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A non-linear combination of the forecasts of rainfall-runoff models by the first-order Takagi–Sugeno fuzzy system

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Abstract

With a plethora of watershed rainfall-runoff models available for flood forecasting and more than adequate computing power to operate a number of such models simultaneously, we can now combine the simulation results from the different models to produce the combination forecasts. In this paper, the first-order Takagi–Sugeno fuzzy system is introduced and explained as the fourth combination method (besides other three combination methods tested earlier, i.e. the simple average method (SAM), the weighted average method (WAM), and the neural network method (NNM)) to combine together the simulation results of five different conceptual rainfall-runoff models in a flood forecasting study on eleven catchments. The comparison of the forecast simulation efficiency of the first-order Takagi–Sugeno combination method with the other three combination methods demonstrates that the first-order Takagi–Sugeno method is just as efficient as both the WAM and the NNM in enhancing the flood forecasting accuracy. Considering its simplicity and efficiency, the first-order Takagi–Sugeno method is recommended for use as the combination system for flood forecasting.

Keywords: Rainfall-runoff modeling; Flow forecasting; Forecast combination; Takagi–Sugeno fuzzy system

1. Introduction

To date, a wide variety of rainfall-runoff models have been developed and applied for flood forecasting (Franchini and Pacciani, 1991; Singh, 1995). These rainfall-runoff models encompass a broad spectrum of more or less plausible descriptions of rainfall-runoff relations and processes, ranging from the primitive empirical black-box models, such as the unit-graph method (Sherman, 1932), to the lumped conceptual models, such as the Xinanjiang model (Zhao et al., 1980), to the semi-distributed models, such as the TOPMODEL (Beven and Wood, 1983), to the very complicated physically based distributed models, such as the SHE model (Abbott et al., 1986a,b).

Regardless of complexity and sophistication, however, no single model has been found to work satisfactorily for simulating and forecasting all flood events in all kinds of watersheds. The regular emergence of new models, across the whole spectrum of models, is testament to the fact that such a single superior model does not yet exist, and indeed will never be produced, despite continuing advances and enhancing of our modeling techniques (Beven, 1996a,b). As noted ruefully by O'Connor (1995), every model has its 'plateau of maximum efficiency', which falls substantially short of perfection, and even when we do get it right (i.e. achieve a good fit) it 'can

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be right for the wrong reasons'. The only possible reason for this situation is that the true runoff-rainfall processes of a watershed are time varying and involve a spectrum of different runoff generation mechanisms and are never as unchanging as assumed by any single preconceived rainfall-runoff model (Beven and Binley, 1992). Assuming that each single model can best describe only one or more particular stages, phases, or mechanisms of the rainfall-runoff processes, it is only to be expected that the discharge estimates which are obtained by combining the results from a number of different models together through some appropriate weighting procedures are more comprehensive and accurate in representing the response of the catchment to rainfall than those from any single model used in the combination. The above concept provides the main justification and the initiative for combining the flood forecasts from the different rainfall-runoff models, as described in this paper, as it was also in the paper of Shamseldin et al. (1997).

Mathematically, if we have p different rainfall-runoff models, this combination process is generally expressed as (Shamseldin et al., 1997)

$$\hat{Q}_{c,i} = F(\hat{Q}_{1,i}, \hat{Q}_{2,i}, \ldots, \hat{Q}_{p,i}), \quad (1)$$

where $\hat{Q}_{1,i}, \hat{Q}_{2,i}, \ldots, \hat{Q}_{p,i}$ are the forecasts of p different models at the time step i, respectively, and $\hat{Q}_{c,i}$ is the combination forecast at time i, $F(\cdot)$ being the combining function or method. A schematic diagram of the process of combining different rainfall-runoff models is shown in Fig. 1.

The work and research on the combination of forecasts from different models or methods was pioneered mainly by Reid (1968) and by Bates and Granger (1969), and other substantial works include those of Dickson (1973, 1975), Newbold and Granger (1974), Thompson (1976), etc. Although Clemen (1989) has cited many studies to show the advantages of combining forecasts in fields as diverse as financial management, statistics, and weather forecasting, it seems that Shamseldin et al. (1997) were the first to introduce the concept of model combination for flood forecasting in hydrology. More recent published work on combination forecasting in hydrology is that of See and Openshaw (2000), who used four approaches (the average model, a Bayesian approach, and two fuzzy models) to construct the hybrid model for river level forecasting.

Shamseldin et al. (1997) have examined three different combination methods in the context of flood forecasting, namely, the simple average method (SAM), the weighted average method (WAM), and the neural network method (NNM). Five rainfall-runoff models, also applied in the present study (see Section 6), were selected for inclusion in the combination, namely, the naïve simple linear model (SLM), the linear perturbation model (LPM), the linearly varying gain factor model (LVGFM), the constrained linear systems with a single threshold (CLS-T), and the soil moisture accounting and routing procedure (SMAR) (see also Section 6). The simulation results for the eleven watersheds have shown that in general the flood estimates obtained from the above

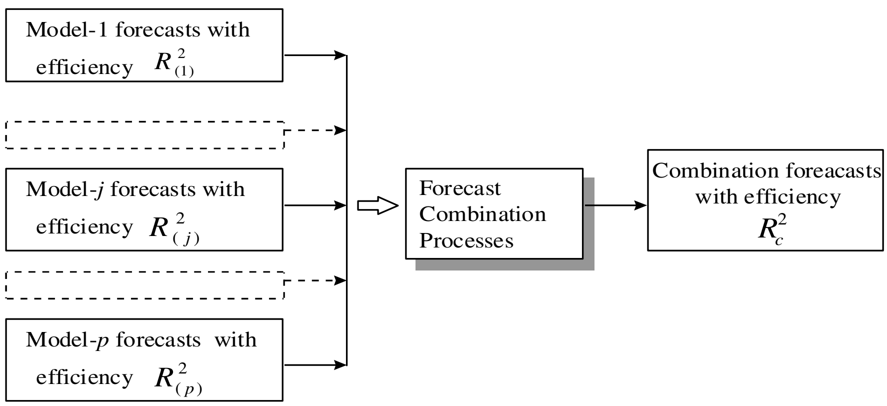

Fig. 1. Schematic diagram for the procedure of combining forecasts of different rainfall-runoff models ($R_{(j)^2}$ denotes the Nash–Sutcliffe simulation efficiency of the Model-p, and R_c^2 denotes the Nash–Sutcliffe simulation efficiency of the combination method).

combination methods are better than those of the best of individual rainfall-runoff models.

In contrast to the three combination methods used by Shamseldin et al. (1997) in the hydrological context, Fiordaliso (1998) presented a quite different method to build the combination framework in the context of time series analysis, i.e. the first-order Takagi–Sugeno fuzzy system (TS1). The TS1 method was originally proposed by Takagi and Sugeno (1985) as a special class of fuzzy systems, and it has been widely used in fuzzy control systems (Driankove et al., 1993; Kruse et al., 1994; Nakamori, 1994). In this paper, the first-order Takagi–Sugeno fuzzy system is introduced and applied as a new combination framework for the forecasts of rainfall-runoff models, as an extension of the study on the three combination methods used by Shamseldin et al. (1997) in flood forecasting.

The present paper is organized in the following manner. Firstly, the basic concepts of the fuzzy theory and of the first-order Takagi–Sugeno fuzzy system are introduced. Secondly, the combination framework suggested by Fiordaliso (1998) is explained and analyzed in considerable detail. Thirdly, the forecasts of the five rainfall-runoff models for the 11 catchments, which were previously used by Shamseldin et al. (1997), are incorporated into the first-order Takagi–Sugeno fuzzy system to test its combination forecasting efficiency. Finally, conclusions on the TS1 combination system as well as the SAM, WAM, and NNM methods are presented on the basis of comparisons of the simulation results.

2. Review of the fuzzy theory and application

Since Zadeh (1965) published the fuzzy set theory as an extension of the classical set theory, the fuzzy set theory has been widely used in many fields of application, such as pattern recognition, data analysis, system control, etc. (Cannon et al., 1986; Driankove et al., 1993; Kruse et al., 1994; Klir and Yuan, 1995; Theodoridis and Koutroumbas, 1999). In hydrology also, the concept of the fuzzy theory and its applications have found expression in many papers (Bardossy et al., 1990; Beven and Binley, 1992; Franks and Beven, 1997; Franks et al., 1998; Schulz and Huwe, 1997; Dou et al., 1999; Pongracz et al., 1999; Schulz et al., 1999; See and Openshaw, 2000; Yu and Yang, 2000). The main advantages of the fuzzy applications are that the fuzzy theory is more logical and scientific in describing the properties of objects as well as relationships that are not completely known to us. Since hydrologists are still uncertain about so many aspects of the physical processes in the watershed, the fuzzy theory has proved to be a very attractive tool enabling them to investigate such problems.

The application of fuzzy theory normally includes three procedures, i.e. fuzzification, logic decision, and defuzzification (see Appendix A). Fuzzification involves the identification of the input variables and the control variable (i.e. the output), the division of both the input and the control variable into different domains; and choosing the membership function. Logic decision involves the design of the IF–THEN inference rules, the calculation of the degree of applicability of each IF–THEN rule, and the determination of the output fuzzy set. Defuzzification involves the determination of the crisp output from the fuzzy outputs of the IF–THEN inference system.

