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Undiscovered Petroleum Accumulation Mapping Using Model-Based Stochastic Simulation[1]

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Stochastic simulation has been proven to be a useful tool for revealing uncertainties in petroleum exploration and exploitation. The application to petroleum resource assessment would result in predicted potential accumulations with geographic locations, a desirable feature for improving both resource management and exploration efficiency. The associated uncertainties with the prediction provide information useful for exploration risk analysis. This attempt has been encumbered by two typical technical difficulties: biased observation data and lack of information with respect to the undiscovered accumulation locations. In this paper we propose a model-based simulation approach, in which models are used to perform unbiased parameter estimation from biased data and to facilitate the location of undiscovered petroleum accumulations based on reasoning of available geological and geophysical observations. The Fourier transform algorithm is chosen for the simulation because the spatial correlation and location-specific features can be studied separately from different data sources and integrated in the simulation in a frequency domain. The proposed approach is illustrated by an example from the Rainbow petroleum play in the West Canadian Sedimentary Basin. In the application example, a pre-1994 exploration history data set was used as input, and the predictions are then checked against the locations of post-1993 exploratory drilling results. The comparison of the predictions from the proposed approach and the traditional conditional simulation shows that the model-based approach captures the essentials of geological controls on the spatial distribution of petroleum accumulation, thus improving the projections of undiscovered petroleum accumulations.

KEY WORDS: Fourier transform approach; resource assessment; spatial distribution.

INTRODUCTION

Petroleum exploration uses modern technologies to reduce risk. However no matter what effort has been made, the risk cannot be eliminated because uncertainties remain as a result of limited data coverage and inferences of petroleum occurrence from subsurface indirect observations. Petroleum accumulations are natural products of geological processes that have occurred over the past millions or hundred millions of years. The complexity of spatial variation and interactions among

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geological processes, as compared with the spare data coverage and multi-possibility of data interpretation, makes it difficult for a deterministic calculation of the undiscovered petroleum accumulations in a petroleum system. Stochastic simulation has been proven a useful tool for revealing uncertainties in petroleum exploration and exploitation (Chen and others, 2004; Deutsch and Tran, 2002; Gao and others, 2000; Georgsen and others, 1994; Hegstad and others, 1994; Holden and others 1998). The application to petroleum resource assessment could generate equal-probable realizations of potential petroleum accumulations with geographic locations and reveal the associated uncertainties with the modeled accumulations, providing useful information for better natural resource management and exploration risk visualization.

There are two technical obstacles in the construction of such a stochastic model. Firstly, the available data, such as the sizes of the discovery obtained from exploration drilling, are biased because of the biased data generating process. Typically, exploration programs test prospects inferred to have a greater potential (commonly the larger features) with higher priority. As a result, the data obtained through such a data collecting process represent a biased sample of the parent distribution. The statistical bias of the discovered accumulation sizes in the sample as compared to its natural population is a well-known phenomenon (e.g., Kaufman, Balcer and Kruyt, 1975; Lee and Wang, 1985; Scheunemeyer and Drew, 1983). This sampling bias prohibits a direct estimation of model parameters. Secondly, no direct information is available with respect to the locations of the undiscovered oil and gas accumulations. If we have had direct information, no prediction would be necessary. To tackle the two obstacles, we propose a model-based stochastic simulation approach, in which established models are used to perform parameter estimation according to sampling characteristics, and to facilitate the location of potential petroleum accumulations based on geological reasoning, thus enhancing the performance of conditional simulation.

Among different stochastic simulation algorithms, Fourier transform approaches, such as the spectrum simulation algorithm (Pardo-Iguzquiza and Chica-Olmo, 1993) and phase identification approach (Yao, 1998) appear to be ideal for predicting the undiscovered petroleum accumulation with geographic locations. There are two major advantages in using the Fourier transform method when simulating undiscovered petroleum accumulations. In addition to the computational advantage, it is possible to study the spatial correlation and location-specific characteristics separately. Models for spatial correlation and locations can be conveniently extracted from different geoscience data sources and subsequently integrated in the simulation in a frequency domain.

In a direct application of the Fourier transform approach, a discovered hydrocarbon accumulation map is transformed into the frequency domain using a fast Fourier transform (FFT), which results in amplitude and phase maps. The amplitude map contains information associated with spatial correlation structure, whereas the phase map contains location-specific information. Only when both the phase and the amplitude maps are completely specified, the complete petroleum accumulation set can be mapped. Unfortunately, neither the amplitude map nor the phase can be completely specified using the discovered petroleum accumulations alone due to a biased data sampling and lack of location-specific information for the undiscovered accumulations. Thus, the inference of an unbiased covariance function and implementation of additional constraints from observations and geological model are necessary for a successful application of stochastic simulation to project the undiscovered petroleum resources.

