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GYROTROPY OF GRANULAR AND THIN-LAYER MEDIA

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The mechanism of the origin of gyrotropy in rocks has been simulated for media with granular and with thin-layer dissymetrical microstructures. Calculation of displacement and stress associated with shear-wave propagation is reduced to a static problem for each element of the medium structure (a grain or a thin layer), as they are negligibly small relative to the wavelength. The models are "rotating" gyrotropic, i.e., the displacement vector of a shear wave in such a medium "turns" at an angle which is the sum of rotations at each element; these elementary rotation angles are equal to the dissymmetry parameter of the medium. A numerical experiment revealed the main features of shear wave propagation in gyrotropic media. As shown by calculations, cross-line (additional) components of displacement may occur in gyrotropic media as well as in media with azimuthal anisotropy, for instance, in transversely isotropic media with vertical symmetry.

Spatial dispersion, gyrotropy, Hooke's law, rotation of polarization plane, dissymmetry, microstructure

INTRODUCTION

This work is in the field of gyrotropy, a new line in seismic studies [1–4], and aims to confirm that geological media can be gyrotropic, i.e., rotating shear-wave polarization planes. The suggested models simulate a micro-inhomogeneous gyrotropic medium with a dissymmetric structure (positioning of micro-objects in space) that permits gyrotropy and, moreover, makes the medium left- or right-enantiomorphic, or rotating clockwise or counterclockwise.

As gyrotropy was experimentally discovered in the uppermost sand-clay section [5–8], the gyrotropic models below simulate granular (sands) and thinly layered rocks. A model of a granular gyrotropic medium was earlier developed in [4, 9–12] and investigated the effect of radial forces applied to a grain. This paper presents another gyrotropic model, for media with layered microstructure. In both models, the forces applied to an elementary unit of the medium are tangential and produced by stress associated with propagation of shear waves.

GYROTROPIC MODELS OF DSSYMETRICAL MICRO-INHOMOGENEOUS MEDIA

Principles of gyrotropic modeling

The concept of seismic gyrotropy was introduced by analogy with optical [13–15] and acoustic [16–23] gyrotropy. Unlike the atom- and molecule-ordered materials with relatively stable physical properties investigated by electromagnetic or acoustic waves of GHz frequencies, rocks appear rather chaotic structures, with ordering of a quite different scale and the physics defined by their "geological" nature. Nevertheless, there is more similarity than there might seem, as (i) rocks generally have an ordered structure and can be considered homogeneous as a first approximation within large volumes, and (ii) their elements may have the same size-wavelength ratio, which is the main gyrotropic parameter. As in optical and acoustic gyrotropy, the scale of microstructure should be within one hundredth of a wavelength. Seismic frequencies are much lower than those used in the optics and acoustics of crystals (hundreds of meters or a few kilometers in seismology

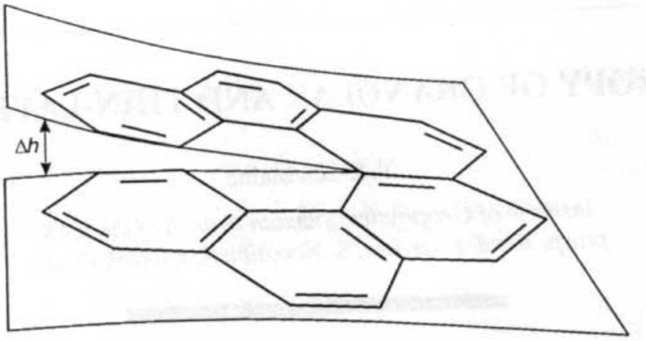

Fig. 1. Hexagelicene molecule: chain of benzene rings arranged on a spiral coil. The coil surface is shown as a plane with separated edges, Δh is coil spacing.

and DSS against fractions of a centimeter in geoacoustic soundings [4]). Then, the elementary units of a gyrotropic microstructure are grains, thin layers, inclusions, or microcracks instead of atoms and molecules.

The effect when the microstructural ordering of oriented layers, cracks and grains of rock is reflected in its properties on a macroscale is known as anisotropy. In this sense gyrotropy and anisotropy are phenomena of the same type. Anisotropy arises in media having axes or planes of symmetry, whereas gyrotropy of crystals, solutions or rocks is associated with the absence of a symmetry center and the presence of enantiomorphism. The absence of a symmetry center is a necessary condition for gyrotropy, and enantiomorphism provides the principal gyrotropic effect, which is rotation of the polarization plane. Rocks may possess both these properties of symmetry, and thus may in principle be gyrotropic.

As in the previous studies [4, 10, 11], construction of gyrotropic models will be based on the Curie principle [24] according to which a dissymmetry relevant to a phenomenon is associated with the same dissymmetry in its causes. In other words, rotation of the polarization plane of a shear wave (rotation of the displacement vector) must be caused, anyway, by a "rotational" structure of the medium.

