Available online at www.sciencedirect.com

SCIENCE DIRECT®

Journal of Applied Geophysics 59 (2006) 47–62

JOURNAL OF APPLIED GEOPHYSICS

www.elsevier.com/locate/jappgeo

Sensitivity analysis of Gassmann's fluid substitution equations: Some implications in feasibility studies of time-lapse seismic reservoir monitoring

Fredy A.V. Artola [a,*], Vladimir Alvarado [b,1]

[a] *Grupo de Tecnologia e Engenharia de Petróleo, Pontifícia Universidade Católica do Rio de Janeiro, Rua Marquês de São Vicente, 225 - Edifício Padre Leonel Franca, 6 andar - CEP - 22453-900, Gávea - Rio de Janeiro - RJ - Brazil*

[b] *Interdepartmental Oil and Gas Program, Pontifícia Universidade Católica do Rio de Janeiro, Rua Marquês de São Vicente, 225 - CEP - 22453-900, Gávea - Rio de Janeiro - RJ - Brazil*

Received 6 December 2004; accepted 20 July 2005

Abstract

Here, we discuss the sensitivity of the seismic response to uncertainties in the physical parameters of the reservoir rock. For this purpose, a probabilistic sensitivity analysis of Gassmann's fluid substitution equations using a Monte Carlo approach was carried out. We represented uncertainties related to each parameter as probability density functions to evaluate the contribution of each parameter uncertainty to the variance of the seismic response (V_p), calculated by means of the Monte Carlo approach. We show that uncertainties related to grain density (ρ_{gr}), dry shear modulus (G_d) and dry bulk modulus (K_d) contribute more significantly on the variance of V_p, if all parameters are uncorrelated. This outcome changes, when physical dependencies are represented as correlations in the Monte Carlo sampling of some of the parameters. In this sense, correlations distribute more evenly the contributions to uncertainty in V_p. On the other hand, we also evaluated scenarios of fluid substitution, in which fluid 1 is replaced by fluid 2, with the corresponding variations in seismic response. In this case, V_{p2} is the P-wave velocity of rock saturated with a fluid 2. If V_{p2} were forecasted from an initial set of parameters of the rock saturated with fluid 1 (V_{p1}, V_{s1}, etc.) the uncertainties related to V_{p1}, V_{s1} and K_{gr} would contribute more significantly to the variance of V_{p2}. From these three initial parameters, the most important contributions come form V_{p1} and V_{s1}. Concomitantly, we evaluated the contribution of possible variations in fluid phase density and bulk modulus and of a pore pressure perturbation (4MPa) for several scenarios of connate and injection fluids on the variance of V_p. We did this for several values of initial differential pressure. Results indicate that the contribution of the elastic piezosensitivity and possible changes in the fluid phase properties depend not only on the initial differential

* Corresponding author. Fax: +55 21 3114 1459.
E-mail address: fre777@zipmail.com.br (F.A.V. Artola).
[1] Fax: +55 21 3114 1165.

doi:10.1016/j.jappgeo.2005.07.006

pressure, but also on the type of fluids involved in substitution process. We conclude that sensitivity information, limited in this case to Gassman's equations, can be used as a tool to improve feasibility studies in time-lapse seismic reservoir monitoring and as a priori qualitative knowledge. The latter can guide the inversion process or help to diminish the uncertainties due to poorly constrained inversion schemes.

Keywords: Sensitivity analysis; Feasibility studies; Time-lapse; Gassmann equations

1. Introduction

The key elements of a successful 4D seismic project consist of feasibility (Lumey et al., 1997), acquisition, processing and interpretation (Lumey and Behrens, 1998). According to Behrens et al. (2002), feasibility comprises detectability and repeatability. Detectability is the ability to detect changes in the seismic response due to alterations in pressure and saturation during production. An appropriate rock physics model is a critical to assessing detectability (Behrens et al., 2002). On the other hand, repeatability is a measure of the similarity of the seismic response between two or more seismic surveys.

In this work, we concentrate on issues of uncertainty propagation for detectability in time-lapse seismic monitoring, through the analysis of Rock Physics equations, as part of feasibility studies. Our results might impact mostly decisions for purposely designed 4D seismic surveys, rather than studies based on legacy seismic data. In general, high or improved detectability, by mitigation of uncertainties, might result relevant in a large number of cases. We focus on quantitative uses of time-lapse seismic. In this regard, the correct use of rock physics models is a must. Understanding of uncertainty propagation is, therefore, a matter of necessity. Feasibility studies are tied to the particular production scenario in a given reservoir, that is, whether being a primary or a secondary production mechanism. A myriad of different data sources at different scales impact the result of these studies usually represented through equations. In this context, Gassmann (1951) equations, whose applicability in porous media is limited to homogeneous isotropic rocks under isobaric conditions, are frequently used to link the seismic response to changes in reservoir properties. A typical form for Gassmann equation is the following:

$$K_s = K_d + \frac{\left(1 - K_d/K_{gr}\right)^2}{\dfrac{\phi}{K_f} + \dfrac{1-\phi}{K_{gr}} - \dfrac{K_d}{K_{gr}^2}} \quad (1)$$

where K_s is the saturated-rock modulus, K_d the frame (dry) bulk modulus, while K_{gr} and K_f correspond to the grain and fluid bulk moduli, respectively, and ϕ is porosity.

On the other hand, Brown and Korringa (1975) generalized equations represent an alternative for anisotropic cases. These equations can consider arbitrary symmetries (VTI, HTI, orthorhombic, monoclinic, etc.) to carry out fluid substitution in rocks. Although the equations are not explicit functions of thermodynamic conditions in a reservoir, pressure and temperature effects can be accounted for through empirical equations. Batzle and Wang (1992) expresses effects of pressure and temperature of the fluid phase, while an elastic piezosensitivity relationship can be used for the dry-rock bulk modulus (MacBeth, 2004; Shapiro, 2003, etc). This latter relationship relates the rock elastic moduli with differential pressure (confining minus pore pressure).

In conventional time-lapse feasibility studies, synthetic monitors are used to estimate likely changes in the seismic response. These estimates are employed to help with the decision-making process of production strategies. Studies start with an initial estimate of physical parameters that control the seismic response, upon which updating of the reservoir conditions follow.

Parameters for the application of Gassmann equation generally come from well and core data, plus empirical equations. The values of V_p and V_s are, in general, estimated from full-wave sonic logs. Saturated-rock moduli can be obtained from estimated V_p and V_s values, while reservoir density, in turn, can be calculated from sonic logs and core measurements. From all these parameters, the frame (dry) bulk modulus (K_d) can be computed, but knowledge of fluid properties (saturation, fluid bulk modulus and density at reservoir conditions), porosity and grain bulk modulus are necessary. Some of these parameters can be estimated from well or core data, while others can be found in tables. There is an issue with Rock Physics measurements that Landrø (2002) associates with repeatability, but one can certainly mention as a problem of uncertainty in rock properties. It has been shown that attempts to restore original conditions of samples do not yield original properties of the reservoir rock. On the other hand, well logs, lab experiments and field seismic measurements operate at different length scales and frequencies. In this sense, there are uncertainties associated with the scale and frequency transformations. Corrective actions are usually taken to deal with sources of errors but even with all the care taken in the estimation of reservoir and fluid parameters, one cannot avoid significant sources of uncertainty associated with those parameters. In addition, if the microstructural complexity of the rock is taken into consideration, that is, the number of mineralogic constituents, grain geometry and arrangement, then the uncertainty associated with the grain effective bulk modulus can be considerable. In truth, only bounds, upper and lower limits, for this modulus can be calculated by means of mixing rules.

