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Fractal characteristics of dense stream networks

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Abstract

In the past, a great deal of research has been conducted to determine the fractal properties which quantify the sinuosity of individual streams and the branching configuration of stream channel networks in watersheds. Much of this work has been conducted with sparse rather than dense stream networks. It seems appropriate that characteristics should be determined for dense networks to decide if the ranges of values of the fractal characteristics computed by previous researchers hold for the dense networks. The first objective of the present study is to compute the fractal measures for several watersheds, which have dense network data, and to compare them to the values in the literature estimated by using sparse network data. The second objective is to compare the different fractal measures for different watersheds and examine their variability. If there is considerable variability in these measures, then the question of which measures to use arises. The third objective is to examine whether these watersheds are self-similar or self-affine. The results indicate that the fractal dimensions vary widely, depending on the definitions used. The watersheds have self-affine rather than self-similar characteristics. © 2001 Published by Elsevier Science B.V.

Keywords: Euclidean; Fractal dimensions; Watersheds

1. Introduction

Euclidean or classical geometry consists of describing physical objects using lines, circles, ellipses, etc. This type of geometry is appropriate for discussing man-made technological objects; however, patterns found in nature are significantly more complex. To describe natural entities, Mandelbrot (1983) developed fractal geometry — the so-called 'geometry of nature'. Fractal geometry is useful for describing irregular and fragmented patterns found in many disciplines. Fractal shapes are characterized by their detail. A magnified view of a fractal displays more detail than the unmagnified view. Accordingly, it is impossible to draw a tangent to a fractal because it is not smooth. Although fractals can be either continuous or fragmented, they are not differentiable.

A fractal which has received significant attention in the literature is the coastline. When measuring the length of a coast on a map with a compass, researchers find that the total length depends on the selected spacing of the compass points. The smaller the unit of measure, the longer the coast. Each time the unit of measure is decreased, one measures more detail of the coastline, thus increasing the apparent total length. This phenomenon occurs due to the fractal nature of the coastline, and hence it is impossible to assign a single value as the length of a coastline. For a closed, non-fractal object, such as a circle, the total length of the outline will converge to a constant value as the unit of measure decreases.

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1.1. General overview of self-similarity

Fractals are distinguished by their self-similarity. The term self-similarity, which is used interchangeably with *scaling* and *scale invariant* in the literature, simply means that the pieces of the object resemble the whole object. For example, consider a cauliflower (Peitgen et al. 1992). Magnified, the small pieces of a cauliflower look similar to the whole cauliflower head. In a mathematical idealization, this dissection of *infinitely detailed* objects could be carried on indefinitely (Peitgen et al., 1992). The cauliflower may be considered *imperfectly self-similar* since, at some point, the structure becomes too small to continue. The metric system is another example of self-similarity. A meter with divisions at each decimeter cannot be distinguished from an enlarged view of a decimeter with divisions at each centimeter.

Self-similarity takes many forms. *Strict self-similarity*, the purest form of self-similarity, implies that each piece is a perfect miniature copy of the whole. *Statistical self-similarity* is the term used when small pieces of the whole look similar to the whole, but slightly varied (Takayasu, 1990). A fractal is termed *self-affine* when a smaller piece of the whole appear to have undergone different scale reductions in the longitudinal and transverse directions (Peitgen et al., 1992). This unequal scaling effectively skews the smaller replica. Although all fractals display some variation of self-similarity, self-similarity in itself does not define a fractal. For example, a cube can be divided into smaller cubes which resemble the original; however, a cube is not a fractal.

1.2. General overview of fractal dimensions

A line is one-dimensional, a plane is two-dimensional, and a cube is three-dimensional; however, the physical forms of fractals are more complicated than these conventional geometric objects. Fractals have dimensions which are fractional, and these real valued dimensions are termed *fractal dimensions*. The fractal dimension quantifies the complexity or irregularity of a fractal object, but not the shape of a fractal. An object with a low fractal dimension is less complex than an object with a higher fractal dimension. According to Barnsley (1993), the fractal dimension ‘attempts to quantify a subjective feeling we have about how densely the fractal occupies the metric space in which it lies’. Although it is perceived that lines are one-dimensional and planes are two-dimensional, Peano and Hilbert (cited in Mandelbrot, 1983) complicated the common thinking by introducing the concept of space-filling curves which have dimensions between one and two.

The following relationship exists between the length, L, the area, A, and the volume, V, of a non-fractal object.

$$L \propto A^{1/2} \propto V^{1/3} \tag{1}$$

This relationship implies that if the side of a cube is increased by a factor of 2, the area of one side is increased by a factor of 2^2, and the volume is increased by a factor of 2^3. In other words, each quantity is increased by a factor equal to two raised to the power of its dimension. If there existed a quantity, X, which were increased by a factor of 2^D when the length of the side doubled, one would say that X is D-dimensional (Takayasu, 1990). In that case, the following relationship would be satisfied.

$$L \propto A^{1/2} \propto V^{1/3} \propto X^{1/D} \tag{2}$$

Following the description by Takayasu (1990), if a one unit cube is divided into similar cubes of half sizes with sides of length $\delta = \frac{1}{2}$, a single segment becomes two smaller ones, a square becomes four smaller ones, and a cube becomes eight smaller ones. Since the numbers 2, 4, and 8 can be written as 2^1, 2^2, and 2^3, the following power–law relationship can be derived to relate the number of pieces N and the size δ for the side, the square, and the cube.

$$N(\delta) = \left(\frac{1}{\delta}\right)^{D_s} \tag{3}$$

In this case, the generic D is replaced by D_s, the *similarity dimension*. The similarity dimension is the fractal dimension for strictly self-similar fractals. It is not limited to integer values; however, for common shapes it will be identical to the empirical dimension (Takayasu, 1990).

Since the similarity dimension is only defined for strictly self-similar fractals, it is necessary to define additional fractal dimensions which are appropriate for all fractals. Hausdorff (Takayasu, 1990) began the development of such a dimension in 1919. Although the so-called Hausdorff dimension can be defined for any fractal, a rigorous calculation of the Hausdorff dimension is difficult (Takayasu, 1990). The following two methods were developed to avoid the computational complexities of the Hausdorff method.

The coastline scenario mentioned previously is used to introduce the line segment method of computing the fractal dimension. Peitgen et al. (1992) suggested that this method is an effort "to measure the degree of complexity by evaluating how fast length … increases if we measure with respect to smaller and smaller scales". For several different compass settings, the number of compass lengths, $N(\delta)$, to traverse the coast is plotted against the corresponding compass setting, δ, on log–log paper. The plotted points fall on a relatively straight line with a negative slope. The observed relationship can be represented as a power law in the following form.

$$N(\delta) \propto \delta^{-D} \quad (4)$$

In Eq. (4), D is the fractal dimension of the coastline, and it is determined by computing the slope of the plotted relationship. For the coast of Britain, the fractal dimension is equal to approximately 1.3. The line segment method is appropriate for non-branching fractals such as individual rivers or coastlines. This method is not applicable for stream networks. The line segment or compass method is sometimes referred to as the Richardson Method as it is attributed to Richardson (1961).