Different geoscience data contain unique information with respect to the properties of petroleum accumulations. At least four types of data carry information regarding the spatial characteristics of hydrocarbon occurrence in a mature play, which include: (1) geological data; (2) exploration drilling results; (3) geophysical data; and (4) location and data quality information regarding geoscience surveys (Chen and others, 2000). Geological information is genetic in character. Available geological information indicates the necessary conditions for hydrocarbon occurrence and it allows, in principle, the inference of petroleum occurrence spatial characteristics. Spatial variation in geology reflects a variation in geological favorability condition on petroleum accumulation in space. Methods have been proposed to integrate geological information for inference of possible undiscovered petroleum accumulation locations (e.g., Chen and others, 2000, 2002; Rostirolla, Mattana, and Bartoszeck, 2003). Other types of geoscience information are also useful. For example, discovery records contain information desirable for estimating accumulation size distribution and aggregated resource potential (Baker and others, 1986; Lee and Wang, 1983, 1985). A seismic grid map contains information permitting the calculation of probability that a sizeable prospect could be missing at a specific location (Chen and others, 2000; Kaufman, 1994). Such information can be conditioned in a simulated outcome.

In this paper, we discuss the model-based conditional simulation and illustrate how the proposed approach can be used to project the spatial occurrences of petroleum resources using an example from the Rainbow petroleum play in Western Canada Sedimentary Basin (WCSB). In the application example, the pre-1994 exploration data set was used as input. The post-1993 petroleum exploration results in the same play were employed to check the predicted results.

METHOD DESCRIPTIONS

Fractal Model of Petroleum Accumulations

Barton and others (1991), Barton and Scholz (1995), and La Pointe (1995) studied the data from well-explored petroleum basins in the United States and

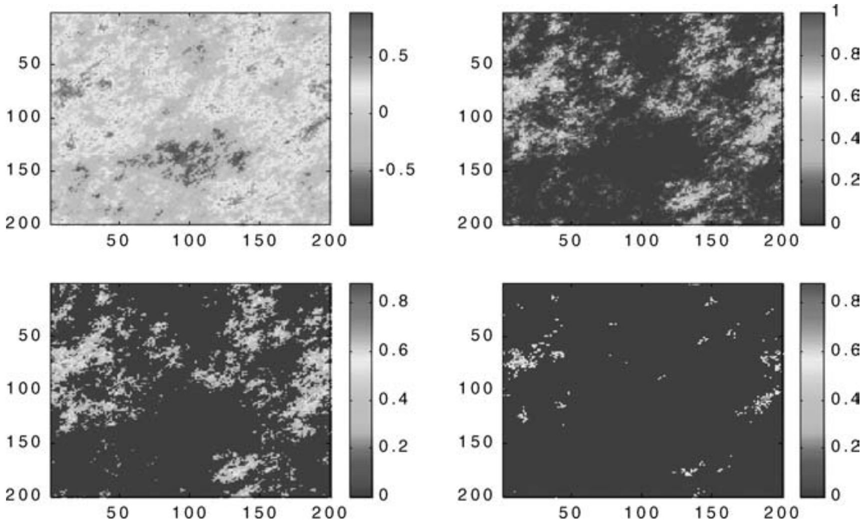

Figure 1. Fractal images present conceptual models of petroleum accumulations. The spatial distribution characteristics of petroleum accumulations are assumed to be fractal and are a function of accumulation sizes.

concluded that the hydrocarbon accumulation spatial distribution is fractal. Our studies in WCSB also indicate self-affine characteristics of petroleum accumulations on resource maps. This characteristic motivates the examination of a fractal model for a quantitative description of hydrocarbon resource spatial distribution. The scaling property of a fractal model implies that the spatial characteristics of large petroleum accumulations could be used to infer the spatial characteristics of the smaller accumulations, which are typically underrepresented in an exploration data set. Thus, the spatial characteristics of petroleum accumulations can be inferred from the biased observations resulting from exploration.

In our model, petroleum resource is described by a fractal image map, on which the value of each pixel represents the average magnitude of petroleum accumulation within that pixel. Figure 1 (top-left image) illustrates a conceptual model of the spatial distribution of petroleum accumulations. Since the location where petroleum is generated is not necessarily the location where petroleum is trapped, the value at each pixel represents the net effect of petroleum accumulation. Negative values signify petroleum migrating away from the pixel, whereas positive values represent a net accumulation. Because the primary objective of petroleum exploration is to find economically recoverable accumulations, only those accumulations exceeding a certain size are significant. The economic size of the petroleum accumulation could be a variable in time and space, depending on economic and infrastructure conditions. It is obvious that spatial distribution pattern of hydrocarbon accumulations may vary with economic accumulation size, while the spatial correlation structure remains. The remaining images in Figure 1 show various versions of the same image with different accumulation size threshold values. The top right image depicts only those pixels with net accumulations. The lower left image has an arbitrary threshold value, >0.2, whereas the lower right one has a threshold value, >0.5. From this series of images, it is clear that even though spatial pattern of petroleum accumulation looks different, the spatial correlation structure for any one of the images in Figure 1 can be inferred from the original one.

