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Cold region river discharge uncertainty—estimates from large Russian rivers

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Abstract

We develop an error model to understand the reliability and accuracy of river discharge datasets that are now being used for a variety of important global change questions. The developed error model for cold region river discharge uses standard hydrometric data along with information on the frequency and precision of measurements, characteristics of river channel capacity, and method of discharge computation. The uncertainties of daily, monthly and annual discharge data for the downstream gauges of the six largest Eurasian rivers (Severnaya Dvina, Pechora, Ob', Yenisei, Lena and Kolyma) in the pan-Arctic drainage along with uncertainty of aggregated annual time series are evaluated using the suggested methodology. The study shows that uncertainties associated with discharge determination significantly change from year to year and strongly depend on the computational methods used and frequency of discharge measurements.

Recent work by Peterson et al. (2002) has shown increases in river discharge to the Arctic Ocean of the six largest Eurasian rivers of 7% from 1936 to 1999. This paper focuses on determination of reliability in the discharge data which provided such conclusion. The obtained results further confirm the findings of Peterson et al. (2002) concerning the rise in river discharge. We found that errors of the total annual discharge for the six rivers over the period 1950–2000 are in the range 1.5–3.5%. The long-term trend of the observed discharge from these six rivers into the Arctic Ocean for 1936–2000, along with uncertainty associated with discharge data, is 2.0 ± 0.4 km^3/year.

Keywords: River discharge; Hydrometric data; Streamflow data; Stage-discharge Rating curve; Discharge uncertainty; Error

1. Introduction

Few of the major measured components of the hydrological cycle are considered to have sufficient accuracy to support a full closure of the water budget. Groundwater fluxes tend to be poorly known and

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Nomenclature

a	absolute maximum value for q_t
b	parameter of the equation approximating the rating curve, which denotes the degree of curvature or slope of the relationship
C_{ice}	correction coefficient for ice-affected discharge
d	parameter of the equation approximating the rating curve, which reflects the scales being used for stage and discharge
H	observed daily stage heightvalue
H_0	parameter of the equation approximating the rating curve, which can be defined as the virtual stage at zero discharge
H_i	stage height at time i
$H_{max,min}$, $\bar{H}$	maximum, minimum and mean water stage values
k_i, k_j	parameters characterizing the pair correlation of the independent variables (water stage) in the polynomial approximation equation for the rating curve
m	order of the polynomial approximation equation for the rating curve
n	number of discharge measurements used for approximation of the rating curve
Q	measured discharge
Q'	estimated daily discharge
$Q=f(H)$	stage-discharge relationship (rating curve)
Q_{dd}	computed daily discharge value
Q_{ice}	discharge measured under the ice conditions
$\bar{Q}$	mean of measured discharge
$\bar{Q}_{T_i}$	averaged daily discharge estimates over the period $?_i$ when the same computational technique to daily discharge was applied
$\tilde{q}_{mean}$	mean value of $\tilde{q}_t$ for period of unstable stage-discharge relationship
q_t; $\tilde{q}_t$	absolute and relative deviations of measured discharge from the rating curve
α_i	relative deviation of water stage value from the mean
$\beta_1 \ldots \beta_m$	fitted parameters of the polynomial approximation equation for the rating curve
$\tilde{\varepsilon}_{an}$	relative error of annual discharge estimate
ε_{apr}	approximation error for stage-discharge relationship
ε_{dd}, $\tilde{\varepsilon}_{dd}$	absolute and relative errors of daily discharge estimates
$\tilde{\varepsilon}_{in}$	relative interpolation error
ε_{mes}, $\tilde{\varepsilon}_{mes}$	absolute and relative random errors of discharge measurements
$\tilde{\varepsilon}_T$	relative error of averaged discharge characteristic over T days
$\varepsilon_{\sum Q}$; $\tilde{\varepsilon}_{\sum Q}$	absolute and relative errors of summarized discharge estimate of several river gauges
σ_q, $\tilde{\sigma}_q$	absolute and relative standard deviation of measured discharge from the rating curve
$\tilde{\sigma}_{\Delta Q}$	relative deviation of the measured discharge from mean measured discharge over the interpolated period
σ_{x_i}, σ_{x_j}	standard deviation of the independent variables (water stage) in the polynomial approximation equation for the rating curve
s	number of gauges used to get the summarized discharge estimate

evapotranspiration is rarely measured in dense sampling over large regions. Measurement of rainfall is carried out more regularly, but in cold regions this only accounts for a fraction of total precipitation. Solid precipitation measurement is known to have large systematic and random errors, often with large uncertainties arising from the use of imperfect gauging techniques (Goodison et al., 1998). The errors for solid precipitation can reach 70% and higher in northern regions under windy conditions (Golubev, 1969; Goodison, 1978; Groisman and Easterling, 1994; Goodison et al., 1998). At the same time, river discharge is believed to be one of the most accurately measured components of the hydrological cycle (Dingman, 2001; Vörösmarty et al., 2001), integrating drainage basin behavior over a range of scales. The accuracy of discharge data has been the subject of a limited number of studies. The earlier error models were either qualitative (Ogievsky, 1937) or they considered only uncertainties of discharge estimates computed under open water conditions (Dickinson, 1967; Herschy, 1985; Alexeev, 1975; Manley, 1977; ISO, 1982). The accuracy for Russian gauges is usually based on hydrometric standards (Methodical Guidance, 1987), which provide average errors but do not take into account the particular conditions of measurement or the data processing techniques.

Recent work by Peterson et al. (2002) shows a mean increase in river discharge to the Arctic Ocean of the six largest Eurasian Rivers ('Eurasian 6') of 7% from 1936 to 1999. While the annual trend estimate is instructive, this increase is the byproduct of a highly variable time series containing annual, seasonal, and daily cycles. The underlying error of any river discharge time series will originate from a number of sources. Initially, there are errors introduced by measurement instrumentation and techniques used in a single stream-gauging observation. Dickinson (1967) discussed 16 possible sources of systematic and random errors related to an individual measurement. The most significant ones were the precision of current-meter calibration, the difference between true and assumed velocity distributions in the vertical and horizontal directions, pulsation in the flow regime defined by the distribution of point velocity in time, and the difference between true and assumed stream bed configuration. Any discharge measurement includes a large set of elementary measurements such as depth, width and velocity in each individual point. Consequently, random error in an aggregate discharge measurement can be estimated by taking into account the random errors of elementary measurements (Zheleznikov and Danilevitch, 1966; Herschy, 1985). Alternatively, we can define the random error of discharge measurement through statistical processing of series of accurate discharge measurements organized at a single site (Rozhdestvensky et al., 1990). Both approaches are based on a set of special reference measurements and cannot be applied to routine discharge measurements. The test measurements are usually made to define uncertainties due to one of the error sources (Pelletier, 1988), for example: (i) error of sampling the cross-section area (Wahl, 1977); (ii) error of sampling the mean velocity in time and in space (Dement'ev, 1962; Carter and Anderson, 1963); (iii) uncertainties in the current meter (Smoot and Carter, 1968) and other error sources. The accuracy of discharge measurement is also affected by human subjective error, depending mainly on personal skills (Walker, 1991). This error source cannot be well controlled or evaluated except for gross errors producing anomalies in the data. In practice, therefore, the error of discharge measurement is often not evaluated for each individual measurement and accepted as a normalized value. Such normalized errors for various gauging conditions are based on a series of test measurements for different types of river and measurement techniques, and are usually given in hydrometric manuals (MHS, 1978; RD 5208318–91, 1991; ISO, 1998).

Another approach to the assessment of error in discharge data is based on the investigation of hydraulic characteristics of river channel capacity and the statistical interpretation of hydrometric data (Dymond and Christian, 1982; Herschy, 1985; 1999). The basis of this method involves a number of assumptions: (i) river control may be considered to be stable during a particular period of time; (ii) there is a true stage-discharge relationship, or at least a mean relationship about which the true one varies randomly; and (iii) most of the variability of point measurements arise from measurement error (Dickinson, 1967). This approach can be applied both to uncertainty in time-aggregated river discharge data (daily, monthly, yearly) as well as to errors related to the direct measurements.

This paper considers the uncertainties associated with time-aggregated discharge data. Daily discharge records are a basic hydrometric characteristic widely used in hydrological research which underlies all other averaged discharge data. It is therefore important to know the expected error of daily discharge estimates to ensure correct interpretation of the data. It should be noted that not all accurately made discharge measurements are used to estimate daily discharge values. There are many conditions when a measurement will deviate from a stage-discharge relationship, such as shifting controls caused by scour, fill, ice, or aquatic growths at the gauging station; or the gauge being affected by variable slope caused by variable backwater or by changing discharge (Pelletier, 1988). In general the accuracy of discharge data will depend on the

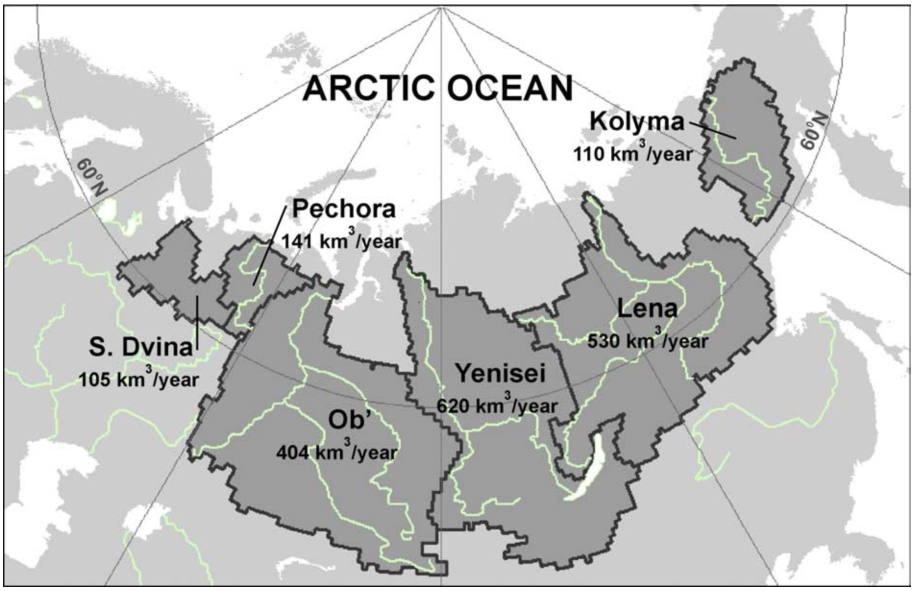


Fig. 1. Map of pan-Arctic watershed showing catchments and average annual discharge of the six largest Eurasian Arctic rivers. The presented error model was applied to the discharge data of the downstream gauges of these six rivers.

accuracy of the individual measurement, the frequency of measurements, their distribution in time and stage, and the computational procedure (Grover and Hoyt, 1916; Karasev, 1980).

We seek here to provide an assessment of the accumulated error found in river discharge records. The analysis of errors will focus on the large downstream gauges of the major Russian rivers draining into the Arctic Ocean (Fig. 1), which have an adequate volume of published, detailed, long-term hydrometric data. However, the error model considered here is also applicable to river discharge data for gauges where similar measuring and computational methods are used. This paper presents a consistent methodology for the objective evaluation of error in river discharge estimates for different computational techniques used in cold region rivers.