As for the fluid phases, since in a real reservoir there are several fluid components occupying the pore space, it is then necessary to estimate effective fluid density and bulk modulus. However, the estimates do not depend only on the saturation value, but also on the way fluids are distributed within the pore space, i.e. the saturation distribution pattern (Mavko and Mukerji, 1998). Therefore, the value of K_f cannot be uniquely estimated, since it is not possible to know accurately a priori the saturation pattern. A mixing rule can be used to calculate K_f, but its selection depends on previous knowledge on the complexity of the microstructural arrangement in the rock and fluid properties, even at initial stages of production. In consequence, the seismic response can be predicted only within bounds.

From the above discussion, it is inevitable to face uncertained scenarios, and hence it becomes necessary to attempt to reduce uncertainties or the least to manage them. Consequently, the use of Gassmann equations for feasibility and inversion studies, within the framework of time-lapse seismic, demands sensitivity analyses of the seismic response with respect to uncertainty in the input parameters. This evaluation could help us to quantify the forecast variances and compare them with the expected time-lapse changes (free of uncertainty). Now, since variations in the seismic response can be in some cases subtle, or more important in other instances, it is required that possible changes be analyzed in view of the forecast variance due to uncertainty in the input parameters. Seen under this light, feasibility studies can turn into useful tools for the decision-making process. Moreover, feasibility studies can let us establish a hierarchy in terms of the impact of uncertainties (in terms of their contribution) on the forecast variance.

The importance of rock physics considerations in the context of time-lapse seismic reservoir monitoring can be better appreciated by looking at initiatives to quantitatively include time-lapse seismic data into history-matching exercises (Aanonsen et al., 2003; Gosselin et al., 2003; Falcone et al., 2004). The European Commission funded an industrial joint project to value 4D seismic results for history matching purposes. Given that elastic properties, and not seismically derived saturation and pressure, were the basis for history match in the workflow chart, the so-called petro-elastic model (PME) or rock physics model is key element in this type of analysis (summarized in terms of Gassmann equations).

On the other hand, changes in the seismic response are linked to changes in both the solid and fluid phases. Therefore, it is important to understand how changes in fluid and solid properties contribute to those changes in the seismic response, for the different stress and production scenarios. Our analysis, however, will disregard geomechanical effects such as subsidence or compaction that can be relevant in certain scenarios of production, but considers the type of fluids and reservoir dynamics. However, the importance of Geomechanics in a 4D seismic workflow cannot be sufficiently stressed here, although it is not being incorporated in our analysis.

One more point to consider is that of the type of sensitivity analysis, either probabilistic or deterministic. Wang (2000) performed a relatively simple deterministic sensitivity calculation on some seismic attributes, due to errors (±10%) in Gassmann equations parameters, for hypothetical sandstone. Sengupta et al. (1998) carried out a sensitivity study in forward AVO modeling to evaluate effects of the fluid substitution in the reflectivity by means of Monte Carlo simulation. On the other hand, Sengupta and Mavko (1999) performed a sensitivity analysis of fluid substitution equations in terms of differential error that was obtained from the partial derivatives of V_{p2} related to each input parameter. In their approach, the input parameters were considered uncorrelated. To extent Wang and Sengupta's findings, we performed a probabilistic analysis of the effect of uncertainties. Contributions of uncertainties of the physical parameters (correlated and uncorrelated parameters) to the variance of the predicted compressional velocity are shown by means of Monte Carlo simulations, through evaluation of Gassmann equations. Two types of rock models are analyzed, a clean and a shaly sandstone, for an interval of porosity between 10% and 30%.

For Monte Carlo simulations, we used Crystal Ball, software from Decisioneering Risk Analysis (Evans and Olson, 2002). Two types of sensitivity charts can be displayed in Crystal Ball. In the first type, sensitivities are measured by rank correlation coefficients. In the second type, used here, approximate percentage of contributions to the variance from input variables are produced. The method is only approximated and does not correspond to a full variance analysis.

Fluid substitution is carried out by considering an initial stage of a reservoir saturated with fluid 1 and later with fluid 2. In correspondence with the initial stage, there exist the initial compressional and shear velocities (V_{p1} and V_{s1}), porosity, saturated-rock bulk density, and grain bulk modulus. Similarly, the same parameters associated with stage 2 are calculated. In consequence, effects of fluid substitution are evaluated in terms of parameter contributions to the variances in the Monte Carlo simulation.

Finally, we evaluate the contribution of bulk moduli and density of the fluid phases variability, in terms of degrees of freedom for some injection scenarios (for several combinations of connate and injection fluids). These contributions are compared to those caused by pore pressure perturbation of 4 Mpa.

2. Sensitivity analysis of Gassmann Equation

The low-frequency approximation for computation wave propagation velocity in Rocks is based on Gassmann (1951) equations. This approximation assumes that the interconnected pores in the rock are saturated with a non-viscous fluid. To compute V_p in Gassmann equations, it is necessary to know the following parameters: porosity (ϕ), frame bulk and shear moduli (K_d and G_d, respectively), grain bulk modulus (K_{gr}), grain density (ρ_{gr}), fluid bulk modulus (K_{fl}) and the fluid density. This way, V_p results in a function of those parameters as:

$$V_p = f(\phi, K_d, G_d, K_{gr}, \rho_{gr}, K_{fl}, \rho_{fl}). \tag{2}$$

It will be shown, by using Gassmann equation (Eq. (1)), that the value of V_p is not equally affected by the uncertainty associated with each parameter. The sensitivity of V_p on Gassmann equation parameters is attained by performing Monte Carlo simulations. Each parameter was modeled as a random variable, whose probability density function (pdf) corresponds to a Gaussian function, characterized by a mean value μ and standard deviation (σ). Although not shown here, other symmetrical distributions yielded similar results, to those to be presented here. However, to evaluate the effect of possible asymmetry in real pdf, triangular density functions with non-zero skewness were also used to model uncertainty sources. The idea being, that the triangular distribution is simple, but flexible enough, to allow a control source of uncertainty, so that results from differently skewed sampling could be compared on the same statistic basis.

Values for parameters used in the sensitivity analysis are not readily available, except perhaps for porosity. To avoid unnecessary inconsistencies in the simulation process, some empirical equations that link K_d to ϕ and K_{gr} were used. A thorough discussion on moduli-porosity relationships can be found in Vernik (1998), and Nolen-Hoeksema (2000).