Power Spectrum Representation of the Fractal Model

The principle and procedure of Fourier integral method and phase identification algorithm for stochastic simulations have been described by Pardo-Iguzquiza and Chica-Olmo (1993) and Yao (1998) in a great detail. Any second stationary discrete stochastic process, $y(k)$, can be expressed as a series of Fourier coefficients a_j and b_j. In the one-dimensional case, the series is written either as:

$$y(k) = \sum_{j=0}^{N-1} \left[a_j \cos\left(\frac{2\pi jk}{N}\right) + b_j \sin\left(\frac{2\pi jk}{N}\right) \right], \quad \text{for} \quad j = 0, 1, 2, \ldots N-1 \tag{1}$$

or using a complex exponential Fourier series:

$$y(k) = \sum_{j=0}^{N-1} A(j) e^{i2\pi kj/N} \tag{2}$$

where $A(j) = a_j - ib_j = |A(j)| \exp\{-i\varphi(j)\}$ is the jth complex Fourier coefficient, $|A(j)| = (a_j^2 + b_j^2)^{1/2}$ is the amplitude spectrum, $\varphi(j) = \tan^{-1}(-b_j/a_j)$ is the phase, and $S(j) = |A(j)|^2,\ j = 0, 1, \ldots, N-1$, is the power spectrum.

A covariance function is related to a power spectrum $S(\omega)$ by the Wiener–Khintchine theorem (Pardo-Iguzquiza and Chica-Olmo, 1993) stating that any stationary process has a covariance function $C(h)$ of the form:

$$C(h) = \int_{-\pi}^{\pi} S(\omega) e^{i\omega h} d\omega \tag{3}$$

where ω is the angular frequency. In fact, the power spectrum function is the Fourier transform of the covariance function.

Self-affine series defined by a power spectrum can be logarithmically transformed into a self-similar fractal time series. For a self-similar fractal time series, the power spectrum density has a power law dependency on frequency (Turcotte, 1997, p. 148):

$$S(k) \propto f^{-\beta} \tag{4}$$

where f is frequency, and β is an exponential coefficient. In the fractal model, the spatial correlation of objects is fully specified by the logarithmically transformed spectrum density function.

Simulation Procedure

The procedure for the proposed approach is similar to that in the phase identification approach proposed by Yao (1998) with two additional procedures in the simulation: (a) using a fractal model to infer the unbiased spatial structure from the biased data set; and (b) employing a geological model, in the form of conditional probability or geological favorability that describes the necessary conditions for petroleum accumulation, to facilitate the location of undiscovered petroleum accumulations. The proposed approach has the following steps:

1. Prepare a petroleum accumulation image map from exploration results;
2. Estimate fractal parameters from the image map according to sampling characteristics;
3. FFT the image map to obtain amplitude and phase maps;
4. Calibrate the amplitude map using the estimated fractal parameters by adjusting the high frequency portion of the original amplitude map to obtain a Modified Fractal Amplitude Map (MFAM);
5. Generate a random phase map in incorporation with the geological constraints from geological favorablity map or probability map of petroleum occurrence, so that the location-specific information is included in the phase map;
6. Generate a fractal image (accumulation map) using the calibrated amplitude and the obtained phase by inverse FFT;
7. Check the fractal image against observations and geological constraints. Calculate the difference between simulated values with those at conditioning pixels [Eq. (5)]. If the difference is below the pre-set tolerance value, accept the results.

$$\text{obj} = \sum_{\alpha=1}^{n} \left| \frac{Z_m(i_\alpha, j_\alpha) - Z_o(i_\alpha, j_\alpha)}{Z_m(i_\alpha, j_\alpha)} \right| \tag{5}$$

where $Z_m(i_\alpha, j_\alpha)$ is the modeled value at grid node (i, j) corresponding to the closest node at the αth observation, α=1, ..., n, and $Z_o(i_\alpha, j_\alpha)$ is the value of αth observation at grid node (i, j);

8. If the difference is greater than the tolerance, modify the accumulation map by replacing the simulated values with the observed values at the conditioning pixels.
9. FFT the modified accumulation map, and get new amplitude and phase maps;
10. Replacing the new amplitude map with the MFAM and keeping the new phase map in step 9, inverse FFT using the MFAM and the new phase. Repeating steps 7–9, this procedure is performed iteratively until a desired tolerance in Eq. (5) is reached.