The box counting method is similar to the line segment method. In this method, a grid with spacing δ is placed over the fractal object, and the number of boxes, $N(\delta)$, in which a part of the fractal falls is counted. This routine is performed at varying grid spacings, and points representing the corresponding box counts and grid spacings are plotted on a log–log plot. The resulting relationship can again be represented as Eq. (4). The box counting method can be used for branching fractals like stream networks as well as for discontinuous fractals such as dusts and galaxies.

1.3. Fractal dimensions of individual streams and stream networks

Research has shown that individual streams and the networks which they comprise are fractals. Hydrologists are interested in calculating two fractal dimensions for streams — the fractal dimension of individual streams, d, and the fractal dimension of the stream network, D. The fractal dimension for an individual stream is a measure of its irregularity; it is a measure of the extent of a stream's meanderings. The fractal dimension for the network is a measure of the ability of a network to fill a plane, and it arises from the branching nature of the network and from the sinuosity of individual streams. If a stream network were truly space-filling, as is the case with a topologically random network, one would expect to compute a stream network fractal dimension of 2.0. Certain researchers, including Mandelbrot (1983) and Tarboton et al. (1988) believe that this may be the case. Most studies have shown that networks are not space-filling — at some level, the stream network stops and the hillslopes begin. The mechanisms, which govern overland flow, channel flow, and erosion, prevent the formation of a space-filling network. Accordingly, it is generally anticipated that the fractal dimension of a stream network is less than 2.0, and it is further acknowledged that the fractal dimensions vary from one location to another.

The fractal nature of stream networks is important to hydrologists for several reasons. A fractal dimension characterizes the scaling properties and indicates how an associated measure changes with changes in scale (La Barbera and Rosso, 1989). Geomorphologic measures such as stream length, drainage density, and slope are typically measured from maps and used in hydraulic and hydrologic modeling. In his paper on the connection between research and practice, Pilgrim (1986) emphasized that different values of these measurements are determined when maps of different scales are used. McDermott and Pilgrim (1982) found that mainstream length measurements can vary up to 80% between different scale maps. Unfortunately, these scale effects have significant implications for the resulting flood predictions. Al-Wagdany (1993) suggested that the fractal dimensions of a watershed are important to better understand the scaling properties of these measures. La Barbera and Rosso (1989) indicated that fractal geometry “can be used to investigate the scaling properties of the attributes and parameters describing drainage basin form and process.”

Takayasu (1990) suggested that the fractal dimension of a meandering line can be determined by examining the relationship between the straight line length and the meandering length for several segments of the line. If this approach is applied to a stream network, it is possible to determine the average fractal dimension of the individual streams, d, in the following manner. For each logical stream, determine the mainstream length, L, by measuring along the curve, and determine the Euclidean mainstream length, L', by measuring the straight line distance between the endpoints of each logical stream. According to Eq. (2), it is expected that the relationship between these two measures will be of the following form if individual streams can be assumed to be strictly self-similar.

$$L' \propto L^{1/d} \tag{5}$$

The fractal dimension can then be computed by determining the slope of the line formed when the mainstream length is plotted against the Euclidean mainstream length on a log–log graph.

Assuming that streams and stream networks are strictly self-similar, a second relationship to estimate the average fractal dimension of an individual stream in a watershed can be derived from Eq. (2), where A is the drainage area.

$$L \propto A^{d/2} \tag{6}$$

If $d = 2\beta$, Eq. (6) can be re-written as the following equation rather than as a proportional relationship.

$$L = \alpha A^{\beta} \tag{7}$$

Eq. (7) is commonly called Hack’s law.

In this case, the average fractal dimension for individual streams in a network can be determined by computing two times the slope of the line formed on the log–log plot of mainstream length versus drainage area (Mandelbrot, 1983). Hack (1957) determined α to be 1.4 and β to be 0.6 ($d = 1.2$) for several rivers in Virginia and Maryland. Hack’s work was performed on non-overlapping basins, rather than investigating sub-basins within a larger basin. Hack also examined data from Langbein (1947) for 400 streams in the northeastern United States and found the same relationship. For two regions in the western United States, Hack found β to be 0.7 ($d = 1.4$), thus proving that fractal dimensions vary from region to region. Gray (1961) found α to be 1.4 and β to be 0.568, and thus the individual stream fractal dimension is twice 0.568 or 1.136. Often 1.136 is rounded to 1.14, and this number is used as the standard individual stream fractal dimension. For eight rivers in Missouri, Hjelmfelt (1988) found the fractal dimension of the mainstream length to vary from 1.036 to 1.219, with an average of 1.158. Muller (1973) reported that β is 0.6 ($d = 1.2$) for basins less than 8000 mi^2, β is 0.5 ($d = 1.0$) for basins between 8000 and 100,000 mi^2, and β is 0.47 ($d = 0.94$) for basins larger than 100,000 mi^2. One may question the smallest value of β, since it implies that the fractal dimension of the mainstream length is less than one. In other words, it suggests that a stream can have a lower dimension than a straight line. This contradiction can be explained, in part, by recalling that defining the fractal dimension in terms of the exponent of the length-area relationship assumes that streams are self-similar. If the streams are not self-similar, then twice the exponent of the length–area relationship is not necessarily an accurate estimate of the fractal dimension. Moreover, Mesa and Gupta (1987) claimed that for a network which behaves according to the random topological model, β should asymptotically tend toward 0.5 as the basin increases in size.

Prior to the development of fractal mathematics, researchers believed that β values greater than 0.5 indicated the tendency of basins to elongate as their sizes increase. Mandelbrot was the first to attribute β values greater than 0.5 to fractal properties which cause the measured length to vary with spatial scale. Rigon et al. (1996) examined Hack’s law in detail and concluded that the β value is primarily due to fractal sinuosity of streams and, to a much lesser extent, is due to elongation of the basins.