In the applications of the fuzzy system in control and forecasting, there are mainly two approaches, the first one being the Mamdani approach and the other the Takagi–Sugeno approach (Kruse et al., 1994). For the Mamdani approach, which has been used in some hydrological applications (Schulz and Huwe, 1997; Schulz et al., 1999), there are three clear procedures, i.e. fuzzification, logic decision, and defuzzification, as described in the previous paragraph. The Takagi–Sugeno approach (Takagi and Sugeo, 1985), however, does not have an explicit defuzzification procedure, or rather, it amalgamates two procedures, the logic decision and defuzzification procedures, into one composite procedure. For the Mamdani approach, the outcome of each IF–THEN rule will be a fuzzy set for the control variable, so that the step of deffuzification is indispensable in order to get a crisp value for the control variable in the final decision. However, in the Takagi–Sugeno method, the conclusion of each IF–THEN inference rule is a scalar rather than a fuzzy set for the control variable.

As far as the present authors are concerned, it would appear that the Takagi–Sugeno fuzzy system has not yet been applied in the hydrological field for flood forecast combination. Indeed, this is one of the main reasons for undertaking the present investigation of the prospects of the Takagi–Sugeno fuzzy system in this paper.

3. The first-order Takagi–Sugeno fuzzy system

The basic concepts about the fuzzy theory and its applications, such as the fuzzy set, membership functions, the domain partitions, and fuzzy IF–THEN inference rules, which have been introduced in numerous hydrological papers, are not reproduced in the body of this paper. They are however included, for the sake of completeness and for those hydrologists unfamiliar with the topic, as an appendix to this paper. A detailed explanation of the Takagi–Sugeno fuzzy system, which is quite different from the widely used Mamdani approach, begins with how to make decisions on the control variable according to the IF–THEN rules.

For the Takagi–Sugeno fuzzy system (Takagi and Sugeno, 1985), the IF–THEN control rules are given in the form of

$$R_r: \text{ IF } (x_1 \text{ is } A_r^{(1)},\ x_2 \text{ is } A_r^{(2)}, \ldots, x_p \text{ is } A_r^{(p)}) \quad \text{THEN } y_r = f_r(x_1, x_2, \ldots, x_p), \tag{2}$$

where $A_r^{(j)}$ is a fuzzy set corresponding to a partitioned domain of the input variable x_j in the rth IF–THEN rule, p the number of the input variables, $f_r(\cdot)$ a function of the p input variables, and y_r is the output of the rth IF–THEN inference rule R_r.

In the first-order Takagi–Sugeno fuzzy system (TS1), the output function $f_r(\cdot)$ is a first order polynomial of the input variables $x_1,\ldots,x_p$ and the corresponding output y_r is determined by

$$y_r = f_r(x_1, x_2, \ldots, x_p) = b_r(0) + b_r(1)x_1 + \ldots + b_r(p)x_p = b_r(0) + \sum_{j=1}^{p} b_r(j)x_j. \tag{3}$$

The final output (control variable) y of the Takagi–Sugeno fuzzy system (TS) having k IF–THEN rules is given by

$$y = \frac{\sum_{r=1}^{k} \alpha_r y_r}{\sum_{r=1}^{k} \alpha_r} = \frac{\sum_{r=1}^{k} \alpha_r f_r(x_1, x_2, \ldots, x_p)}{\sum_{r=1}^{k} \alpha_r}, \tag{4}$$

where α_r is the applicability degree of the IF–THEN rule (see Appendix A). By Eq. (4), the degree of applicability α in the Takagi–Sugeno approach is not only the measure of fulfillment of the premise in the IF–THEN rules, but also the weighting factor assigned to the corresponding scalar output on the control variable y. Hence, the final result y of the control variable is obtained as a weighted average of these scalar outputs from each IF–THEN rule.

Following the example given in the Appendix A.3, we add a decision equation of the control variable $Q^{(0)}$ into each IF–THEN rule, i.e. Eqs. (A13.1)–(A13.4), respectively, giving

$$R_1: \text{ IF } (Q^{(1)} \text{ is “low flow”},\ Q^{(2)} \text{ is “low flow”})$$

$$\text{THEN } Q_1^{(0)} = 0.8Q^{(1)} + 0.9Q^{(2)}, \tag{5.1}$$

$$R_2: \text{ IF } (Q^{(1)} \text{ is “low flow”},\ Q^{(2)} \text{ is “medium flow”})$$

$$\text{THEN } Q_2^{(0)} = 0.85Q^{(1)} + 0.8Q^{(2)}, \tag{5.2}$$

$$R_3: \textit{ IF } (Q^{(1)} \text{ is “medium flow”},\ Q^{(2)} \text{ is “low flow”})$$

$$\text{THEN } Q_3^{(0)} = 0.9Q^{(1)} + 0.85Q^{(2)}, \tag{5.3}$$

$$R_4: \text{ IF } (Q^{(1)} \text{ is “medium flow”},\ Q^{(2)} \text{ is “medium flow”})$$

$$\text{THEN } Q_4^{(0)} = 0.7Q^{(1)} + 0.95Q^{(2)}. \tag{5.4}$$

For the prior information that $Q^{(1)} = 110\ \text{m}^3\ \text{s}^{-1}$ and $Q^{(2)} = 490\ \text{m}^3\ \text{s}^{-1}$, we have determined that $\alpha_1 = 0.8775$, $\alpha_2 = 0.1225$, $\alpha_3 = 0.1$ and $\alpha_4 = 0.1$ (see Appendix A.3). By the IF–THEN rules (5.1)–(5.4), the corresponding values of $Q^{(0)}$ are: $Q_1^{(0)} = 529\ \text{m}^3\ \text{s}^{-1}$, $Q_2^{(0)} = 485.5\ \text{m}^3\ \text{s}^{-1}$, $Q_3^{(0)} = 515.5\ \text{m}^3\ \text{s}^{-1}$, and $Q_4^{(0)} =$

542.5 $m^3 s^{-1}$. So, the final result for $Q^{(0)}$ is determined according to Eq. (4), i.e.

$$Q^{(0)} = \frac{\sum_{r=1}^{4} \alpha_r Q_r^{(0)}}{\sum_{r=1}^{4} \alpha_r} = \frac{0.8775 \times 529 + 0.1225 \times 485.5 + 0.1 \times 515.5 + 0.1 \times 542.5}{0.8775 + 0.1225 + 0.1 + 0.1 = 524.6} = 524.6 \text{ m}^3 \text{ s}^{-1}. \quad (6)$$

It is obvious that the final result $Q^{(0)} = 524.6 \text{ m}^3 \text{ s}^{-1}$ is closest to $Q_1^{(0)} = 529 \text{ m}^3 \text{ s}^{-1}$, since the value of α_1 for the IF–THEN rule R_1, which is 0.8775, is far larger than the other three values of the degree of applicability, i.e. those of α_2, α_3, and α_4.

4. The first-order Takagi–Sugeno fuzzy combination framework

In Section 3, we have demonstrated by a very simple example how to implement the first-order Takagi–Sugeno approach in fuzzy control. In the same way, but in a more general context, the first-order Takagi–Sugeno approach can be employed to constitute a framework for the combination flood forecast.

Assume that there are p rainfall-runoff models simultaneously used for flood forecasting on a watershed so that we then can get the p different estimates of an unknown discharge. Denoting these p different estimates at the time step i of a discrete discharge series by $\hat{Q}_{1,i}, \hat{Q}_{2,i}, \ldots, \hat{Q}_{p,i}$, we can then construct a vector $\hat{Q}_i = [\hat{Q}_{1,i}, \hat{Q}_{2,i}, \ldots, \hat{Q}_{p,i}]^T$, where the superscript T denotes the transpose of the vector, and the corresponding combination forecast is denoted by $\hat{Q}_{c,i}$. In the terminology of the fuzzy control theory, the vector $\hat{Q}_i = [\hat{Q}_{1,i}, \hat{Q}_{2,i}, \ldots, \hat{Q}_{p,i}]^T$ is the input variable, and $\hat{Q}_{c,i}$ is control variable, i.e. the output.

Four main topics on the TS1 combination framework are now explained, i.e. the domain partition of the input variables; the design of IF–THEN rules; the determination of the degree of applicability, and the nature of the TS1 combination method.

4.1. The domain partition of the discharge series

Firstly, we should divide the discharge series of the watersheds into some different domains and, as will be discussed later, it is better not to make the partitions too fine, or it will finally introduce too many parameters. In this paper, the methods of pattern recognition are used to cluster the discharge series into several different groups, each of which is considered to be the kind of flow domain that has the arithmetic mean value of the group components as the group representative (Theodoridis and Koutroumbas, 1999). To make these domains distinguishable, each domain or group is labeled with a name or linguistic term. For instance, when we classify the whole discharge series into three different groups/domains, then we can call these groups by the linguistic terms low flow, medium flow, and flood, respectively.

To find the domain representative, denoted by $\boldsymbol{\mu}_r$, for each of three domains, i.e. low flow, medium flow, and flood, some unsupervised pattern recognition methods can be used (Theodoridis and Koutroumbas, 1999). For example, either the *C-means algorithm* or the *K-means algorithm* is chosen to find the arithmetic mean value of the group as $\boldsymbol{\mu}_r$ because of their computational simplicity (Lloyd, 1982; Bishop, 1995; Theodoridis and Koutroumbas, 1999). Likewise, the *fuzzy C-means algorithm* can also be employed in determining the value of $\boldsymbol{\mu}_r$ (Cannon et al., 1986; Theodoridis and Koutroumbas, 1999). For the comparison of these two clustering methods, the clustering results on the five-year discharge series of the Halda catchment in Bangladesh (from 1 April 1980, see Table 2) are listed in Table 1. From Table 1, it is found that the two

Table 1
The fuzzy set center $\boldsymbol{\mu}$ of the discharges on the Halda catchment (the fuzzifier in the fuzzy C-means method is 1.2; the unit of discharge is $m^3 s^{-1}$)

$\boldsymbol{\mu}$ (the fuzzy set center)	Small flow	Medium flow	Flood
The C-means method	16.94	136.21	384.17
The fuzzy C-means method	15.52	130.25	374.03

Table 2
Summary description of the watersheds (Shamseldin et al., 1997) (these mean values are calculated by using the data of the calibration period)

Watershed	Country	Area (km^2)	Mean rainfall (mm day^{-1})	Mean pan evaporation (mm day^{-1})	Mean discharge (mm day^{-1})	Calibration period (year)	Verification period (year)	Starting date
Sunkosi-1	Nepal	18,000	4.65	3.30	3.63	6	2	1 January 1975
Yanbian	China	2350	3.28	5.79	2.55	6	2	1 January 1978
Nan	Thailand	4609	3.89	3.33	1.82	6	3	1 April 1978
Brosna	Ireland	1207	2.20	1.31	0.98	8	2	1 January 1969
Sg. Bernam	Malaysia	1090	7.08	5.06	2.86	5	2	1 January 1977
Kelantan	Malaysia	12,867	6.58	4.84	3.50	6	2	1 January 1975
Halda	Bangladesh	779	6.75	2.77	4.84	5	2	1 April 1980
Shiquan-3	China	3092	2.30	2.41	0.98	6	2	1 January 1973
Baihe	China	61,780	2.59	2.89	1.04	6	2	1 January 1972
Chu	Vietnam	2370	3.78	2.54	1.64	8	2	1 January 1965
Bird Creek	Australia	2344	2.66	3.58	0.61	6	2	1 October 1955

clustering methods will give nearly the same results for $\boldsymbol{\mu}$.