In this work, we selected an empirical equation that applies to clean and shaly sandstones, subjected to the

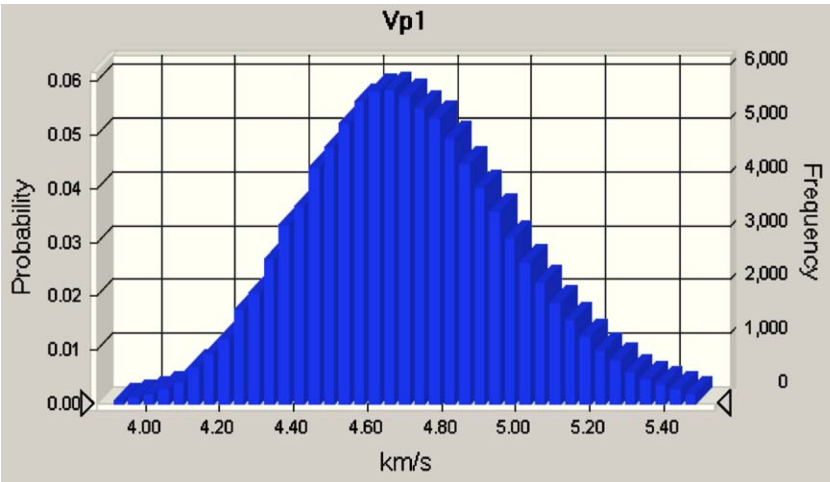

Fig. 1. V_p forecast considering contributions of the uncertainties in the input parameters (10% of the mean value for each parameter).

same pressure differential conditions. The equations were taken from Batzle and De-hua (2004), written in general form as:

$$K_d = \left(1 - A \times \phi + B \times \phi^2 - C \times \phi^3\right) \times K_{gr} \qquad (3)$$

where, A=3.206, B=3.349 and C=1.143, in the case of clean sandstone, and A=3.053, B=3.070 and C=1.016 for shaly sandstone.

Two situations are considered, in terms of the possible mutual dependencies among parameters in the equation. In the first one, Monte Carlo (MC) samples for each parameter are drawn independently, meaning that no correlation exists among parameters. This is common ground with reported applications of Gassmann equations, in which the parameters are used as if they were completely independent, although some are. However, the assumption on absolute independence is not entirely physically sound. In this sense, mean values of the sample pdf for K_d and G_d are computed from Eq. (3). This type of results is referred to as uncorrelated random variables. In the second set of MC simulations, in addition to pre-establishing the mean values of the pdfs, the sampling for K_d and G_d are correlated to those of ϕ and K_{gr}. In contrast, these results are referred to the Section on correlated random variables.

In all the simulations, 10% of the mean value of each parameter was used as uncertainty source. This in turn corresponds to 2σ. The seismic response (V_p) is given as a pdf, depicted in Fig. 1, where results of a Monte Carlo simulation exemplify this type of evaluation. In general, the dispersion around the mean value of the distribution is used to determine the influences of the different parameters.

Falcone et al. (2004) illustrated the importance of the PEM for the real case of the Girassol field, offshore Angola. In their work, values of elastic properties and associated uncertainties were calculated by inversion. Uncertainty distributions were not explicitly shown, but the features of the input V_p and V_s values can be extracted from error calculations, depicted in Fig. 2 in their article, derived from acoustic log data. V_p and V_s show a clear positively skewed trend, indicating departure from symmetric distributions. This motivated exploration of the effect of

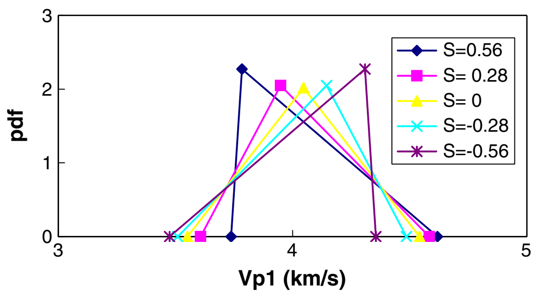

Fig. 2. Triangular pdfs for V_{p1} for different values of the relative skewness. The mean value and the variance are identical for all pdfs.

asymmetry in the probability density function on the contribution of input parameters in Gassmann equations, through the use of triangular pdfs. Special care was taken to avoid misleading comparisons. To control uncertainty sources, the mean value and standard deviation were kept constant for all triangular density functions for each of the parameters in the Monte Carlo simulation. The skewness, S, was varied, but properly normalized, $S^*=S/(\sigma^2)^{3/2}$, so that the asymmetry of the probability density function was ade-

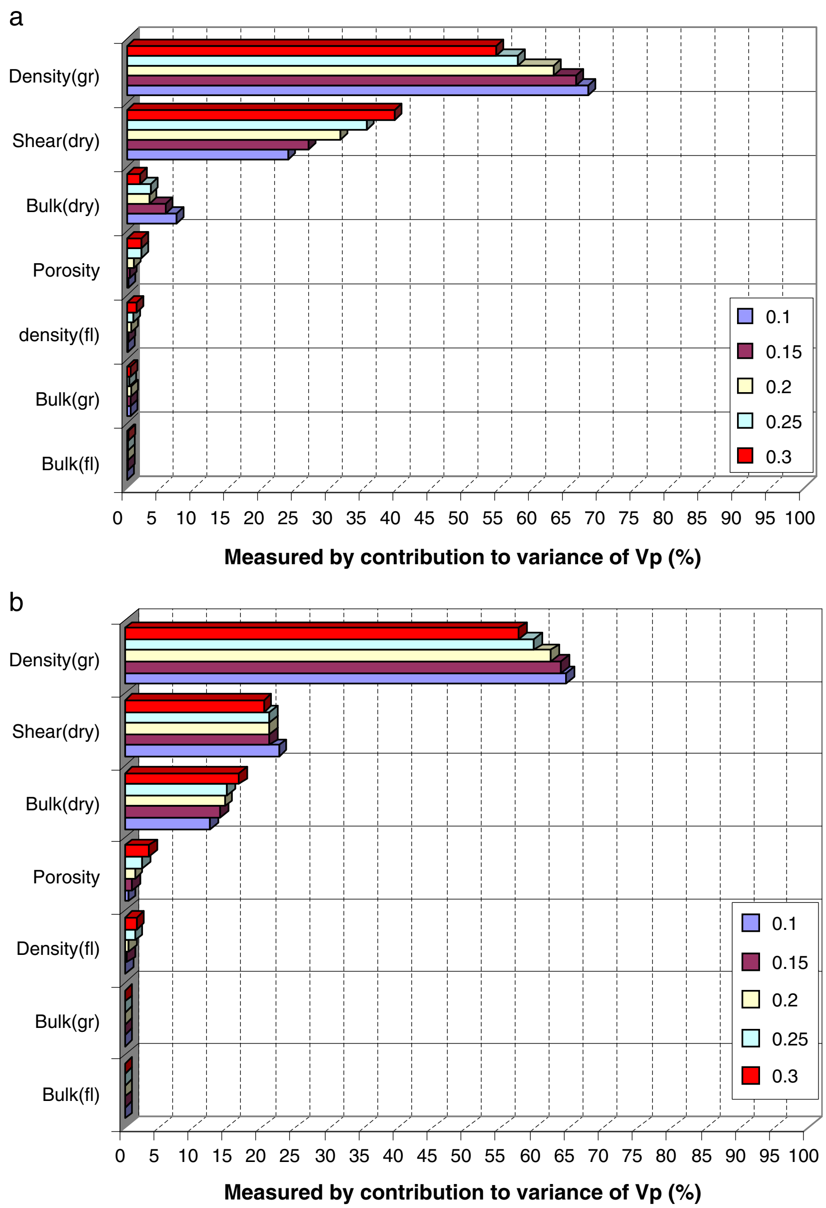

Fig. 3. Contributions to the variance of V_p from uncertainties associated with input uncorrelated parameters. (a) Clean sandstone. (b) Shaly sandstone.

