Evidence for the existence of a simple relation between earthquake magnitude and the fractal dimension of seismogenic faults: a case study from central Italy

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Abstract: Fault data from the central Apennines (Italy) were integrated with earthquake information from seismic catalogues in order to derive an empirical relation between the magnitude of the strongest historical earthquake and the fractal dimension of active fault patterns. We show that the assessment of earthquake magnitude from fault data has given good results, hence suggesting that the relation may be used to evaluate the potential hazard of seismic source areas in the Apennines using a low-cost methodology. We also suggest that a similar approach may be used in other seismic belts worldwide, provided that the basic seismological and geological information needed is adequate to constrain the appropriate relation between these two size parameters.

Our current knowledge of earthquake faulting (e.g. Cello & Tondi 2000; Scholz 2002) suggests that the assessment of the scaling properties of fault zones by means of fractal statistics, and the simulation of their growth by means of Self Organized Criticality (SOC) models (e.g. Cowie *et al.* 1993; Bak & Tang 1995) provide interesting perspectives for seismic hazard evaluation (e.g. Main 1995; Sherman & Gladkov 1999; Cowie & Roberts 2001; Tondi & Cello 2003).

In the last couple of decades, systematic studies of fault zone characteristics have significantly improved our understanding of the factors controlling their spatial arrangement and geometric complexity (e.g. Caine *et al.* 1996; Cello *et al.* 2000*a*, *b*; Aydin *et al.* 2005). At the same time, SOC models have emphasized that fault growth occurs, over long time-scales, through the coalescence of distributed lower-rank structural features that eventually link together and localize strain on a few dominant fractal structures that control most of the associated seismic energy release (e.g. Sornette & Sornette 1989; Sornette *et al.* 1990*a*; Cowie *et al.* 1993). Furthermore, the transition from distributed to localized deformation within a brittle shear zone seems to be marked by its tendency to change from an Euclidean to a fractal geometry (Sornette *et al.* 1990*b*; Cello 1997). As a result, the degree of complexity of a finite fault zone (which can be quantified by measuring its fractal dimension, *D*) is considered to be an indicator of fault size at any given evolutionary stage in the space–time domain. Accordingly, we suggest that measuring the spatial pattern of active faults within a seismic belt may be used to predict earthquake size, as it is well established that fault dimension controls the seismic moment release, and hence earthquake magnitude.

Fault and earthquake data

The central sectors of peninsular Italy are part of the Apennines fold and thrust belt, and include a few major tectonic elements of the peri-Mediterranean mountain system (Fig. 1). These sectors of the Apennines, which formed in response to the convergent motion of the African and European plates (Dewey *et al.* 1989; Goes *et al.* 2004), are made up of different tectonic units derived from the deformation of the various palaeogeographic domains of the southern sectors of the Afro-Adriatic continental margin (see e.g. Calamita *et al.* 1994*a*; Deiana & Pialli 1994).

In central Italy, a Plio-Quaternary fault system dissects older structural fabrics of the former (Late Miocene–Early Pliocene) fold and thrust belt (Fig. 2, see also Cello *et al.* 1997; Barchi *et al.* 2000; Boncio *et al.* 2000). The faults belonging to this system, showing different kinematic behaviour and variable ages, include newly generated and reactivated features of the Tyrrhenian–Apennines domain that formed during previous evolutionary stages in the long deformation history of the area (see Calamita *et al.* 1994*b*; Cello *et al.* 1997).

Field work and remote sensing analysis of the major faults exposed in the axial zones of the central Apennines allowed us to recognize several active segments and to derive a detailed fault