4.2. Design of IF–THEN rules

For three flow domains and p different input variables, theoretically we could design 3^p different IF–THEN rules, such as such as 'IF ($\hat{Q}_{1,i}$ is flood, $\hat{Q}_{2,i}$ is low flow, …,$\hat{Q}_{p,i}$ is medium flow) THEN…'. However, we must always bear in mind that the number of the IF–THEN rules should be determined carefully in order to prevent the over-parameterization of the TS1 combination method. For each IF–THEN rule, we have $(p + 1)$ parameters, i.e. $b(0)$, $b(1)$, …,$b(p)$, so if we have one more rule, we will have $(p + 1)$ more parameters. To achieve a balance between the combination efficiency and the over-parameterization, the use of too many IF–THEN rules is not encouraged as otherwise too many parameters will probably depress the verification efficiency (Fiordaliso, 1998).

To eliminate most of the trivial and unnecessary IF–THEN rules, we assume that the simulated discharge series from each individual rainfall-runoff model produces a similar trend of the observed discharge. This means that, since each individual rainfall-runoff model has attempted to simulate the same observed discharge Q_i with maximum efficiency, then it is reasonable, at least in theory, that its estimates from different models, denoted by $\hat{Q}_{1,i},\ \hat{Q}_{2,i},\ldots,\hat{Q}_{p,i}$, do not differ from each other too much *in the order of magnitude*, although naturally they are not of the same value. With this assumption, for the three discharge domains, three IF–THEN rules ($k = 3$) are reasonably sufficient to form the whole IF–THEN control system. These are

$$R_1: \text{ IF } (\hat{Q}_{j,i} \text{ is "low flow"}, j = 1,\ldots,p)$$
$$\text{THEN } \hat{Q}_{c,i} = b_1(0) + \sum_{j=1}^{p} b_1(j)\cdot\hat{Q}_{j,i}, \tag{7.1}$$

$$R_2: \text{ IF } (\hat{Q}_{j,i} \text{ is "medium flow"}, j = 1,\ldots,p)$$
$$\text{THEN } \hat{Q}_{c,i} = b_2(0) + \sum_{j=1}^{p} b_2(j)\cdot\hat{Q}_{j,i}, \tag{7.2}$$

$$R_3: \ IF \ (\hat{Q}_{j,i} \text{ is "flood"}, j = 1,\ \ldots,p)$$
$$\text{THEN } \hat{Q}_{c,i} = b_3(0) + \sum_{j=1}^{p} b_3(j)\cdot\hat{Q}_{j,i}. \tag{7.3}$$

According to this logic, if we divide the observed discharge series into two domains, then we will need only two IF–THEN rules ($k = 2$), whereas if we treat the whole discharge series as a group, then we just need one IF–THEN rule ($k = 1$).

4.3. Determination of the degree of applicability

At each time step, for each IF–THEN rule, we need to determine the degree of applicability, denoted by $\alpha_r(\hat{Q}_i)$, according to the input vector $\hat{Q}_i$. Firodaliso (1998) defined the degree of applicability $\alpha_r(\hat{Q}_i)$ by the radial basis function (RBF) of Gaussian form, i.e.

$$\alpha_r(\hat{Q}_i) = \exp(-\|\hat{Q}_i - \boldsymbol{\mu}_r\|^2_{\mathbf{S}_r}), \tag{8}$$

where $\boldsymbol{\mu}_r$ is a p-dimensional vector that is the rth domain representative as well as the center of the fuzzy set defining that domain, and $\|\mathbf{X}\|_{\mathbf{S}_r}$ is the weighted norm of the p-dimensional vector $\mathbf{X}$ defined by $\|\mathbf{X}\|^2_{\mathbf{S}_r} = \mathbf{X}^{\mathrm{T}}\mathbf{S}_r^{\mathrm{T}}\mathbf{S}_r\mathbf{X}$, where $\mathbf{S}_r$ is a $p \times p$ square matrix. In this paper, $\mathbf{S}_r$ is taken as the identity matrix, so that $\|\bullet\|_{\mathbf{S}_r}$ just corresponds to the Euclidean norm $\|\bullet\|_2$.

Just as the fuzzy sets for the linguistic terms can be defined by different empirical functions, so the determination of the degree of applicability α_r is rather arbitrary, being dependent on some empirical knowledge and prior information. Besides the definition of Eq. (8), a linear form of RBF, the argument of which is the Euclidean distance of the input vector $\mathbf{X}$ from a center $\boldsymbol{\mu}$, is also used in this paper to define the degree of applicability $\alpha_r(\hat{Q}_i)$ for the IF–THEN rules, having the form

$$\alpha_r(\hat{Q}_i) = 1 - \|\hat{Q}_i - \boldsymbol{\mu}_r\|^2_{\mathbf{S}_r}. \tag{9}$$

The TS1 fuzzy system using Eq. (9) for defining the degree of applicability α_r is denoted by TS1*. The meaning of both definitions of Eqs. (8) and (9) is that, when the input vector $\hat{Q}_i$ is more closely approaching the fuzzy set center and the domain representative $\boldsymbol{\mu}_r$, the degree of applicability $\alpha_r(\hat{Q}_i)$ will be closer to unity and the premises in the IF–THEN rules are more closely fulfilled.

4.4. The nature of the TS1 fuzzy combination framework

In fact, the TS1 fuzzy combination method is one realization of the linear mixture of the individual forecasts (Fiordaliso, 1998), which can be demonstrated as follows.

Substituting the linear Eq. (3) into (4) and rearranging it, yields

$$\hat{Q}_{c,i} = \frac{\sum_{r=1}^{k} \alpha_r(\hat{Q}_i)\cdot[b_r(0) + \sum_{j=1}^{p} b_r(j)\cdot\hat{Q}_{j,i}]}{\sum_{r=1}^{k} \alpha_r(\hat{Q}_i)}$$

$$= \frac{\sum_{r=1}^{k} \alpha_r(\hat{Q}_i)\cdot b_r(0)}{\sum_{r=1}^{k} \alpha(\hat{Q}_i)_r} + \frac{\sum_{r=1}^{k} \alpha_r(\hat{Q}_i)\cdot b_r(1)}{\sum_{r=1}^{k} \alpha(\hat{Q}_i)}\hat{Q}_{1,i}$$

$$+ \cdots + \frac{\sum_{r=1}^{k} \alpha(\hat{Q}_i)\cdot b_r(p)}{\sum_{r=1}^{k} \alpha(\hat{Q}_i)}\hat{Q}_{p,i} \tag{10.1}$$

$$= w_{0,i} + w_{1,i}\cdot\hat{Q}_{1,i} + w_{2,i}\cdot\hat{Q}_{2,i} + \ldots + w_{p,i}\cdot\hat{Q}_{p,i}, \tag{10.2}$$

where

$$w_{0,i} = \frac{\sum_{r=1}^{k} \alpha(\hat{Q}_i)\cdot b_r(0)}{\sum_{r=1}^{k} \alpha(\hat{Q}_i)}, \tag{11}$$

$$w_{j,i} = \frac{\sum_{r=1}^{k} \alpha_r(\hat{Q}_i)\cdot b_r(j)}{\sum_{r=1}^{k} \alpha(\hat{Q}_i)}, \qquad j = 1, 2, \ldots, p \tag{12}$$

and where $w_{0,i}, w_{1,i}, \ldots, w_{p,i}$ are the combination weights, which are varying at each time step i, $i = 1, 2, \ldots, N$, N being the length of the data used. From Eq. (10.2), it can be seen that the TS1 fuzzy combination method is indeed one realization of the linear weighting of the individual forecasts at each time step. Note that the weights vary from one time step to the next, depending on the magnitude of the discharge.

From Eq. (10.2), it is also seen that both the SAM and the WAM are special limiting cases of the TS1 combination method. For example, in Eq. (10.2), if $w_{0,i} = 0$ and $w_{j,i} = 1/p$ $(j = 1, 2, \ldots, p)$ in the TS1, then it reduces to the SAM. Likewise, if $w_{0,i} = 0$ and the $w_{j,i}$ ($j = 1, 2, \ldots, p$) are all constants, then the TS1 reduces to the WAM.