APPLICATION EXAMPLE

The Rainbow petroleum play, located in northwestern Alberta, WCSB, is a mature exploration play with an areal extent of about 5000 km^2. Major geological controls on this play and its petroleum system have been well described (Barss, Copland, and Ritchie, 1970, Fowler and others, 2002; Li and others, 1999; Podruski and others, 1988; Reinson and others, 1993;). Exploration for oil and gas in this play began in the early 1950s. By 1994, 22 gas pools, 87 oil pools, and 77 oil and gas pools were found with a total oil and gas reserve of 269.1 $\times$ 10^6 m^3 (in place) oil equivalent (o.e.). In that period, 409 wild cats (new field wildcat and new pool wildcat in Lahee class) were drilled, amongst which, 186 intersected hydrocarbon and 223 were dry (EUB, 2001). In the subsequent period from 1994 to 2000, 52 additional exploratory wells were drilled, in which 32 are oil/gas pools or wells with oil/gas flows. Figure 2 shows the location of Rainbow Sub-basin and the locations of the exploratory wells (discovery and dry holes).

In the application example, the pre-94 date set was used to estimate model parameters and conditioning the simulation, and the post-93 data set was served as a test data set to check whether or not the output from the simulation can predict the locations of undiscovered petroleum accumulations. A discovered petroleum pool map (Fig. 3) was prepared from EUB's annual reserve report (EUB, 2001), on which the location of a pool is represented by the location of its discovery well at the center and areal extent of a pool is approximated by the estimated pool area, and size of the pool is indicated by the magnitude of the pixel value. In the simulation, the dry wells were used as areal constraints excluding any occurrence of petroleum accumulation. A rectangular area of 0.36 km^2 is assumed to be exhausted of petroleum potential by an exploratory well in this study.

An original amplitude map (OAM) and phase map are derived by a Fourier transform of the discovered petroleum accumulation map. Figure 4 shows amplitude profiles of the OAM in easting and northing directions. The deviations from a linear relationship in high frequency parts of both directions on the profiles are interpreted as the sampling bias due to selective drilling, suggesting small pools are under-represented on the discovered petroleum pool map. A modified

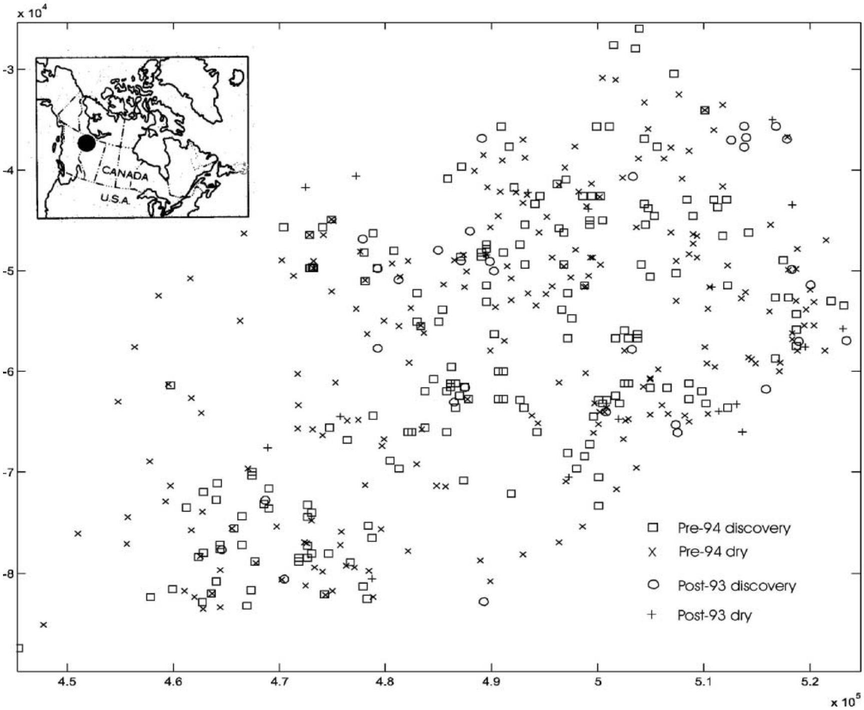

Figure 2. The study area and the locations of discovery and dry wells in the Rainbow petroleum play, WCSB.

amplitude map (MFAM), derived by calibrating the original amplitude map using estimated fractal parameters, represents the spatial correlation for all petroleum accumulations in the size range indicated by the data. The phase map derived from the discovered gas accumulations contains no information regarding the locations of the undiscovered petroleum accumulations. The use of the phase identification approach with the MFAM results in a random realization of a stochastic simulation conditioned on discovered petroleum accumulations and dry wells. In such a realization, the spatial correlation in the amplitude map is well retained, but the locations of the undiscovered petroleum pools could be anywhere except from the conditioned pixels. Figure 5 shows a probability map calculated based on 3000 realizations of the conditional simulations with MFAM using the phase identification approach.