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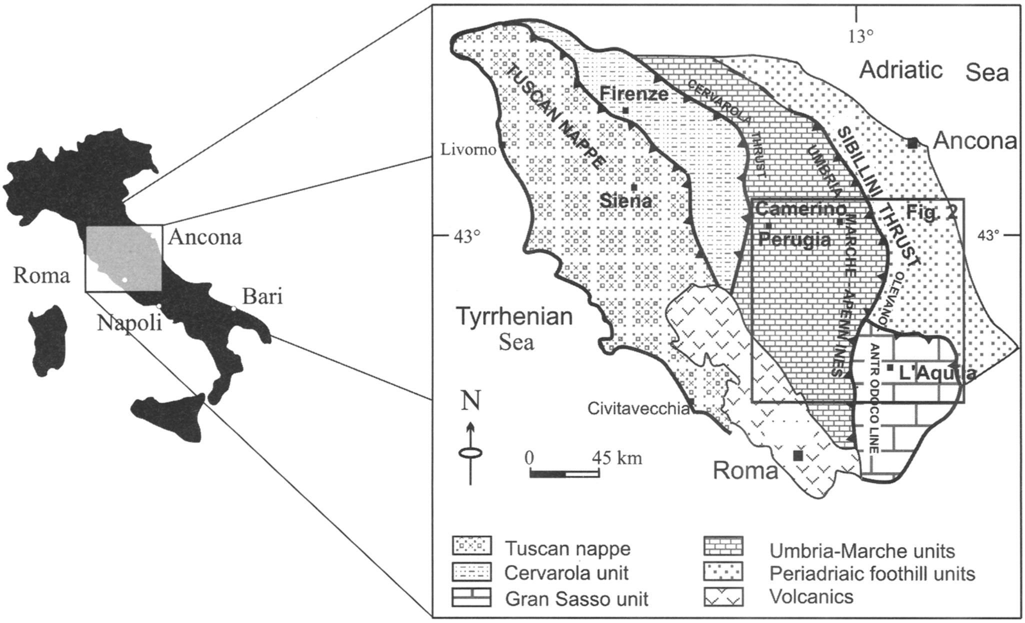

Fig. 1. Geo-tectonic setting of the study area, in Italy.

map of the area. This includes seven kinematically coherent arrays, which were interpreted as the surface expression of the well-known seismogenic structures thought to be responsible for most of the earthquakes occurring in this sector of the mountain belt (Fig. 3). These arrays, as a whole, make up the so-called Central Apennines Fault System (CAFS) and are the causative structures generating earthquakes with magnitudes in the range 5.0–7.0 (Table 1). They also control the evolving fragmentation pattern of the brittle crustal volume undergoing deformation in response to the current stress regime acting in the area from about 700,000 years ago (Cello *et al.* 1997; Di Bucci & Mazzoli 2002; Goes *et al.* 2004).

The record of the major historical earthquakes that struck this sector of the Apennines includes mostly shallow (<20 km deep) and a few intermediate (<100 km deep) events (Amato & Selvaggi 1991). Destructive earthquakes in the area are mainly confined within intramontane Plio-Pleistocene basin areas located within the CAFS, and most of them are generated at shallow depth in the crust.

The scaling relation between computed seismic moments and length of the inferred CAFS-related seismogenic faults (Fig. 4) shows that most of the events may be considered as 'small' earthquakes (Scholz 2002) following the expression $M_0 \sim L^3$ (with M_0 = seismic moment and L = fault length). This result suggests therefore that the analysed earthquakes are characterized by source dimensions that are smaller than the inferred thickness of the seismogenic layer (about 12 km; Deschamps *et al.* 1984).

The log N v. M relation (with N = number of earthquake and M = earthquake magnitude) is characterized by a b value of 0.8 (Fig. 5). This suggests that the magnitude of the maximum expected event, in the area, is of the same order as the largest historical event recorded in central Italy in the last millennium.

In order to derive an empirical relation between earthquake magnitude and the D-value of fault map patterns, we analysed in detail the geological database available for the CAFS (see Tondi & Cello 2003) and the most recent seismic catalogues compiled for this sector of the Italian Peninsula (Boschi *et al.* 1997; Camassi & Stucchi 1998; C.P.T.I. Gruppo di Lavoro 1999). This allowed us to identify, for each of the seven seismogenic zones of Figure 3, the strongest historical earthquake and to assess its equivalent magnitude (*Me*).

The fractal dimension (D) of the different fault arrays (each corresponding to a single earthquake source area) was obtained by using the