So, Eqs. (7)–(12) define the construction of the TS1 framework to combine the forecasts from the different rainfall-runoff models. To make this combination method work, we need to know the coefficients $b_r(j)$, $j = 0, 1, \ldots, p$, for each rule. For the k rules, we have the total $(p+1) \times k$ unknown parameters to be estimated. These $(p+1) \times k$ parameters can be found by minimizing the quadratic error function of the form

$$E = \sum_{i=1}^{N} (\hat{Q}_{c,i} - Q_i)^2, \tag{13}$$

where $\hat{Q}_{c,i}$ is just the control variable of the TS1 combination method, i.e. the combination forecast of the observed discharges Q_i.

5. Evaluation of performances of the single models and the combination method

For rainfall-runoff models, the main criterion used for assessing simulation efficiency is the Nash–Sutcliffe efficiency index R^2 (Nash and Sutcliffe, 1970), defined as

$$R^2 = \frac{F_0 - F}{F_0}, \tag{14}$$

where F is the sum of squares of differences between estimated and observed discharges, and F_0 is the sum of squares of differences between the observed discharges and the mean discharge during the calibration period.

For a combination system or method (see Fig. 1), the Nash–Sutcliffe efficiency R^2 is also used to judge the accuracy of the combination forecasts against the observed discharges. Since a combination system is supposed to take advantage of the merits of each component model, we should intuitively expect a successful combination system to behave better than any of individual models in flood forecasting. In terms of the Nash–Sutcliffe efficiency index R^2, this expectancy for the combination method is expressed as

$$R_{\mathrm{c}}^2 \geq \max_{j=1}^{p} (R_{(j)}^2), \tag{15}$$

where the $R_{(j)}^2$ is Nash–Sutcliffe efficiency value for each single model and R_{c}^2 is Nash–Sutcliffe efficiency value for the combination forecasts.

This inequality (15) in facts demands that the combination system should have the ability to retain the best simulation results from the several different models. If the combination method produced combination forecasts worse than the forecasts of the best one among the several combined models, then we would not use that combination method at all and would adopt the forecast of that best model.

In practice, to take account of the probability of a combination system to satisfy Eq. (15), a new index is introduced to assess the combination method, which is called the combination capacity, denoted by ψ and defined as

$$\psi = \frac{\prod [R_{\mathrm{c}}^2 \geq \max_{j=1}^{p} (R_{(j)}^2)]}{M}, \tag{16}$$

where M represents the number of catchments to which the combination method has been applied, and $\prod [R_{\mathrm{c}}^2 \geq \max_{j=1}^{p} (R_{(j)}^2)]$ represents the number of catchments where the inequality (15) holds. The values of both R^2 and ψ will be less than or equal to unity.

To assess the influence of each individual model on the performance of the combination system, we use the correlation coefficient between the R^2 values of the individual models and the R^2 values of the combination system as an index. For the L catchments, the correlation coefficient, denoted by ξ, between the individual model j and the combination system is defined as

$$\xi_{\mathrm{c}}(j) = \frac{\sum_{l=1}^{L} [R_{(j)}^2(l) - \overline{R_{(j)}^2}] \cdot [R_{\mathrm{c}}^2(l) - \overline{R_{\mathrm{c}}^2}]}{\sqrt{\sum_{l=1}^{L} [R_{(j)}^2(l) - \overline{R_{(j)}^2}]^2} \cdot \sqrt{\sum_{l=1}^{L} [R_{\mathrm{c}}^2(l) - \overline{R_{\mathrm{c}}^2}]^2}}, \tag{17}$$

where $R_{(j)}^2(l)$ represents the Nash–Sutcliffe coefficient of the single model j in simulating the floods of the catchment l, $R_{\mathrm{c}}^2(l)$ represents the Nash–Sutcliffe coefficient of the combination system in simulating the floods of the catchment l, $\overline{R_{(j)}^2}$ and $\overline{R_{\mathrm{c}}^2}$ are the mean R^2 values of the individual model and the combination system, respectively, over the total of L catchments.

6. Applications of the combination method based on the TS1 fuzzy system

As noted in Section 1, Shamseldin et al. (1997) have combined the forecasts of five different models by three different methods. These five rainfall-runoff models are

1. The SLM (Nash and Foley, 1982; Kachroo and Liang, 1992).
2. The LPM (Liang and Nash, 1988; Liang and Guo, 1994).
3. The LVGFM (Ashan and O'Connor, 1994).
4. The constrained linear system model (CLS) (Todini and Wallis, 1977; Kachroo and Natale, 1992).
5. The SMAR model (Kachroo, 1992; Liang, 1992).

The fifth model is a conceptual model, the first four being black-box models.

The three forecast combination methods are (Shamseldin et al., 1997)

1. The SAM.
2. The WAM.
3. The NNM.

All these five models and the three combination methods are not described in this paper since they have been presented in some detail by Shamseldin et al. (1997). Shamseldin et al. (1997) firstly applied the five rainfall-runoff model to the daily rainfall and runoff data of 11 selected catchments and then combined the forecast results of the five models by the three combination methods to obtain the required combination forecasts, finally concluding that the combination forecasts are better than those from each individual model according to the values of the Nash–Sutcliffe model efficiency index R^2.

The Takagi–Sugeno first-order combination method (TS1) is now used as the fourth approach to combine the forecasts from the five models, on eleven catchment, and then the TS1 combination results are compared with the corresponding combination results of SAM, WAM, and NNM. A brief summary of those 11 watersheds is listed in Table 2, the data interval being one day in every case.

In the TS1 combination method, is the total number of parameters is $(p+1)\times k$, where p the number of the models and k is the number of IF–THEN inference rules. As we now have five models used for combination forecasts, i.e. $p=5$, then any one additional rule would introduce six more unknown parameters to be optimized. So, it is sensible to keep the number of IF–THEN inference rules in the TS1 fuzzy system as small as possible without seriously compromising the accuracy of the combination forecasts from the TS1 method (Fiordaliso, 1998). The TS1 method is run in three cases, i.e. for $k=1$, $k=2$, and $k=3$, with $p=5$ in every case. For the case that $k=1$, this TS1 method just reduces to the standard linear regression method having six constant coefficients or parameters, which may expressed as

$$\hat{Q}_{c,i} = b_0 + \sum_{j=1}^{5} b_j \cdot \hat{Q}_{j,i}. \tag{18}$$

For the cases that $k=2$ and $k=3$, those coefficients $w_{0,i}$ and $w_{j,i}$ in Eq. (10.2) are not constant but keep changing with each time step. These results for the TS1 combination, as well as corresponding results presented by Shamseldin et al. (1997), are listed together in Table 3. The results of the TS1* ($k=2$) fuzzy combination system are also listed in Table 3. For comparisons between the TS1 method and the other combination methods and the single models, the TS1 results of $k=2$ are chosen. Four figures (Figs. 2–5) are plotted to demonstrate the performance of the TS1 ($k=2$) in flood forecasting.

6.1. Calibration period

For the 11 catchments, the values of R^2 for the NNM method always rank No.1 (see Table 3), which means that, in the calibration period, the NNM combination system method can always make the inequality (15) hold, that is, $\psi = 11/11$. There is no doubt that the NNM combination system has the ability to retain the best simulation results from the several different models for the calibration period.

In this period, the values of R^2 for the WAM are very similar to those of R^2 for the TS1 ($k=2$) combination system, although on the Halda catchment, the value of R^2 for the TS1 is ranked just No. 5. So, it is still true that both the WAM and TS1 combination systems can also render the inequality

Table 3
The R^2 (unit:%) efficiency results of the five models and the four combination methods