Table 1
Descriptive information about discharge gauging stations

Station name	Drainage area (km^2)	Average annual discharge		Distance from gauge to basin outlet (km)	Datum (m)	Daily discharge data available (Years)	Measured discharge records available (Years)
		m^3/s	$km^3/year$				
Severnaya Dvina at Ust-Pinega	348000	3330	105	137	1.57	1881–2001	1946–1975
Pechora at Ust-Tsilma	248000	3440	108	425	10.54	1932–2001	1946–1975
Ob' at Salekhard	2950000[a]	12600	398	287	0.44	1930–2000	1951–1975
Yenisei at Igarka	2440000	18400	581	697	0.03	1936–2000	1955–1975
Lena at Kusur	2430000	16700	527	211	−1.41	1934–2001	1949–1973, 1994–2001
Kolyma at Srednekolymsk	361000	2200	69.4	641	8.00	1927–2001	1952–1964

[a] Total drainage area not including endorheic (internal) basins is 2,430,000 km^2.

2. Selection of sample data

High latitude rivers have a number of distinctive features complicating otherwise reliable discharge estimates. Long ice-covered periods interfere with the use of an open channel rating curve to estimate discharge for up to 7–8 months of each year. Substantial ice thickness, cold weather, and low river flux velocity under the ice reduce the accuracy of such measurements (Prowse and Ommaney, 1990).

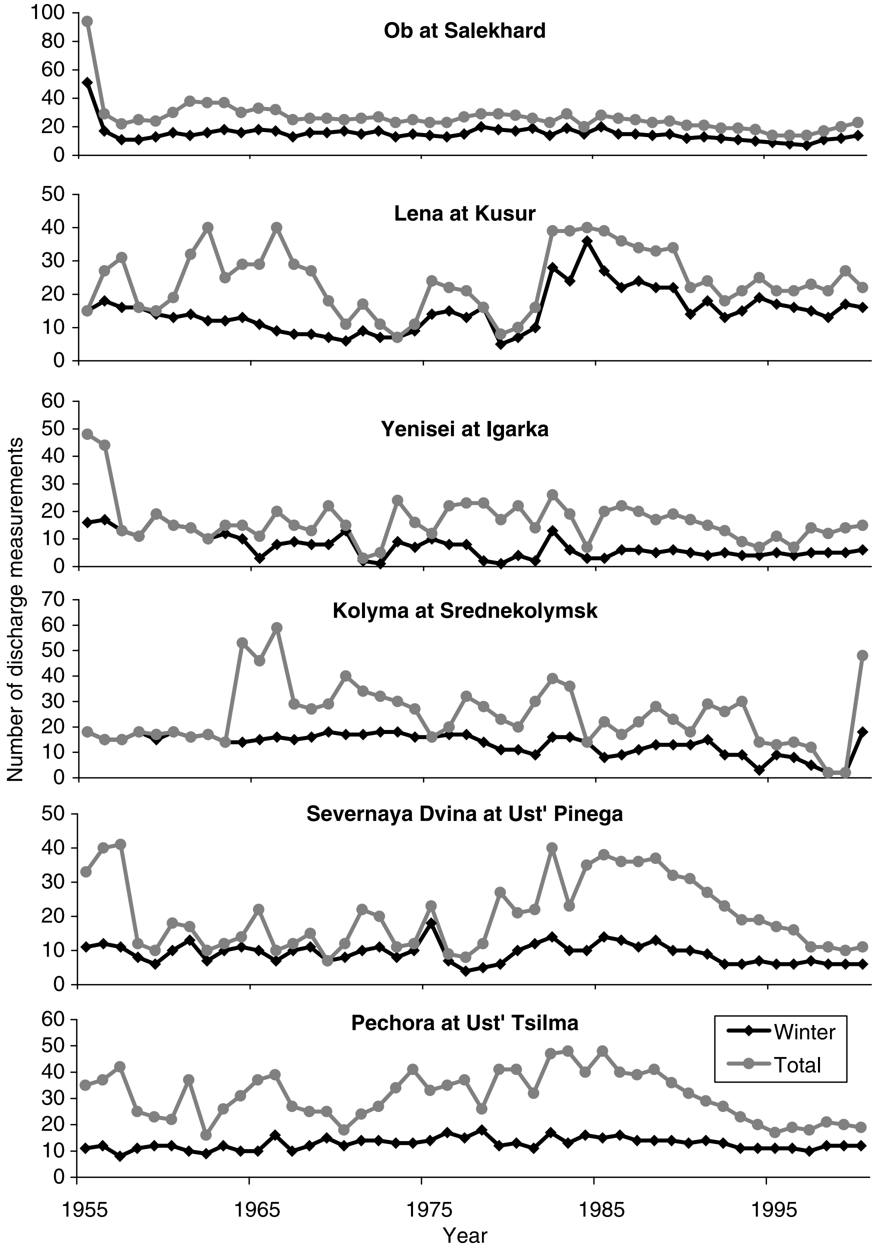
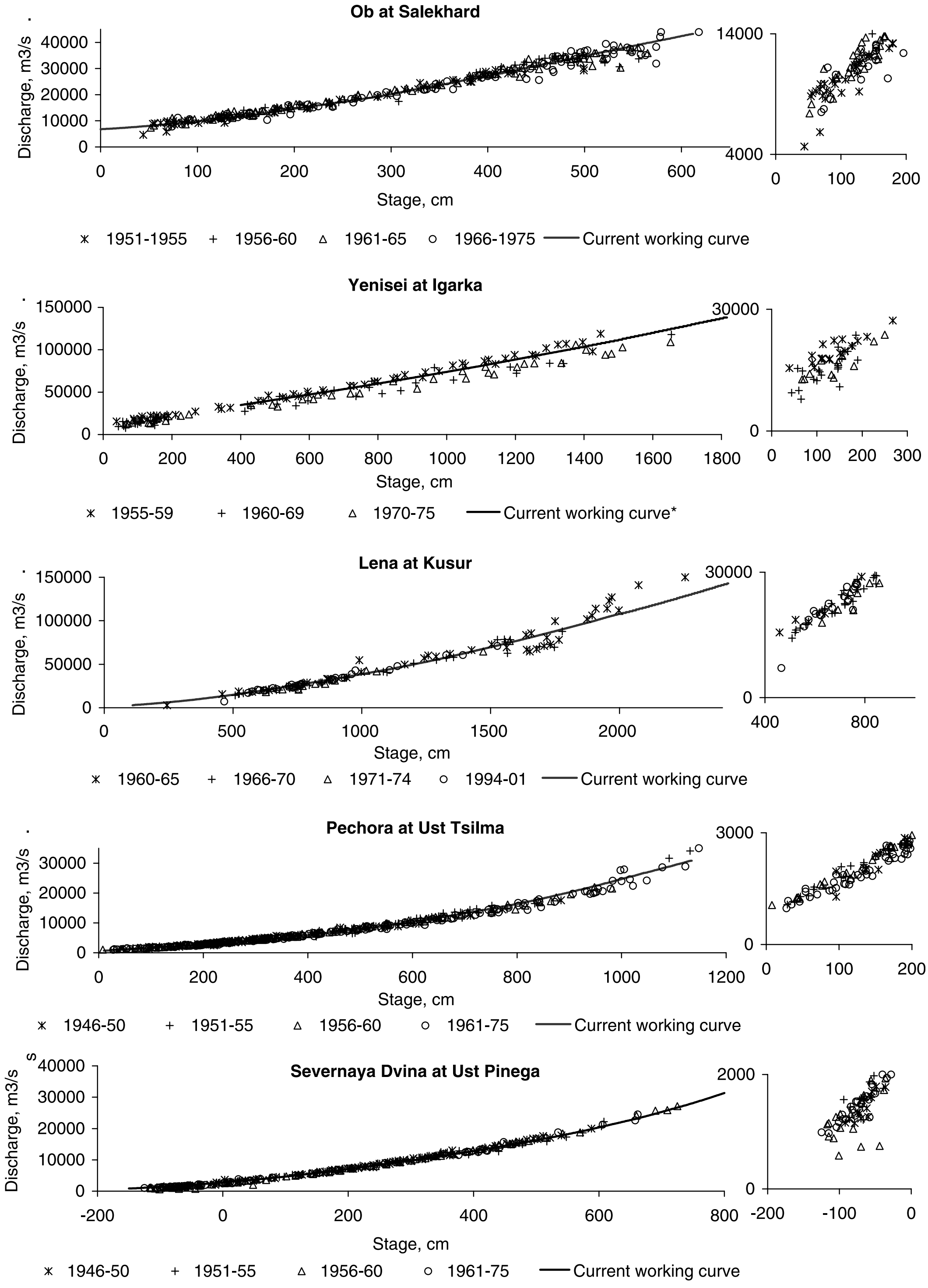


Fig. 2. The total number of discharge measurements per year used both to confirm long-term rating curve and to estimate the values of discharge under ice conditions for 1955–2000.


Ob at Salekhard
Discharge, m3/s
Stage, cm
1951-1955
1956-60
1961-65
1966-1975
Current working curve
Yenisei at Igarka
Discharge, m3/s
Stage, cm
1955-59
1960-69
1970-75
Current working curve*
Lena at Kusur
Discharge, m3/s
Stage, cm
1960-65
1966-70
1971-74
1994-01
Current working curve
Pechora at Ust Tsilma
Discharge, m3/s
Stage, cm
1946-50
1951-55
1956-60
1961-75
Current working curve
Severnaya Dvina at Ust Pinega
Discharge, m3/s
Stage, cm
1946-50
1951-55
1956-60
1961-75
Current working curve

The complex water regime after ice breakup, with large amounts of drifting ice, ice jams and high velocities, make discharge measurements both complicated and dangerous. Low winter stage, often below the minimal open channel stage does not allow for the use of the summer rating curve and this requires interpolation between measurements as the only routine way to estimate daily discharge. In this case, the accuracy of discharge estimates will strongly depend on the variability of discharge and on the frequency and reliability of their measurements. All of these complications related to cold region river discharge estimation require the monitoring agency to use several computational techniques throughout the year. Each technique has varying accuracy and this is reflected in our error model. The least accurate discharge estimates occur in the periods of ice affected river flow.

The six largest gauged rivers in the Eurasian Arctic drainage basin were selected for this analysis (the same as in Peterson et al., 2002). These gauges are the most important for reliable estimates of river discharge to the Arctic Ocean and cover about 70% of total Eurasian drainage area (Shiklomanov and Shiklomanov, 2003). Descriptive information regarding these stations from R-ArcticNet (Lammers et al., 2001; Shiklomanov et al., 2002) is given in Table 1, and their contributing watersheds are shown in Fig. 1.

To estimate the accuracy of daily, monthly and annual discharge records the data of discharge measurements, rating curves and information about used techniques of daily discharge computations during a year were compiled. Some discharge measurement data were published in Russia for 1950–74 and since 1975 the only information about reliability of discharge computation is accessible through regularly compendia (Hydrological yearbooks, 1936–2001). This includes the number of measurements, the method of daily discharge determination and the coverage of discharge variation with measurements. The techniques of measurement, the equipment, and the methods of discharge computation have not significantly changed over the last 50 years (personal communications with Karasev—Head of Hygrometry Department in SHI), and therefore the same error model can be applied to the entire observational period.