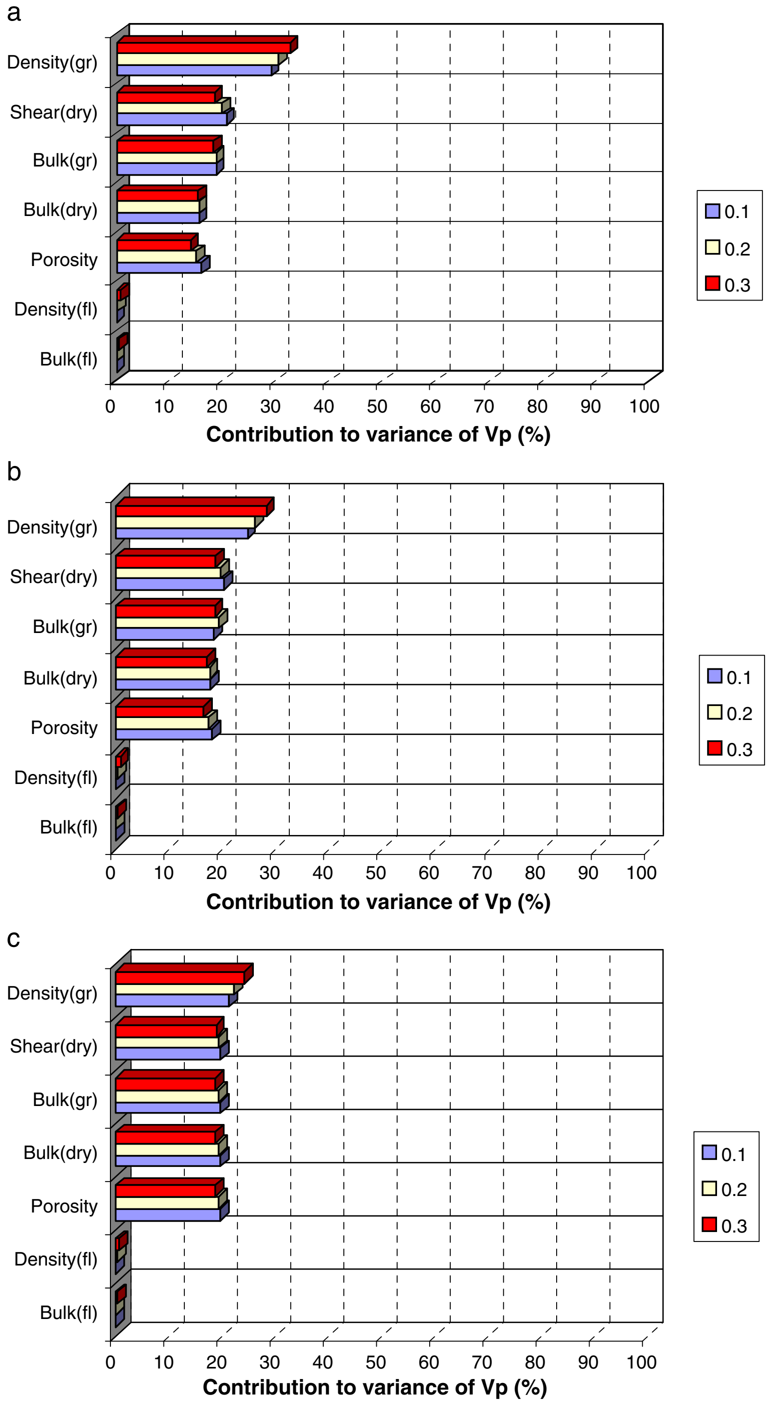

Fig. 4. Contributions to the variance of V_p from uncertainties associated with input correlated parameters for shaly sandstone.

quately controlled. Fig. 2 shows pdfs for one input parameter for the five values of normalized skewness tested. The relative asymmetry for all other parameters pdfs, for the same value of S^*, reproduces the same shapes shown in Fig. 2.

3. Uncorrelated random variables

Fig. 3a and b show the parameters uncertainty contributions to the variance of V_p, obtained in the MC approach. The sensitivity results shown were performed for porosity values between 0.1 and 0.3. From the figures, it is apparent that ρ_{gr}, G_d and K_d are the dominant parameters on the variance of V_p. Contributions from uncertainties in the remaining parameters are practically negligible, unless a much larger error in those parameters were introduced (not shown). Additionally, the results corroborate that uncertainties with respect to ρ_{gr} grow inversely with ϕ, being more evident in the case of the clean sandstone. On the other hand, uncertainty contributions from G_d are greater for clean sandstone, diminishing with decreasing porosity. As opposed to ρ_{gr} and G_d, uncertainty contributions arising from K_d are significantly larger for the case of shaly systems, as can be clearly seen in Fig. 3a and b. The trend with respect to porosity shifts from decreasing contributions in clean systems to increasing contributions in shaly sandstone. From these results, critical parameters that could compromise a reliable estimate of V_p turned out to be ρ_{gr}, G_d and K_d. These conclusions on the impact of the different input parameters on the uncertainty of V_p agree with results obtained with a deterministic analysis, published by Wang (2000).

A number of simulations with triangular pdfs were completed, starting for a whole set of symmetrical pdfs for all input parameters and continuing with various combinations of skewed pdfs for the different parameters. However, it was found that the relative contributions to variance differ insignificantly as a result of these choices. It might be that the shape of the resulting pdf for the predicted seismic monitor could change, but from the point of view of uncertainty propagation, all the results fall within sampling error.

3.1. Correlated random variables

For the results in this section, MC simulations were carried out considering correlations among certain parameters, such that the outcomes of ϕ and K_{gr} conditioned the sampling of K_d and G_d, and vice versa. Fig. 4a–c show results for the case of the shaly sandstone. Three correlation values were used: 0.8

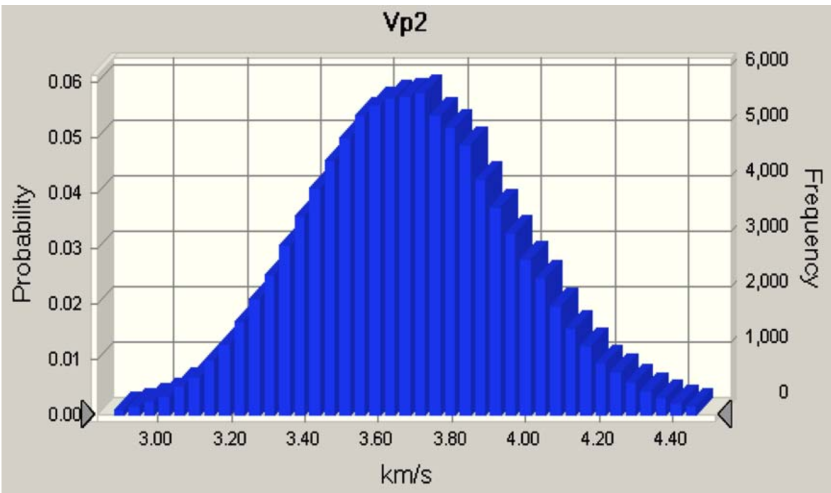

Fig. 5. V_{p2} forecast considering contributions of the uncertainties in the input parameters (10% of the mean value for each parameter).

(3a), 0.9 (3b) and 1.0 (3c), being negative with respect to porosity and positive in terms of grain bulk modulus. As in the uncorrelated simulations, ρ_{gr}, G_d and K_d are still the most contributing parameters to the variance of V_p. As opposed to uncorrelated cases, though, the remaining parameters gain importance, even exhibiting roughly the same order of importance, with their contributions increasing with higher correlation. This result should be expected because the correlations included K_d and G_d, which are two of the most relevant parameters. Similar results were observed for clean sandstones.