Model	R^2 (rank)										
	Sunkosi-1	Yanbian	Nan	Brosna	Sg.Bernam	Kelantan	Halda	Shiquan-3	Baihe	Chu	Bird Creek
Calibration period											
SLM	85.78 (9)	73.66 (9)	65.87 (9)	40.12 (9)	65.27 (9)	62.81 (9)	81.70 (9)	72.01 (9)	70.40 (9)	59.63 (9)	59.52 (9)
LPM	91.96 (4)	83.00 (6)	75.81 (7)	70.28 (6)	74.87 (5)	76.71 (8)	84.58 (6)	76.85 (7)	74.26 (8)	63.10 (8)	63.56 (8)
LVGFM	88.63 (8)	78.05 (8)	76.53 (6)	41.37 (8)	72.15 (7)	78.91 (7)	83.62 (8)	87.63 (5)	87.16 (4)	83.18 (4)	86.10 (5)
CLS	90.52 (6)	81.59 (7)	74.33 (8)	46.95 (7)	72.48 (6)	79.49 (6)	83.99 (7)	75.76 (8)	83.27 (7)	77.54 (6)	65.35 (7)
SMAR	89.81 (7)	85.87 (4)	84.02 (4)	85.83 (4)	71.01 (8)	89.24 (4)	85.52 (3)	89.74 (4)	83.29 (6)	75.90 (7)	88.69 (4)
SAM	*91.25 (5)*	*83.69 (5)*	*80.97 (5)*	*71.00 (5)*	*75.59 (4)*	*84.44 (5)*	*85.47 (4)*	*84.83 (6)*	*84.65 (5)*	*77.65 (5)*	*80.28 (6)*
WAM	*92.93 (3)*	*87.66 (2)*	*85.69 (2)*	*89.03 (3)*	*79.35 (2)*	*91.34 (2)*	*87.23 (2)*	*91.78 (2)*	*90.02 (2)*	*85.86 (2)*	*90.18 (2)*
NNM	*93.12 (1)*	*90.54 (1)*	*87.58 (1)*	*92.62 (1)*	*82.09 (1)*	*91.47 (1)*	*87.71 (1)*	*93.25 (1)*	*90.96 (1)*	*89.95 (1)*	*91.24 (1)*
TS1 ($k = 2$)	**93.07 (2)**	**87.43 (3)**	**84.92 (3)**	**89.10 (2)**	**78.47 (3)**	**89.83 (3)**	**84.70 (5)**	**91.38 (3)**	**89.85 (3)**	**84.92 (3)**	**90.01 (3)**
TS1[a] (k = 2)	93.09	87.55	84.97	89.13	79.88	89.89	84.73	91.26	89.80	84.95	90.05
TS1 (k = 1)	92.47	86.44	84.87	88.76	76.62	89.74	84.59	91.21	89.13	84.65	89.97
TS1 (k = 3)	93.14	87.87	85.29	89.49	79.12	90.78	84.87	91.34	90.40	85.26	90.10
Verification period											
SLM	83.37 (9)	75.74 (9)	60.48 (9)	45.68 (9)	47.08 (2)	37.49 (8)	72.38 (8)	54.04 (9)	70.52 (6)	69.69 (8)	−53.21 (9)
LPM	90.49 (1)	79.21 (7)	75.73 (7)	77.53 (5)	47.92 (1)	37.89 (7)	77.18 (6)	56.16 (8)	73.17 (3)	70.28 (7)	−38.99 (7)
LVGFM	83.59 (8)	76.38 (8)	69.56 (8)	48.06 (8)	20.60 (8)	43.84 (6)	75.31 (7)	79.01 (1)	61.45 (9)	75.34 (3)	24.90 (5)
CLS	84.62 (7)	81.10 (6)	79.12 (6)	60.25 (7)	31.65 (4)	37.49 (8)	66.53 (9)	57.30 (7)	65.50 (8)	66.36 (9)	−43.72 (8)
SMAR	85.19 (6)	83.93 (4)	83.70 (2)	85.39 (4)	−18.30 (9)	50.55 (2)	84.55 (2)	68.16 (5)	72.79 (4)	71.82 (6)	73.31 (1)
SAM	*86.70 (5)*	*83.02 (5)*	*79.29 (5)*	*71.58 (6)*	*37.63 (3)*	*47.82 (5)*	*78.55 (5)*	*67.47 (6)*	*80.27 (1)*	*77.98 (1)*	*23.81 (6)*
WAM	*88.95 (2)*	*86.06 (2)*	*84.34 (1)*	*88.17 (2)*	*21.85 (6)*	*48.48 (3)*	*82.25 (3)*	*77.08 (3)*	*66.48 (7)*	*73.99 (4)*	*71.53 (2)*
NNM	*88.05 (4)*	*86.49 (1)*	*81.66 (4)*	*91.54 (1)*	*20.68 (7)*	*48.10 (4)*	*85.50 (1)*	*78.09 (2)*	*74.15 (2)*	*71.93 (5)*	*70.02 (4)*
TS1 ($k = 2$)	**88.70 (3)**	**85.40 (3)**	**83.52 (3)**	**86.73 (3)**	**29.64 (5)**	**50.74 (1)**	**81.18 (4)**	**76.18 (4)**	**71.55 (5)**	**76.30 (2)**	**70.87 (3)**
TS1[a] ($k = 2$)	88.56	85.82	83.29	86.74	20.25	50.74	81.48	75.71	67.48	76.33	71.11
TS1 ($k = 1$)	89.25	84.49	83.31	86.15	33.61	50.62	80.75	75.06	72.65	76.28	70.86
TS1 ($k = 3$)	88.33	86.13	83.78	87.23	27.10	50.10	82.08	76.17	66.89	76.53	70.39

[a] Means that the membership function is $\alpha_r(\hat{Q}_i) = 1\|\hat{Q}_i - \mu_r\|^2_{s_r}$.

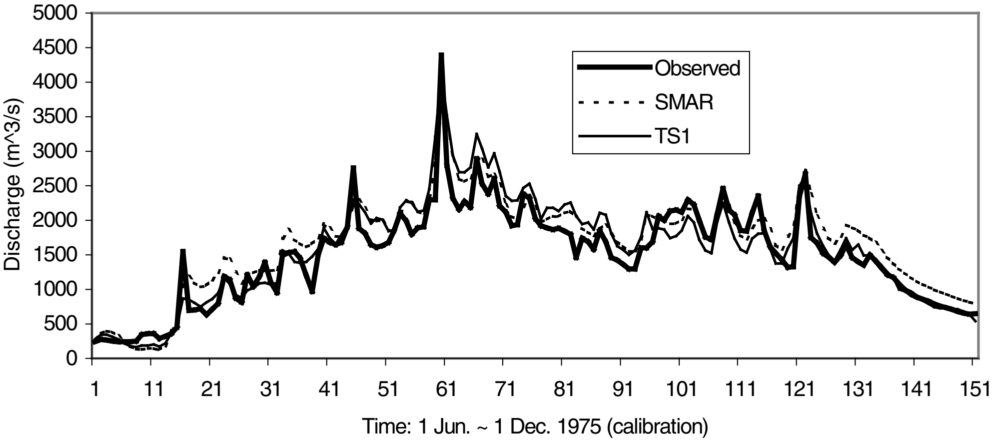

Fig. 2. Comparisons of simulated hydrographs on the Sunkosi-01 catchment (1 June–1 December 1975).

(15) valid, i.e. they can retain the best simulation results from several different models.

For the SAM, the values of R^2 are larger than all those of each individual model only in the case of two catchments (Sg. Bernam and Halda), and $\psi = 2/11$. If the inequality (15) is used to select efficient combination methods, then clearly the SAM method should be excluded.

For the TS1 combination method, it is demonstrated that the TS1 ($k = 3$) method has the best efficiency, the TS1 ($k = 2$) the second best, and the TS1 ($k = 1$) the worst. The average value of R^2 of the TS1 method, for $k = 1, 2$ and 3 are 87.12, 87.61, and 87.96%, respectively. Although the efficiency of the TS1 increases slightly when the inference rules and the number of parameters are increased, it would seem that three rules are enough to make the TS1 system work well, if we do not want too many parameters to be optimized.

Generally, in the calibration period, the NNM, WAM, and TS1 methods have performed better than any single rainfall-runoff model in the combination, which is only to be expected for successful combination systems.

6.2. Verification period

In the verification period, the R^2 values of the four combination methods are not always better than the

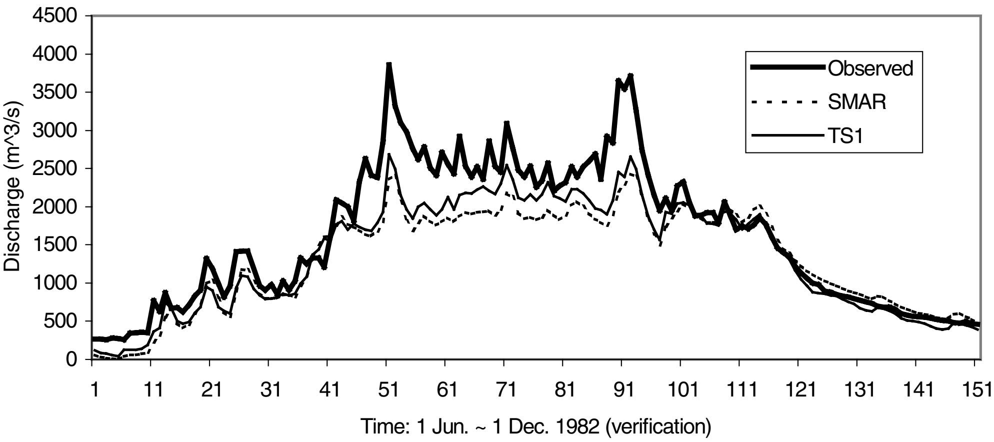

Fig. 3. Comparisons of simulated hydrographs on the Sunkosi-01 catchment (1 June–1 December 1982).

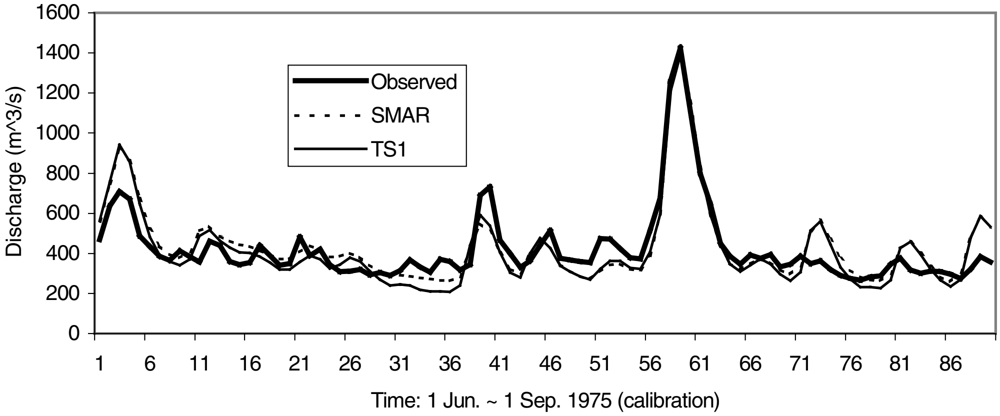

Fig. 4. Comparisons of simulated hydrographs on the Kelantan catchment (1 June–1 September 1975).

maximum of the five single models (see Table 3). For the SAM combination method, its combination capacity $\psi = 2/11$, for the WAM, $\psi = 3/11$, for the NNM, $\psi = 4/11$, and for the TS1 ($k = 2$), $\psi = 4/11$.

For the SAM method, only on two catchments out of the 11 tested is the value of R^2 larger than that from any of the five single models, i.e. in verification, the SAM combination method can improve the simulation results only on the Baihe and Chu catchments. For the WAM method, only in the case of three catchments is the value of R^2 larger than that from any of the five single models, these three catchments being the Yanbian, Nan, and Brosna catchments. For the NNM method, its combination forecasts are better, in terms of R^2, than the simulation results of each single model on the Yanbian, Brosna, Halda, and Baihe catchments. For the TS1 method, its combination forecasts are better than the simulation results of each single model also on four catchments, i.e. the Yanbian, Brosna, Kelantan, and Chu.