Of all gyrotropic bodies, crystals most truly correspond to geological media. The gyrotropic properties of crystals are caused either by a dissymmetrical structure of molecules or by dissymetrical distribution of molecules (or atoms) in the lattice [14, 15, 17, 20]. The hexagelicene molecule (Fig. 1) consisting of a spiral chain of benzene rings is a classical example of an optically active molecule. Figures 2 and 3 show structural elements of the gyrotropic crystals of α-quartz and tellurium. The siliceous tetrahedrons of quartz and tellurium atoms make spiral-shaped chains. The idea of a spiral is used in radiophysics and physical optics to create electromagnetic materials which are artificial gyrotropic (chiral) media [25, 26]. In these materials, wire elements screwed as shown in Fig. 4 act as dissymmetrical molecules. An artificial chiral medium is either an ordered structure or a chaotic mixture of chiral elements embedded in an ordinary dielectric.

A gyrotropic model of a granular medium

This model was described in detail in [9–11], and the consideration below is restricted to its features relevant to tangential stress. The model of a granular medium (Fig. 5) simulates gyrotropic effects caused by dissymmetrical distribution of grains (spheres) in each column of a cubic packing: The centers of the spheres are aligned along a spiral, and their projections onto a horizontal plane lie on a circumference (Fig. 5, *B*). The model simulates the structure of hexagelicene, quartz, and tellurium (Figs. 1–3), as grains in columns make chains similar to the chains of benzene rings in hexagelicene, siliceous (SiO_4) tetrahedrons of quartz, and atoms in the tellurium lattice. The model of sandy rocks implies an infinite by large number of grains in a period; so, a gyrotropic packing of grains shows an azimuthal departure from a regular one (in which all the sphere centers lie on the same straight line) only for part of a spiral coil.

A gyrotropic model of a thin-layered medium

Unlike the previous model in which grains are microelements of sandy rocks, this model includes elongate elements, planar thin layers ($h_k \ll \lambda$, λ is wavelength) whose thicknesses are as a rule greater than a grain

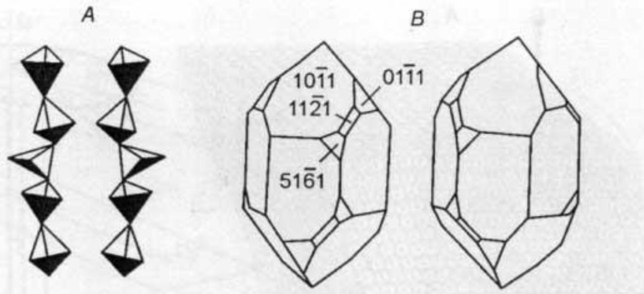

Fig. 2. **α-quartz in two enantiomorphic modifications.** ***A*** **is right- and left-oriented chains of SiO_4 tetrahedrons of quartz,** ***B*** **is right- and left-oriented quartz crystals.**

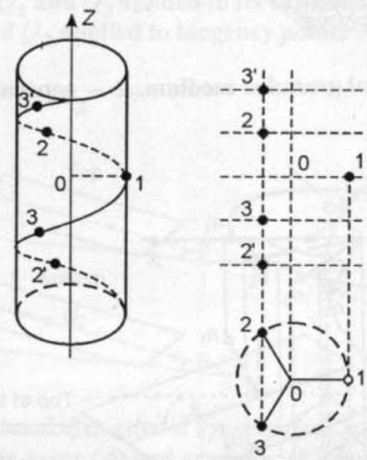

Fig. 3. Position of atoms in tellurium cell. Te atoms (filled circles) make a spiral about an axis parallel to the optical axis ***Z*****. Atoms 1, 2, 3 make a coil. On the right is the same chain of atoms projected onto a vertical (above) and a horizontal (below) plane.**

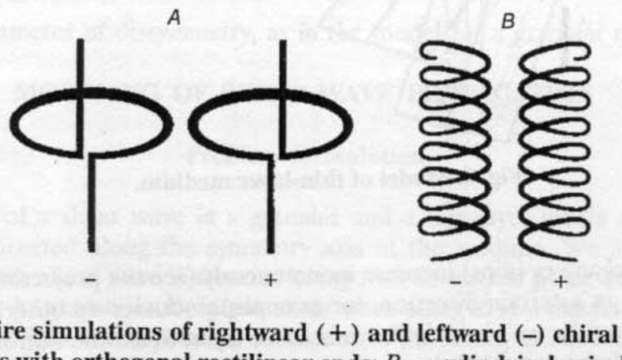

Fig. 4. Wire simulations of rightward (+) and leftward (–) chiral bodies. ***A*** **— rings with orthogonal rectilinear ends;** ***B*** **— cylindrical spirals.**

size. Therefore, gyrotropy in such media is caused by thin layering. (Cf. anisotropy in sediments associated with lamination, or quasi-anisotropy).

Like the model of a granular medium, a gyrotropic model of a thin-layer medium can apparently be based on "azimuthal rotation plus translation". In the latter, this principle is realized (Fig. 6) in similar slope but different dip directions of layers. Elastic moduli are different in different layers, and densities may also be dissimilar. For simplicity assume that a thin-layer medium is alternately two-component.