The stream network fractal dimension, D, measures the fractal characteristic of the total stream length, Z. Assuming that the stream network is strictly self-similar, the following relationship to determine the fractal dimension of the total stream length can be derived from Eq. (2).

$$Z \propto A^{D/2} \tag{8}$$

If $D = 2\epsilon$, the proportional relationship can be rewritten as the following equation.

$$Z = \phi A^{\epsilon} \tag{9}$$

If a log–log plot of the total stream length versus the drainage area is developed, the stream network fractal dimension can be determined by doubling the slope of the straight line formed by the plotted points. Typical values of network fractal dimensions for total stream length vary from 1.5 to 2.0. Takayasu (1990) computed total stream length fractal dimensions for the Amazon and the Nile using the box counting method. He determined that D is 1.85 for the Amazon and 1.40 for the Nile. Takayasu suggested that the fractal dimension is greater in regions of greater rainfall. As mentioned previously, some researchers believe that the stream network fractal dimension is actually 2.0. Mandelbrot (1983) described fractal geometrical patterns which resemble river networks, and he suggested that these space-filling fractals are models of river networks. In his patterns, the fractal dimension of individual streams is 1.1 while the network fractal dimension is 2.0, since the patterns completely fill the space. Tarboton et al. (1988) asserted that if the fractal dimension is 2.0, then the fractal description of river scaling is consistent with the random topology model.

Since natural stream networks are not strictly self-similar, it should be emphasized that the fractal dimensions computed from the empirically derived exponents of the length–area relationships are only estimates of the actual fractal dimensions. Several researchers, however, use the fractal dimensions calculated in this manner as the actual fractal dimensions when attempting to investigate the utility of fractal dimension formulas (Rosso et al., 1991). All the proposed methods of determining the fractal dimensions are simply estimates of the fractal dimensions.

According to Rosso et al. (1991), Horton's laws of network composition (i.e. the Laws of Stream Numbers, Lengths, and Areas) are geometric-scaling relationships because they hold regardless of the order or resolution at which the network is viewed and because they yield self-similarity of the catchment–stream system. These laws typically hold for a wide range of scales in nature, with the exception of the largest watersheds. The Horton ratios can, thus, be employed to determine the fractal dimensions of individual streams and stream networks. Several researchers have followed this approach.

Hack (1957) suggested that the drainage density is constant throughout a watershed, or alternatively, that the overland flow distance to each stream is the same. Based upon this hypothesis, Feder (1988) derived the following relationship to relate the Horton ratios to the fractal dimension of the mainstream length which arises due to a stream's sinuosity.

$$d = 2\frac{\ln R_L}{\ln R_B} \tag{10}$$

La Barbera and Rosso (1987, 1989) proposed that the network fractal dimension can be computed as the maximum of the ratio of the logarithm of the bifurcation ratio to the logarithm of the length ratio and 1.0.

$$D = \max\left(\frac{\log R_B}{\log R_L}, 1\right) \tag{11}$$

They claimed that this equation permits values of D between 1.0 and 2.0 with the mean value falling in the range of 1.6–1.7. They also claimed that empirical results demonstrating decreasing drainage densities with increasing area imply that D should not equal 2.0.

In a published comment, Tarboton et al. (1990) referred to the fact that La Barbera and Rosso (1989) assumed that individual streams, especially first order, were linear measures with a fractal dimension of 1.0. Taking into account the effects of the individual fractal streams, Tarboton et al. derived the following formulation of the network fractal dimension.

$$D = d\frac{\log R_B}{\log R_L} \tag{12}$$

Tarboton et al. argued that, when using the stream fractal dimension of 1.14, this formulation produces network fractal dimensions closer to 2.0. They contended that the dimension should be 2.0 since, at high resolutions, one could imagine a network that drains every point and thus fills the area it drains. Tarboton et al. suggested that the phenomenon whereby the drainage density decreases with increasing area may be due to the fact that higher resolution maps are typically used when examining smaller catchments.

In a reply to the published comment by Tarboton et al. (1990); La Barbera and Rosso (1990) derived the following network fractal dimension equation which includes the sinuosity of individual streams.

$$D = \left(\frac{1}{2 - d}\right)\frac{\log R_B}{\log R_L} \tag{13}$$

La Barbera and Rosso acknowledged that this equation provides fractal dimensions very similar to those of Eq. (12) when the fractal dimension of individual streams is close to 1.0. The researchers also refer to additional research indicating that stream networks do not have fractal dimensions of 2.0.

Rosso et al. (1991) proposed the following two equations for computing the individual stream and network fractal dimensions.

$$d = \max\left(1, 2\frac{\log R_L}{\log R_A}\right) \tag{14}$$

and

$$D = \min\left(2, 2\frac{\log R_B}{\log R_A}\right) \tag{15}$$

In deriving the first equation, they relaxed Hack's (1957) suggestion regarding constant drainage density and instead opted to permit drainage density to vary inversely with area as suggested by Horton (1945). For five Italian watersheds, they computed the individual stream fractal dimensions using Eqs. (10) and (14), and the measured value determined via the box counting method. The authors also investigated data from Hjelmfelt's (1988) Missouri watersheds by estimating the d value using these two equations and comparing the results to the values which Hjelmfelt measured. They found that the fractal dimensions computed by using Eq. (14) compare quite favorably with the measured values for both the Italian and Missouri watersheds. They concluded that Eq. (10) overpredicts the fractal dimension for individual streams. In another segment of this study, they investigated the fractal characteristics of individual streams in 60 sub-basins of the Alta Liri watershed. They found that the d values estimated using Eq. (14) compare quite favorably with the fractal dimension calculated as twice the value of the empirical exponent in the length–area relationship (1.12 ± 0.08 compared to 1.16 ± 0.07, respectively). They computed d values for 30 river basins around the world using Eq. (14). For these basins, the d values range from 1.0 to 1.3 with the bulk of the values between 1.1 and 1.2. From this investigation, they concluded that the range of values observed precludes the arbitrary assignment of one particular individual stream fractal dimension to all rivers.

For the Alta Liri basin, Rosso et al. also determined the network fractal dimension via the length–area relationship and compared the result to the value determined using Eq. (15). They found close agreement between these two fractal dimensions. Both methods resulted in D values of the order of 1.9. They computed D values for 30 rivers around the world using Eq. (15). For these basins, the D values range from 1.2 to 2.0 with the majority of the values falling between 1.7 and 1.8. From this investigation, they concluded that the range of values observed precludes the arbitrary assignment of a particular network fractal dimension to all river networks.

1.4. Self-similarity of stream networks

Although the concept of self-similarity has been briefly discussed previously, it is worthwhile to focus specifically on the meaning of self-similarity of a stream network. Much of the present discussion is derived from the work of Tokanaga (1978) and Peckham (1995). For stream networks, self-similarity describes the relationship between a stream and its side channels of lower order (i.e. those channels of lower order which enter a stream from the side, not those which enter at the upstream end). In a truly self-similar network, the number of third-order streams, which enter a fourth-order stream from the side, is the same as the number of second-order streams which enter a third-order stream from the side. This relationship continues throughout the basin. In a self-similar or scale invariant network, one cannot decipher a second-order to third-order confluence from a first-order to a second-order confluence. As a consequence of self-similarity, if one prunes the exterior links off of a fifth-order self-similar tree (SST), the resulting tree is topologically identical to the fourth-order sub-trees in the original network (Peckham, 1995). In other words, the pruned tree, which is now fourth order, will have the same number of first-, second-, and third-order side channels as the fourth-order sub-tree in the unpruned network.