We also evaluated the combined effect of asymmetrical pdfs with correlation. However, it seems that the

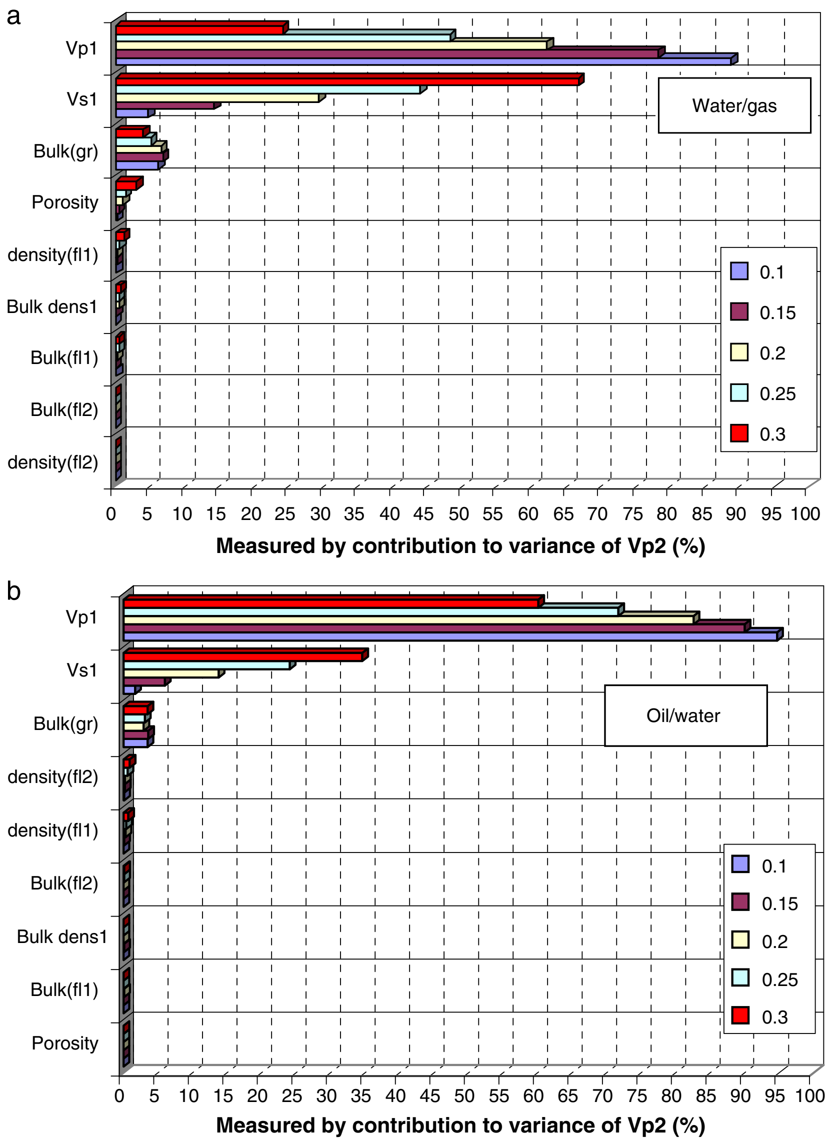

Fig. 6. Contributions to the variance of V_{p2} for a clean sandstone from uncertainties associated with input parameters. (a) Connate fluid:Gas/ Injection fluid: Water. (b) Connate fluid:Oil, Injection fluid:Water, (c) Connate fluid:Oil, Injection fluid:Gas.

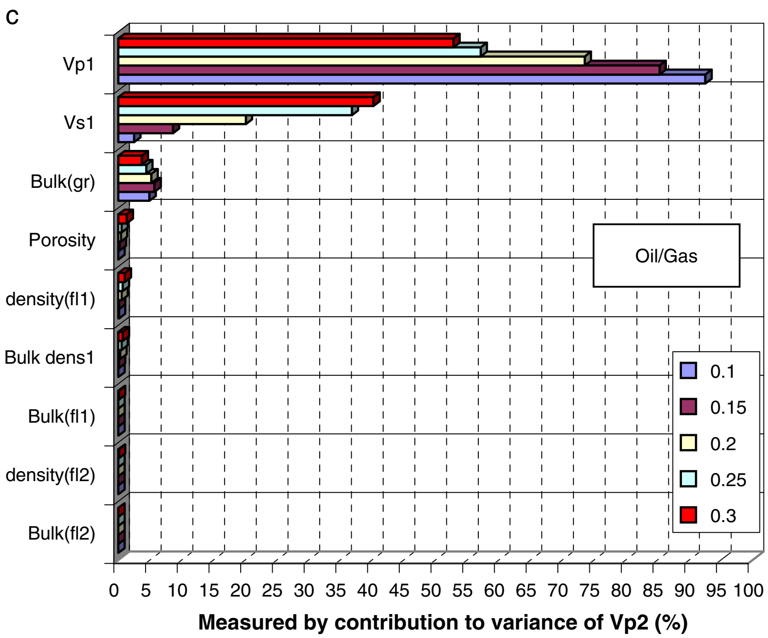

Fig. 6 (*continued*).

yield is mostly a result of correlations, exhibiting very similar behavior as that shown in Fig. 4.

4. Fluid substitution effects

Now, let us consider a rock initially saturated with fluid 1, whether partially or completely. For this condition, there is a number of rock and fluid properties associated that lead to an initial velocity V_{p1}. For the prediction of the wave velocity (V_{p2}) at a synthetic monitor, after a fluid substitution with fluid 2, a total of nine parameters are required. These parameters consist of V_{p1}, V_{s1}, K_{gr}, ϕ, ρ_{fl1}, ρ_{fl2}, K_{fl1}, K_{fl2} and ρ_b (initial bulk density). Upon simple manipulation of Gassmann equations, an equation for V_{p2} is obtained:

$$V_{p2} = \left(\frac{\Im}{\rho_{b_2}}\right)^{1/2} \quad (4)$$

where:

$$\Im = \frac{K_0}{1-\alpha^{-1}} + \frac{4}{3}V_{s1}^2\rho_{b_1};$$

$$\rho_{b_2} = \rho_{b_1} + \phi\left(\rho_{fl_2} - \rho_{fl_1}\right);$$

and,

$$\alpha = \frac{\rho_{b_1}\left(V_{p1}^2 - \frac{4}{3}V_{s1}^2\right)}{\rho_{b_1}\left(V_{p1}^2 - \frac{4}{3}V_{s1}^2\right) - K_{gr}} - \frac{K_{fl_1}}{\phi\left(K_{fl_1} - K_{gr}\right)} + \frac{K_{fl_2}}{\phi\left(K_{fl_2} - K_{gr}\right)}.$$

As before, provided that some of the input parameters in Eq. (4) are random variables, V_{p2} is given by a pdf. For instance, Fig. 5 shows a MC simulation result for V_{p2}.

In the present scheme of fluid substitution, V_{p1}, V_{s1} and K_{gr} contribute the most to the variance of V_{p2}, in decreasing order of importance. The result holds for both clean and shaly sandstones (see Figs. 6a–c and 7a–c). It is clear that the type of substitution event (water/gas, oil/water, oil/gas) and the mean value of porosity impact considerably the variance of V_{p2}.