The position of a layer in space will be determined by the angle from the normal n_k to its top (polar angle θ of layer's slope) and the azimuthal angle φ of the layer's dip. The polar angles (slopes) are the same

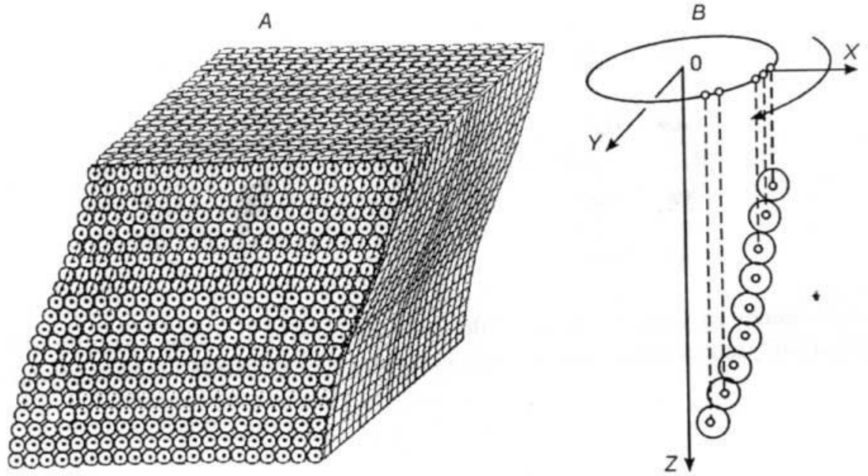

Fig. 5. Model of dissymmetrical granular medium. A **– general view,** B **– single column.**

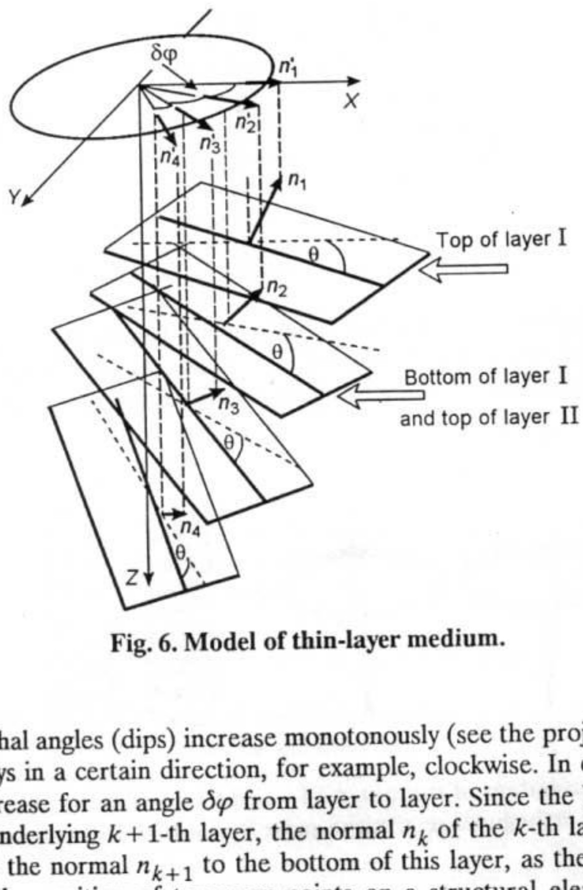

Fig. 6. Model of thin-layer medium.

for all layers but the azimuthal angles (dips) increase monotonously (see the projections of the normals relative to the previous layer) always in a certain direction, for example, clockwise. In other words, the slopes (θ) are invariable and the dips increase for an angle $\delta\varphi$ from layer to layer. Since the bottom of a k-th layer is at the same time the top of the underlying $k+1$-th layer, the normal n_k of the k-th layer defining the position of its top does not coincide with the normal n_{k+1} to the bottom of this layer, as the dip azimuths differ by $\delta\varphi$.

The dissymmetry in the position of tangency points on a structural element (layer or grain) in these models causes their disequilibrium within this element. The equilibrium problem in a model of a granular medium was solved as follows [10]. Two possibilities were suggested to balance the dissymmetrical forces applied to grains. (1) The dissymmetrical granular medium was represented as a selected volume of an "infinite" geological medium. Then, the disequilibrium forces on each grain of the selected volume are equilibrated by response forces from the enclosing medium. (2) Equilibrium was assumed to arise within an individual grain with regard to compensating inelastic forces from the presence of clay particles near the grains of the medium.

The problem of force balance can be solved in the same way in the model of a dissymmetrical layered medium. If the geological medium is assumed infinite, the top and bottom surfaces will intersect in infinity as

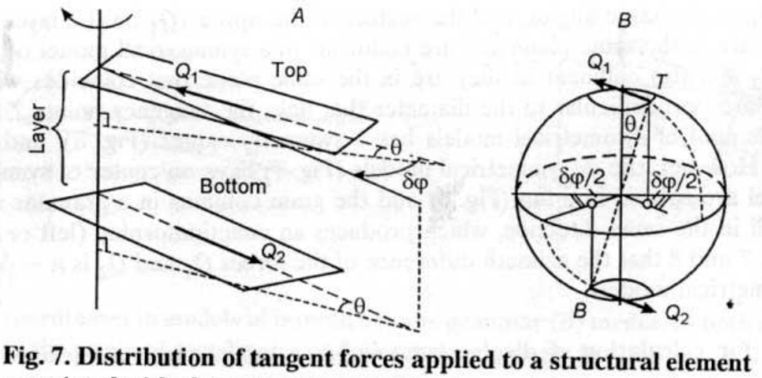

Fig. 7. Distribution of tangent forces applied to a structural element associated with shear-wave propagation in gyrotropic models. *A* — layer and forces Q_1 and Q_2 applied to its top and bottom; *B* — grain and forces Q_1 and Q_2 applied to tangency points *T* and *B*.