A previous study of the spatial distribution characteristics of petroleum accumulations in the same area resulted in a conditional probability map (Fig. 6) of hydrocarbon occurrence (see Chen and others, 2001 for details in data and method). This map integrated information from geological factors describing petroleum

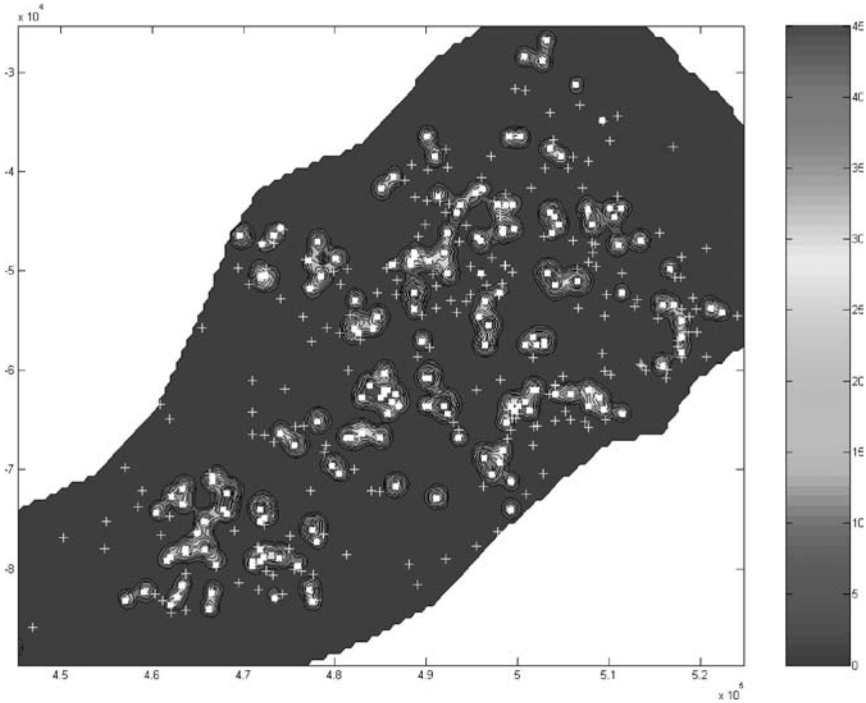

Figure 3. A discovered petroleum pool map prepared using the pre-94 oil and gas discoveries in the Rainbow petroleum play. *Square* indicates the discovery well location and *cross* indicates the dry well location.

accumulations in the region. With this independently determined conditional probability map, the simulated petroleum occurrence realizations are validated not only considering discovered petroleum accumulations, dry well locations, the exhaustion of potential by previous activity, but also geological conditions for petroleum accumulations. Figure 7 is a probability map of petroleum occurrence based on 3000 conditional realizations, representing the uncertainty associated with the predicted locations of undiscovered accumulations in the play. The likely sizes of accumulations (discovered and undiscovered petroleum) with geographic locations, predicted by the model-based simulation, are presented in Figure 8.

DISCUSSION AND CONCLUSIONS

In the subsequent period from 1994 to 2000, 52 wells were drilled, among which 32 wells indicate new oil/gas discoveries in the Rainbow Sub-basin. The 52 wells are superimposed on the calculated probability and accumulation size