The contribution to uncertainty in V_{p2} coming from V_{p1} is larger than that originated from V_{s1}, except when the value of porosity is high (0.3) and gas is substituted with water. In this latter case, V_{s1} is a greater contributing source of uncertainty than V_{p1}, for both clean and shaly sandstone. In general, for all types of fluid substitution considered here, contributions to the uncertainty of V_{p2} coming from V_{p1} grow as porosity decreases. The opposite occurs with V_{s1}, since its contribution positively relates to porosity.

For clean and shaly sandstones, contributions to uncertainty of V_{p2} are decreasingly important for oil/water, oil/gas, and water/gas substitution process, respectively, for the whole range of porosity values tested in this work. On the other hand, the contributions from V_{s1} come first for water/gas substitution, then oil/gas case and finally oil/water. For K_{gr}, although its contribution to the predicted V_{p2} stays

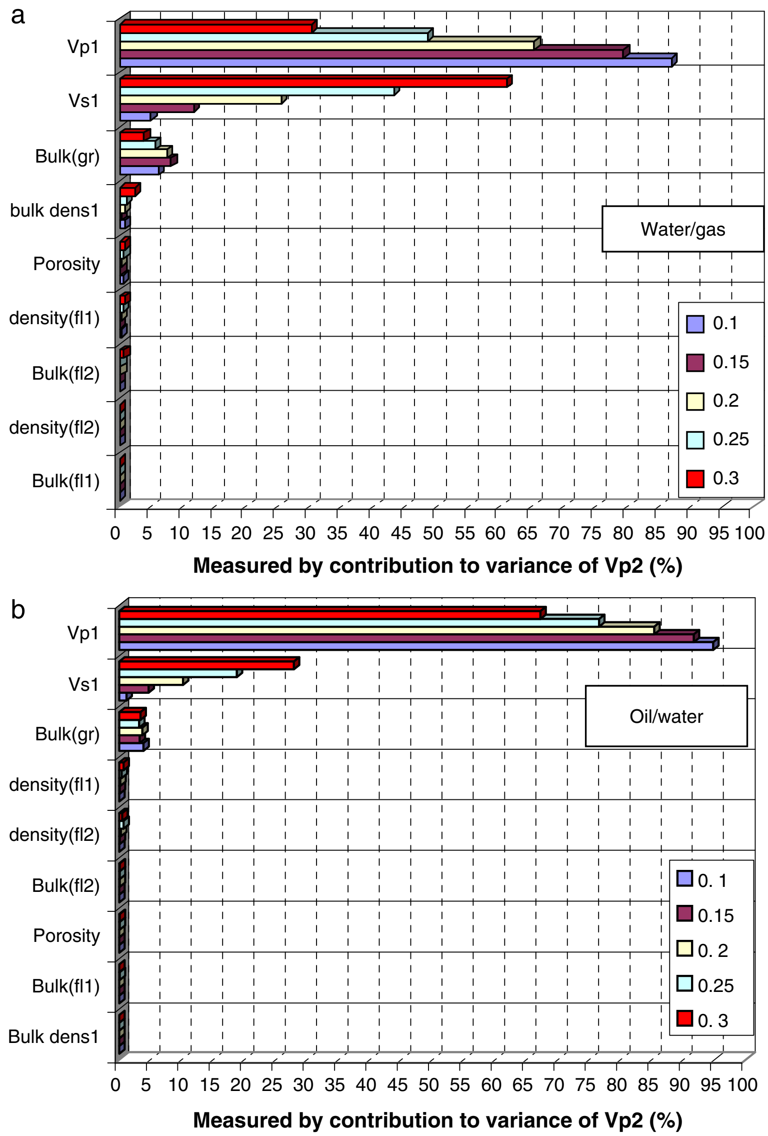

Fig. 7. Contributions to the variance of V_{p2} for shaly sandstone from uncertainties associates with input parameters. (a) Connate fluid:Gas, Injection fluid:Water. (b) Connate fluid:Oil, Injection fluid:Water. (c) Connate fluid:Oil, Injection fluid:Gas.

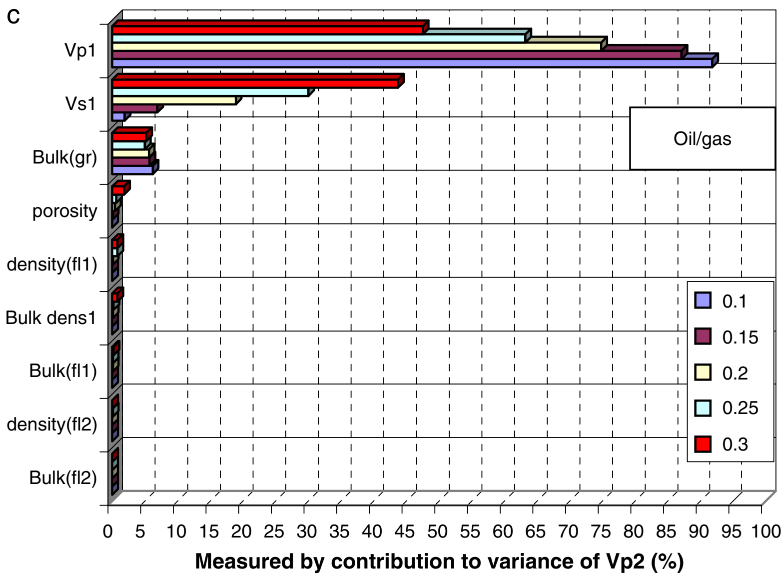

Fig. 7 (*continued*).

small, its value is significantly larger than those of the remaining parameters. This contribution varies between 3% (for oil/water substitution) and 7% (for water/gas substitution).

5. Fluid properties and elastic piezosensitivity effects

Changes in seismic attributes are due in good part to two different types of effects. On one hand, frame properties, K_d and G_d, are prompt to change as a result of production in an oil field (mainly by changes in the stress conditions). On the other hand, fluid properties such as K_{fl} and ρ_f, are also affected by production and depend on saturation, pressure and temperature distributions. Predominance of either type of effect, in terms of their contribution to variations in the seismic response depends on a myriad of factors, mainly reservoir depth, initial differential pressure (P_d), temperature, in-situ (connate) and injection fluids.

In the last few years, important contributions to the understanding of saturation and pressure effects on characteristics of the seismic response have emerged. Landrø (2001) and Landrø et al. (2003) proposed approximations that relate reflectivity with saturation and pressure. Based on these approximated formulations, the author discussed the separability between saturation and pressure effects by means of PP or simultaneous PP and PS reflectivity inversion, in addition to quantification of uncertainty. Discernment of saturation effects (directly associated to changes in K_{fl} and ρ_f), from those related to pressure can be realized by comparative analysis of the seismic response sensitivity in well-defined production scenarios.