One of the main characteristics affecting accuracy is frequency of discharge measurements or the number of measured discharge values used to estimate daily discharge. The number of discharge measurements for the six Eurasian rivers varies greatly from year to year and from gauge to gauge (Fig. 2). In general, the frequency of winter discharge measurements is more consistent than open channel measurements because they are directly used to estimate daily discharge in the winter period. The open channel discharge measurements may be completely absent during some years, with heavy reliance on long-term stage-discharge rating curves. In spite of the fact that this absence negatively effects the accuracy of daily discharge estimates for these years, it is acceptable if there is a stable long-term stage-discharge relationship and discharge variations during these years fall within the part of the rating curve which is well covered by discharge measurements from previous years. To ascertain the stability of the river channel cross-section for a given station and hence the stability of the rating curves we used results from discharge measurements for the long-term periods along with the current (contemporary) rating curves. The stage and discharge values were plotted sequentially in time for each gauge and this provided a simple method by which to define major shifts in the rating curves. Fig. 3 illustrates the current working rating curves (as of 2001) received from the local Roshydromet offices and the measured discharge for the previous periods. In practice not all measurement values are accepted in support of the rating curve. Many are rejected because of reduced accuracy and large deviations from the long-term relationship. We plotted all available discharge measurements without such screening (Fig. 3). The plots nonetheless clearly demonstrate that the rating curves have been relatively stable over the long-term. The high stage/ discharge parts of the rating curves show more

Fig. 3. The current rating curves (relationship between relative stage height and discharge) used to compute discharge for major Eurasian pan-Arctic rivers and measured discharge for the long-term period. Right plots show the low portions of stage-discharge relationships. The Rating Curve for Kolyma at Srednekolymsk is not given because of limited availability of discharge measurement data. * Another rating curve is used for Yenisey at Igarka for stage lower 400 cm.

variability for the Ob, Lena, Yenisei and Pechora and therefore indicate reduced reliability during high water. The measurements for this period are often made under drifting ice conditions with simplified gauging techniques, including a reduced number of gauging verticals and points of velocity measurements, shorter gauging time, or measurements of only surface stream velocity using floating objects. Nevertheless, all shown rating curves are relatively stable over the long-term period and the error analysis based on the stable long-term stage-discharge relationships can be applied to all these gauges.

There are specific conditions when streamflow at a gauge may not follow the stable rating curve. We found that at the Ob-Salekhard gauge the left bank floodplain starts flooding when the stage value exceeds 550 cm. Discharge through the floodplain is usually not measured and the stability (uniqueness) of the rating curve may be broken resulting in a significant decrease in the accuracy of the discharge estimates. The gauge at Yenisei-Igarka is annually under the influence of backwater (water backed up in this case due to wind tides when the stage height is lower than 400 cm). Similar conditions are observed in downstream of other large rivers (Meade et al., 1991). To eliminate the backwater effect the local hydrological station uses a less reliable relationship between stage at the next upstream gauge Yenisei-Selevanikha located about 200 km up stream and discharge at Igarka to estimate daily discharge values. In both cases, these conditions contributing to higher uncertainty discharge estimates were taken into account when estimating the discharge error for these gauges.

3. Accuracy of discharge estimates

3.1. Under condition of stable stage-discharge relationship

The methodology for computing mean daily discharge involves the estimation of mean daily stage from observational records and the application of this mean stage value to the rating curve to obtain a mean daily discharge data point. The errors inherent in the determination of mean daily stage can be better controlled and rendered minimal because stage measurement tends to be much more accurate relative to discharge measurement. Uncertainties in different types of stage measurements do not usually exceed 1 cm (Herschy, 1985). Moreover, stage observation is a single measurement of a height above some datum whereas a discharge measurement requires sampling at several different depths along the cross-section of the river. Therefore, the uncertainty associated with discharge measurement is significantly higher than that for stage measurement. For the purposes of this discussion, we assume errors in the measurement of water stage height are insignificant and all measurement error is contained within the related discharge term.

To represent the stage-discharge relationship mathematically, it is necessary to choose a regression equation that adequately describes the relationship with a minimum number of parameters. The following exponential equation is most often used in practice to represent the rating curve (Dickinson, 1967, USA; Dymond and Christian, 1982, New Zealand; Kennedy, 1984, USA; Herschy, 1985, Great Britain; Ivanov, 1989, Russia; Serkov et al., 1989, Russia; ISO, 1983, International).

$$Q' = a(H-H_0)^b \tag{1}$$

where Q' is the estimated discharge from an observed stage height H and a, b, H_0 are the estimated parameters of the equation. The first parameter a, reflects the numerical scale being used for stage and discharge; b denotes the degree of curvature or slope of the estimated relationship; and H_0 may be defined as the virtual stage at zero discharge. Most often H_0 has been assumed to be zero (Dickinson, 1967).

Eq. (1) is not always a suitable approximation to the rating curve over the entire range of stage heights. The specific characteristics of the stage-discharge relationship are substantially defined by the shape of the channel cross-section (Kennedy, 1984). Changes in slope angle of the rating curve usually take place at those stage values when a large expansion or narrowing of the cross-section is observed (e.g. when the floodplains are inundated or draining). Eq. (1) normally has only two fitted parameters a, b and does not allow the approximation of a complex shape relationship $Q=f(H)$ over the entire range. In such cases, it is recommended to carry out a piecewise approximation (Schmidt and Yen, 2002) when a compound rating curve consists of different segments for different flow ranges or to artificially apply a correction ΔQ to minimize the deviation of the approximating curve from the measured points (Ivanov, 1989). More universal approximation to the stage-discharge relationship is in the form of an mth order polynomial (Herschy, 1985; Krashnikov, 1987):

$$Q' = \beta_0 + \beta_1 H^1 + \beta_2 H^2 + \cdots + \beta_m H^m \quad (2)$$

where β_0, $\beta_1 \ldots \beta_m$ are fitted parameters. Theoretically, any shape of rating curve could be approximated by this high order polynomial. The principal advantage of this approximation type is in fitting stage-discharge curves having break points or inflexions which cannot be treated by other means (Grigorjev et al., 1977; Gavrin, 1982; Lesnikova, 1973; Herschy, 1985).

In reality, discharge in a channel is a function of not only the stage but also of water-surface slope, channel geometry, unsteadiness of the flow, and other factors. The stage-discharge relationship is thus not unique but multi-valued, which is often seen as discontinuities or loops in the rating curve (Schmidt, 2002). Moreover, from the methodological point of view, it is advisable to assume instability in the rating curve to identify its causes and to develop effective methods to adjust discharge from stage to account for these factors (Karasev and Kovalenko, 1992). Conditions of instability are the basic prerequisites for choosing a computational technique and estimating the accuracy of daily discharge (Karasev, 1980). Stability of rating curves can be defined based on statistical testing of deviation q_t of measured discharge Q from the corresponding rating curve value Q'. If there are nonrandom associations between water stage and discharge data the polynomial rating curve $Q=f(H)$ or regression Eq. (2) should be considered to be unstable, reflecting only a first approximation of daily discharge. The plotting of residuals q_t versus measured discharge Q and Student t-test were applied to evaluate randomness of deviations q_t for six considered gauges. The analysis showed that all stage-discharge relationships can be assumed to be stable (unique). The daily discharge values can therefore be directly evaluated from the rating curve or its approximating equation and the error analysis is based on the statistical estimate of parameters in (1) or (2). There could be significant deviation between the observed and estimated discharge values due to (1) error in the discharge measurements, (2) incomplete information about control conditions, and (3) unsteady nature of river flow. If the approximation of the stable stage-discharge relationship was made such that the deviations of measured discharge from the rating curve are uncorrelated (random), then these three components are difficult to define individually and their aggregate effect is estimated as a random error of measurement (Karasev and Shumkov, 1985).

Parameters β_0, $\beta_1 \ldots \beta_m$ in the fitted regression Eq. (2) used to describe the stage-discharge relationship are subject to error, which will influence Q'. Uncertainty in the predicted value Q' at any given point $x_1 \ldots x_m$ is evaluated as a sum of variances of uncertainties for regression parameters in (2): (Cramer, 1946; Rozhdestvensky and Chebotarev, 1974). The uncertainty characterizes inaccuracy of approximating equation and can be denoted as ε_{apr},—the approximation error:

$$\varepsilon_{apr} = \pm \frac{\sigma_q}{\sqrt{n-m}} \sqrt{1 + \frac{\sum_{i,j=1}^{m} k_i k_j (x_i - \bar{x}_i)(x_j - \bar{x}_j)}{\sigma_{x_i} \sigma_{x_j}}} \quad (3)$$

where n is the number of measurements, which usually equals 10–30 per year for the typical hydrometric network (MHS, Manual for Hydrometeorological Stations, 1975; 1978), σ_q standard deviation of q_t, k_j, k_i are the parameters depending on the paired correlation coefficients of the variables x_i; x_j and $\bar{x}_i$, $\bar{x}_j$ are mean values of arguments x in the range their changes from x_{min} to x_{max}. In our case, water stage is an independent variable, that is $x_{1i} = H_i,\ x_{2i} = H_i^2, \ldots, x_{mi} = H_i^m, \quad i = 1, 2, \ldots, m.$ The parameters k_i, k_j are defined from well-known statistical relationships (Cramer, 1946; Seber and Lee, 2003; Rozhdestvensky and Chebotarev, 1974) (see Appendix A).

For each individual river gauge the parameters in (3) and ε_{apr} can be estimated based on a set of discharge measurements. However, in practice it is important to have the generalized form of the equations, which will allow estimation of error with a minimal number of input data. The majority of relatively stable stage-discharge rating curves can be approximated with the third or even second order polynomials (Karasev, 1982). Karasev and Yakovleva (2001), using numerous Russian observational data, obtained the mean values of the parameters k_i and k_j for typical hydrographs of lowland rivers with spring snow flood and derived the simplified equations for ε_{apr}. When the daily discharge values were computed from Eq. (2) the approximation error ε_{apr} was found to be equivalent to the absolute error of the daily discharge estimates (ε_{dd}) and can be estimated for a third order polynomial in Eq. (2) as:

$$\varepsilon_{dd} = \pm \frac{\sigma_q}{\sqrt{n-3}} \sqrt{1 + 352.5\alpha_i^2} \quad (4)$$

where α_i characterizes the relative deviation of particular water stage value from the mean.

$$\alpha_i = \frac{H_i - \overline{H}}{H_{i\ \max,\ \min} - \overline{H}} \quad (5)$$

If the stage-discharge relationship is stable, the standard deviation of measured discharge from the rating curve σ_q is assumed to be equal to the absolute value of error for the mean discharge measurement $\bar{Q}$ over the computational period; that is

$$\sigma_q \cong \tilde{\varepsilon}_{mes} \bar{Q}. \quad (6)$$

The relative error in discharge measurement $\tilde{\varepsilon}_{mes}$ is normalized and is usually equal to 5–10% depending on the measurement technique (MR, 1977). Hence, if we do not have data to estimate the standard deviation, the normalized relative error of measurements $\tilde{\varepsilon}_{mes}$ may be applied instead. Eq. (4) can be adjusted to estimate the relative error of daily discharge $\tilde{\varepsilon}_{dd}$:

$$\tilde{\varepsilon}_{dd} = \pm \frac{\tilde{\sigma}_q}{K_i\sqrt{n-3}} \sqrt{1 + 352.5\alpha_i^2} \quad (7)$$

where $K_i = Q_i/\bar{Q}$ is the rate of measured discharge or the ratio of individual measured discharge Q_i to the mean measured discharge $\bar{Q}$; $\tilde{\sigma}_q$is relative standard deviation of measured discharge from the rating curve.