For the TS1 combination method, the average value of R^2 of the TS1 method for $k = 1, 2$, and 3 are 73.00, 72.80, and 72.25%, respectively, which are slightly decreasing with an increase in the number of parameters. This trend indicates that, increasing the number of parameters could depress the verification efficiency because of the uncertainty introduced by the extra parameters, although it would naturally improve the calibration efficiency as shown above. In this sense, the adoption of two rules are suggested in this paper for the TS1 combination system, the same as that recommended by Fiordaliso (1998), in

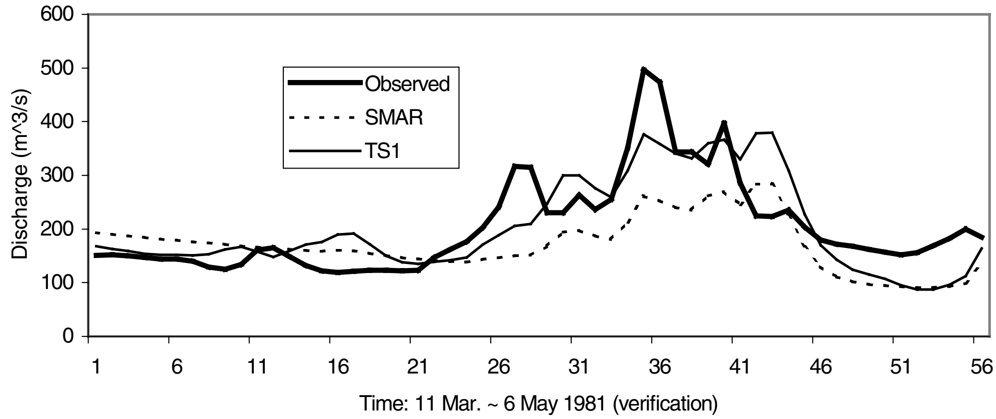

Fig. 5. Comparisons of simulated hydrographs on the Kelantan catchment (11 March–6 May 1981).

order to achieve the appropriate balance between the calibration efficiency and the verification efficiency, and between over-parameterization and the degree of uncertainty in the optimization of the parameter set.

6.3. Results of the TS1* ($k = 2$) combination

The average values of R^2 for the TS1 ($k = 2$) system are 87.61 and 72.80%, for the calibration and verification periods, respectively, while for the TS1* ($k = 2$) system, they are 87.75 and 71.59%. The only obvious difference occurs in the case of the verification period for the Sg. Bernam catchment, the value of R^2 for the TS1 ($k = 2$) being 29.64%, while $R^2 = 20.25\%$ for the TS1* ($k = 2$). On the other watersheds, the values of R^2 are nearly the same, although the values of the 12 parameters, $b_r(j)$ (($j = 0, 1, \ldots, 5$), $r = 1, 2$), will be different when using the different degree function of applicability.

6.4. Summary

In general, among the four combination methods (SAM, WAM, NNM, and TS1), the WAM, NNM, and TS1 combination systems have behaved similarly, according to the combination capacity index ψ, although the NNM can obviously enhance the simulation efficiency in the calibration period. It is noteworthy that, although the WAM, NNM, and TS1 methods all perform satisfactorily in the calibration period, they have not performed equally well in the verification period.

7. Influence of the component models on the combination efficiency

The results of the TS1 fuzzy combination presented in this paper, as well as those results presented by Shamseldin et al. (1997) demonstrate that the combination forecasts of the WAM, NNM, and TS1 methods are better than the corresponding forecasts of each individual model, especially in the calibration period, in terms of the value of R^2. Here, we further analyze those combination methods by examining the contribution of each individual model to the combination forecasts. From the results presented in Table 3, we calculate the correlation coefficients ξ between the R^2 values of each of five the individual models j and the R^2 values of each of the four combination methods over the 11 tested catchments, these results being presented in Table 4.

7.1. Calibration period

The SAM has demonstrated a fairly strong relationship to each individual model, the smallest value of $\xi = 0.54$, being obtained for the SMAR model, which is consistent with the concept that the SAM method has paid equal attention to the estimates of each single model. The rejection of the SAM method as a good combination system by the criterion ψ, the combination capacity, has been corroborated also by its average R^2 value of 81.80%, which is even less than that of the SMAR model, which is 84.45% and also the maximum among those of the five single models. As noted earlier, if a combination method cannot work as well as the best one among those individual models used for combination, then we would rather choose that best model rather than waste time on that unsuccessful combination method.

For the WAM, NNM, and TS1 ($k = 2$) methods, it is found that only the SMAR model has by far the largest correlation coefficient values, i.e. $\xi = 0.90$, 0.80, and 0.87, respectively, reflecting its strongest influence on the combination efficiency for all three combination methods. The other four models (SLM, LPM, LVFGM, and CLS) have just very small or even negative correlation coefficients to the WAM, NNM, and TS1 ($k = 2$) methods. For the WAM, NNM, and TS1 methods, it is seen that the estimates of the SMAR model have been assigned much larger coefficients or weights in the combination forecasts than the estimates of the other four models. This finding confirms that the WAM, NNM and TS1 methods are flexible in identifying and assigning different weights to different models, which are different from that of the SAM method that assigns an equal coefficient to each individual model.

7.2. Verification period

Once again, the average R^2 value of the SAM method, 66.74%, is smaller than that of the SMAR model, which is 67.37% and the best among those of the five single models, while the average R^2 values of the WAM, NNM, and TS1 ($k = 2$) methods, are 4 or 5% greater than that of the SMAR model.

Table 4
The correlation coefficient between the R^2 values of the component models and the R^2 values of the combination method (the bold values are the mean R^2values (%) of each single model and each combination method; $TS1^{(4)}$ indicates that the SMAR model is not used for this combination)

		SAM	WAM	NNM	TS1 (k = 2)	$TS1^{(4)}$ (k = 2)
Calibration						
		81.80	**88.28**	**90.05**	**87.61**	**84.59**
SLM	**66.98**	0.89	0.17	− 0.05	0.11	0.71
LPM	**75.91**	0.71	0.24	0.06	0.20	0.42
LVGFM	**78.48**	0.78	0.25	0.06	0.20	0.82
CLS	**75.57**	0.86	0.15	− 0.04	0.09	0.76
SMAR	**84.45**	0.54	0.90	0.80	0.87	–
Verification						
		66.74	**71.74**	**72.38**	**72.80**	**66.22**
SLM	**51.20**	0.86	0.22	0.25	0.26	0.81
LPM	**58.78**	0.90	0.35	0.38	0.39	0.87
LVGFM	**59.82**	0.90	0.71	0.70	0.73	0.91
CLS	**53.29**	0.93	0.40	0.42	0.44	0.90
SMAR	**67.37**	0.60	0.95	0.96	0.95	–

From the results of the correlation coefficients of each single model to the combination method (Table 4), it is found that the WAM, NNM, and TS1 ($k = 2$) methods still have the strongest relationships to the SMAR model, which is the best of the five models.

7.3. Combination results without the SMAR model

Since the conceptual SMAR model has been the best of five single rainfall-runoff models, the strong correlation coefficients of the WAM, NNM, and TS1 ($k = 2$) methods with only the SMAR model can be explained by recalling that the function of the combination methods is to identify the best single model and then improve its estimates by information of other combined models. When this objective is realized, then we can conclude that the efficiency of the combination methods is greatly influenced by the efficiency of the best one of those single models used in the combination. To further explore this association of strong correlation coefficients of a successful combination method with the best individual models used in the combination, we removed the SMAR model from the combination and used only the other four models in the Takagi–Sugeno fuzzy system, which is denoted by $TS1^{(4)}$ ($k = 2$). The results of R^2on the 11 catchments by the $TS1^{(4)}$ ($k = 2$) combination method are listed in the Table 5, and the correlation coefficients ξ between the R^2 values of each of four the individual models and the R^2 value of the $TS1^{(4)}$ ($k = 2$) method are also listed in Table 4.

From the results listed in Table 5, we can quickly identify that, in general, the $TS1^{(4)}$ ($k = 2$) combination method provides better forecast results than any of the four models used in the combination, i.e. the SLM, LPM, LVGFM, and CLS models. Among these four models, the best one is the LVGFM, with the average calibration efficiency of 78.48% and the average verification efficiency of 59.85%. For the $TS1^{(4)}$ ($k = 2$) combination method, the average values of R^2 for both calibration and verification are 84.59 and 66.22%, respectively, which are nearly 6% greater than the corresponding values of the LVGFM.

As to the correlation coefficients ξ between each of the R^2 values of four of the individual models and the R^2 value of the $TS1^{(4)}$ ($k = 2$) method, given in the last column of Table 4, these show that the LVGFM model has the largest value of ξ in both the calibration and the verification period, these values being 0.82 and 0.91, respectively. Once again, we have the result that successful combination methods involve strong correlation coefficients with the best individual model, recalling that the objective of the successful combination method is to identify the best single model and then to improve its estimates by absorbing some independent information reflected in the forecasts of the other models used in the combination.