According to Tokanaga and Peckham, a stream network can be thought of as a collection of streams in which every stream of order ω has two or more upstream tributaries of order $(\omega - 1)$ and $T_{\omega,k}$ side tributaries of order k, where ω varies from 2 to Ω, and k varies from 1 to $(\omega - 1)$. The numbers $T_{\omega,k}$ can be arranged in a square, lower triangular 'generator' matrix as shown below.

$$\begin{bmatrix} T_{2,1} & 0 & 0 & \cdots & 0 \\ T_{3,1} & T_{3,2} & 0 & \cdots & 0 \\ T_{4,1} & T_{4,2} & T_{4,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ T_{\Omega,1} & T_{\Omega,2} & T_{\Omega,3} & \cdots & T_{\Omega,\Omega-1} \end{bmatrix} \tag{16}$$

The generator matrix partially specifies a tree of order Ω. A SST is one in which $T_{\omega,\omega-k} = T_k$ for all ω. T_k is a number which depends on k, not ω, and gives the number of side tributaries of order $\omega - k$. If the numbers on each diagonal of the generator matrix are constant, then the network is a SST. Different networks may have identical generator matrices because the tributaries may enter the main stream from either the right or the left and tributaries of different orders may be interspersed differently.

For natural stream networks, the $T_{\omega,\, k}$ values in the generator matrix are averages of the $T_{\omega,\, k}$ values for each stream of order ω. The generator matrix provides an immediate indication of basin self-similarity or lack thereof. By simply noting if each diagonal is constant or varied, one can tell if the network is, in general, self-similar.

1.5. Self-affine nature of watersheds

Nikora and Sapozhnikov (1993) claimed that most stream networks are self-affine fractals. In other words, the scaling exponents in the longitudinal and transverse directions are not equal. The term self-affine implies that two parameters are required to describe the fractal nature of the network. They began with the following two relationships which relate the characteristic longitudinal and transverse sizes of the network, l and w, to the total stream length, Z.

$$l \propto Z^{v_l} \tag{17}$$

$$w \propto Z^{v_w} \tag{18}$$

In these equations, the exponents v_l and v_w are the scaling exponents in the longitudinal and transverse directions, respectively. If the scaling exponents are equal, then the network is self-similar, and the fractal dimension of the network, D, is numerically equal to the inverse of the value of either scaling exponent. If the exponents are not equal, the network is self-affine, and the lacunarity dimension, $D_{G,}$ is a substitute for the fractal dimension. The lacunarity dimension is computed according to the following equation.

$$D_G = \frac{2}{v_l + v_w} \tag{19}$$

Nikora and Sapozhnikov (1993) interpreted the ratio of the exponents, v_w/v_l, as Hurst's exponent, H, as suggested by Mandelbrot (1986). Hurst's exponent characterizes the degree of self-affinity of the network. The greater the departure of H from 1.0, the more self-affine the network. If the network is self-similar, then D is equal to D_G. If Eqs. (17) and (18) are combined, the following relationship results.

$$w \propto l^{v_w/v_l} \tag{20}$$

If the catchment area is proportional to w multiplied by l ($A \propto lw$) then the following two relationships result from Eq. (20).

$$l \propto A^{v_l/(v_l+v_w)} \tag{21}$$

and

$$w \propto A^{v_w/(v_l+v_w)} \tag{22}$$

Substituting Eq. (17) into Eq. (21) produces the following relationship.

$$Z \propto A^{1/(v_l+v_w)} \tag{23}$$

Nikora et al. (1993) investigated the self-affine relationship for individual streams represented by Eq. (24) in which L is the mainstream length and v'_l is the scaling exponent of the channel pattern in the longitudinal direction.

$$l \propto L^{v'_l} \tag{24}$$

Combining this relationship with Eq. (21) results in Eq. (25).

$$L \propto A^{v_l/(v_l+v_w)v'_l} \tag{25}$$

Table 1
Equations used for individual stream fractal dimension computations

Estimate	Authors	Equation	Equation no.
$d1$	Mandelbrot (1983)	$L \propto A^{d/2}$	6
$d2$	Takayasu (1990)	$L' \propto A^{1/2}$	5
$d3$	Feder (1988)	$d = 2\frac{\ln R_L}{\ln R_B}$	10
$d4$	Rosso et al. (1991)	$d = \max\left(1, 2\frac{\log R_B}{\log R_A}\right)$	14

Since Nikora et al. determined that v_l' is nearly 1.0, this relationship can be viewed as a self-affine interpretation of Hack's law with:

$$\beta = \frac{v_l}{v_l + v_w} = \frac{1}{1 + H} \tag{26}$$

Additionally Eq. (23) can be interpreted as a total length–area relationship with

$$Z \propto A^{\epsilon} = A^{D_G/2} \qquad \epsilon = \frac{1}{v_l + v_w} = \frac{D_G}{2} \tag{27}$$

From these relationships, it is clear that v_l and v_w can be determined by determining the slopes of the mainstream length vs. area plot (β) and the total stream length vs. area plot (ϵ) and then simultaneously solving Eqs. (26) and (27).

Table 2
Equations used for network fractal dimension computations

Estimate	Authors	Equation	Equation no.
$D1$	Mandelbrot (1983)	$Z \propto A^{D/2}$	8
$D2$	Nikora et al. (1993)	$Z \propto A^{D_G/2}$	23 and 27
$D3$	La Barbera and Rosso (1987, 1989)	$D = \max\left(\frac{\log R_B}{\log R_L}, 1\right)$	11
$D4$	Rosso et al. (1991)	$D = \min\left(2, 2\frac{\log R_B}{\log R_A}\right)$	15
$D5$	Tarboton et al. (1990)	$D = \max\left(d\frac{\log R_B}{\log R_L}\right)$	12
$D6$	La Barbera and Rosso (1990)	$D = \left(\frac{1}{2-d}\right)\frac{\log R_B}{\log R_L}$	13

For the 60 sub-basins of the Alto Lira basin discussed in Rosso et al. (1991), Nikora and Sapozhnikov determined the following values of the two scaling parameters: $v_l = 0.61$ and $v_w = 0.44$. As these values are unequal, they concluded that the network is self-affine rather than self-similar. The authors computed a lacunarity dimension equal to 1.90. Note that this is numerically equal to the network fractal dimension computed by Rosso et al. Nikora and Sapozhnikov found similar values of the scaling exponents and the lacunarity dimension for rivers in Moldova. They concluded that their scaling exponent estimates indicate that rivers exhibit self-affine fractal behavior.