Correspondingly, for the boundary values of daily discharge (Q_{ddmax} and Q_{ddmin}), when $\alpha^2_{\max,\ \min} = 1$, Eq. (7) can be transformed in the following way:

$$\tilde{\varepsilon}_{dd\ \max,\ \min} = \pm \frac{18.8\tilde{\sigma}_q}{K_{i\ \max,\min}\sqrt{n-3}} \quad (8)$$

Because $K_{\max} > 1$ and $K_{\min} < 1$, the greater approximation error takes place for minimal daily discharge. At low velocities and shallow depth, the accuracy of discharge measurement significantly decreases if the same gauging technique is applied (Karasev, 1980; Herschy, 1999). Therefore, a wide scattering of measurements at the low water stage values increase uncertainty in the lower part of the rating curve. Absolute errors in the low discharge estimates are less than those of high discharge but the relative error can often be substantial. The least-squares method, based on a minimization of the sum in the square deviations of the measured points, is usually applied to approximate $Q = f(H)$. Small weights of the deviations at low stage may lead to the wrong approximating curve, which could lie outside of the band of the measured points. The method of constrained regression is often applied to avoid that. In practice, observational data from previous years and additional hydraulic channel characteristics, for example the Chezy formula, are used to define more accurately a correspondence between minimal water stage $H_{\min}$ and some mean minimum discharge $Q_{\min}$ (Kennedy, 1984). This is defined based on an analysis of the river cross-section for low stage and long-term observational data. The value $Q_{\min}$ serves as a reference point for the lower branch of the rating curve and regression parameters for $Q = f(H)$ are computed by constraining the regression to $Q_{\min}$ at $H_{\min}$. The implication of such assumption is hard to estimate and could be different for various hydrological sites. Karasev and Yakovleva (2001) postulated that at $H_{\min}$ the relative error $\tilde{\varepsilon}_{dd}$ is $\sqrt{(n-3)}$ times lower than computed by Eq. (8) because of additional averaging and Eq. (8) can be written as:

$$\tilde{\varepsilon}_{dd\ \min} = \pm \frac{18.8\tilde{\sigma}_q}{K_{\min}(n-3)}. \quad (9)$$

Thus, the errors of daily discharge can be evaluated over the whole range of discharge changes based on the frequency of measurements and discharge variability if the standard deviation of measured discharge or mean relative error of measurements are

Table 2
Relative errors in daily discharge estimates under different numbers of discharge measurements for constant $\tilde{\varepsilon}_{mes} = \sigma_q = 10\%$ computed from simplified Eqs. (7)–(9)

Relative error	$Q/\bar{Q}$	Number of discharge measurements									
		Long-term relationship $Q(H)$[a]						Annual relationship $Q(H)$[a]			
		150	130	110	100	80	60	40	30	20	10
$\tilde{\varepsilon}_{dd_{max}}$	$Q/\bar{Q}=5$	0.03	0.03	0.04	0.04	0.04	0.05	0.06	0.07	0.09	0.14
$\tilde{\varepsilon}_{dd}$	$Q/\bar{Q}=1$	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.04
$\tilde{\varepsilon}_{dd_{min}}$	$Q/\bar{Q}=0.2$	0.06	0.07	0.09	0.10	0.12	0.16	0.25	0.35	0.55	1.34

[a] Subdivision between long-term relationship and annual is subjective.

known. Table 2 demonstrates the relative errors of daily discharge estimates computed for $\tilde{\varepsilon}_{mes} = \sigma_q = 10\%$. The errors associated with lower discharge $Q/\bar{Q}=0.2$ are significantly greater than those for high and average discharge values because of higher uncertainties in lower portion of the stage-discharge relationship.

Relative errors given in Table 2 were computed for the stable long-term stage-discharge relationship and the division between the annual and the long-term relationship is very subjective and is only based on the long-term mean number of discharge measurements used to confirm the rating curve. In reality, to correctly estimate the error of daily discharge estimates for an individual year it is necessary to take into account the standard deviation of all discharge values measured for the year from the long-term rating curve by:

$$\varepsilon_{dd_i} = \pm\sqrt{\varepsilon_{dd}^2 + \sigma_{year}^2} \quad (10)$$

where $\varepsilon_{dd} = \varepsilon_{apr}$ and was defined in Eq. (3). σ_{year} is the standard deviation of measured discharge for the individual year from the long-term $Q=f(H)$ relationship. Because of added uncertainty in Eq. (10) errors for the individual years will be higher than average error in Table 2 and will strongly depend on the deviation of measured discharge for the individual years from the long-term rating curve.

3.1.1. Application to Eurasian data

To approximate the rating curves for the six selected gauges, second order polynomials were used. Comparison of the approximations, based on analysis of regression residuals, showed that increasing the order of polynomial does not provide any additional substantial improvement. Moreover, transformation of the rating curve under specified limitations in the area of minimal water stage values allows linearizing the second order polynomial and defining the coefficients of linear regression by least-squares method. This approach increases the reliability of these coefficients.

The error in daily discharges computed from the long-term stable rating curves significantly depends on the magnitude of discharge. The greatest relative error takes place during the minimal water stage period, which is observed before the beginning of ice conditions for all gauges. Fig. 4 demonstrates the variation of errors as a function of discharge. Large uncertainties at the lowest discharge values, varying from 8% for Ob at Salekhard to up to 25% for Yenisei at Igarka, are associated with large uncertainties in the low branches of the rating curves (Fig. 3). Additionally, as mentioned above, each year Yenisei at Igarka was affected by tides at low stage which introduce additional uncertainty. The lowest free channel discharge values corresponding to 25% error on Fig. 4 were observed before construction of the large dams in the Yenisei basin (until mid 60's). As result of reservoir construction and flow regulation, the appearance of such low discharge values is now practically impossible. Current discharge at this gauge before freeze-up is about 60–80% higher than it was before 1957 (Shiklomanov, 1994; Yang et al., 2004); under these conditions the error of daily discharge does not exceed 15%.

Absolute error for the high discharge portions of the rating curves are 2–5% if the stage-discharge relationships are stable and their positions are confirmed by actual measurements. Such small relative errors are explained by relatively small

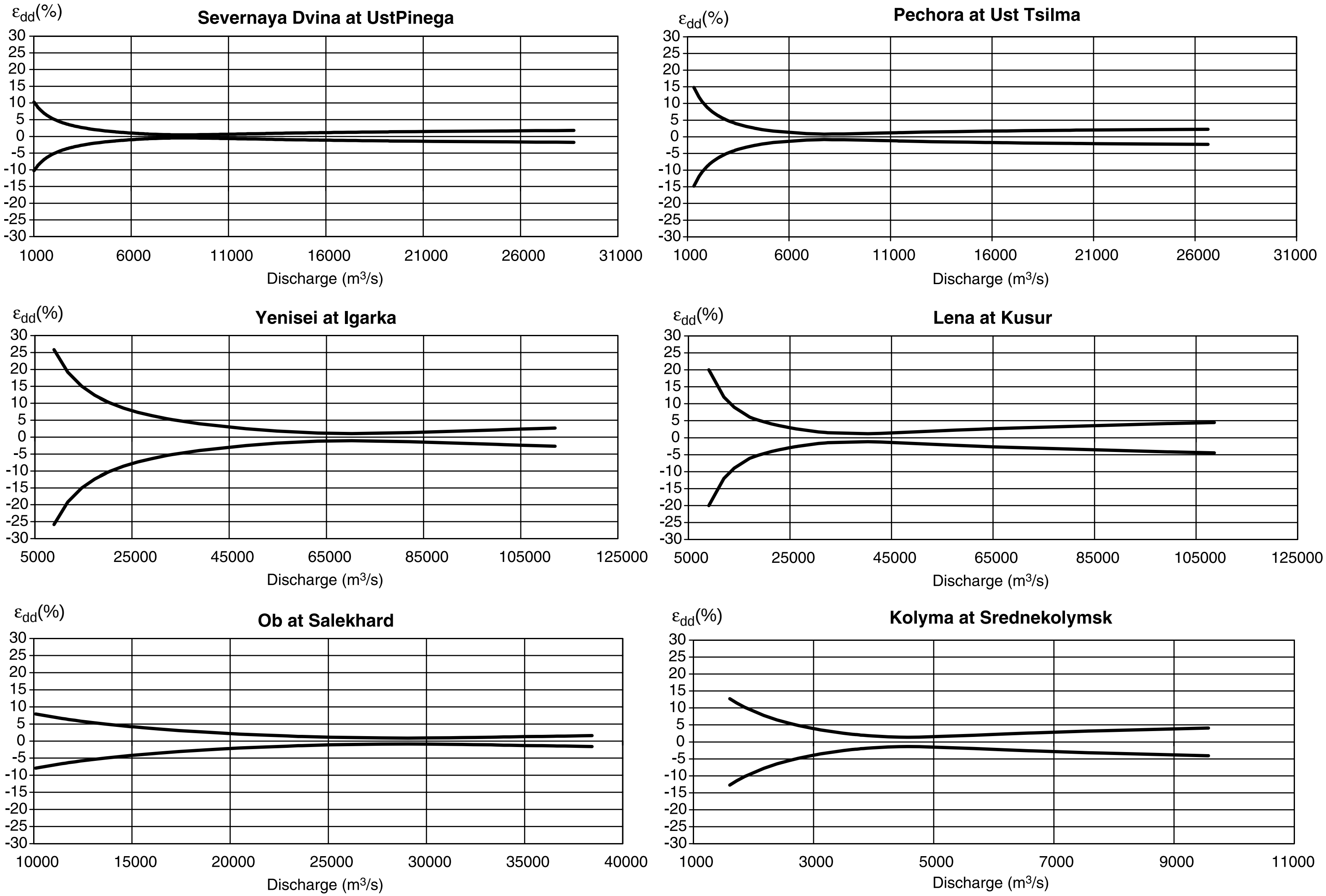


Fig. 4. Relationships of relative error ($\tilde{\varepsilon}_{dd}$) in daily discharge estimates of the six Eurasian gauges computed using best fit polynomial for approximation of long-term stable rating curve (Eq. 3) from the daily discharge value.

variations in discharge measurements for these stage values over the long-term period (Fig. 3). This suggests that the each of the rivers has a relatively stable channel at the gauging cross-section. Generally, these results show good correspondence with estimates of daily discharge errors made earlier using of other error models for rivers in Great Britain (Herschy, 1985) and the US (Dickinson, 1967).