As an example, a sensitivity analysis on V_p with respect to changes in the fluid system and pressure was carried out. To pursue this, synthetic, hypothetical production scenarios were simulated by setting initial value of P_d and type of fluid, whether injection or in-situ fluids. To evaluate the piezosensitivity, equations developed by MacBeth (2004) were used. The equations are applicable to sandstone and are given by:

$$K(P) = \frac{K_\infty}{1 + E_k e^{-P/P_k}} \tag{5}$$

and

$$G(P) = \frac{G_\infty}{1 + E_G e^{-P/P_G}} \tag{6}$$

Table 1
Fluid properties used in modeling of seismic velocities

Fluid substitution type	Bulk modulus (Gpa)	Density (gr/cm^3)
Heavy oil (API = 15)/water	1.7856–2.25	0.965–1
Medium oil (API = 30)/water	1.338–2.25	0.876–1
Light oil (API = 45)/water	1.1106–2.25	0.80–1
Heavy oil (API = 15)/gas	1.7856–0.0404	0.9665–0.126
Medium oil (API = 30)/gas	1.3338–0.0404	0.876–0.126
Light Oil (API = 45)/Gas	1.1106–0.0404	0.80–0.126

where,

$$E_k = \frac{K_\infty - K(0)}{K(0)} = \frac{S_k}{1 - S_k} \tag{7}$$

$$E_G = \frac{G_\infty - G(0)}{G(0)} = \frac{S_G}{1 - S_G}. \tag{8}$$

K_∞ and G_∞ represent asymptotes for high pressures associated with elastic moduli; P_K and P_G are the characteristic pressure constants that define the rollover point, beyond which the rock frame becomes relatively insensitive to pressure and S_K and S_G represent likely overall possible variation in K and G. The experiment is performed using the following parameter values: $\phi = 0.19$, $K_\infty = 25.77$, $G_\infty = 14.44$, $S_K = 0.64$, $S_G = 0.59$, $P_K = 12.73$ and $P_G = 11.0$ (MacBeth, 2004). Here, we evaluate the effect of a small perturbation (4 Mpa) in the pore pressure on velocity for initial Pd of 10, 30, 50 and 70 Mpa, when the in-situ fluids are heavy, medium and light oil and injection fluids are either water or gas. For these isothermal production scenarios, we evaluate contributions to variations of the pore pressure and possible changes in the bulk modulus of the fluid system and density. The evaluation is carried out for the whole range of the parameters that characterize the fluid system. For the simulated scenarios, the bulk modulus and density of the system can vary within intervals, as shown in Table 1.

For each system, the degrees of freedom for K_{fl} and ρ_{fl} are treated in terms of uncertainty and can be introduced into Gassmann equation in the form of a pdf. Similarly, a pore pressure perturbation can be introduced as a pdf.

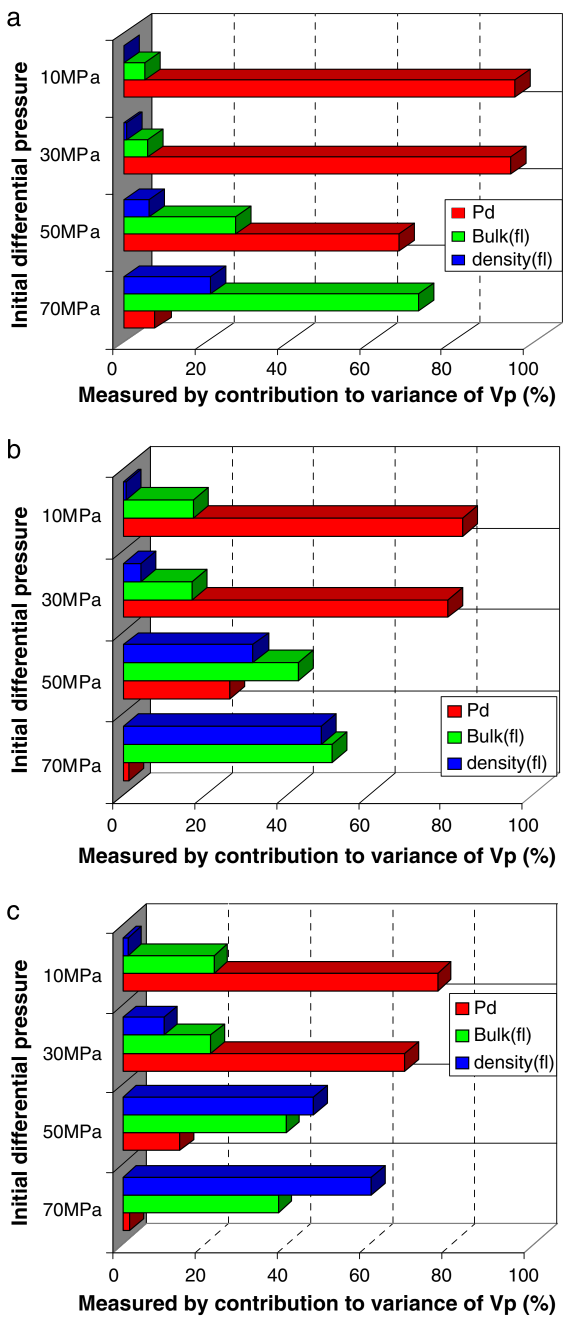

Fig. 8. Contributions of the density and bulk modulus of the fluid system variability and pore pressure disturbance. (a) Connate fluid:Heavy Oil, Injection fluid:Water. (b) Connate fluid:Medium oil, Injection fluid:Water. (c) Connate fluid: Light oil, Injection fluid:Water.

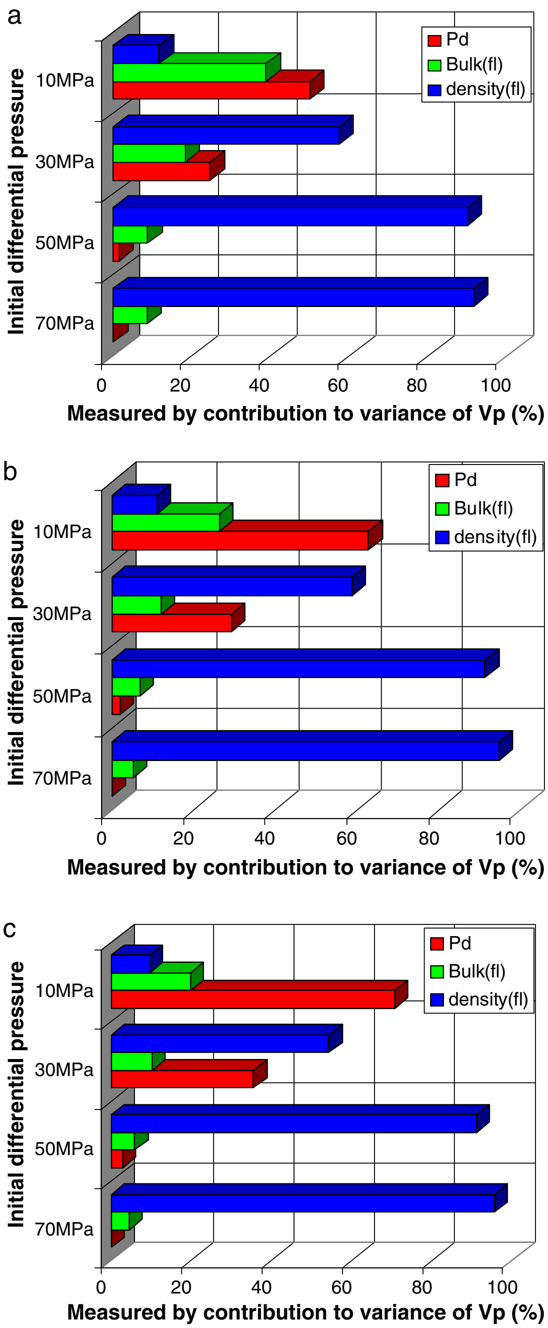

Fig. 9. Contribution of density and bulk modulus of the fluid system variability and pore pressure disturbance. (a) Connate fluid:Heavy oil, Injection fluid:Gas. (b) Connate fluid:Medium oil, Injection fluid:Gas. (c) Connate fluid:Light oil, Injection fluid:Gas.