1.7. Summary of fractal dimension computations

Tables 1 and 2 provide a summary of the equations suggested by different authors for individual stream and network fractal dimension computations.

1.8. Objectives of the present study

The focus of this study is to investigate the fractal nature of Indiana stream networks. In doing so, several questions are examined and discussed. These investigations fall under the following three categories.

1. Differences in the fractal characteristics computed by using dense and sparse streams network data.
2. The variability of fractal dimension estimates in the study watersheds.
3. The self-affine nature of the study watersheds.

Several aspects are investigated under the first category. First of all, fractal characteristics of the individual streams and stream networks are investigated. The ranges of fractal dimensions obtained for the individual streams and stream networks, and the agreement of these ranges with those suggested in the research literature are studied. The implications of the values of the fractal dimensions about the streams and stream networks are considered in the second category. Another aspect of the study is whether some techniques for fractal dimension computations are better than others. For the third category, the focus of the study is whether the selected networks are self-similar or self-affine.

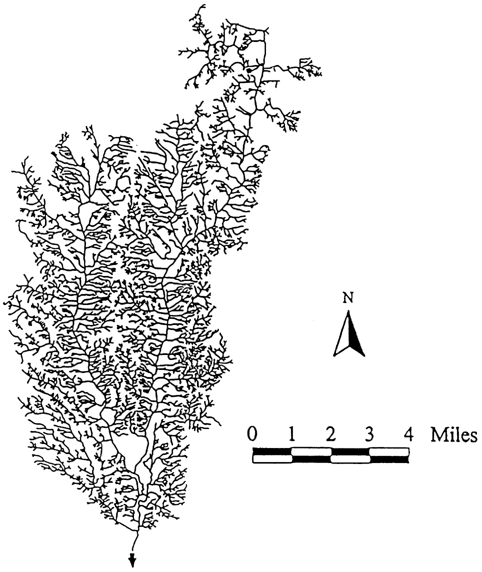

Fig. 1. Stream Network Map for the Whitewater River near Hagerstown.

The individual stream and network fractal dimensions were computed by using the formulas listed in Tables 1 and 2. It is noted that all of these techniques, with the exception of *D*2, assume self-similarity of stream networks.

To investigate the self-affinity characteristics of watersheds, the self-similarity and self-affinity characteristics were analyzed as recommended by Peckham (1995) and Nikora and Sapozhnikov (1993), respectively. Generator matrices were developed for each watershed and the diagonals were inspected to determine if the networks are self-similar. To investigate self-affinity, the longitudinal and transverse scaling exponents were computed for each network. Hurst's exponents were determined to quantify the degree of self-affinity of the self-affine networks.

Table 3
Individual stream fractal dimensions for the study watersheds

No.	Watershed	*d*1	*d*2	*d*3	*d*4
1	Plum Creek	1.18	1.05	1.15	1.08
2	Little Mississinewa River	1.18	1.04	1.22	1.00
3	Rimmell Branch	1.18	1.07	1.20	1.00
4	Galena River	1.18	1.06	1.24	1.04
5	Back Creek	1.18	1.04	1.05	1.01
6	Indian-Kentuck Creek	1.18	1.03	1.09	1.07
7	White Lick Creek	1.18	1.07	1.10	1.00
8	Pine Creek	1.18	1.07	1.13	1.00
9	Lost River	1.18	1.04	1.16	1.08
10	Iroquois River	1.18	1.06	1.23	1.00
11	Little Pine Creek	1.18	1.07	1.20	1.10
12	Whitewater River	1.18	1.05	1.14	1.07
	Average	1.18	1.05	1.16	1.04
	Standard deviation	0.00	0.014	0.058	0.038

2. Data used in the study

All the computations and analyses in this study use data from 12 rural Indiana (USA) watersheds. These watersheds range in size from 7.78 to 150.4 km^2 and are located throughout the state. The stream network data for this study were obtained from the *Atlas of County Drainage Maps — Indiana* (Joint Highway Research Project, 1959). The paper map data were converted to a digital vector form through scanning and digitizing procedures described in Schuller et al. (1999).

These drainage network data were developed at Purdue University by analyzing air photo maps. Each map depicts a very fine drainage network as well as other features. The level of stream network detail on these maps is orders of magnitude higher than that given by standard USGS quadrangle maps.

A disadvantage in these maps is the absence of elevation data. Consequently the sub-watersheds were delineated by using the drainage maps. The sub-basin areas and mainstream lengths were delineated by using these drainage network maps. An example of the drainage network map is shown in Fig. 1.

3. Individual stream fractal dimension results

Fractal dimensions for the individual streams were computed by using several methods described earlier and listed in Table 1. The results are tabulated in Table 3. The values of the stream fractal dimensions vary from 1.0 to 1.24. The validity of this range and the various computational methods are discussed below.

The mainstream length and area relationships are shown in Fig. 2. The length (L) vs. area (A) relationships are represented as $L = \alpha A^{\beta}$. The coefficients derived for each of the 12 basins given in Table 3 ranged from 1.43 to 1.65 whereas the exponents β

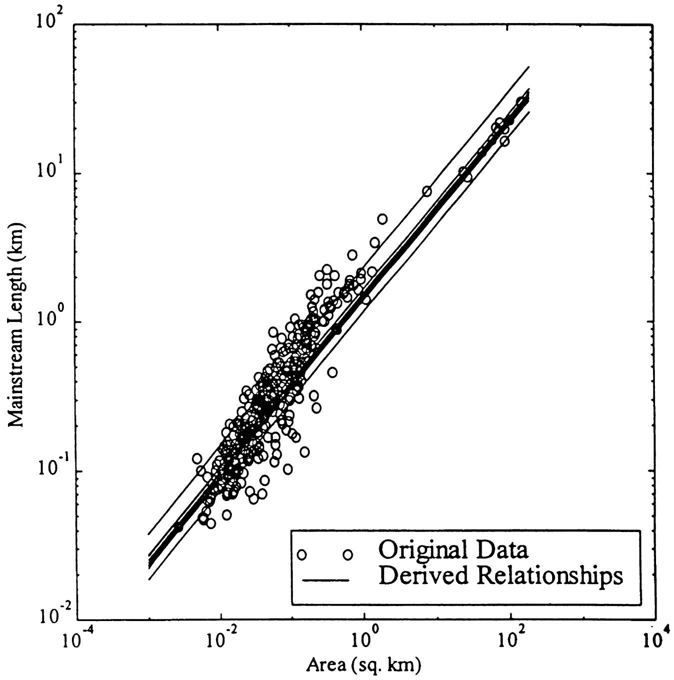

Fig. 2. Plot of Mainstream Length vs. Area Relationships. There are 12 lines, one each per watershed fitted to the data. The original data were collected from first order to the maximum order of the basin.

varied very narrowly from 0.58 to 0.61. Consequently these exponent values were averaged to 0.592 and the lines were plotted as in Fig. 2. The scatter of the data in Fig. 2 clearly allows this approximation.