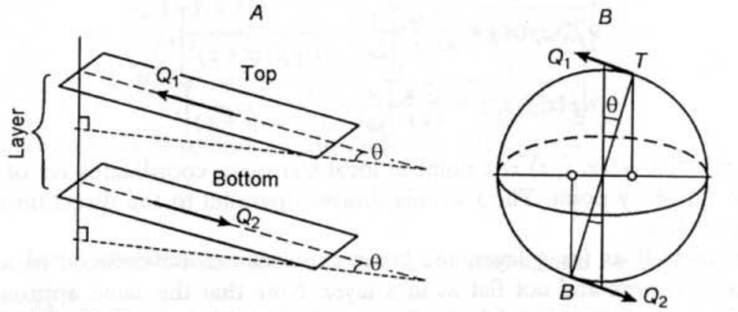

Fig. 8. Structural elements of symmetrical (nongyrotropic) models of thin-layer (*A*) and granular (*B*) media.

two nonparallel planes, and compensating response forces and their moments will arise on this line. In reality, the compensation will occur earlier than in infinity because of outpinching or horizontal replacement of the layers.

The angle $\delta\varphi$ is a parameter of dissymmetry, as in the model for a granular medium [10].

MODELING OF SHEAR WAVE PROPAGATION

Problem formulation

Consider propagation of a shear wave in a granular and a thin-layer media along the Z axis, which in the global coordinates is directed along the symmetry axis of the medium. We assume that at $Z = 0$ the displacement vector in a plane shear wave is directed along X. The vertical plane XOZ will be the observation plane. It is to be shown that propagation of a plane shear wave along Z in a dissymetrical model is associated with the appearance of cross-line Y components additional to the X displacement, and the rotation angle of the displacement vector increases with wavelength.

The problem of elastic wave propagation can be reduced to a static problem with respect to an element of the model (grain or layer), as their size is much smaller than the wavelength ($l << \lambda$), where l is the radius of a grain or the thickness of a layer. Displacements inside an element of the model from known stress on its surface corresponding to the stress associated with propagation of a shear wave are sought.

Consider the distribution of forces applied to a single structural element during propagation of a shear wave. In the model of a layered medium, two tangential forces of equal magnitudes, Q_1 and Q_2 ($|Q_1| = |Q_2| = Q$) applied to each point of the top and bottom of a layer cause its stress (Fig. 7, *A*). The tangential forces Q_1 and Q_2 applied to a grain are shown in Fig. 7, *B*.

Consider also symmetrical models of layered and granular media corresponding to the dissymmetrical ones. The distribution of forces in these models is shown in Fig. 8. In a symmetrical model of a layered medium, all layers slope and dip at the same angles, and the vectors of the upper (Q_1 on the layer top) and lower (Q_2 on the bottom) forces are in the same plane and are collinear. In a symmetrical model of a granular medium, the vectors Q_1 and Q_2 are also collinear as they are in the same plane that coincides with the plane of the figure (Fig. 8, *B*) and are perpendicular to the diameter that links the tangency points *T* and *B*.

Note that any element of symmetrical models has a symmetry center (Fig. 8), and such a model thus cannot be gyrotropic. However, the dissymmetrical models (Fig. 7) have no center of symmetry: Layers in the layered medium model are twisted in a fan (Fig. 6) and the grain columns in a granular medium are twisted on a spiral (Fig. 5), all in the same direction, which produces an enantiomorphic (left or right) structure.

It is seen in Figs. 7 and 8 that the azimuth difference of the forces Q_1 and Q_2 is $\pi - \delta\varphi$ in dissymmetrical models and π in symmetrical models.

Algorithm for calculation of displacement inside a model element (grain or layer)

Displacement can be estimated by Cerrutti's formula [27]:

$$u_x(x,y,z) = \frac{Q}{4\pi\mu}\left(\frac{\lambda+3\mu}{\lambda+\mu}+\frac{x^2}{r^2}\right)\frac{1}{r} - \frac{Q}{2\pi(\lambda+\mu)r} + \frac{Q}{4\pi(\lambda+\mu)}\left[1-\frac{x^2}{r(r+z)}\right]\frac{1}{r+z},$$

$$u_y(x,y,z) = \frac{Q}{4\pi}\frac{xy}{r}\left[\frac{1}{\mu r^2} - \frac{1}{(\lambda+\mu)(r+z)^2}\right], \tag{1}$$

$$u_z(x,y,z,) = \frac{Q}{4\pi}\frac{x}{r}\left[\frac{z}{\mu r^2} + \frac{1}{(\lambda+\mu)(r+z)}\right],$$

where $r = (x^2 + y^2 + z^2)^{1/2}$, $r = (x, y, z)$ is a point in local Cartesian coordinates *xyz* of a structural element with the origin *O* at the tangency point. The *x* axis is directed parallel to the direction of the force and *z* is normal to the boundary surface.