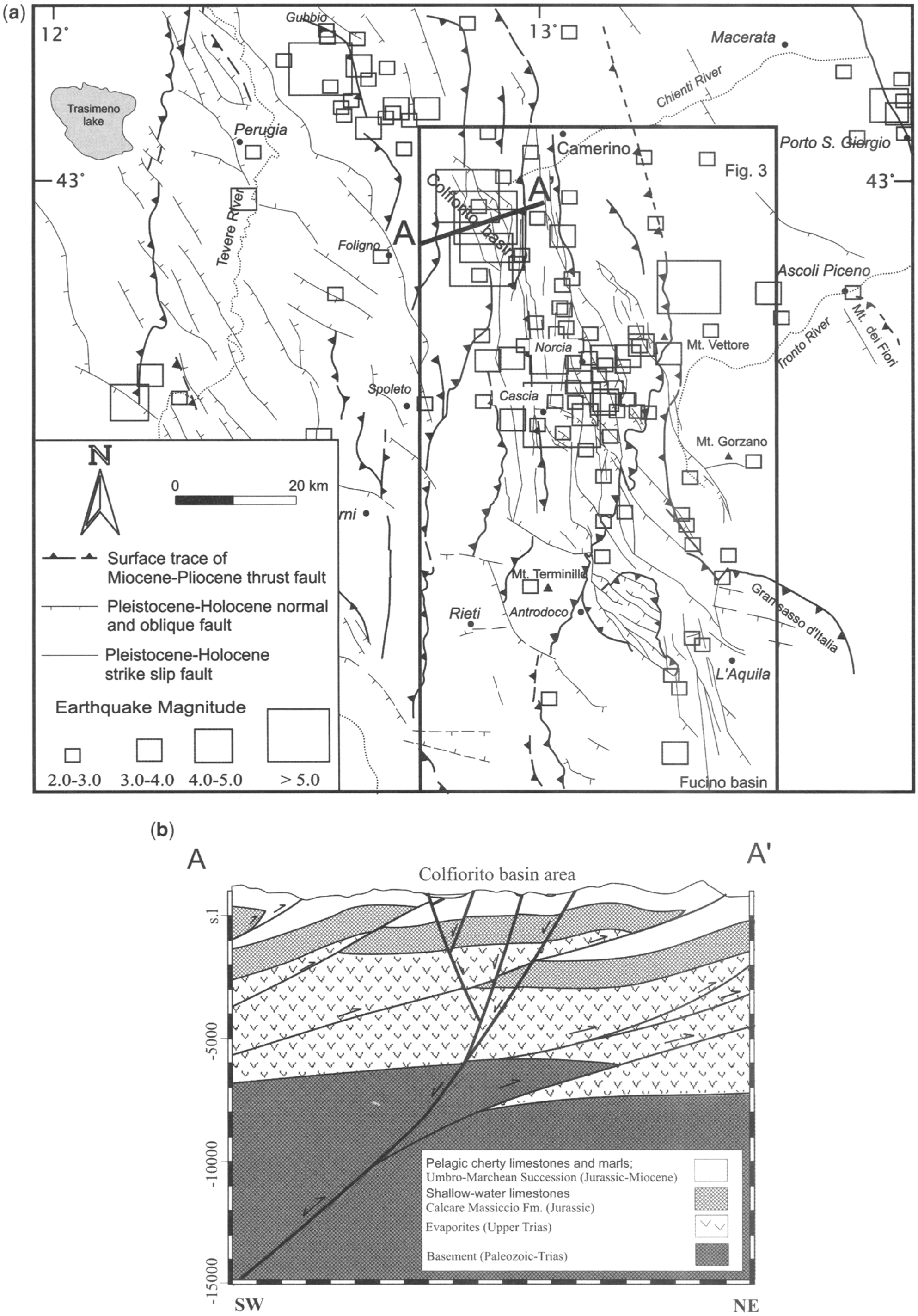

Fig. 2. Structural features of the Central Apennines Fault System: (**a**) Simplified fault map and seismic activity (1985–2000) in the study area; (**b**) representative geological profile and fault structure across the Colfiorito seismogenic zone.