For each study watershed, the fractal dimension computed from the slope of the mainstream length versus area plot (estimate $d1$) was 1.18. This result was expected since the same exponent (0.592) was used in all cases to compute the sub-basin areas from the sub-basin mainstream lengths. The resulting fractal dimension is close to the fractal dimension of 1.14 suggested in the literature by several researchers. The individual stream fractal dimensions were also determined for each watershed as twice the slope of the mainstream length versus Euclidean mainstream length plot (estimate $d2$). By the L versus L' computation, the resulting $d2$ values fall into a narrow band ranging from 1.03 to 1.07, with an average value of 1.05. Two facts about these results are reassuring. First, in this case, there exists variability in the $d2$ values from watershed to watershed. Although such a variability exists for $d1$ also, it has been suppressed in this case by selecting the same exponent value of 0.592. It is not expected that the individual stream fractal dimension would be constant from watershed to watershed, since some watersheds contain short straight streams while others have long serpentine streams. Second, this method is independent of underlying techniques which may introduce numerical error. Both L and L' are measured directly from the digitized data. There are 12 lines, one each per watershed fitted to the data. The original data were collected from first-order basins to the maximum order of the basin.

Furthermore, this fractal value computation does not depend on the Horton Ratios, which are derived from regression lines through scattered data. Accordingly, it is believed that estimating the fractal dimension in this manner has significant merit. Interestingly, the fractal dimension values computed in this manner are significantly lower than the value of 1.14 suggested in the literature.

Individual stream fractal dimensions were also computed using estimates $d3$ and $d4$. Both of these estimates use equations based on the Horton Ratios. The resulting $d3$ values display a great deal of

Table 4
Steam network fractal dimensions for the study watersheds

No.	Watershed	$D1$	$D2$	$D3$	$D4$	$D5$ ($w/d = d3$)	$D5$ ($w/d = d4$)	$D5$ ($w/d = 1.14$)	$D6$ ($w/d = d3$)	$D6$ ($w/d = d4$)	$D6$ ($w/d = 1.14$)
1	Plum Creek	1.56	1.56	1.73	1.87	2.00	1.87	1.98	2.00	1.89	2.00
2	Little Mississinewa River	1.50	1.50	1.64	1.61	2.00	1.64	1.87	2.00	1.64	1.90
3	Rimmell Branch	1.47	1.47	1.67	1.55	2.00	1.67	1.91	2.00	1.67	1.95
4	Galena River	1.50	1.50	1.61	1.68	2.00	1.68	1.83	2.00	1.68	1.87
5	Back Creek	1.67	1.67	1.91	1.93	2.00	1.93	2.00	2.00	1.93	2.00
6	Indian-Kentuck Creek	1.71	1.71	1.83	1.96	2.00	1.96	2.00	2.00	1.97	2.00
7	White Lick Creek	1.54	1.54	1.82	1.71	2.00	1.82	2.00	2.00	1.82	2.00
8	Pine Creek	1.49	1.49	1.78	1.75	2.00	1.78	2.00	2.00	1.78	2.00
9	Lost River	1.49	1.49	1.73	1.87	2.00	1.87	1.97	2.00	1.88	2.00
10	Iroquois River	1.52	1.52	1.63	1.51	2.00	1.63	1.85	2.00	1.63	1.89
11	Little Pine Creek	1.49	1.49	1.67	1.84	2.00	1.84	1.91	2.00	1.86	1.95
12	Whitewater River	1.57	1.57	1.76	1.88	2.00	1.88	2.00	2.00	1.89	2.00
	Average	1.54	1.54	1.73	1.76	2.00	1.80	1.94	2.00	1.80	1.96
	Standard deviation	0.072	0.072	0.088	0.145	0.00	0.111	0.063	0.00	0.115	0.048

variability from watershed to watershed. They vary from 1.05 to 1.24, with an average of 1.16. Interestingly, the average value falls very close to the suggested value of 1.14 and to the value of 1.18 from the mainstream length–area relationships. The average $d4$ value is 1.04. Intrinsic to the underlying equation is the lower bound $d4$ value of 1.0. This lower bound limitation resulted in five of the twelve watersheds having individual stream fractal dimension values of 1.0. In other words, the results indicate that the individual streams in these cases are not fractals. This conclusion is debatable, as all of the other techniques produce fractal dimensions which imply that the individual streams are indeed fractals. Since the Horton Ratios are determined by fitting a regression line through an imperfectly aligned scatter of points, the accuracy of fractal dimension estimates which use the Horton Ratios, such as $d3$ and $d4$, is questionable.

4. Network fractal dimension results

Network fractal dimensions were determined by a variety of methods described previously in this study and listed in Table 2. The entire resulting network fractal dimension estimates are listed in Table 4. First, D values were computed as twice the slope of the sub-basin total stream length versus sub-basin area plots (estimate $D1$). The fractal dimensions were computed from plots which did not include first-order streams. The first-order streams were neglected because including them introduced a downward bias in the resulting $D1$ values. The downward bias occurs because first-order main-stream lengths and total stream lengths are the same, and thus the total stream length and watershed area for these sub-basins are related through the exponent 0.592, as discussed in Schuller and Rao (1999). The network fractal dimension values computed according to estimate $D1$ vary between 1.47 and 1.71 with a mean value of 1.54. With the exceptions of Back Creek, Indian-Kentuck Creek, and the Whitewater River, most of the $D1$ values are approximately 1.5. Interestingly, these three watersheds have the highest drainage densities, and thus are expected to have higher fractal dimensions as they are more nearly 'space-filling'. Neglecting these three basins, there is little variability in the $D1$ values from basin to basin. The scatter of the data in Fig. 2 and

Table 5
Longitudinal and transverse scaling exponents, Lacunarity dimensions, and Hurst's exponents for the study watersheds

Watershed	ν_l	ν_w	$D2 = D_G$	H
Plum Creek	0.76	0.52	1.56	0.684
Little Mississinewa River	0.79	0.54	1.50	0.684
Rimmell Branch	0.80	0.55	1.47	0.688
Galena River	0.79	0.54	1.50	0.684
Back Creek	0.71	0.49	1.67	0.690
Indian-Kentuck Creek	0.69	0.48	1.71	0.696
White Lick Creek	0.77	0.53	1.54	0.688
Pine Creek	0.79	0.55	1.49	0.696
Lost River	0.79	0.55	1.49	0.696
Iroquois River	0.78	0.54	1.52	0.692
Little Pine Creek	0.79	0.55	1.49	0.696
Whitewater River	0.75	0.52	1.57	0.693
Average	0.77	0.53	1.54	0.691
Standard deviation	0.033	0.023	0.072	0.005

the fit of the regression lines in this work is similar to that observed in the work of Rosso et al. (1991).