For a single grain, as well as for a layer, the static problem can be reduced to a Cerrutti's problem, though the grain surface is convex and not flat as in a layer. Note that the same approach was used in [10], where normal compressive and not tangential forces were applied to the grain surface and Boussinesq formulas were employed.

In [10] it was shown that the problem with a source on a spherical surface can be replaced by a similar problem for a flat surface, as the main contribution to the stress inside a grain is from the stress in the near-contact area. Therefore, radial displacement in the immediate vicinity of the applied force can be estimated using Cerrutti's problem.

Consider displacements u_x and u_y in the plane *xoz*. It is obvious that neither the lower force along *x* nor the upper force along $-x$ yield *y* displacement. Indeed, from Cerrutti's formulas (1) the *y* displacement component becomes zero at $y = 0$: $u_y = 0$. Therefore, in a symmetric model, the rotation angle defined as arctg u_y/u_x is zero.

Displacement from a dissymmetrical force can be obtained as follows. The vector of the force *Q* directed at an angle φ from *x* can be decomposed into two components: $Q^{||}$ along *x* and $Q^{\perp}$ along *y*:

$$Q = Q^{||} + Q^{\perp}$$

where $Q^{||} = Q\cos\varphi e_1$ and $Q^{\perp} = Q\sin\varphi e_2$. The solution for $Q^{||}$ directed along *x* already exists, we are only to take into account that the magnitude of force is $Q\cos\varphi$ in Cerrutti's formula (1) for u_x, i.e.,

$$u_x^{||} = u_x\cos\varphi;$$

$$u_y^{||} = u_y\cos\varphi; \tag{2}$$

$$u_z^{||} = u_z\cos\varphi.$$

The solution for $Q^{\perp}$ directed along *y* is obtained by substitution of *y* for *x* in Cerrutti's formula (1). Taking into account that the magnitude of the force is $Q\sin\varphi$, obtain:

$$u_y^{\perp}(x,y,z) = \frac{Q\sin\varphi}{4\pi\mu}\left(\frac{\lambda+3\mu}{\lambda+\mu}+\frac{y^2}{r^2}\right)\frac{1}{r} - \frac{Q\sin\varphi}{2\pi(\lambda+\mu)r} +$$

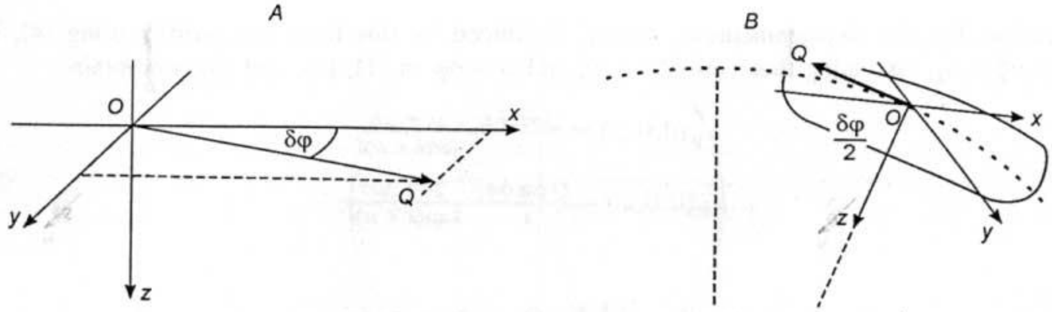

Fig. 9. Local coordinates in models of layered (*A*) and granular (*B*) media. In both models force Q is applied to the origin of coordinates and directed at an angle to x.

$$+\frac{Q\sin\varphi}{4\pi(\lambda+\mu)}\left[1-\frac{y^2}{r(r+z)}\right]\frac{1}{r+z},$$

$$u_x^{\perp}(x,y,z)=\frac{Q\sin\varphi}{4\pi}\frac{xy}{r}\left[\frac{1}{\mu r^2}-\frac{1}{(\lambda+\mu)(r+z)^2}\right], \tag{3}$$

$$u_z^{\perp}(x,y,z)=\frac{Q\sin\varphi}{4\pi}\frac{y}{r}\left[\frac{z}{\mu r^2}+\frac{1}{(\lambda+\mu)(r+z)}\right].$$

The solution u^{φ} is a superposition of solutions represented by (2) and (3):

$$u^{\varphi}=u^{||}+u^{\perp}. \tag{4}$$

Local coordinates of a structural element (grain or layer)

Cross-line (additional) displacement components u_y in the model of a layered gyrotropic medium are produced by the dissymmetry of the layer bottom ($\delta\varphi$ departure from its "regular" position in the symmetrical model). Let in the first layer ($k = 1$) X coincide with x, which is parallel to the dip direction of the first layer top*. Assume that on the surface ($Z = 0$) the force Q_1 on the top of the first layer (and, hence, the polarization vector $\boldsymbol{u}$) is at an angle π from $X = x$, i.e., $Q_1 = -Q_1 e_1$, where e_1 is a unit vector of X. The problem is to find cross-line displacement components produced by the dissymetrical force Q_2 on the layer bottom, where the origin of local coordinates is placed. Then x is at an angle $\delta\varphi$ from the direction of force on the bottom (Fig. 9, *A*).