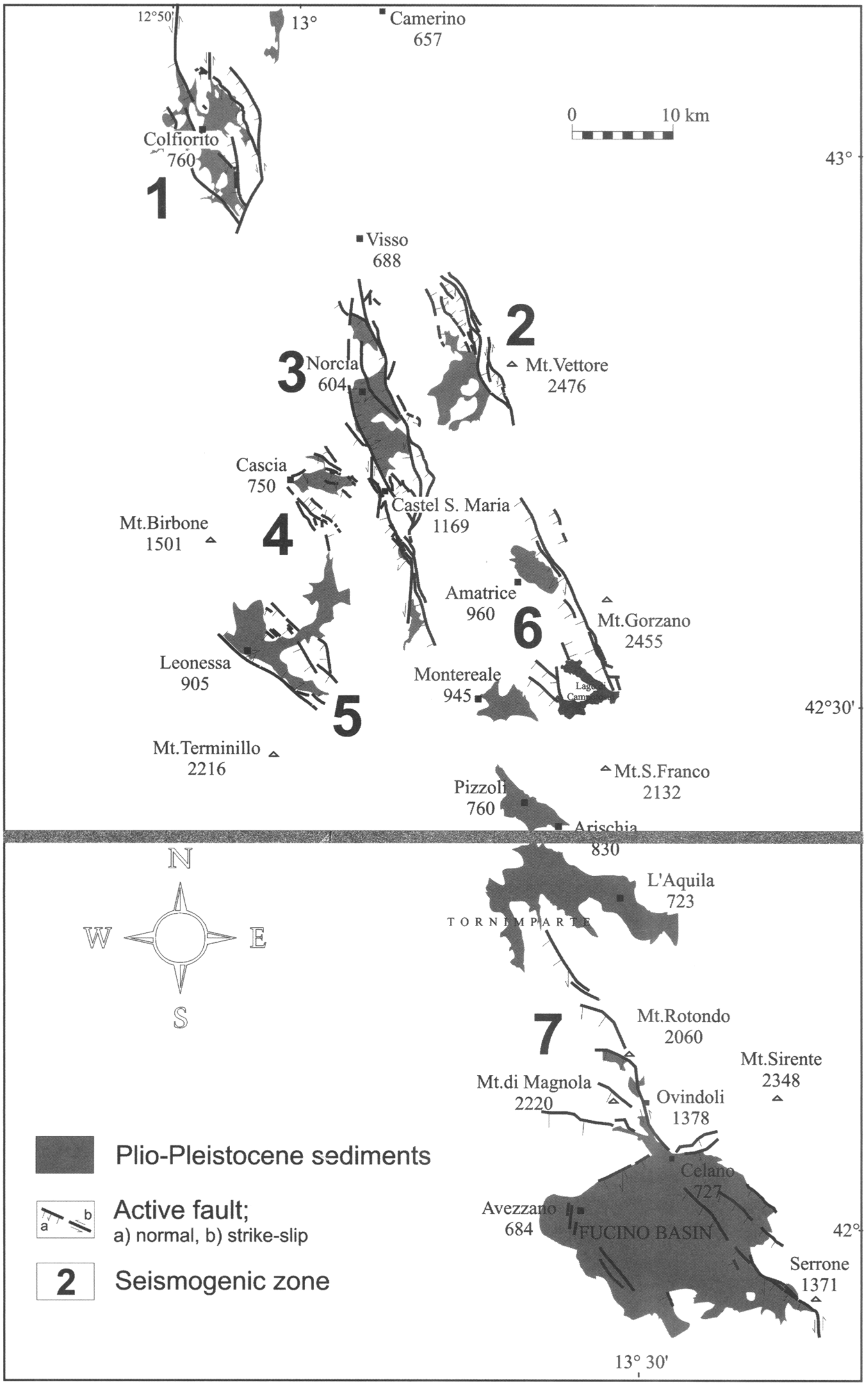

Fig. 3. Map of active faults in the axial zones of the central Apennines, Italy.

Table 1. *Historical earthquakes from Boschi et al. 1997: Characteristic parameters of Central Apennines Fault System-related earthquakes and associated seismogenic zones*

Year	Epicentral area	I_{max}	Me	Seismogenic fault zone
1599	Cascia	8.0	5.5	4
1639	Amatrice	9.0	5.4	6
1703	Norcia	11.0	6.7	3
1730	Leonessa	7.5	5.0	5
1859	Vettore	7.0	5.0	2
1979	Norcia	8.5	5.9	3
1915	Fucino	11.0	6.9	7
1997	Colfiorito	8.0	5.9	1

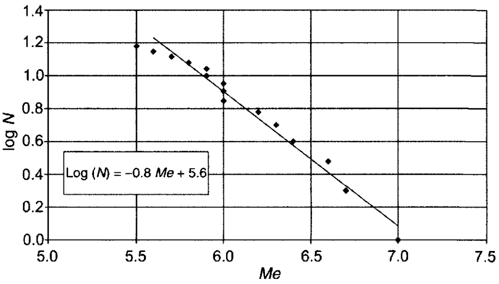

Fig. 5. Cumulative distribution of the equivalent magnitudes (*Me*) of the historical earthquakes occurring within the Central Appennines Fault System (data from Tondi & Cello 2003).

box-counting technique, a conventional method for analysing map patterns, that is, the two-dimensional spatial properties of a given structure (Mandelbrot 1983). This technique allows one to derive log/log diagrams with *Ns* (number of boxes containing the pattern) plotted as a function of *s* (where *s* is the size of the measuring grid) and to construct box-counting curves whose slope values give the appropriate fractal dimension of the analysed structure.