Table 5
The R^2 (unit:%) efficiency results of the four models and the $TS1^{(4)}$ ($k = 2$) combination method ($TS1^{(4)}$ ($k = 2$)means that only four models, without the SMAR model, are included in the combination system)

Model	R^2 (rank)										
	Sunkosi-1	Yanbian	Nan	Brosna	Sg.Bernam	Kelantan	Halda	Shiquan-3	Baihe	Chu	Bird Creek
Calibration period											
SLM	85.78 (5)	73.66 (5)	65.87 (5)	40.12 (5)	65.27 (5)	62.81 (5)	81.70 (5)	72.01 (5)	70.40 (5)	59.63 (5)	59.52 (5)
LPM	91.96 (2)	83.00 (2)	75.81 (3)	70.28 (2)	74.87 (2)	76.71 (4)	84.58 (1)	76.85 (3)	74.26 (4)	63.10 (4)	63.56 (4)
LVGFM	88.63 (4)	78.05 (4)	76.53 (2)	41.37 (4)	72.15 (4)	78.91 (3)	83.62 (4)	87.63 (2)	87.16 (2)	83.18 (2)	86.10 (1)
CLS	90.52 (3)	81.59 (3)	74.33 (4)	46.95 (3)	72.48 (3)	79.49 (2)	83.99 (3)	75.76 (4)	83.27 (3)	77.54 (3)	65.35 (3)
TS1$^{(4)}$ ($k = 2$)	**92.89 (1)**	**85.71 (1)**	**81.00 (1)**	**75.80 (1)**	**77.96 (1)**	**85.52 (1)**	**84.06 (2)**	**88.08 (1)**	**89.14 (1)**	**84.27 (1)**	**86.04 (2)**
Verification period											
SLM	83.37 (5)	75.74 (5)	60.48 (5)	45.68 (5)	47.08 (2)	37.49 (4)	72.38 (4)	54.04 (5)	70.52 (2)	69.69 (4)	− 53.21 (5)
LPM	90.49 (1)	79.21 (3)	75.73 (3)	77.53 (1)	47.92 (1)	37.89 (3)	77.18 (2)	56.16 (4)	73.17 (1)	70.28 (3)	− 38.99 (3)
LVGFM	83.59 (4)	76.38 (4)	69.56 (4)	48.06 (4)	20.60 (5)	43.84 (2)	75.31 (3)	79.01 (1)	61.45 (5)	75.34 (1)	24.90 (1)
CLS	84.62 (3)	81.10 (2)	79.12 (1)	60.25 (3)	31.65 (4)	37.49 (4)	66.53 (5)	57.30 (3)	65.50 (3)	66.36 (5)	− 43.72 (4)
TS1$^{(4)}$ ($k = 2$)	**89.25 (2)**	**83.93 (1)**	**78.46 (2)**	**77.06 (2)**	**37.68 (3)**	**45.07 (1)**	**77.44 (1)**	**78.73 (2)**	**63.06 (4)**	**74.33 (2)**	**23.47 (2)**

Table 6
The combination efficiency R^2 (%) of the TS1 ($k = 2$) on the Yanbian catchment with the increasing values of $\boldsymbol{\mu}_1^*$ or $\boldsymbol{\mu}_2^*$

Scenario 1	R^2(%)	$S = 0$	$s = 0.2$	$S = 0.3$	$s = 0.4$	$s = 0.5$
	Calibration	**87.43**	87.44	87.43	87.42	87.37
	Verification	**85.40**	85.44	85.47	85.48	85.45
Scenario 2	R^2(%)	$t = 1$	$t = 1.2$	$t = 1.4$	$t = 1.6$	$t = 1.8$
	Calibration	**87.43**	87.65	87.79	87.87	87.90
	Verification	**85.40**	85.68	85.90	86.06	86.20

8. Effect of the domain representative $\boldsymbol{\mu}$ on the combination efficiency

In the above discussion, the domain representative $\boldsymbol{\mu}$ is determined by using the clustering algorithms (mainly the C-means algorithm). In this section, we investigate how the change of the value of the flow domain representative $\boldsymbol{\mu}$ will affect the combination efficiency of the TS1 fuzzy system. For example, for the TS1 ($k = 2$) system, which involves two domain representatives, i.e. $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, two simple scenarios are considered. One scenario is to keep the value of $\boldsymbol{\mu}_2$ unchanged but to increase the value of $\boldsymbol{\mu}_1$, while the other one is to keep the value of $\boldsymbol{\mu}_1$ unchanged but to increase the value of $\boldsymbol{\mu}_2$. Mathematically, the first scenario for changing the values of the flow domain representative is expressed as

$$\boldsymbol{\mu}_1^* = \boldsymbol{\mu}_1 + s\cdot(\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1), \tag{19}$$

$$\boldsymbol{\mu}_2^* = \boldsymbol{\mu}_2,$$

where $\boldsymbol{\mu}_1^*$ and $\boldsymbol{\mu}_2^*$ represent the new values for the flow domain representative, $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are the values of the flow domain representative determined by the C-means algorithm, and s is a coefficient whose value is between 0 and 1. When the value of s approaches to 1, the value of $\boldsymbol{\mu}_1^*$ will approach that of $\boldsymbol{\mu}_2^*$.The second scenario for changing the values of the flow domain representative is expressed as

$$\boldsymbol{\mu}_1^* = \boldsymbol{\mu}_1 \tag{20}$$

$$\boldsymbol{\mu}_2^* = t\cdot\ \boldsymbol{\mu}_2$$

where t is another coefficient assumed to vary between 1.0 to 2.0. When the value of t increases, the value of $\boldsymbol{\mu}_1^*$ will move away from that of $\boldsymbol{\mu}_2^*$.

When $\boldsymbol{\mu}_1^*$ and $\boldsymbol{\mu}_2^*$ used as the domain representatives, the Gaussian equation, i.e. Eq. (8), is still employed to calculate the degree of applicability of the IF–THEN rules in the TS1 ($k = 2$) system. For the different values of $\boldsymbol{\mu}_1^*$ and $\boldsymbol{\mu}_2^*$, the results of the combination efficiency R^2 (%) of the TS1 ($k = 2$) on the Yanbian catchment are presented in Table 6. From Table 6, it is found that, in the scenario 1, in which the value of $\boldsymbol{\mu}_1^*$ is increasing while the value of $\boldsymbol{\mu}_2^*$ is kept the same, the values of R^2 have a negligible variation in both the calibration period and the verification period. For the scenario 2, in which the value of $\boldsymbol{\mu}_1^*$ is kept the same while the value of $\boldsymbol{\mu}_2^*$ increases, the values of R^2 increase gradually in both the calibration period and the verification period. For the other ten catchments, nearly the same conclusion about the impact of changing the values of $\boldsymbol{\mu}_1^*$ and $\boldsymbol{\mu}_2^*$ on the combination efficiency R^2 can be reached.

The reason why the value of R^2 is increased gradually with the increasing $\boldsymbol{\mu}_2^*$ (while $\boldsymbol{\mu}_1^*$ is constant) is explained as follows. When Eq. (8) is employed to calculate the degree of applicability α_2 for the second IF–THEN rule (associated with large flow), the larger the value of $\boldsymbol{\mu}_2^*$, the larger will be the value of the degree of applicability α_2 for the big discharge inputs. Since $\boldsymbol{\mu}_1^*$ is kept constant, for the same discharge inputs, the degree of applicability α_1 for the first IF–THEN rule (associated with small flow) is unchanged. So, in producing the output of the TS1 ($k = 2$) system with the large discharge inputs, the weight assigned to the output from the second IF–THEN rule will become larger and larger compared to the weight assigned to the output from the first rule, when t is increased in the scenario 2. In this way, the whole TS1 fuzzy system will become more sensitive and effective in simulating the large discharges than in simulating the small discharges. When large floods are simulated more effectively and accurately by the TS1 fuzzy system, the values of the combination efficiency R^2 will have a chance to be enhanced.

In general, within the context of the TS1 ($k = 2$) system, it is found that changing values of the flow domain representative will indeed have some impact on the combination efficiency value R^2. In particular, increasing the value of $\boldsymbol{\mu}_2^*$ (while $\boldsymbol{\mu}_1^*$ is held constant) can lead to a slight increase in the value of the combination efficiency R^2. However, such an improvement on the model efficiency is still not that significant.

9. Conclusions and discussions

Proposed as the fourth combination method (in addition to the SAM, WAM, and NNM methods) for organizing the forecasts of different rainfall-runoff models in a systematic way in order to obtain better forecasts of catchment flows, the first-order Takagi–Sugeno fuzzy system (TS1) is employed to incorporate the forecasts of five rainfall-runoff models, using the data sets of eleven catchments. The comparison of model efficiency R^2 values of the TS1 combination with the other three methods (SAM, WAM, and NNM) has shown that the TS1 system has behaved almost the same as the WAM and NNM methods.

Considering that the inputs to any combination system are the simulation results from the individual rainfall-runoff models, the performance of the combination method will naturally be influenced by the efficiency of each single model. One of the conclusions of this study is that, based on the correlation analysis, the forecasting efficiency of those combination methods such as the WAM, NNM, and TS1, is greatly influenced by the corresponding efficiency of the best one of those single models used in the combination. When the forecast series of the individual models are largely independent (i.e. considerably different) from each other, we can expect that the combination system will very easily identify and strongly reflect the forecast of the best single model. Under the extreme case, when individual models are very similar to each other in their flood forecasting ability, the combination system will produce forecasts only marginally better than those of the individual models. Also, it is likely that the combination of the different forecasts might lead to bigger errors than the individual ones at some particular time steps, because any single optimization objective or criterion, such as R^2, can only reflect a form of efficiency trend of the data series rather than at each single time step. A more detailed analysis of forecasting efficiency at different discharge levels or parts of the flood wave might facilitate the ranking of different combination methods of near-equal R^2 values.

Two points about the TS1 fussy system should be briefly discussed. One is why (or when) a combination forecast might be needed and the other one concerns the limitation of the TS1 system. There are probably many reasons leading to the failure of a single model, such as data distortion and model structure inadequacy. The scenario for the combination method to work is that when the simple model structure, adopted for the combination, while reasonably efficient, is still not flexible enough to reflect the full spectrum of hydrological processes, by combining such different models the structural inadequacy of any single model used in the combination might be least partially overcome. If (and only if) a watershed could always be perfectly simulated by one single model, would a good combination method fail to produce some improvement, however marginal. As to the weakness of the TS1 system, it is clearly the danger of overparameterization. With the danger of overparameterization, the number of IF–THEN rules must be decided very carefully, i.e. those should not be too many. However, the choice of a small number of IF–THEN rules will undoubtedly make the whole TS1 fuzzy system less flexible.