In the dissymmetrical model of a granular medium, the departures $\delta\varphi$ from the «regular» position in the symmetrical model are assumed similar for the upper (*T*) and the lower (*B*) tangency points (compare the symmetrical and dissymmetrical models in Figs. 8, *B* and 7, *B*).

Thus, the cross-line displacement components produced by the two forces Q_2 and Q_1 inside a grain are equal, and the problem is reduced to a static problem for one force, in which the found rotation angle of the displacement vector is to be duplicated. (Below it is shown that the rotation of the polarization vector occurs in the same sense from both forces, so these rotations can be added.)

Figure 9, *B* shows local coordinates of a grain with the origin at the upper tangency point. Like in the local coordinates for a layer**, x is parallel (or, collinear) to the force vector in the symmetrical model $Q_1^{\rm sim}$, i.e., $e_1 = -Q_1^{\rm sim}/Q$, where e_1 is a unit vector of x.

Calculations for rotation of displacement vector for a structural element

The rotation angle of the displacement vector for the dissymmetrical model can be found from the ratio of cross-line (u_y) and in-line (u_x) components on z (beneath the force point). Consider a model of a layered medium in which a force Q_2 on the layer bottom is directed at an angle $\delta\varphi$ from x.

* For simplicity, slope is neglected, more so that θ is as small as 1°.

** The grain surface being nonflat, the applicability of this approach is restricted to small θ, as it is assumed that the tangential forces $Q_1^{\rm sim}$ and $Q_1^{\rm dis}$ make an angle $\delta\varphi/2$ equal to the angle between two longitudinal sections, one through the tangency point in the symmetrical model and the other through the tangency point in the dissymmetrical model.

Expressions for the displacements u_y and u_x produced by this force are written using (4), in which $u_y^\varphi \equiv u_y$ and $u_x^\varphi \equiv u_x$. Assuming that $x = 0$, $y = 0$, and $\varphi = \delta\varphi$ in (1), (2), and (3), we obtain

$$u_y(0, 0, z) = \frac{Q \sin \delta\varphi}{z} \cdot \frac{2\lambda + 3\mu}{8\pi(\lambda + \mu)},$$

$$u_x(0, 0, z) = \frac{Q \cos \delta\varphi}{z} \cdot \frac{2\lambda + 3\mu}{8\pi\mu(\lambda + \mu)}.$$

Then,

$$\mathbf{u} = \frac{1}{z} \cdot \frac{2\lambda + 3\mu}{8\pi\mu(\lambda + \mu)} \mathbf{Q}, \quad u_y/u_x = \operatorname{tg} \delta\varphi.$$

Hence, $\operatorname{arctg}(u_y/u_x) = \delta\varphi$, i.e., the rotation angle equals the dissymmetry parameter $\delta\varphi$.

In a granular medium, the rotation angle at the lower tangency point on a grain is $\delta\varphi/2$, as the force is directed at $\delta\varphi/2$ from x. At the upper tangency point, the force is directed at $\pi - \delta\varphi/2$. From Cerrutti's formula (1) it follows that in the plane xoz ($y = 0$) the u_y displacement is zero and the displacement u_x produced by a force directed along positive x coincides with the x displacement produced by a force that acts in the opposite sense. Therefore, the force directed at $\pi - \varphi$ from x produces the same u_x and u_y displacements at $y = 0$ as the force directed at φ. Hence, the rotation angle at the upper tangency point also equals $\delta\varphi/2$, and the rotation angle for a grain is $\delta\varphi$, as well as for a layer.

Thus, the Curie principle fulfills exactly: the dissymmetry of a medium to an angle $\delta\varphi$ provides the same rotation of the displacement vector associated with propagation of a shear wave in this medium.

Total rotation of a displacement vector in the dissymmetrical model

The rotation of the polarization vector is determined by the u_Y/u_X ratio in the global coordinates XOZ. Local coordinates for a k-th element can be transformed into the global coordinates (to the slope θ of the plane xy) by $(k - 1)\delta\varphi$ counter-clockwise rotation of the plane xy about Z (Fig. 10). Note that the ratio u_y/u_x is independent of θ, because it is cancelled in the u_Y/u_X ratio as both displacements u_x and u_y are multiplied by $\cos\theta$ on transition to the global coordinates.

The local coordinates of the first layer coincide with the global system, and the rotation of the displacement vector relative to X equals $\delta\varphi$, the dissymmetry parameter of a structural element of the model (see above). At the second layer, the displacement vector also rotates at $\delta\varphi$ from the first layer, etc. The total rotation of the wave polarization vector over a path of a length H is obtained by multiplying a single rotation $\delta\varphi$ for a structural element by their number $n = H/l$ (where l is the characteristic size of the element):

$$\alpha = n\,\delta\varphi.$$

According to the available experimental data [3, 5, 7], the total rotation over 1 m is about 0.1–1°. Then, the dissymmetry parameter $\delta\varphi$ coinciding with the rotation angle for an element of the size $l = 1$ cm will range as 0.001–0.01°.