As has been noted, the errors in daily discharge data for individual years defined from Eq. (15), taking into account the deviation of discharge measurements for the particular year from the long-term rating curve, will be greater than those based on the long-term rating curve and shown in Fig. 4. For all years which detailed measurement data were available (Table 1) we computed the averaged errors of daily discharge records as well as the errors of aggregated monthly and yearly values. The results for the six gauges are given in Table 3. The errors computed using actual discharge data for the six gauges are significantly higher than mean errors for the long-term rating curve given in Table 2 because the deviations of measured discharge values for individual years were taken into account. This deviation could be significant for some years as a result of both changes in hydraulic characteristics of the river channel at the gauge and other factors affecting the quality of discharge measurements (e.g. weather, skills, instruments and so on).

3.2. Under the absence of stable stage-discharge relationship

The stability of the rating curve may be broken if the hydraulic character of river control changes. For large rivers this usually occurs because of floodplain flooding, deformation of the river bed, effects of water vegetation, ice and backwater conditions. The most common event affecting the stage-discharge relationship for cold region rivers is the presence of ice at a river control causing the gauge height to falsely indicate a greater than actual discharge. The ice conditions and other factors affecting the relationship are the physical causes of the relative deviation q_t in the measured discharge from the rating curve. It should be noted that the series of deviations q_t involves two components: the first reflects the changes in hydraulic-morphological conditions of the river flow and the second is due to the random errors of discharge measurement. When the rating curve is stable only the second component is present. If the criterion Eq. (3) is not observed then the rating curve should be considered unstable, giving only a first approximation for daily discharge.

There are several methods for winter discharge computations that vary across the northern countries (Pelletier, 1990). The most widely used technique in Russia for rivers with stable ice-cover during the winter consists of computation of a correction coefficient C_{ice} for each winter discharge measurement:

$$C_{ice} = \frac{Q_{ice}}{Q'} \tag{11}$$

Where Q_{ice} is the discharge measured under ice conditions and Q' is the discharge at the same gauge height computed from an open water stable stage-discharge relationship. The values of C_{ice} are generally less than unity. The C_{ice} values for all measurements are plotted and interpolated to generate a daily time series. In those gauges where actual discharge measurement are sparse (e.g. remote telemetric gauges) air temperature and other meteorological data as well as ice thickness variation are often included to improve the interpolation (Luchsheva, 1983; Pelletier, 1990; Rantz, 1982; Melcher and Walker, 1990).

Serkov et al. (1989) suggests using the relative deviation of measured winter discharge from the open water rating curve $\tilde{q}_t$ instead of the correction coefficient C_{ice} to facilitate this computation.

$$\tilde{q}_t = \frac{Q_{ice} - Q'}{Q'} = C_{ice} - 1 \tag{12}$$

C_{ice} can be simply transformed to q_t and vice versa. The relative deviation $\tilde{q}_t$ is computed for each measurement and then interpolated for the period between measurements. The time series of $\tilde{q}_t$ is used to define the daily discharge values Q_{dd}:

$$Q_{dd} = Q'(1 + \tilde{q}_t). \tag{13}$$

It should be noted that this approach can be applied not only to the winter discharge but to any conditions causing a break in the stable stage discharge relationship. Therefore, the determination of uncertainty in daily discharge during these periods is related

Table 3
The long-term mean errors for 1955–2000 of daily, monthly and annual discharge data for the six Eurasian gauges

River	Discharge	Annual	January	February	March	April	May	June	July	August	September	October	November	December
Ob at	Daily		28.1	28.1	28.1	28.1	11.3	10.3	4.7	4.7	4.7	6.8	27.3	27.3
Salekhard	Monthly		19.8	19.8	19.8	19.8	5.7	7.3	3.3	3.3	3.3	5.1	18.3	18.3
	Annual	5.7												
Yenisei at	Daily		25.2	25.2	25.2	25.2	16.8	5.7	5.7	8.0	12.7	12.7	25.2	25.2
Igarka	Monthly		17.9	17.9	17.9	17.9	11.9	2.9	2.9	4.0	9.0	9.0	17.9	17.9
	Annual	6.1												
Lena at	Daily		28.2	28.2	28.2	28.2	22.1	5.6	6.2	6.2	6.2	17.2	28.2	28.2
Kusur	Monthly		17.0	17.0	17.0	17.0	12.8	4.0	4.4	4.4	4.4	12.2	17.0	17.0
	Annual	4.3												
Pechora at	Daily		10.0	10.0	10.0	10.0	8.7	5.5	7.7	7.7	7.7	9.6	10.0	10.0
Ust-Tsilma	Monthly		7.1	7.1	7.1	7.1	5.5	2.8	3.1	3.1	3.1	5.6	7.1	7.1
	Annual	2.3												
Severnaya	Daily		17.4	17.4	17.4	17.4	15.4	5.8	5.8	5.8	9.4	12.2	17.4	17.4
Dvina at	Monthly		12.8	12.8	12.8	12.8	8.6	4.1	4.1	4.1	6.7	7.5	12.8	12.8
Ust-Pinega	Annual	4.3												
Kolyma at	Daily		38.5	38.5	38.5	38.5	27.8	8.2	8.2	8.2	19.4	19.4	38.5	38.5
Sredneko-	Monthly		28.8	28.8	28.8	28.8	20.8	3.9	3.9	3.9	13.8	13.8	28.8	28.8
lymsk	Annual	7.1												

to the computation of uncertainty for q_t in the range between discharge measurements. Average uncertainty or relative error of the linear interpolation of $\tilde{q}_t$ in the range between discharge measurements can be computed as:

$$\tilde{\varepsilon}_{in} = \pm\sqrt{0.83a^2 e^{-0.22(n-1)} + 0.5\tilde{\varepsilon}^2_{mes}} \quad (14)$$

where $\tilde{\varepsilon}_{mes}$ is the relative error of discharge measurement, e is the base of natural logarithm, a is an absolute maximum value for $\tilde{q}_t$ and n is the number of the discharge measurements for the period of unstable rating curve T (see Appendix B).

Interpolation error serves to estimate the accuracy of daily discharge data, which is characterized by a combination of uncertainties for the arguments in Eq. (13):

$$\tilde{\varepsilon}_{dd} = \pm\sqrt{\frac{\sigma^2_q + Q'^2 \varepsilon^2_{in} + \sigma^2_q \tilde{q}^2_t}{Q'^2(1+\tilde{q}_t)^2}} \quad (15)$$

when $\tilde{q}_t < 1$, that is typically observed during ice conditions, the third term in the numerator can be omitted as negligible and the equation for relative daily discharge can be simplified:

$$\tilde{\varepsilon}_{dd} = \pm\frac{1}{1+\tilde{q}_{mean}}\sqrt{\tilde{\sigma}^2_q + \tilde{\varepsilon}^2_{in}} \quad (16)$$

where $\tilde{q}_{mean}$ is the mean value of $\tilde{q}_t$ for the period of unstable stage-discharge relationship $\tilde{\sigma}_q$ is the dispersion measure of the rating curve or relative deviations of measured discharges from the regression equation in Eq. (2).

For many northern rivers, the minimal water discharge and stage are observed during ice cover. There may be no reliable free-channel rating curve for the low stage values (confirmed by measurements) and therefore the correction coefficient cannot be defined. In this case, the method of linear interpolation between measured discharges or recession curve method can be used to compute daily discharge for most of the winter. In practice the combination of different computational techniques is often used during the winter. For example, the correction coefficient curve method is applied at the beginning and end of winter when the water stage values are still high enough to be used for discharge computation with the rating curve while the method of interpolated discharge measurements or recession curve method are used during the rest of the winter period (Pelletier, 1990). The gauges analyzed here are affected by reservoirs (Ob', Yenisei, Lena, Kolyma) in their catchments with greater influence on winter discharge (Shiklomanov, 1994; Ye et al., 2003) or have large winter runoff variability (Severnaya Dvina, Pechora). The interpolation between discharge measurements is therefore the most widely used method for this period because the approach based on the recession curve which provides good results for large natural rivers with small discharge variability cannot be used as a primary method for daily discharge estimation. It should be noted that these three methods with some variations are mainly used for daily discharge computations during ice-affected period in all other northern countries (Pelletier, 1990).

To estimate the error of the linear interpolation between measurements we suggest using the approach similar to the interpolation of deviations q_t. In this case, instead of relative deviations of measured discharge from the rating curve, the relative deviations from mean measured discharge over the interpolated period is used (ΔQ_{mes}). The expression for the errors of daily discharge estimates can then be written as:

$$\tilde{\varepsilon}_{dd} = \pm\frac{1}{1+\tilde{q}_{mean}}\sqrt{\tilde{\sigma}^2_{\Delta Q} + \tilde{\varepsilon}^2_{in}}. \quad (17)$$

The magnitude of relative interpolation error $\tilde{\varepsilon}_{in}$ is defined similarly to the correction coefficient method from Eq. (16). The value of $\tilde{\sigma}^2_{\Delta Q}$ in Eq. (17) is significantly higher than σ^2_q in Eq. (16) and strongly depends on the discharge variation during the period when this technique applies. It follows that the errors of daily discharge determination under the linear interpolation method will therefore be greater. As has been mentioned the interpolation between discharge measurements as well as between gauged correction coefficient C_{ice} or q_t is usually carried out graphically (Luchsheva, 1983; Rantz, 1982). This introduces additional subjectivity to the daily discharge data but in cases of experienced staff the errors of daily discharge will be lower than those computed in Eqs. (16) and (17) based on a linear interpolation between existing points.

3.2.1. Application to Eurasian data

Analysis of water stage and discharge data during the winter period for the six Eurasian gauges shows that minimal water stage and discharge are typically observed near the end of the frozen period. Consequently, two techniques are used to define the daily discharge records. The winter correction coefficient method can usually be applied to Severnaya Dvina, Pechora and Ob' during the entire winter period excluding some years with low winter stage values and for the Yenisei, Lena and Kolyma only during transition periods and in the beginning and the end of frozen period. The method of interpolation between discharge measurements is used for all rivers when daily discharge cannot be defined from the rating curve because of very low stage values. The period of application of the method is different for all rivers and varies from several days for Severnaya Dvina and Pechora to 6 months for the Kolyma River.

The accuracy of daily discharge estimates for both methods applied during the winter depends on discharge variation for the period and the frequency of discharge measurements. At the same time the correction coefficient method is more accurate and gives better results with reliable and frequent measurements for rivers with relatively small deviations in winter discharge measurements from the long-term rating curve. Under such conditions, the errors in daily discharge estimates for the Ob', Severnaya Dvina and Pechora are asymptotic to approximately 10% (Fig. 5, panel (a)). For rivers with thick ice cover and higher winter discharge variation, the errors in daily discharge records remain above 21% even with frequent discharge measurement. The method of interpolation between measurements usually has lower accuracy, although when frequent measurements (greater than 15 per winter) are made the accuracies of both methods are close (Fig. 5).