The lacunarity dimension, likewise termed D_G or estimate $D2$, was also computed for the study watersheds. The lacunarity dimension is numerically equal to the fractal dimension computed from the *Z–A* plot; however, it carries a different name to emphasize the fact that it is based on self-affinity rather than self-similarity. The longitudinal and transverse scaling exponents for the study watersheds are listed in Table 5. As shown in this table, the two scaling exponents are not equal in any of the cases. The average value of ν_l is 0.77, and the average value of ν_w is 0.53. Consequently, it can be concluded that the study networks are self-affine rather than self-similar. This result is discussed in greater detail later in this paper.

Fractal dimensions computed according to estimates $D3$ and $D4$ also exhibit great variability from basin to basin. The $D3$ values fall between 1.63 and 1.91 with a mean value of 1.73. La Barbera and Rosso (1989) suggested that the D estimates from this method would fall between 1.5 and 2.0 with a mean value between 1.6 and 1.7. In light of their predictions, the mean value of 1.73 seems reasonable. The resulting $D4$ values are between 1.51 and 1.96 with a mean value of 1.76.

For each watershed, three network fractal dimension values were computed according to estimate $D5$. These three values were determined by using three different estimates of the individual stream fractal dimension: $d3$, $d4$, and 1.14. Using $d3$ as the d estimate, $D5$ values of 2.0 resulted for all watersheds due to algebraic cancellations. Next, d was estimated by $d4$, and a range of $D5$ values varying from 1.63 to 1.96 with a mean of 1.80 resulted. Finally, the d value of 1.14, as suggested in previous literature, was used in the $D5$ equation. This combination resulted in $D5$ values ranging from 1.83 to 2.00 with a mean of 1.94. With d equal to 1.14, the network fractal dimension often took a value of 2.0 as set by the upper bound limitation. The network fractal dimensions computed using $d4$ as the estimate of the individual stream fractal dimension exhibit the highest variability from watershed to watershed.

For each watershed, the equation for $D6$ was used to determine three values of the network fractal dimension corresponding to three different estimates of d: $d3$, $d4$, and 1.14. First, d was approximated by $d3$, which resulted in $D6$ values limited by the upper bound of 2.0 for all twelve watersheds. Second, d was computed according to estimate $d4$, and a range of $D6$ values varying from 1.63 to 1.97 with a mean of 1.80 resulted. Finally, the d value of 1.14 from previous literature was used in the $D6$ computation. This combination resulted in $D6$ values ranging from 1.87 to 2.00 with a mean of 1.96. With d equal to 1.14, the $D6$ value was limited by the upper bound of 2.0 for several of the study watersheds. The network fractal dimension values computed using the d value estimated by $d4$ exhibit the highest variability from watershed to watershed.

It should be emphasized that $D5$ and $D6$ are nearly identical for a given watershed when the same individual stream fractal dimension estimate was used. This is not surprising when the individual terms in the two underlying equations are examined while considering that the d values are near unity. Significantly more important in determining the resulting D value was the selected method of estimating the d value. When $d4$ values were used, the resulting network fractal dimensions are on the order of 1.80. If one of the other two means of estimating the d values was employed, then the resulting network fractal dimensions are on the order of 1.95–2.00. Since $d4$ appears to be a low estimate of the individual stream fractal dimension, its use in $D5$ and $D6$ consequently produces lower estimates of the network fractal dimension.

Table 6
Box counting data for the Plum Creek network

Grid spacing	Number of boxes intersected
0.065	1211
0.165	390
0.412	114

Finally, the network fractal dimension for Plum Creek was computed using the box counting technique. The selected grid spacings and the number of boxes intersected are listed in Table 6. When these points were plotted on a log–log plot, the best fit regression line had a slope of -1.28, thus indicating a network fractal dimension of 1.28. The data points plotted nearly on a straight line; however, the low valued fractal dimension is surprising. The next lowest D value determined for this watershed is 1.55 and was found via the total length–area relationship. The other methods yielded significantly higher values.

5. Self-similarity and self-affinity characteristics

Peckham (1995) proposed that the generator matrix for a stream network is an indicator of the network's self-similarity or lack thereof. If each diagonal of the generator matrix contains constant values, then the network is self-similar. Eqs. (30)–(34) are the generator matrices for 5 of the 12 study watersheds. After examination of the values in the generator matrices, it is evident that the values along each diagonal are quite variable, therefore indicating a lack of self-similarity among the twelve study watersheds. The following paragraphs describe a few of the resulting matrices in greater detail.

Many interesting network qualities can be observed by carefully examining the generator matrices. For instance, in the Plum Creek generator matrix, the second diagonal has values of 2.5 and 19.0. This fact indicates that the network is definitely not self-similar. Examining the structure of the network sheds additional light on these numerical results. There are only two short third-order streams at the upstream end of the network. As such, very few lower order streams flow into these third-order streams from the sides. On the other hand, these two streams drain into an exceptionally long fourth-order stream which receives numerous first- and second-order side tributaries. The fact that the Plum Creek network is not "balanced" serves as a strong suggestion of its lack of self-similarity.

Often the generator matrices indicate an inordinate number of lower order streams feeding into the highest order stream. One example is the Pine Creek watershed. In the Pine Creek matrix, the numbers in the bottom row are considerably higher than other numbers on their respective diagonals. Again this indicates that the highest order stream is relatively long, and thus has more tributaries, on average, than the lower order streams in the network.

$$\text{Plum Creek Generator} = \begin{bmatrix} 1.3 & 0 & 0 \\ 2.5 & 1.0 & 0 \\ 27.0 & 19.0 & 0 \end{bmatrix} \tag{28}$$

Back Creek Generator

$$= \begin{bmatrix} 1.2 & 0 & 0 & 0 & 0 \\ 4.0 & 1.2 & 0 & 0 & 0 \\ 8.4 & 3.8 & 1.0 & 0 & 0 \\ 3.3 & 2.7 & 0.6 & 0.1 & 0 \\ 33.0 & 30.0 & 9.0 & 8.0 & 5.0 \end{bmatrix} \tag{29}$$

Indian-Kentuck Creek Generator

$$= \begin{bmatrix} 1.5 & 0 & 0 & 0 & 0 \\ 3.4 & 1.3 & 0 & 0 & 0 \\ 7.9 & 3.3 & 0.9 & 0 & 0 \\ 16.8 & 6.3 & 3.0 & 0.8 & 0 \\ 68.0 & 42.0 & 17.0 & 4.0 & 2.0 \end{bmatrix} \tag{30}$$