Fig. 8a–c show contributions of K_{fl}, ρ_{fl} and K_d to the variance of V_p for the scenario where the connate fluid is heavy, medium or light oil and the injected fluid is water. Note that the contribution from pressure decreases not only with the initial P_d, but also with the API gravity of the connate fluid. The latter means that the contribution is greatest for heavy oil, followed by a medium oil and the least for light one. Contributions from K_{fl} and ρ_{fl} grow with initial P_d. For P_d<30 Mpa, the contribution from density is small when compared to the more significant contribution from K_{fl}. Contributions from ρ_{fl} increase with decreasing API gravity of the in-situ fluid.

Fig. 9a–c also depict contributions from K_{fl}, ρ_{fl} and K_d on the variance of V_p, but now the injection fluid is gas. It can be seen that P_d contributions decrease with the effective initial pressure and grow with decreasing API gravity of the connate fluid. As opposed to the case where water is used as the injection fluid, for gas injection, contributions from K_{fl} decrease with depth. In general, it holds that for high P_d values, contributions to the variance of V_p, being more predominant for gas as compared to water injection. On the other hand, for gas injection, ρ_{fl} contributions are markedly dominant. In general, contributions from the variability of density can be dramatically larger on the variance of V_p, when the injection fluid is gas as compared to water injection.

6. Conclusions

We evaluated the effect of uncertainty of the reservoir rock properties on the seismic response (compressional velocity) in a Monte Carlo simulation framework. First, simulation results for Gassmann equations showed that the variance of the compressional velocity (V_p) turns out to be more sensitive to uncertainties in grain density (ρ_{gr}), shear (G_d) and bulk (K_d) moduli, in decreasing order of contribution. The remaining parameters contribute negligibly for uncorrelated simulations. However, the existence of physical correlations among some input parameters, when represented as correlated Monte Carlo sampling, makes more even the contributions to variance. Our results for asymmetric triangular distributions indicate that in terms of relative contributions to monitor variables in Gassmann equations, the effect of skewness is negligible.

When fluid substitution is carried out, the uncertainties associated with the compressional and shear velocities and grain bulk modulus contribute the most to the variance of V_{p2}. This was demonstrated only for uncorrelated input parameters. The remaining input parameters yielded insignificant contributions to variance.

On the other hand, we compared the effect of the fluid properties variability with rock frame piezosensitivity on the variance of the compressional velocity. This comparison was performed for various initial differential pressures and several scenarios of connate and injection fluids. Results show that the sensitivity of V_p depends not only on the initial differential pressure and burial depth, but also on the type of connate and injection fluids involved in the fluid substitution process.

We conclude that sensitivity information can be used as a tool to improve feasibility studies in time-lapse seismic reservoir monitoring and as a priori qualitative knowledge. The latter can guide the inversion process or help to diminish the uncertainties due to poorly constrained inversion schemes.

Acknowledgements

We would like to thank Prof. Sergio Fontoura, Coordinator of the Petroleum Engineering and Technology Group (GTEP), for encouragement and support to develop this work. We are indebted to the referee for suggestions that when incorporated, improved the structure of the manuscript and helped to contextualize the presented analysis. Acknowledgements are due to PETROBRAS Advanced Oil Recovery Program (PRAVAP19) and to the Brazilian National Petroleum Agency (ANP) for financial support through the human resources program PRH-7.

References

Aanonsen, S.I., Aavatsmark, I., Barkve, T., Cominelli, A., Gonard, R., Gosselin, O., Kolasinski, M., Reme, H., 2003. Effect of scale dependent data correlations in an integrated history matching loop combining production data and 4D seismic data. Paper SPE 79665, Presented at the SPE Reservoir Simulation Symposium, Houston, Texas.

Batzle, M.L., De-hua, H., 2004. Gassmann's equations and fluid saturation effects on seismic velocities. Geophysics 69, 398–405.

Batzle, M.L., Wang, Z., 1992. Seismic properties of pore fluids. Geophysics 57, 1396–1408.

Behrens, R., Condon, P., Haworth, W., Bergeron, M., Wang, W., Ecker, C., 2002. 4D seismic monitoring of water influx at Bay Marchand: the practical use of 4D in an imperfect word. SPE Reservoir Evaluation and Engineering, 410–420 October.

Brown, R.J.S., Korringa, J., 1975. On the dependence of the elastic properties of a porous rock on compressibility of a pore fluid. Geophysics 40, 608–616.

Evans, J.R., Olson, D.L., 2002. Introduction to Simulation and Risk Analysis. Prentice Hall, New Jersey.

Falcone, G., Gosselin, O., Maire, F., Marrauld, J., Zhakupov, M., 2004. Petroelastic modelling as key element of 4D history matching: a field example. SPE 90466, Presented at the SPE Annual Technical Conference and Exhibition, Houston, Texas.

Gassmann, F., 1951. Über die elastizität poröser medien. Vierteljahresschrift der Naturforschenden Gesellschaft in Zurich 96, 1–23.

Gosselin, O., Aanonsen, S.I., Aavatsmark, I., Cominelli, A., Gonard, R., Kolasinski, M., Ferdinandi, F., Kovacic, L., Neylon, K., 2003. History matching Using Time-lapse Seismic. Paper SPE 84464, Presented at the SPE Annual Technical Conference and Exhibition, Denver, Colorado.

Landrø, M., 2001. Discrimination between pressure and fluid saturation changes from time-lapse seismic data. Geophysics 66, 836–844.

Landrø, M., 2002. Uncertainties in quantitative time-lapse seismic analysis. Geophysical Prospecting 50, 527–538.

Landrø, M., Veire, H.H., Duffaut, K., Najjar, N., 2003. Discrimination between pressure and fluid saturation changes from marine multicomponent time-lapse seismic data. Geophysics 68, 1592–1599.

Lumey, D.E., Behrens, R.A., 1998. Practical issues of 4D seismic reservoir monitoring: what an engineer needs to know. SPE Reservoir Evaluation and Engineering, 528–538 December.

Lumey, D.E., Behrens, R.A., Wang, Z., 1997. Assessing the technical risk of a 4-D seismic project. The Leading Edge 16, 1287–1292.

MacBeth, C., 2004. A classification for the pressure-sensitivity properties of a sandstone rock frame. Geophysics 62, 497–510.

Mavko, G., Mukerji, T., 1998. Bounds on low-frequency seismic velocities in partially saturated rocks. Geophysics 63, 918–924.

Nolen-Hoeksema, R.C., 2000. Modulus-porosity relations, Gassmann's equations, and the low-frequency elastic-wave response to fluids. Geophysics 65, 1355–1363.

Sengupta, M., Mavko, G., 1999. Sensitivity analysis of seismic fluid detection: expanded abstracts. 69th Ann. Int. Meeting, Soc. Exp. Geophys, pp. 180–183.

Sengupta, M., Rai, C., Sondergeld, C., 1998. Sensitivity studies in forward AVO modeling: expanded abstracts. 68th Ann. Int. Meeting, Soc. Exp. Geophys, pp. 204–207.

Shapiro, S.A., 2003. Elastic piezosensitivity of porous and fractures rocks. Geophysics 68, 482–486.

Vernik, L., 1998. Acoustic velocity and porosity systematics in siliciclastics. The Log Analyst, 27–35.

Wang, Z., 2000. The Gassmann Equation revisited: comparing laboratory data with Gassmann's predictions. Geophys (Reprint series, SEG, Seismic and Acoustic velocities in reservoir rocks, vol. 3).