An effective model of the medium

Micromodels have been constructed by repeating two procedures: $\delta\varphi$ azimuthal rotation of a structural element and its translation along Z, therefore, Z will be a spiral vertical symmetry axis in the corresponding effective macro-models. The Z-axis is of the order $n = 2\pi/\delta\varphi$. The angle $\delta\varphi$ in dissymmetrical models can be of the order of a thousandth to tenth fractions of a degree (e.g., $\delta\varphi = 0.06°$ in [10]).

Crystals with hexagonal symmetry and media with infinite symmetry are transversely isotropic with respect to their elastic moduli. Therefore, the effective model of microheterogeneous media (granular and layered) with an axis of n-fold symmetry, $n \to \infty$ is assumed to simulate a transversely isotropic medium (∞ symmetry) described by five elastic modulus tensor components (c_{ijkl}) and four gyration tensor components (b_{ijklm}) (about gyration constants see [2, 3, 23, 28]).

Elastic moduli and density for a transversely isotropic medium can be found from the parameters of the corresponding microheterogeneous (granular or layered) medium [29]. (Note that a granular medium of the type of a cubic packing of spheres is approximated within the limits of a cubic symmetry. In describing it by the constants of a transversely isotropic medium, the transition to the cubic symmetry is achieved assuming

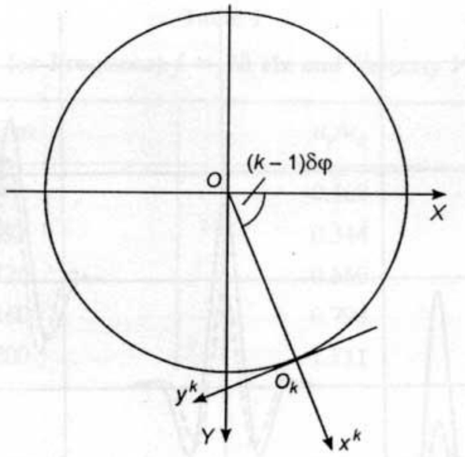

Fig. 10. Local xyz^k and global XYZ coordinates. z and z^k are orthogonal to the plane of the figure.

that $c_{11} = c_{33}$, $c_{44} = c_{66}$, $c_{13} = c_{12}$, $c_{12} \neq c_{11} - 2c_{66}$.). Shear-wave propagation along the symmetry axis can be investigated merely on the basis of two velocities V_{S_1} and V_{S_2}, without knowing all elastic and gyration constants.

THEORETICAL SEISMOGRAMS

Shear-wave propagation in gyrotropic media within the limits of dissymmetrical microinhomogeneity models can be illustrated by a two-component (x, y) theoretical seismogram for a shear wave propagating along the symmetry axis (z) in a transversely isotropic gyrotropic medium. In anisotropic gyrotropic media, rotation of a shear-wave polarization plane – the principle gyrotropic effect – is observed along axes of three-fold or higher symmetry [17].

Medium parameters and observation system

Modeling of theoretical seismograms in a general case (when waves propagate in different directions in space) implies setting reduced* elastic moduli and gyration tensors (matrices of elastic moduli tensor $\mathbf{c}\rho^{-1}$ and gyration pseudo-tensor $\mathbf{g}\rho^{-1}$ [30]). Propagation of a shear wave along the symmetry axis of a transversely isotropic gyrotropic medium (or "rotating" gyrotropic medium) is described using an elastic modulus c_{44} $(c_{44} \to c_{2323})$ and a gyration constant b_{543} $(b_{543} \to b_{13233})$ [23].

On the symmetry axis, the velocities** V_{S_1} and V_{S_2} of two shear waves are

$$V_{S_{1,2}} = V_{S_0} \pm d. \tag{5}$$

Here $V_{S_0} = \sqrt{c_{44}/\rho}$ is the velocity on the Z-axis in the same medium but without gyration; $d = (\omega b_{543})/(2c_{44})$ is a gyrotropic addition to V_{S_0}, and ω is circular frequency.

In mathematical modeling of shear-wave propagation along the symmetry axis it is more convenient to set the gyrotropy constant d than the constant b_{543} and the frequency ω.

The defined values in our case were $V_{S_0} = 300$ m/s, $d/V_{S_0} = 0.01$, i.e.,

$$V_{S_1} = 303 \text{ m/s}, \quad V_{S_2} = 297 \text{ m/s}.$$

* Divided by density ρ.
** On symmetry axes phase and ray velocities are equal.