Pine Creek Generator

$$= \begin{bmatrix} 1.1 & 0 & 0 & 0 & 0 \\ 2.5 & 1.0 & 0 & 0 & 0 \\ 5.8 & 2.0 & 0.6 & 0 & 0 \\ 5.3 & 1.3 & 1.0 & 0 & 0 \\ 42.0 & 16.0 & 8.0 & 11.0 & 1.0 \end{bmatrix} \tag{31}$$

Iroquois River Generator

$$= \begin{bmatrix} 0.8 & 0 & 0 & 0 & 0 \\ 1.1 & 0.5 & 0 & 0 & 0 \\ 2.0 & 1.3 & 0.7 & 0 & 0 \\ 4.5 & 4.5 & 2.0 & 1.0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (32)$$

Interestingly, the generator matrix for the Iroquois River indicates that only one stream flows into the sixth-order stream from the side. Accordingly, one can identify this sixth-order stream as an exceptionally short stream immediately upstream of the outlet into which one first-order side tributary flows. The sixth-order stream for this network is presumably very short because the watershed outlet is determined by the gage location. Had the gage been farther downstream, the sixth-order stream may be longer than the average length of the fifth-order streams as would be expected. The generator simply reemphasizes the shortness of the sixth-order stream by indicating that, due to its length, only one tributary flows into it.

Nikora and Sapozhnikov (1993) proposed that networks be tested for self-similarity by comparing the resulting scaling exponents in the longitudinal and transverse directions. If these two scaling exponents prove to be equal, then the network is self-similar. If they are not equal, then the network is self-affine. The scaling exponents, v_l and v_w, for the twelve study watersheds are listed in Table 5. As shown in the table, the scaling exponents are not equal in any of the cases. The average value of v_l is 0.77, and the average value of v_w is 0.53. These results indicate that the study networks are self-affine rather than self-similar. The Hurst's exponents vary from 0.684 to 0.696 with an average value of 0.691. Since the values are substantially different than 1.0, the Hurst's exponent indicates that the degree of self-affinity in the study networks is significant. Nikora and Sapozhnikov (1993) found values of the longitudinal and transverse scaling exponents to be on the order of 0.62 and 0.45 and Hurst's exponent values varying from 0.69 to 0.76. Their study watersheds, however, were substantially larger and less detailed than those of this study.

In a recent study Veneziano and Nieman (2000a,b) have found that several existing geomorphological relationships should be modified because they were obtained by using quantities involved or inappropriate techniques. These methods proposed by Veneziano and Nieman (2000a,b) may resolve some of the disparities observed in the present study.

6. Conclusions

To summarize the results of the individual stream fractal dimension computations is difficult due to the range of the results. It is reasonable to conclude that the individual streams are indeed fractal in nature due to their sinuosity. Only one of the computational procedures (estimate $d4$) gave evidence to the contrary. Deciding whether the fractal dimension is on the order of 1.05 or 1.15 is impossible. The computation based on the length–Euclidean length relationship ($d2$) indicated that the fractal dimension is on the order of 1.05. The length–area relationship ($d1$) and the results of estimate $d3$ suggested fractal dimensions on the order of 1.15. The shortcoming of the length–area based fractal dimension is the fact that the controlling exponent was determined from a limited set of measured length and approximated area data which was then applied to determine the sub-basin areas. The length–Euclidean length relationship is independent of such a derived relationship. All of the selected methods assume self-similarity of the individual streams. Although this has not been addressed in detail, the limited scatter of the data in the L–L' plots serves as evidence of individual stream self-similarity. Additionally, estimate $d3$ assumes that drainage density is constant with area. As discussed below, this assumption is invalid. Once again, it should be emphasized that all of these results are merely estimates of the actual fractal dimensions of individual streams.

In summary, the Z–A relationships ($D1$) provided network fractal dimensions on the order of 1.54, estimates $D3$ and $D4$ provided fractal dimensions on the order of 1.75, and estimates $D5$ and $D6$ resulted in fractal dimensions on the order of 1.8 or 1.95–2.00, depending on the individual stream fractal dimension estimates used in the computations. The box counting method resulted in a network fractal dimension of 1.28 for the Plum Creek watershed. It should be emphasized that both the length–area relationships and the Horton Ratio based computations presume that the stream networks are self-similar. Interestingly, the Indiana stream networks proved to be self-affine rather than self-similar, and thus perhaps representing the fractal nature with two scaling exponents is a better characterization of the fractal structure than a single network fractal dimension.

If networks can be described by a fractal dimension or a lacunarity dimension, should researchers expect this dimension to be 2.0? A network fractal dimension of 2.0 implies topological randomness. One would expect that the effects of geological, topological, and hydrological restraints reduce the ability of the stream network to develop as a purely branching process, and thus lower the network fractal dimension to something less than 2.0 (LaBarbera and Rosso, 1989). The following argument can be used to support this hypothesis. As mentioned earlier, Mandelbrot (1983) has suggested that the total length of a river system should increase like the power of $D/2$ of its drainage area. In other words, β is equal to $D/2$. This relationship was stated as Eq. (10), and is restated below as Eq. (33).

$$Z \propto A^{D/2} \tag{33}$$

If drainage density is defined as the ratio of total stream length to drainage area, the following relationship can be obtained by dividing both sides of the above relationship by the basin area,

$$\mathrm{DD} \propto A^{D/2-1} \tag{34}$$

where 'DD' is drainage density and is different from $D*D$. Since $D/2-1$ is negative, the drainage density decreases like the power of $1-D/2$ of the drainage area. From inspection of the above relationship, D equal to 2.0 implies that the drainage density is independent of basin area (La Barbera and Rosso, 1989). For the study watersheds, it was found that drainage density decreases with increasing area (for smaller basins within the overall basin); it is not constant. The fact that drainage density is not independent of basin area for the study watersheds indicates that the network fractal dimensions of these watersheds should not be equal to 2.0.

The following conclusions are presented based on the research discussed in this paper:

1. Different methods for determining individual stream and network fractal dimensions result in wide ranges of fractal dimension values. As the 'correct' values for the individual stream and network fractal dimensions are unknown, it is not possible to determine which of the proposed methods provide the best results.
2. The proposed methods for determining network fractal dimensions are founded on the assumption that the stream networks are self-similar fractal objects. The present research indicates that the study networks are self-affine rather than self-similar. As such, the self-similarity assumption used in developing the computational methods is highly questionable, and thus the results based on this assumption are also suspect.
3. As the networks are self-affine, the two parameter fractal characterization suggested by Nikora and Sapozhnikov (1993) is a better representation of the fractal structure of watersheds.

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