Fig. 5 can also be used to identify the optimal number of discharge measurements during the winter period. The necessary frequency of discharge measurements for each hydrological gauge can be identified to maintain the established accuracy in daily

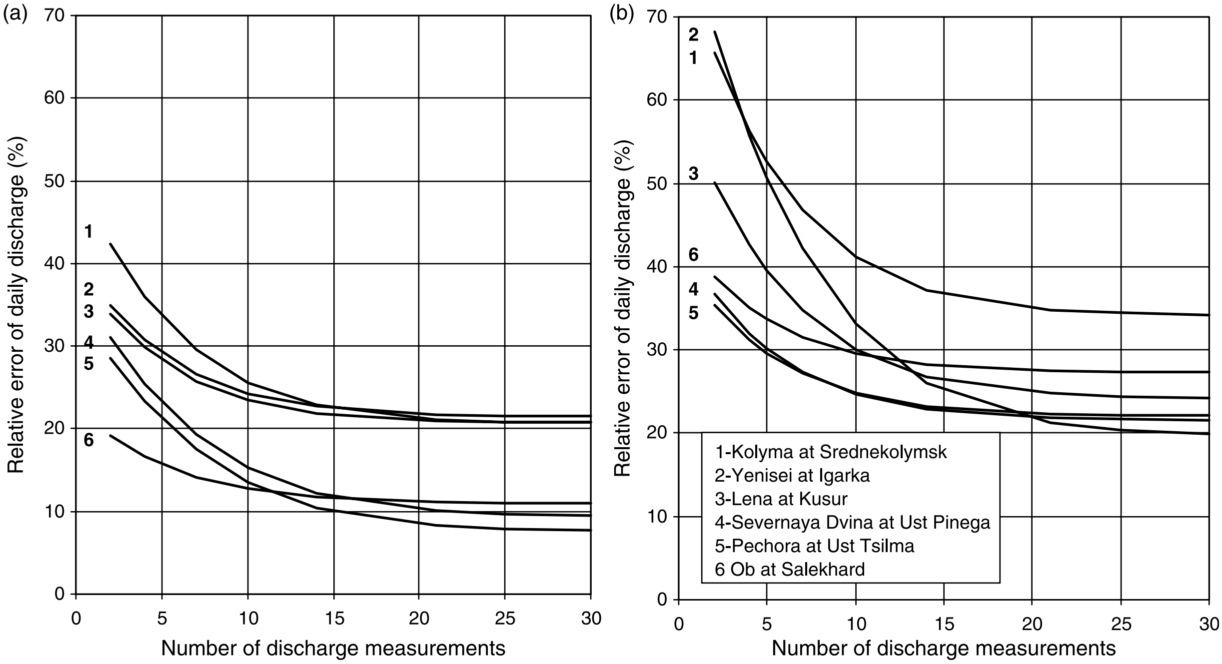


Fig. 5. Relative error of daily discharge estimates ($\tilde{\varepsilon}_{dd}$) from the number of discharge measurements for the entire period when the stable stage-discharge relationship is absent; (a) method of correction coefficients is applied to estimate the daily discharge and Eq. (16) is used to estimate the relative error; (b) daily discharge values are estimated by linear interpolation between measurements and Eq. (17) is used to estimate the relative error.

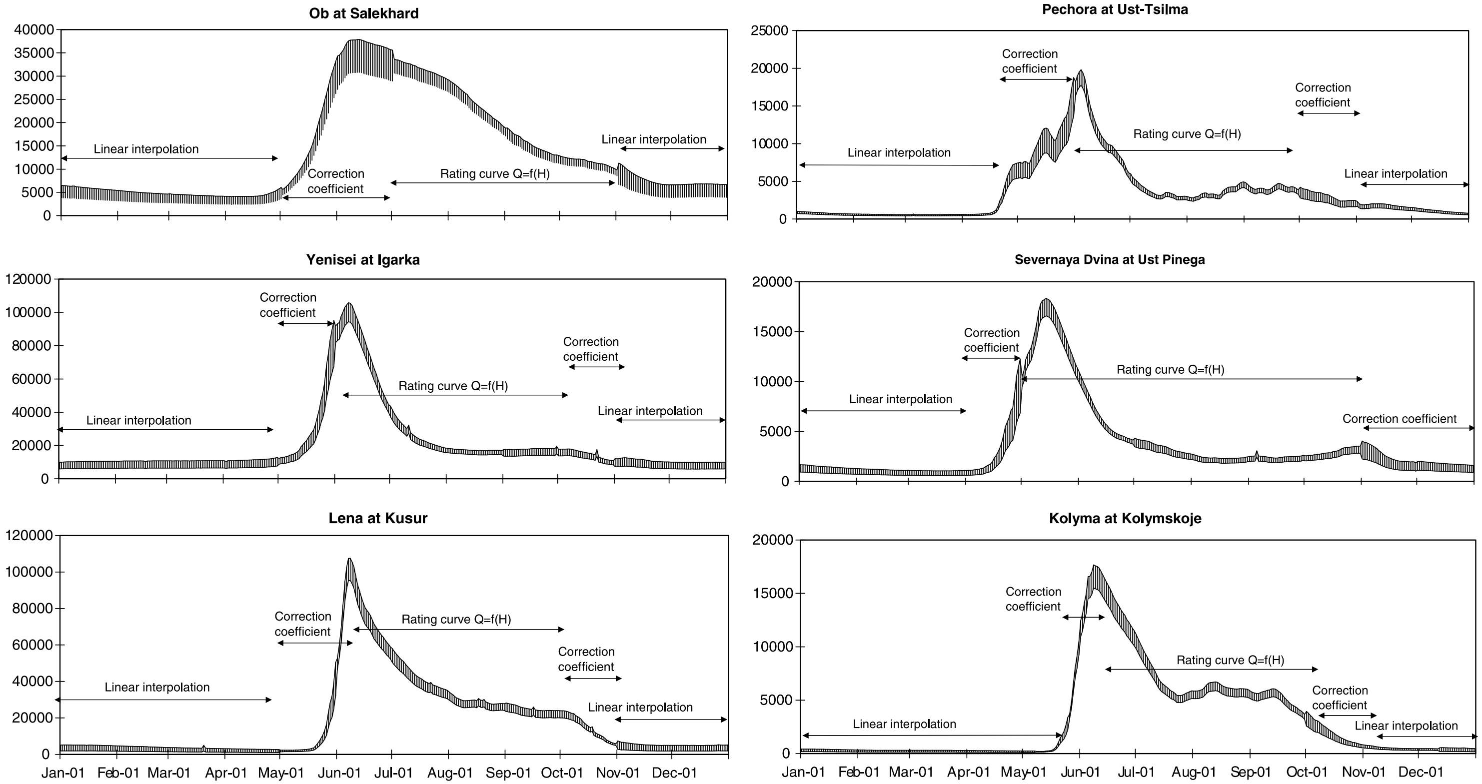


Fig. 6. The long-term mean uncertainties in daily discharge estimates (hatched area) for the annual hydrograph are shown. The arrows demonstrate the duration of mean period within the year in which each error sub-model was used.

discharge records. As seen from Fig. 5 for all rivers the accuracies of daily records are stable after 15–20 measurements for the winter period and therefore more frequent measurements are not effective.

The errors in daily discharge estimates for the entire year have been computed based on the use of actual gauging data for the long-term period and take into account the techniques applied to calculate the daily discharge values. Fig. 6 shows the long-term mean uncertainties associated with daily discharge errors and average period of the year when each computational technique is used. The greatest relative errors for all gauges are observed during fall and winter and reside in the range of 15% for Pechora at Ust Tsilma to 45% for Kolyma at Srednekolymsk. Such a large uncertainty for Kolyma River can be explained by thick ice, relatively low discharge with high variation, and infrequent discharge measurements during the winter. Errors for individual years are much more variable and depend on many factors such as the number of discharge measurements, deviation of discharge values measured for the year from the long-term rating curve, presence of extreme discharge values, the computational techniques used and their duration over the year.

3.3. Uncertainty in time-aggregated discharge estimates

Time-averaged river discharge is defined as a simple mean of daily discharge estimates over the period T (10-days, month, year). An a priori expectation is that when aggregating to coarser time steps there will be a reduction in uncertainty due to lack of bias in the data. However, this is not necessarily the case, especially when interpolation between measurements is used for daily discharge estimates. It is known from Drozdov and Shepelevsky (1946) that the mean of interpolated values x in the range between measurements $\tau_($ does not differ from the mean of the measured values x_i and x_j:

$$\bar{x} = \frac{1}{\tau_\partial}\int_0^{\tau_\partial}\left[x_i\left(1-\frac{\tau}{\tau_\partial}\right)+\frac{x_j\tau}{\tau_\partial}\right]d\tau = 0.5(x_i+x_j). \quad (18)$$

Consequently, Q_T discharge averaged for the period T, including $(n-1)$ equal intervals between measurements, can be represented as:

$$Q_T = \frac{\sum_{i=1}^{i=T} Q_{dd_i}}{T} = \frac{\sum_{k=1}^{k=n-1} Q_{dd_k}}{n-1} \quad (19)$$

where Q_{dd_k} mean daily discharge for the interval between measurements k.

Analysis of statistical characteristics of relative error of mean daily discharge $\tilde{\varepsilon}_{dd_k}$ for the interval between measurements, showed that it can be considered as a random, uncorrelated magnitude (Karasev and Kovalenko, 1992). Thus, the relative errors of the discharge averaged for period T, can be defined by summation of variances and their averaging for the number of intervals $(n-1)$:

$$\tilde{\varepsilon}_T^2 = \frac{\sum_{k=1}^{k=n-1} \tilde{\varepsilon}_{dd_k}^2}{n-1}. \quad (20)$$

The error of mean discharge for equal intervals k is therefore:

$$\tilde{\varepsilon}_T = \pm\frac{\tilde{\varepsilon}_{dd}}{\sqrt{n-1}} \quad (21)$$

where $(n-1)$ is the number of intervals between discharge measurements and ε_{dd} is the daily discharge error. Eq. (21) is true when the period of discharge averaging T is less then the duration of use the same discharge determination technique T_i. For example, the same method of daily discharge computation is usually applied for 10 days; therefore an error in the 10-day discharge can be computed from Eq. (21). If there is one measurement per 10 days, it follows from Eq. (21) that the error of the mean 10-day discharge is consequently equal to the daily discharge error for the 10-day period. It is true if the daily discharge is computed based on linear interpolation between measurements. However, when daily discharge is estimated based on the stable rating curve the error might be slightly less. At the same time, determination of daily discharge values based on the rating curve has a very subjective nature and therefore the Eq. (21) can be also applied for this case.