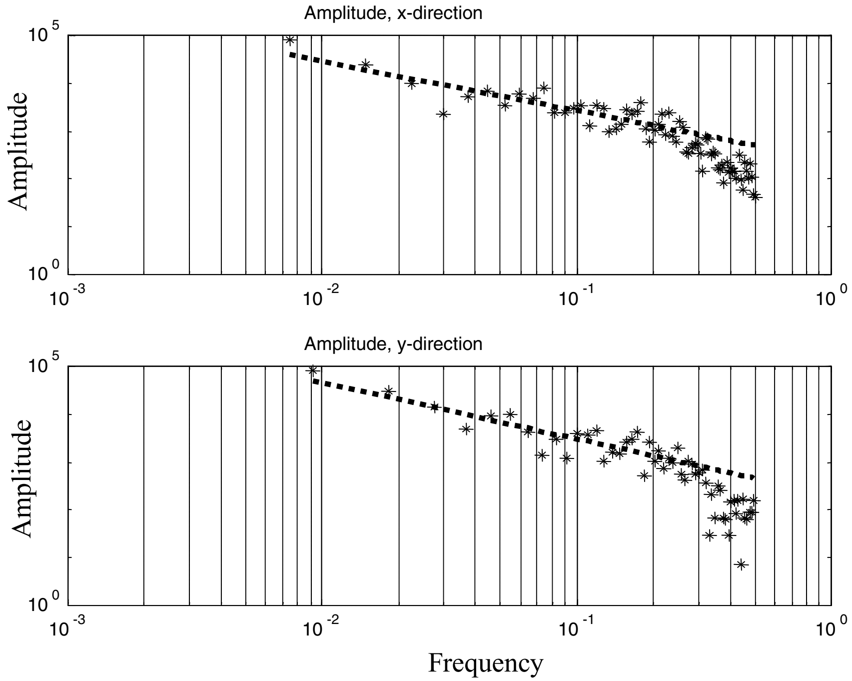

Figure 4. Amplitude profiles in *x* and *y* directions. The deviations of the amplitude from the linear relationships in higher frequency parts are interpreted as sampling bias (i.e., under-presented in smaller pool accumulations due to the selective drilling in petroleum exploration). (Horizontal axis: frequency and Vertical axis: amplitude).

maps, respectively (Figs. 7 and 8). In a comparison of the post-93 discoveries with the predicted probability of petroleum occurrence, it shows that 22 of the 32 new discovery wells are located in areas with probability values higher than 0.5, which gives an average success rate of 68% (Fig. 9). Whereas, for the wells drilled in the areas with probability value less than or equal to 0.5, the success rate is 50%. It is interesting to see that the model-based simulation produces relatively high probabilities in a less explored area in the northeast of the Rainbow play, where only one unsuccessful well was drilled prior to 1994. Seven post-1993 exploratory wells were completed in that part of the play and six are discoveries. The proposed method produces maps (probability and resource maps, Figs. 7 and 8) showing a profound influence of the geological characteristics of the play and the conditioning process did not destroy the high probability outlined on the conditional probability map. In contrast, the traditional conditional simulation did not predict those six post-93 discoveries. Its probability map shows a more random pattern due to lack of information of the undiscovered accumulations.

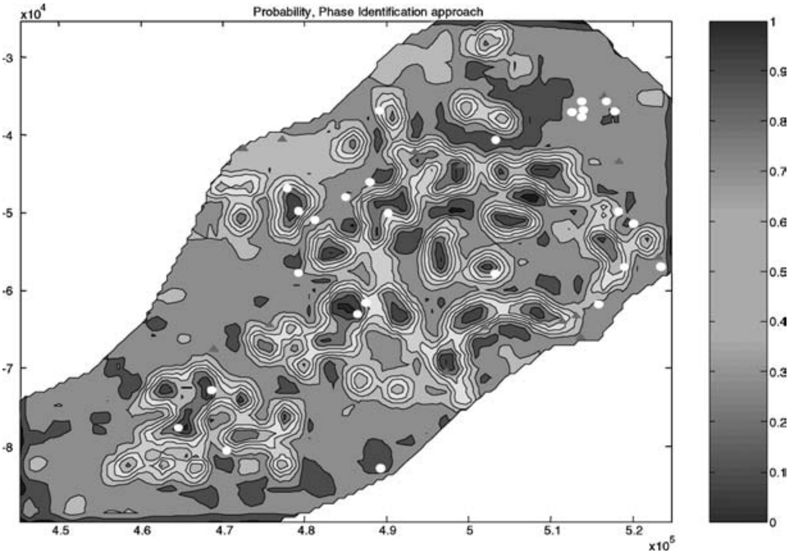

Figure 5. A probability map calculated from 3000 realizations of conditional simulation with MFAM from the phase identification approach.