The *D* values obtained from the box-counting curves derived from the map of the seven seismogenic fault zones of the CAFS are shown in Figure 6. In Figure 7, we plotted the *D* value together with the appropriate *Me* values inferred for each earthquake source area. As may be seen, except for point 3* in Figure 7, the two size parameters display a well-constrained relation. This result is relevant, in our opinion, for the evaluation of earthquake-related hazard, as the relation $Me = 11D - 7$ may lead to accurate predictions of the seismogenic potential of a given fault zone.

The fact that point 3* does not fit the correlation curve and plots below it is also of interest, as it suggests that the 1979 Norcia earthquake ($Me = 5.9$), which occurred within the seismogenic zone 3, did not release all the strain energy available in the system. This is in agreement with available historical information on the seismic activity in the Norcia zone, as there is a lot of evidence that the whole area was struck by a $Me > 6.5$ earthquake in 1703 (Boschi *et al.* 1997; Cello *et al.* 1998). As shown in Figure 7, this latter event (point 3) plots much closer to the *Me–D* correlation curve than point 3*. Therefore, our study not only shows that there exists a linear relation between *Me* and *D*, but also that 'anomalous' values (i.e. point 3* in Fig. 7) indicate that each seismogenic zone may release variable amounts of stored strain energy through the occurrence of earthquakes with *Me* values smaller than that of the maximum expected event. This may be better appreciated by comparing the earthquake potential of the seismogenic zones (3: Norcia) and (1: Colfiorito). In these cases, it may be observed that although the 1979 earthquake in the Norcia area is a low-energy event ($Me = 5.9$) within an active fault zone with a seismic potential of $Me > 6.5$, the 1997 Colfiorito earthquake ($Me = 6$) may be considered as the strongest event that may possibly be generated within the seismogenic zone 1.

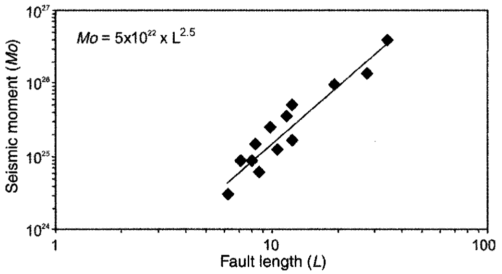

Fig. 4. Fault length v. seismic moment of the Central Apennines Fault System-related historical earthquakes (data from Tondi & Cello 2003).

Conclusions

The results of this study emphasize that our assumption suggesting that it is possible to obtain information on the seismogenic potential of a given area by measuring the fractal dimension of active faults is fundamentally correct. In our opinion, this is not surprising because (1) the fractal dimension of fault map patterns represents a measure of fault complexity and size, and hence of the degree of maturity of a fault zone (Cello 1997); (2) it is well established that fault growth processes occur mainly through linkage between fault segments (Cartwright *et al.* 1995; Cladouhos & Marrett 1996). Consequently, any

