boldsymbol{\tau}_\sigma)$ is our Q function evaluated at each of the preselected frequencies (one for each of the chosen relaxation mechanisms). The variables $\boldsymbol{\tau}_\epsilon$ and $\boldsymbol{\tau}_\sigma$ are L-dimensional vectors of the relaxation times, with respect to which the optimization is done. This cost function is different for each wave mode. The Q function assumes no dependence of Q_S on the P relaxation and, for both P- and S-waves (Blanch et al., 1995), has the form

$$Q(\omega, \boldsymbol{\tau}_\epsilon, \boldsymbol{\tau}_\sigma) = \frac{1 - L + \sum_{\ell=1}^{L} \frac{1 + \omega^2 \boldsymbol{\tau}_{\varepsilon\ell} \boldsymbol{\tau}_{\sigma\ell}}{1 + \omega^2 \boldsymbol{\tau}_{\varepsilon\ell}^2}}{\sum_{\ell=1}^{L} \frac{\omega(\boldsymbol{\tau}_{\varepsilon\ell} - \boldsymbol{\tau}_{\sigma\ell})}{1 + \omega^2 \boldsymbol{\tau}_{\varepsilon\ell}^2}} \qquad (2)$$

where ω is the circular frequency, $\boldsymbol{\tau}_{\varepsilon\ell}$ is the ℓth strain-relaxation time, and $\boldsymbol{\tau}_{\sigma\ell}$ is the ℓth stress-relaxation time. Only a few iterations of the Nelder-Mead algorithm are needed for convergence; other nonlinear optimization algorithms may work just as well.

A comparison between corresponding elastic and viscoelastic results is performed to emphasize the importance of viscoelastic modeling, even in marine experiments. Figures 3 and 4 show the bathymetric relief and streamer seismograms from a simulation of a marine seismic survey. The water depth varies from about 100 to 600 m. There is a plane interface at 1200 m depth (depths are relative to the sea surface). Another plane interface is at 1500 m depth. Parameters of each layer from top (sea) to bottom are the following (the Q values are used only in the viscoelastic simulation): $v_{P_1} = 1480$ m/s, $v_{S_1} = 0$, $Q_{P_1} = Q_{S_1} = 40{,}000$; $v_{P_2} = 2000$ m/s, $v_{S_2} = 1154$ m/s, $Q_{P_2} = 15$, $Q_{S_2} = 10$, $v_{P_3} = 2500$ m/s, $v_{S_3} = 1443$ m/s, $Q_{P_3} = 70$, $Q_{S_3} = 50$, $v_{P_4} = 3000$ m/s, $v_{S_4} = 1732$ m/s, $Q_{P_4} = 140$ and $Q_{S_4} = 100$. The density everywhere is 1800 kg/m^3. A Ricker pressure-point source of central frequency 15 Hz is located in the middle of the x and y dimensions of the model at 10 m depth. The model setup yields an unconventional marine geometry for marine seismic surveys in which the air-gun source is towed near the center of the recording cable. The model is uniformly discretized ($dx = dy = dz = 10$ m) and consists of 200 grid points in each dimension; thus, it is a cube of 2000 m on all sides. The free surface is the planar sea surface; the bathymetry is modeled by stair steps, and all remaining interfaces are planar. Absorbing boundaries (represented by 20 grid points) using both low Q and the exponential damping of Cerjan et al. (1985) are employed along all grid boundaries except on top at the sea surface. The receivers of the west-east profile (Figure 4) are at the middle of the south-north dimension and extend from 500 to 1500 m. An elastic simulation is done (by using the viscoelastic code with very high Q values), and then the seismogram for the corresponding viscoelastic medium in Figure 3 is given as a comparison. All model parameters except Q are kept constant.

A difference can be observed in the seismograms even after waves have propagated through the larger depth extent of the acoustic sea at the top of the model. Apart from the direct arrivals, much detail of the two sea-bottom reflections (as well as diffractions from the stair-step discretization of the see bottom) and, to a larger extent, the reflections from deeper layers and sea-bottom mode conversions are dispersed and attenuated in the viscoelastic simulation. Deeper-layer reflections should occur at about 1.3 s in the seismograms (all times are two-way traveltimes); all arrivals later than this time are indistinguishable in the viscoelastic example because of physical attenuation. This occurs even for marine experiments with receivers close to the sea surface, when such a highly attenuative sea bottom is modeled.

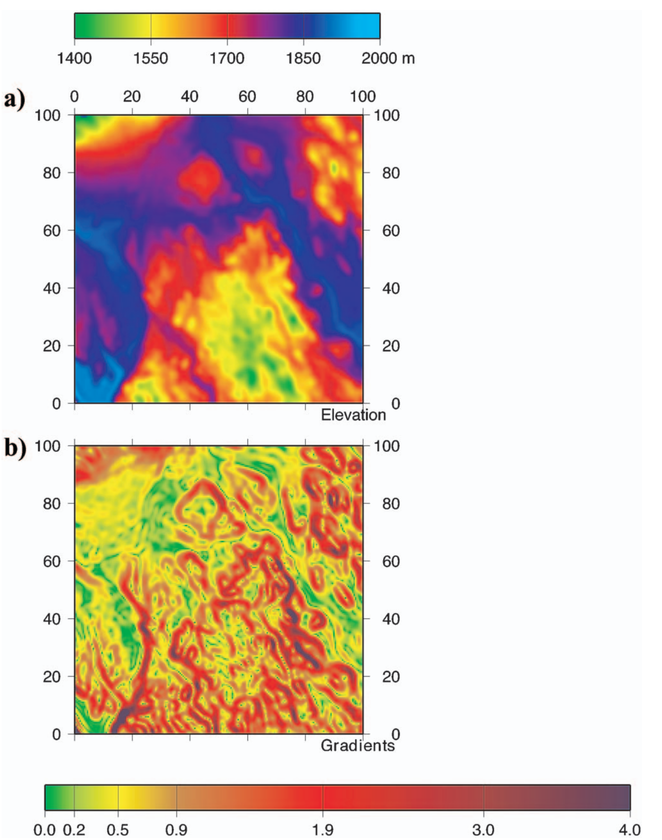

Figure 3. (a) Bathymetry in meters and (b) its absolute gradients for a marine seismic experiment. Axes are in multiples of 20 m to cover the xy-section of 2000×2000 m^2 of the model used in the simulations.