The error of mean monthly discharge is computed from Eq. (21) if it complies with the condition $T<T_i$. When $T>T_i$ it is necessary to use the variance weighting procedure, taking into account the averaged daily discharge estimates $\bar{Q}_{T_i}$ over the time T_i. The

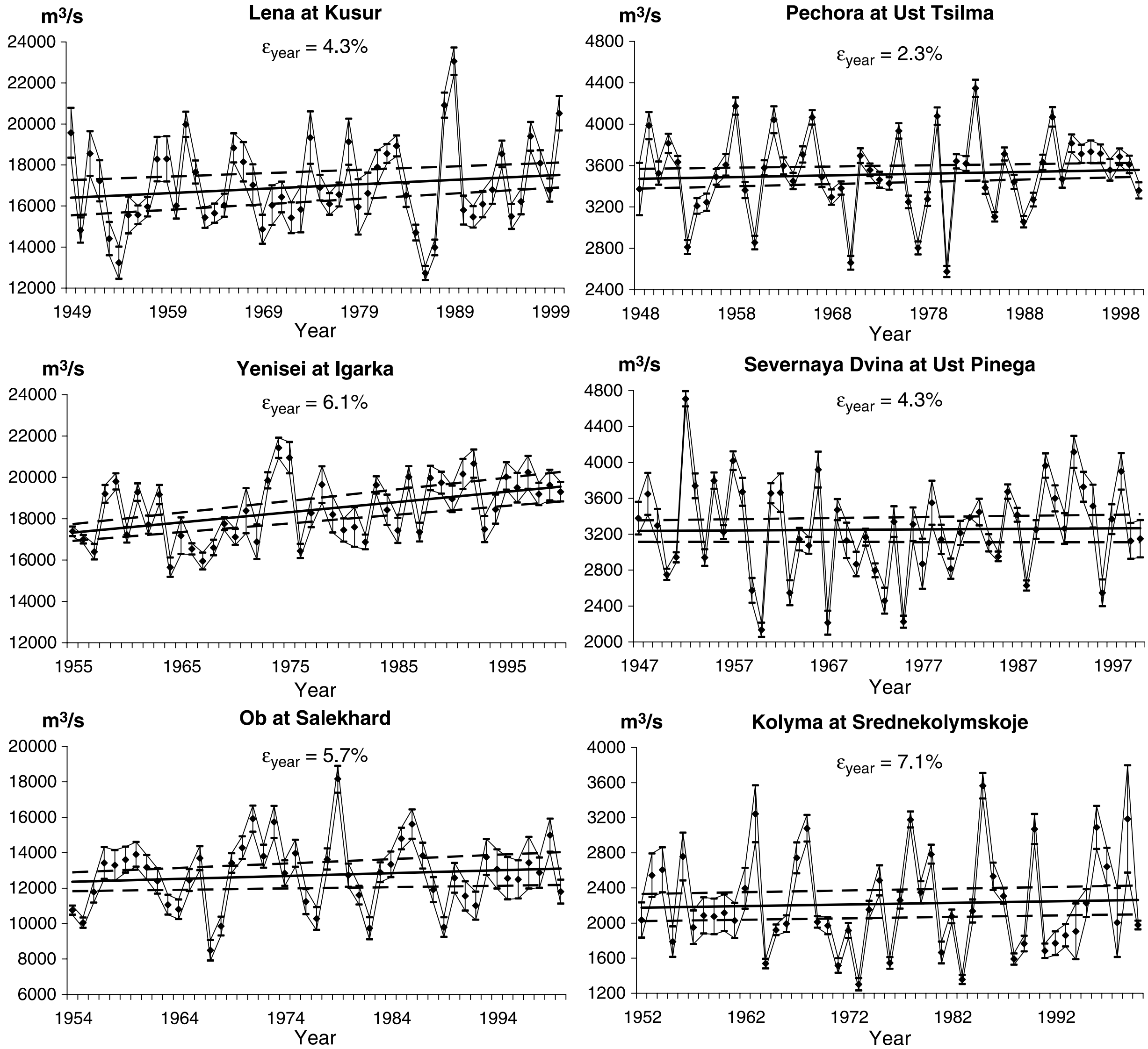


Fig. 7. Long-term variation of annual discharge (points) along with uncertainties associated with annual discharge determination (error bars) computed using Eq. (29). Linear trend lines are shown for annual discharge (solid line) and for annual discharge with $\pm\varepsilon_{\text{year}}$ uncertainties (dash lines) for six largest downstream gauges in the Eurasian pan-Arctic.

relative error of discharge averaged for period T will be computed then as:

$$\tilde{\varepsilon}_T = \pm\sqrt{\frac{\sum_{i=1}^{N} \bar{Q}_{T_i} T_i \tilde{\varepsilon}^2_{T_i}}{\bar{Q}_T T}}, \quad T = \sum_{i=1}^{N} T_i. \tag{22}$$

Mean annual discharge error for $T=365$ or $T=366$ can be calculated as:

$$\tilde{\varepsilon}_{\text{an}} = \pm\sqrt{\frac{\sum \frac{T_i}{T} K_i \tilde{\varepsilon}^2_{\text{dd}}}{n_i - 1}} \tag{23}$$

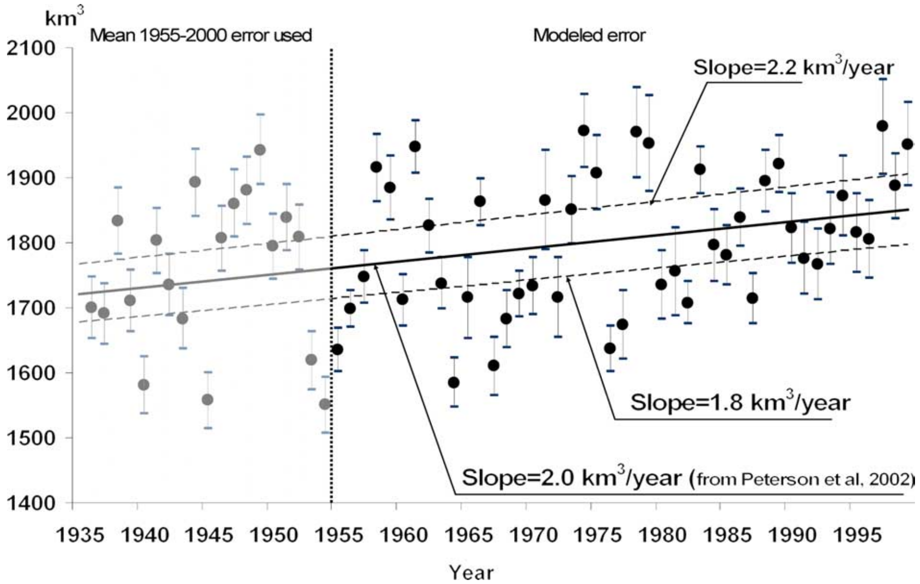


Fig. 8. Combined annual discharge from the six largest Eurasian arctic rivers for the period from 1936 to 1999 (Peterson et al., 2002) and uncertainties associated with discharge data. The trendlines and slopes are from a simple linear regression of time versus discharge. The faded part of the plot represents the time when the mean long-term error computed based on data for 1955–1999 was applied to the period 1936–1954.

where n_i is the number of discharge measurements for period T_i, K_i is the rate of discharge, $K_i = \bar{Q}_i/\bar{Q}_{an}$ where $\bar{Q}_i$ is mean discharge for period T_i, $\bar{Q}_{an}$ is the mean annual discharge, T is the number of days in the year.

3.3.1. Application to 'Eurasian 6'

The long-term mean errors of monthly and annual discharge data for the six Eurasian gauges computed are shown in the Table 3. The greatest uncertainties in monthly streamflow data take place in winter. Mean errors in annual discharge estimates for all gauges do not exceed 7% because the contribution of low flow discharge with high uncertainties is negligible. The errors in annual discharge over the long-term period vary significantly. The maximal errors are observed for years with the minimal number of discharge measurements. The uncertainties in annual discharge have increased for the long-term period for Ob at Salekhard and Yenisei at Igarka, have slightly decreased for Lena at Kusur, and have not practically changed for the other two gauges (Fig. 7).

3.4. Aggregation of the random errors to estimate multi-river continental discharge

We consider the summarized error in the spatially aggregated river discharge of the six largest Eurasian pan-Arctic rivers but the same method can also be used for disaggregated discharge, for example, to determine the lateral inflow or inter-station discharge between two or more gauges. Peterson et al. (2002) discussed the trends in the summarized river discharge and it is important to estimate the uncertainty of this time series. It is reasonable to suggest that random errors of discharge records for different gauges are not related and the random error addition rule can be applied (Rozhdestvensky et al., 1990). The total observed discharge to the ocean over s gauges is: $\sum_{g=1}^{s} Q_g$, and error squared of the aggregated discharge is equal to the sum of errors squared of all individual gauges:

$$\varepsilon^2_{\sum Q} = \sum_{g=1}^{s} \varepsilon^2_{Q_g} \tag{24}$$

where ε_{Q_g} is the absolute discharge error for each individual gauge, which can be defined as daily,

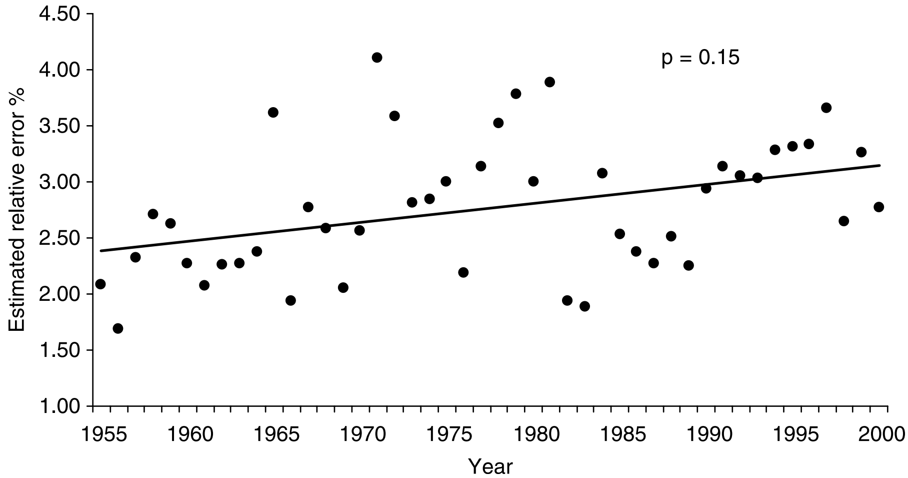


Fig. 9. Temporally and spatially aggregated relative error of combined annual discharge from the six largest Eurasian arctic rivers for the period 1955–2000. The trendline is from a simple linear regression.

monthly, or annual errors. Error of the total discharge can be expressed through the relative discharge errors of the individual gauges:

$$\tilde{\varepsilon}^2_{\sum Q} = \frac{\sum_{g=1}^{s} \tilde{\varepsilon}^2_{Qg} Q_g^2}{s^2 \bar{Q}^2} \qquad (25)$$

where $\bar{Q}$ is mean discharge averaged for s-gauges.

3.4.1. Application to Eurasian data

The estimated errors of the total annual discharge for the six rivers over the period 1950–2000 are in the range 1.5–3.5% or 28–70 km^3/year (Fig. 8). The uncertainties in total discharge until 1955 (shaded area) are shown as the average value computed for 1955–2000 and adjusted for mean annual discharge. The long-term trend of the observed discharge from the six Eurasian Basins into the Arctic Ocean for 1936–2000 with discharge determination uncertainty is 2.0 ± 0.4 km^3/year. The plot of the errors in total discharge of the six rivers over the period (1955–2000), when detailed assessment of discharge was made, demonstrates the increase in uncertainty in discharge data (Fig. 9). The increase in uncertainty is likely a consequence of decline in frequency of discharge measurements and higher discharge variation.