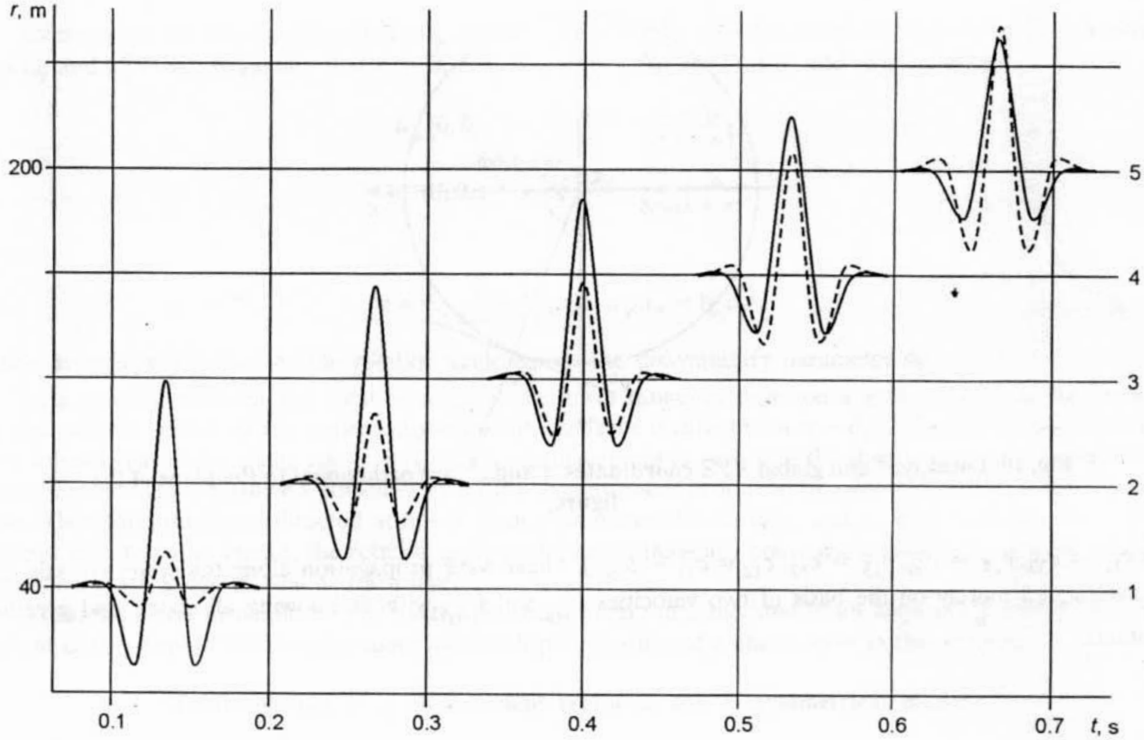

Fig. 11. Two-component (*x*, *y*) seismogram of a shear wave generated by force *X* and propagating along symmetry axis *z*. Solid line shows $u_x(t)$ and dashed line shows $u_y(t)$.

The observation system was configured as follows. Shear waves from a source in the origin of coordinates *Oxyz* propagated along *z* and were recorded by an array of five *x*, *y* geophones at offsets from 40 to 200 m. The force in the source is directed along *x*.

The calculations were run at zero ray approximation. Source signal was set as a symmetrical pulse

$$F(t) = U\exp(-\beta t^2)\cos\omega t, \tag{6}$$

with the following parameters: frequency $f = \omega/(2\pi) = 20$ Hz; attenuation $\beta = f^2 \ln(1/R)$, where $R = F(t_0 \pm 1/f)/F(t_0) = 0.01$; the duration of signal $F(t) = 2.5$ periods, and time in its middle t_0. Since the source acts as an *X*-force, $U = Ue_1$.

Calculation results

Calculated theoretical seismograms are shown in Fig. 11. The component u_x (parallel to the force direction in the source) is the in-line (main) and u_y is the cross-line (additional) component. To facilitate comparison, the two displacements are superposed. Moreover, the records for different offsets are given with no account of scattering.

It is seen from Fig. 11 that the displacement $u_x(t)$ decreases and $u_y(t)$ increases with offset, i.e., the initial displacement vector *U* rotates as far as the wave travels. At offsets of about 200 m, the displacements of the two components become equal.

The rotation α of the displacement vector *U* for harmonic waves is expressed by the formula [2, 3]

$$\alpha = \mathrm{arctg}\,\frac{u_y}{u_x} = \frac{\omega r}{2}\left(\frac{1}{V_{S_2}} - \frac{1}{V_{S_1}}\right). \tag{7}$$

In the case of pulse signals of the type of (6), (7) remains valid for the amplitudes in the center of signals $u_x(t)$ and $u_y(t)$. This can be seen both from a comparative analysis of expressions for harmonic and pulse waves S_1 and S_2 on the *x*- and *y*-components and from calculations. The calculated u_y/u_x ratios and rotation angles are given in Table 1.

Table 1

u_y/u_x Ratios and α for Frequency f = 20 Hz and Velocity $V_{S_{1,2}}$ = 300±3 m/s

Channel	r, m	u_y/u_x	$\alpha°$
1	40	0.169	9.6
2	80	0.344	19.2
3	120	0.550	28.8
4	160	0.793	38.4
5	200	1.111	48.0

It is seen from the table that rotation angles α are proportional to offset r. Specific rotation estimated from these data is 0.24 deg/m, which is of the order of that for sands and clays in the uppermost section [3, 5].

Therefore, the cross-line displacement components arise in gyrotropic transversely isotropic media with vertical symmetry (effective models of microheterogeneous horizontally stratified and granular media) as well as in azimuthally anisotropic media (effective models of media with vertical aligned cracks).

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