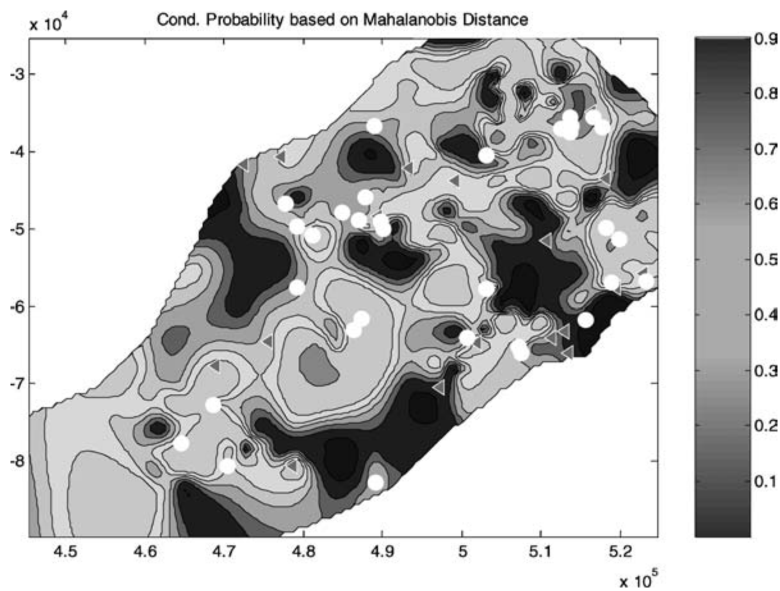

Figure 6. A conditional probability map of petroleum occurrence of the Rainbow petroleum Play, derived from a multivariate statistical method.

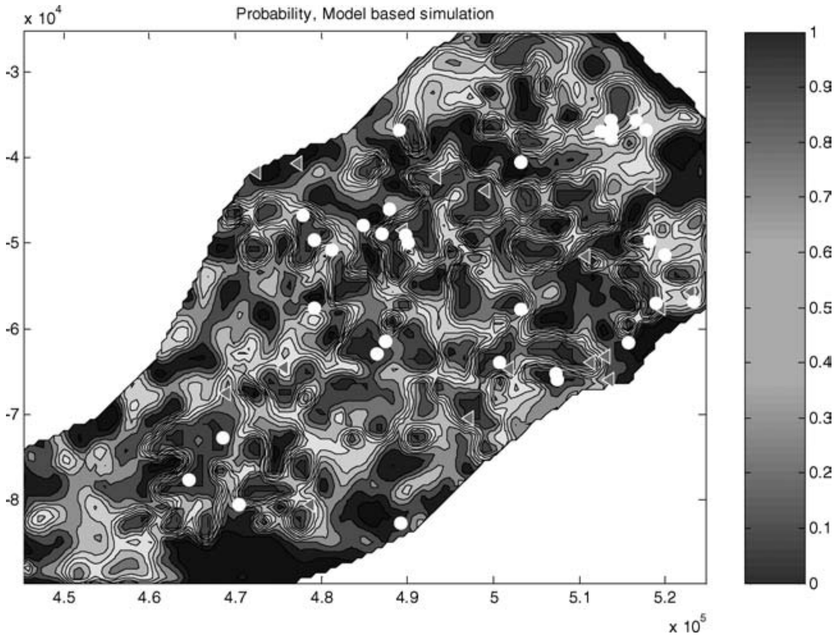

Figure 7. A probability map of petroleum accumulations of the Rainbow petroleum play based on 3000 realizations from the model enhanced simulation. *Solid circles* indicate the locations of success exploratory wells and the *triangles* represent the dry well locations drilled in 1994–2000.

All these suggest that the proposed approach captures the essentials of the spatial features as well as the geological characteristics of the petroleum accumulations in the simulation and provides important information for petroleum exploration decision.

In summary, the proposed approach employs a fractal model to infer a representing spatial correlation structure from the biased observation data set. The modified amplitude map is hypothesized to represent the true spatial correlation structure of petroleum accumulations in the size range indicated by the exploration data. To improve the prediction of undiscovered accumulation locations, we employ a geological model, in the form of conditional probability map derived from an integration of geological information and exploration results, to constrain the unknown locations of undiscovered accumulations in a frequency domain. The results shows that if location is the most important feature in the simulation, the Fourier transform approach is a good choice because the spatial correlation structure and location-specific information can be studied separately using different data sources and subsequently integrated in the simulation in a frequency domain. We have demonstrated through an application example that the proposed