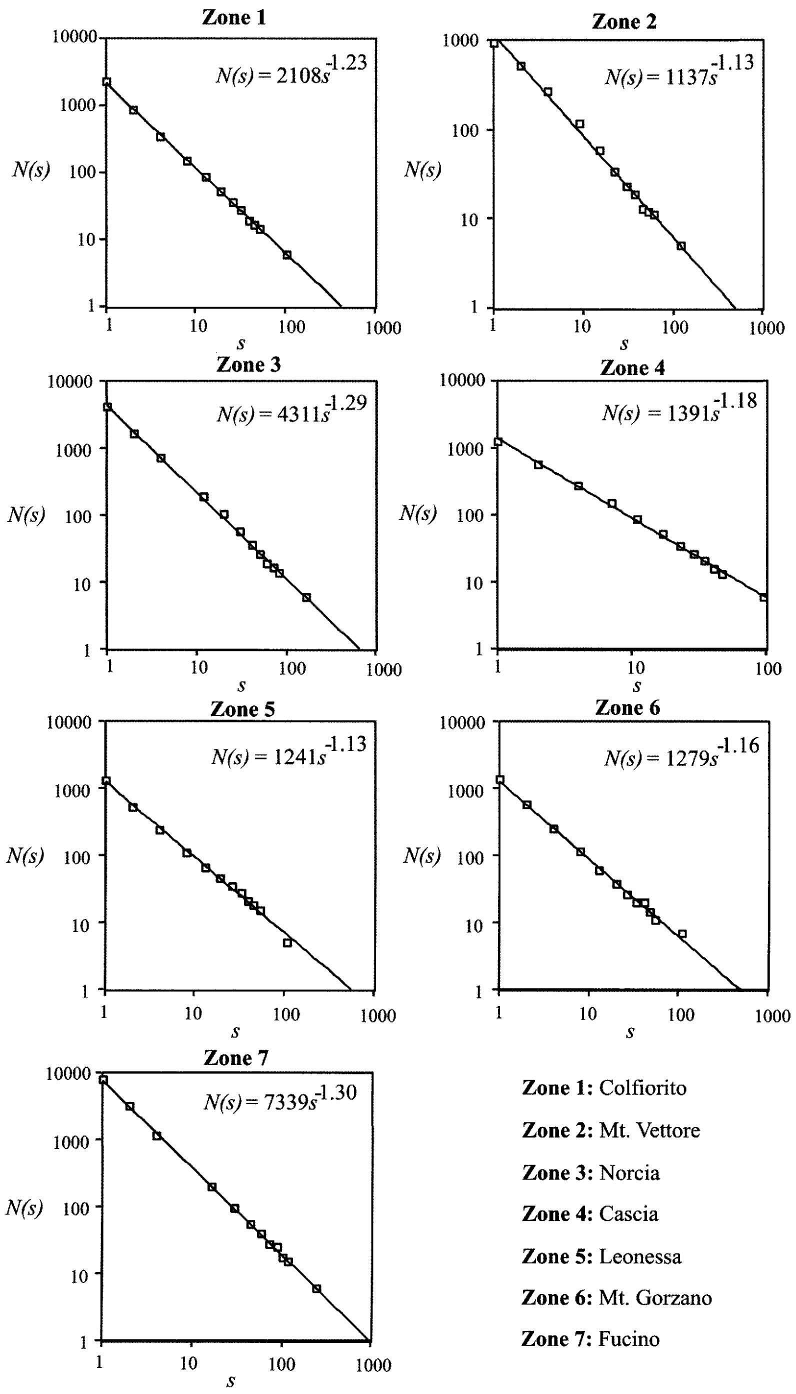

Fig. 6. Box-counting curves (the number of boxes N at a given box size s are plotted as a function of s, on log–log axes) of the analysed active fault patterns shown in Figure 3.

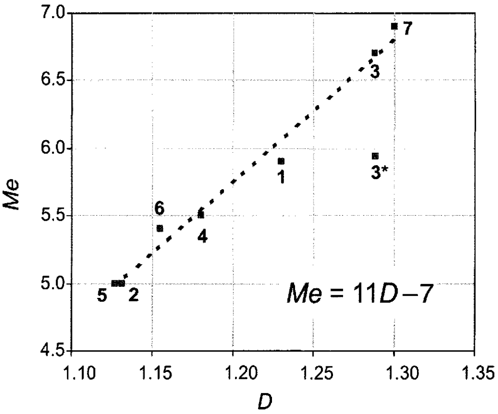

Fig. 7. Diagram showing the relation between the equivalent magnitude (*Me*) of historical earthquakes occurring in the axial zones of the central Apennines and the fractal dimension (*D*) of the seismogenic faults shown in Figure 3.

increment in fault dimensional properties and geometric complexity produces an increase in the *D* value of each fault pattern; (3) any increment in size of an actively faulting rock volume (i.e. of a seismogenic source area) causes an increase of the expected maximum earthquake moment, and hence of its equivalent magnitude (*Me*).

We are aware that in different geostructural contexts one may, however, not be able to collect all the necessary geological and/or seismological information needed to derive an appropriate *Me–D* relation. Possible limitations to a generalized use of the proposed procedure for assessing the earthquake hazard of a given area may in fact come from (1) poor resolution in the identification of seismogenic sources within a seismic belt, (2) scarcity of good historical and/or instrumental records of earthquakes, and (3) our ability to recognize and map active fault segments with the appropriate details required for standard box-counting analysis.

In conclusion, we have shown that predicting earthquake magnitude from fault data has given good results for the axial zones of the central Apennines; we believe, however, that more data from other areas of active faulting worldwide are needed to possibly generalize the empirical relation proposed in this study.

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