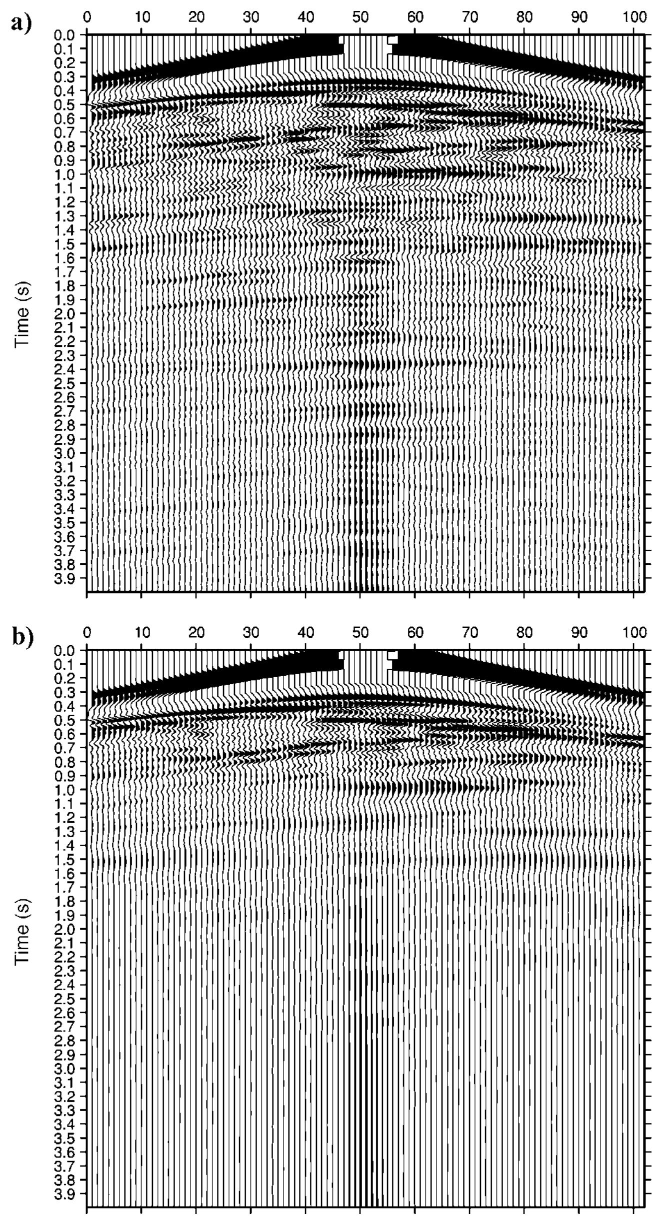

Figure 4. (a) Elastic and (b) viscoelastic pressure seismograms for a profile of 101 receivers at 20 m depth and spaced at 10 m, along the middle of the model ($y = 1000$ m) described in Figure 3. The profile extends from $x = 500$ to $x = 1500$ m. Horizontal axes are receiver numbers.

CONCLUSIONS

For accurate viscoelastic Q parameterization, we have employed a new procedure in which we combine two previously known methods for improved results. The Blanch et al. (1995) τ method was used to produce starting values for stress- and strain-relaxation times that were input into the nonlinear optimization Nelder-Mead algorithm. The latter was used to minimize the square of the difference between the Q expression for our formulation and our desired constant Q to produce well-fitted Q curves with frequency and viable viscoelastic effects. The procedure leads to improved results compared to using either method separately. As such it should lead to an improved state of the art for fitting of any Q function over any chosen frequency band. Boundary conditions for free surfaces are combined with the velocity-stress formulation of the viscoelastic wave equations to model a marine seismic survey with a steep and strongly varying bathymetry, with and without viscoelasticity included. The need to include physical attenuation and dispersion is clearly illustrated even in this marine experiment, when a lossy sea bottom is present. The goal of this work is to improve the accuracy of viscoelastic wave modeling, with consequent increased future motivation to include viscoelasticity in velocity model building, for example.

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APPENDIX A

NELDER-MEAD ALGORITHM AND DESCRIPTION OF ITS MATLAB IMPLEMENTATION

The Nelder-Mead algorithm (Nelder and Mead, 1965) is a direct-search unconstrained optimization method. *Direct search* means that the method is independent of any gradient information and hence can be slower than some methods that use such information. In the applied case, however, convergence is very fast. Because of its direct-search type, the method is particularly useful for discontinuous functions. It is based on evaluating a function at the vertices of a simplex, then iteratively shrinking the simplex as better points are found until some desired bound is obtained. A simplex is the generalization of a tetrahedral region of space to n dimensions. The simplex is so named because it represents the simplest possible polytope in any given space.

The Nelder-Mead algorithm (Nelder and Mead, 1965) is implemented in Matlab through the function *fminsearch*. If n is the dimension of a vector x, a simplex in n-dimensional space is characterized by the $n + 1$ distinct vectors that are its vertices. In 2D space, a simplex is a triangle; in 3D space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function values at the vertices of the simplex, and, usually, one of the vertices is replaced by the new point to give a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance.

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