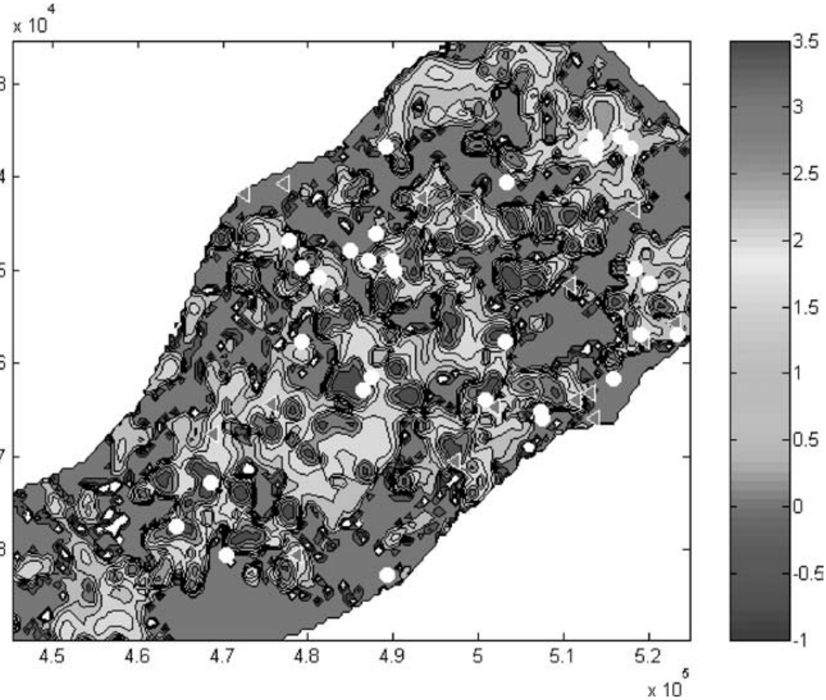

Figure 8. A petroleum resource map represents predicted sizes (logarithmic value) of petroleum accumulations with geographic locations from the model-based simulation. The sizes at each pixel are average value of 3000 realizations. *Solid circles* are the locations of success exploratory wells and the *triangles* represent the dry well locations drilled in 1994–2000.

simulation procedure can handle sampling bias in data and integrates different types of information (including soft-information) in a consistent and efficient manner.

We are able to use the model-based approach to produce a petroleum accumulation map, on which both the size and location for the undiscovered petroleum accumulations are predicted. No other method has the capability of simultaneously predicting both. A probability map from multi-realizations of the simulation highlights the areas with low and high probability values, providing a general view of the exploration risk in the play.

We demonstrated that the use of additional geological and geophysical prospecting data could enhance spatial modeling by adding location-specific information in the phase map. In cases, additional information such as a geological favorability map or a conditional probability map of petroleum occurrence may not be available. The use of the calibrated amplitude map improves the conditional simulation by providing a more complete and appropriate representation

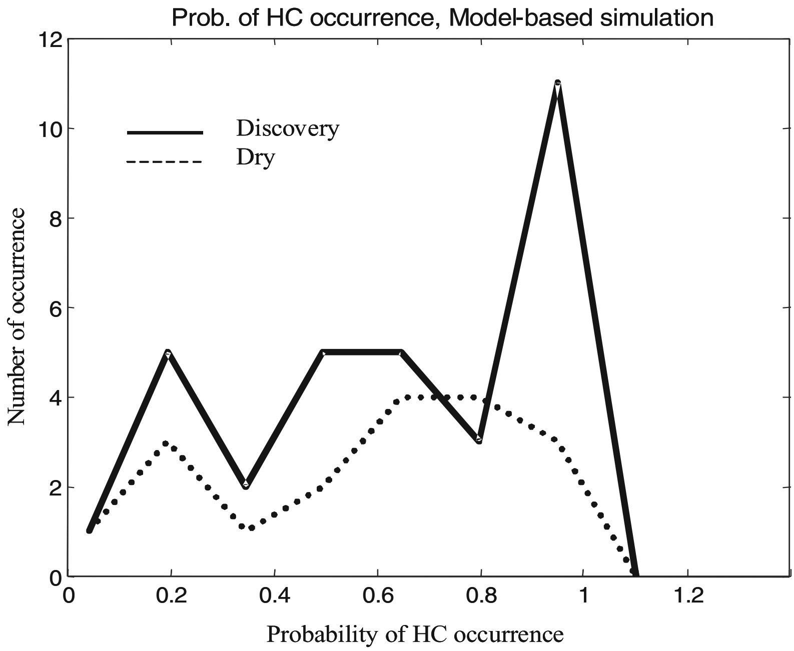

Figure 9. Comparison of the predicted probability values at discovery well locations with the values at the dry exploratory well locations drilled between 1994 and 2000. The average success rate for the post-93 wells located in areas with probability value greater than 0.5 is 68% and the success rate is about 50% in the areas where probability value is less than or equal to 0.5. The overall average of post-93 well success is 61%.

of the spatial correlation structure. The use of a random phase map, followed by iterative variations of the phase map through conditioning against observations (e.g., discovered accumulations, dry well locations, and exploratory exhaustion of potential), leads to a viable conditional simulation. Additional constraints, such as geographic information from geophysical prospecting as illustrated by Chen and others (2000) can also be used to conditional locations without prospectivity, resulting in more geologically sound simulation results.

We have learned through experiments that information concerning both spatial correlation in an amplitude map and locations in a phase map are equally important to the simulation. An appropriate phase map is the key in correctly locating the modeled objects in space given complete information in magnitude map. For many years, the spatial correlation structure has been the main focus of the geostatistic study. More effort may be necessary to better understand the impact of phase and how to use the information associated with phase in a stochastic simulation.

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