4. Conclusions

We have described a river discharge error model applicable to cold region drainage basins. The model uses some hydraulic characteristics of river channel capacity along with a statistic interpretation of hydrometric data and allows us to obtain an objective estimate of river discharge error for individual gauging stations and for different averaging time periods ranging from daily to annual. The approach takes into account different flow conditions and the computational techniques associated with them and can be applied to various type of gauging stations. The error model for discharge estimates during ice-affected periods was based on techniques, which are widely used in Russia for large rivers with consistent ice cover. However, the very similar methods are used in other northern countries for winter discharge estimates. They are usually based on the interpolation between measurements or correction factors and use of additional information such as air temperature, precipitation and discharge of adjacent rivers (Pelletier, 1990). Thus, the error model developed here can be adopted to many large cold region river gauges. The model allows us to evaluate the reliability of river discharge data based on routine hydrometric information and to identify the optimal number of discharge measurements required to maintain a desired level of accuracy in the discharge data.

Adaptation of the error model to six large Eurasian River gauging stations, which are crucial to estimate river discharge into the Arctic Ocean, allowed us to analyze the accuracy of stream flow data over the long-term. Uncertainties associated with discharge determination significantly change from year to year and strongly depend on the computational methods used. Mean relative error of daily discharge can range from less than 3% during the summer months (Pechora at Ust Tsilma, Ob at Salekhard) to more than 40% (Kolyma at Srednekolymsk) during ice-affected periods. For those years where discharge measurements are few the errors associated with winter daily streamflow can exceed 60–70%. The highest relative error of daily discharge correlate strongly with very low discharge periods. The least uncertainties in daily discharge data take place in summer when daily discharge is estimated from central or higher portion of stable rating curve and the errors are in the range of 3–10%. Accuracy is lower when discharge is low due to higher relative uncertainties in the rating curve position because of greater scatter of the discharge measurements (Fig. 3). The errors of monthly and annual discharge estimates are greatly reduced as a result of aggregation.

The suggested error model was adopted for the gauges that have stable, long-term stage-discharge relationship covering the whole amplitude of discharge variation. For all gauges with similar features this model can be used as is. However, there are several conditions in which errors may increase greatly. This will occur when the daily discharge estimates fall out of the range of measured discharge. In this case, the discharge value is determined by extrapolation of rating curve and the error may be significantly higher because these parts of the curve are not confirmed with measurements. The extrapolation error can be estimated by empirical equations depending on the extrapolation step (Karasev, 1982; Karasev and Kovalenko, 1992).

Analysis of error over the long-term period showed the significant variation in accuracy of discharge data from one year to another. The error of annual discharge data can vary from 1 to 10% depending on the number and quality of measurements during the individual year. We found that error of discharge data for the six Eurasian rivers has had a tendency to be increase over the period 1955–2000 (Fig. 9). It is mostly related to the decrease in the number of discharge measurements being carried out. The problem of decreasing discharge measurements in North America has been recognized by the USGS (USGS Memorandum No 94.03, 1994). Thus, the decline in number of hydrological monitoring stations found earlier (Shiklomanov et al., 2002) and deterioration of discharge data demonstrated in this paper for six very important and well-operated gauging stations could lead to irreversible consequences in our ability to monitor a rapidly-changing hydrology of the pan-Arctic.

Acknowledgements

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Appendix A

Parameters k_i, k_j in Eq. (3) can be defined using a correlation matrix D:

$$D = \begin{vmatrix} 1 & \cdots & r_{x_1x_j} & r_{x_1x_m} \\ \cdots & \cdots & \cdots & \cdots \\ r_{x_jx_1} & \cdots & \cdots & r_{x_jx_m} \\ r_{x_mx_1} & \cdots & r_{x_mx_j} & 1 \end{vmatrix} \tag{A1}$$

where $r_{x_my}\ldots r_{x_jx_m}$ are the correlation coefficients between y and x^m, and x^j, and x^m, respectively. In our case, an independent variable y and dependent variables x_{ij} represent the estimated discharge Q' and corresponding water stage H. Equation for k_{ij} can be written using determinant D:

$$k_{ij} = \sqrt{\frac{1 - r^2_{x_ix_j}}{D}}; \quad i,j = 1,\ldots,m. \tag{A2}$$

In practice, the majority of relatively stable stage-discharge rating curves can be approximated with third or even second order polynomials (Krashnikov, 1987). We will therefore consider the third order polynomials as the most widely use to describe the rating curve. In this case, the parameters $k_{i,j}$ will be:

$$k_1 = \sqrt{\frac{1-r^2_{x_2x_3}}{D}}; \qquad k_2 = \sqrt{\frac{1-r^2_{x_1x_3}}{D}}; \qquad k_3 = \sqrt{\frac{1-r^2_{x_1x_2}}{D}}. \tag{A3}$$

It turns out, that $k_{i,j}$ is greater when the correlations between variables $x_{i,j}$ are higher.

Karasev and Yakovleva (2001), using numerous Russian observational data, obtained the mean values of the correlation coefficients for typical hydrograph of lowland rivers with spring snow flood as follow $r_{x_1x_2}=0.97$, $r_{x_1x_3}=0.93$, $r_{x_2x_3}=0.96$. Another term $(x_i-\bar{x}_i)(x_j-\bar{x}_j)/\sigma_{x_i}\sigma_{x_j}$ in Eq. (3), is defined from the condition of evenly distributed x_i and x_j in the range from $x_{i,j\min}$ to $x_{i,j\max}$, then:

$$(x_{i\ \min,\ \max}-\bar{x}_i)(x_{j\ \min,\ \max}-\bar{x}_j) = 3\sigma_{x_i}\sigma_{x_j}. \tag{A4}$$

If we denote that,

$$\alpha_i = \frac{x_{1i}-\bar{x}_1}{x_{1\ \max,\ \min}-\bar{x}_1} \cong \frac{x_{2i}-\bar{x}_2}{x_{2\ \max,\ \min}-\bar{x}_2} \cong \frac{x_{3i}-\bar{x}_3}{x_{3\ \max,\ \min}-\bar{x}_3}\cdots \tag{A5}$$

and accept that approximation error ε_{apr} is an equivalent of the absolute daily discharge error ε_{dd}, then the Eq. (3) can be written as:

$$\varepsilon_{dd} = \pm\frac{\sigma_q}{\sqrt{n-m}}\sqrt{\left(1+3\alpha_i^2\sum_{i,j=1}^{m}k_ik_j\right)}. \tag{A6}$$

The same equation for approximation of rating curve with third order polynomial will then be:

$$\varepsilon_{dd} = \pm\frac{\sigma_q}{\sqrt{n-3}} \times\sqrt{1+3\alpha_i^2\left(k_1^2+k_2^2+k_3^2+k_1k_2+k_1k_3+k_2k_3\right)}. \tag{A7}$$

For the correlation coefficients given above: $k_1=4.11$, $k_2=5.40$, $k_3=3.57$ and we can get the simplified equation for the error of daily discharge estimates:

$$\varepsilon_{dd} = \pm\frac{\sigma_q}{\sqrt{n-3}}\sqrt{1+352.5\alpha_i^2}. \tag{A8}$$

Appendix B

To estimate the interpolation error it is necessary to have information about correlation of the interpolated elements.Karasev and Yakovleva, 2001, reasoning from cyclical fluctuations of q_t, proposed to approximate the variation of the deviations over time using a single wave cosine function:

$$q_t = a\cos\left(\frac{2\pi\tau}{T}+\varphi\right) \tag{B1}$$

with phase: φ and period: $T=2T_{ice}$, where T_{ice} is a duration of period with a non-unique stage-discharge relationship, for example, the duration of freezing-over; a is an absolute maximum value for q_t or $a=|q_{max}|$ and τ is a shift parameter defined through T. The approximation of the random process with the cosine curve allows estimation of statistical characteristics: mean is zero, variance is $a^2/2$ and normalized autocorrelation function (ACF) is defined as:

$$r(\tau) = \cos(2\pi/T)\tau. \tag{B2}$$

Both this normalized ACF and the random process itself are defined by the same type of trigonometric expressions. This facilitates its approximated estimate without any additional discharge measurements. Henceforth, we will use the normalized ACF in the limited range of the shift parameter $0\le\tau\le 0.25?$. The mean correlation time τ_c in this range will then be:

$$\tau_c = \int_0^{0.25T}\cos\left(\frac{2\pi\tau}{T}\right)d\tau = 0.16T \tag{B3}$$

τ_c is one of the most important characteristics of random processes. The ratio of τ_c and the mean interval between discharge measurements τ_d is a discontinuity parameter $\vartheta=\tau_c/\tau_d$, which serves as a deciding characteristic to estimate the reliability of representation of continuous temporal functions by their discontinuous values registrated in individual time moments. If the period of approximating cosine curve coincides with period of the non-unique stage-discharge relationship (winter regime), the discontinuity interval will be:

$$\tau_d = T/n \tag{B4}$$

where n is the number of equidistant measurements for the period T. The discontinuity parameter may be expressed as:

$$\vartheta = 0.16(n-1). \quad \text{(B5)}$$

A priori, we assume the period of the approximating cosine curve coincides with the period T, but it is known from practice that to have better agreement between the observational data and the cosine curve, the different parameters a and T may be used for the individual parts of time series graph q_t.

To estimate the interpolation error of the quasistationary random process q_t the following terms are applied: a measure of averaged error of linear interpolation η_{in}, which is defined as:

$$\eta_{in} = \frac{\varepsilon_{in}^2}{\sigma_{q_t}^2} \quad \text{(B6)}$$

and a measure of averaged error of discharge measurements η_{mes}:

$$\eta_{mes} = \frac{\varepsilon_{mes}^2}{\sigma_{q_t}^2} \quad \text{(B7)}$$

where $\sigma_{q_t}^2$ is a variance of the deviations q_t, ε_{in} is relative error of linear interpolation of the deviations and ε_{mes} is relative error of discharge measurements. The measure of interpolation error is distinguished in different points of the interval between measurements and its maximum value takes place in the middle of the interval. Equations for the measure of interpolation error were derived by Drozdov and Shepelevsky (1946); Karasev (1980) adopted them for q_t series. The equation for the measure of averaged error of linear interpolation obtained by integration of general dependence for measure of interpolation error was derived by Karasev, 1980:

$$\bar{\eta}_{in} = 4(\vartheta^2 - \vartheta) + \left(\frac{1}{3} - 4\vartheta^2\right)e^{-(1/\vartheta)} + \frac{5}{3} + \frac{2\eta_{mes}}{3}. \quad \text{(B8)}$$

The Eq. (8b) is too bulky and can be approximated for discontinuity parameter in the range $\vartheta \leq 4$, as follows:

$$\bar{\eta}_{in} = \frac{5}{3}e^{-1.4(\tau_c/\tau_m)} + 0.5\eta_{mes} \quad \text{(B9)}$$

Using Eqs. (B6) and (B7) and taking into account that for the approximating cosine curve $\sigma_q^2 = (a^2/2)$, and $(=(\tau_c/\tau_d)$ we obtain the equation for relative interpolation error:

$$\tilde{\varepsilon}_{in} = \sqrt{0.83a^2 e^{-1.4\vartheta} + 0.5\tilde{\varepsilon}_{mes}^2}. \quad \text{(B10)}$$

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