There is no doubt that there are many methods that may improve the accuracy of flood forecasting (See and Openshaw, 2000). The first-order Takagi–Sugeno (TS1) fuzzy system for combining the forecasts from different individual models is just one such method. It is the contention of the present authors that the results presented in this study confirm that the first-order Takagi–Sugeno fuzzy system, in the context of forecast combination, is a very simple yet very effective tool for enhancing the accuracy of river flow forecasts.

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Appendix A. Basic concepts of fuzzy theory and its applications

A.1. The fuzzy set and membership function

Zimmermann (1985) has given a comprehensive exposition of the fuzzy set theory. The fuzzy set theory is very flexible in describing the features of objects, which are usually expressed by some linguistic terms such as large, red, etc., compared to the classic set theory. In the classic set theory, the membership function, $m(\cdot)$, which represents the relationship between a component x from a subset X and a reference set $\wp$, has just two crisp results: 1 or 0, where 1 means that x is a member of the reference set $\wp$, while 0 means not. This classic set theory is mathematically defined as

$$m_X : \wp \rightarrow \{0, 1\},$$

$$m_X(x) = \begin{cases} 1, & \text{if } x \in \wp, \\ 0, & \text{otherwise.} \end{cases} \quad \text{(A1)}$$

In practice, this classic set theory is not very suitable for indicating the subtle distinctions between fairly similar objects. For example, to define the height of a man, under the classic set theory, we can have only two opposite conclusions, tall and short. If we think of a man of height 1.80 m as being tall and a man of height 1.60 m as being short, then how do we classify a man of height 1.70 m? Following the classical set theory, we must define such a man of height 1.70 m being either tall or short, neither of which is appropriate in practice, because a man of height 1.70 m is just of medium hight.

Overcoming the limitations of the classic set theory as described above, the fuzzy set theory (Zadeh, 1965; Zimmermann, 1985) allows the membership $m(\cdot)$ to be any real value between 0 and 1 to represent the relationship between a component x and a reference set $\wp$. Put simply, the membership function $m(\cdot)$ is just a definition of the certainty with which we can use a linguistic term to describe the features of an object, with the value 1 indicating the complete certainty and the value 0 indicating nearly zero certainty. For the above example, we can now think of the man of height 1.70 m as being tall, but only with some degree of certainty, or, we can think of that man as being short, also with some degree of certainty. For the linguistic term tall to describe the stature of a man, an empirical membership can be reasonably defined as follows:

$$m_X : \wp \rightarrow [0, 1],$$

$$m_X(x) = \begin{cases} \dfrac{x}{1.8}, & 0 < x < 1.8, \\ 1.0, & 1.8 \le x < 2.0, \end{cases} \quad \text{(A2)}$$

where the value 1.8 is the selected threshold defining the linguistic term tall. According to the Eq. (A2), we can say without doubt that a man of height 1.90 m is tall, for his membership value is 1.0, while we can also say that a man of height 1.70 m is tall, but only with some certainty, for his membership value is 0.94, which is considerably less than 1.0. To take another familiar example, the temperatures of both 35°C and 40°C can sensibly be regarded as being hot. However, we would have more certainty and confidence in saying that the temperature of 40°C is hot than in saying that the temperature of 35°C is hot, according to the membership function defining hot.

In general, compared to the classic set theory, the fuzzy set theory is more advantageous in expressing vague, uncertain, and imprecise information, which is certainly not uncommon in the scientific and engineering fields (Zadeh, 1965; Zimmermann, 1985; Kruse et al., 1994).

A.2. The domain partition

In Section A.1, we have given an example of the fuzzy set defined for the linguistic term tall, i.e. through the Eq. (A2). In practice, we will not just use only the tall or short terms to express our perception of the height of a man. We will also use more delicate linguistic terms, such as rather short, rather tall, and very tall, which need be defined by some fuzzy membership functions, or fuzzy sets. In the context of flood forecasting, some linguistic terms, such as drought, low flow, medium flow, and flood, are also used to generalize the different characteristics of the observed discharges that are time varying. These different linguistic terms divide the whole

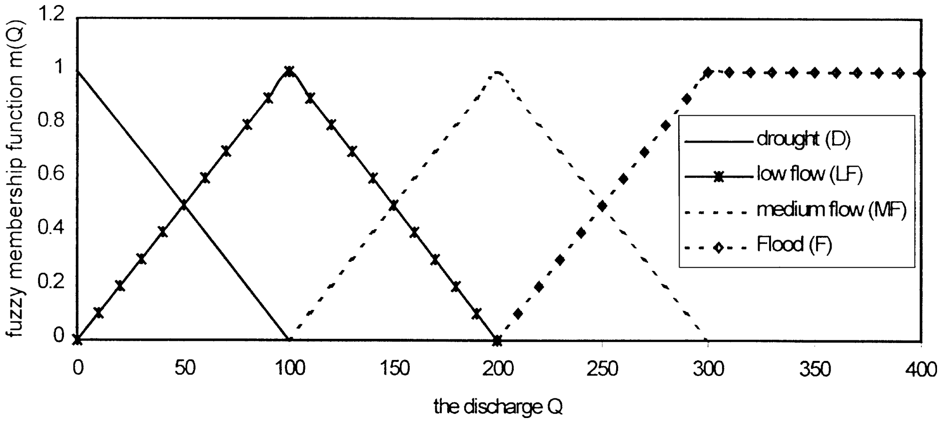

Fig. 6. The fuzzy domain partitions of the discharge series on a watershed.

spectrum of flow conditions into different domains, this division being called the domain partition. Note that those domains are not mutually exclusive. Assume, for example, that there is a watershed for which the observed maximum discharge is 400 $m^3 s^{-1}$, and that we define drought (D), low flow (LF), medium flow (MF), and flood (F) by the following fuzzy membership functions

$$m_{(D)}(Q) = \begin{cases} 1 - \dfrac{Q}{100}, & 0 < Q < 100, \\ 0, & Q \geq 100, \end{cases} \tag{A3}$$

$$m_{(LF)}(Q) = \begin{cases} \dfrac{Q}{100}, & 0 < Q < 100, \\ 2 - \dfrac{Q}{100}, & 100 \leq Q < 200, \end{cases} \tag{A4}$$

$$m_{(MF)}(Q) = \begin{cases} \dfrac{Q}{100} - 1, & 100 \leq Q < 200, \\ 3 - \dfrac{Q}{100}, & 200 \leq Q < 300, \end{cases} \tag{A5}$$

$$m_{(F)}(Q) = \begin{cases} \dfrac{Q}{100} - 2, & 200 \leq Q < 300, \\ 1.0, & 300 \leq Q < 400, \end{cases} \tag{A6}$$

where Q represents the discharge values of the watershed, and $m_{(D)}$, $m_{(LF)}$, $m_{(MF)}$, and $m_{(F)}$ are the fuzzy sets corresponding to the linguistic terms drought, low flow, medium flow, and flood, respectively. This fuzzy partition defined by Eqs. (A3) and (A4) is plotted in Fig. 6.

Under the fuzzy domain partition, we can find that the same object can belong to more than one fuzzy set, and yet have the different membership values. For instance, for the discharge $Q = 80\ m^3\ s^{-1}$, it can be regarded as being drought with the fuzzy membership value $m_{(D)}(x = 80) = 0.2$, but it can also be regarded as being medium flow with $m_{(LF)}(x = 80) = 0.8$. According to the membership values, we can determine that the discharge $Q = 80\ m^3\ s^{-1}$ more probably belongs to medium flow rather than to drought.

A.3. The fuzzy inference rules

In classical control engineering, we specify an accurate mathematical model to control the relevant physical processes. However, under the fuzzy control theory, it is the linguistic IF–THEN rules that are used to control the given process (Takagi and Sugeno, 1985; Kruse et al., 1994; Schulz et al., 1999; Dou et al., 1999; Pongracz et al., 1999). These IF–THEN inference rules are the logic decisions about how the input variables, denoted by $x_1,\ldots,x_2$, affect the control variable, denoted by y, in the fuzzy control systems. In the context of flood forecasting by means of the rainfall-runoff models, the input variables are mainly rainfall and evaporation, and the only control variable (i.e. output) is the river discharge.

In the assessment of how different combinations of the input variables affect the control variable, firstly it is necessary to partition each input variable into different domains and then, for each input variable, one of its domains is selected. Those selected domains, each of which is corresponding to one different input variable, consist of the so-called premise or condition of the IF–THEN rules. With such a premise known, the state of the control variable can be finally determined according to the corresponding IF–THEN rule. Generally, the fuzzy inference system includes a set of linguistic IF–THEN rules, and each single IF–THEN rule has the following general form

$$R_r: \text{ IF } (x_1 \text{ is } A_r^{(1)},\ x_2 \text{ is } A_r^{(2)}, \ldots, x_p \text{ is } A_r^{(p)}) \quad \text{THEN } y \text{ is}\ldots, \tag{A7}$$

where R_r means the rth rule of the fuzzy inference system ($r = 1, 2, \ldots, k$, where k is the number of those IF–THEN control rules), $x_1,\ldots,x_p$ are the input variables, $A_r^{(1)},\ldots,A_r^{(p)}$ are the linguistic terms corresponding to the fuzzy sets $m_r^{(1)},\ldots,m_r^{(p)}$, which are used to partition the input domain of each input variable, respectively, and y is the output control variable.

As to each IF–THEN rule, firstly the decision logic determines the degree to which the measured inputs $x_1,\ldots,x_k$ fulfil the premise of the rule. This is called the degree of applicability and it is denoted by α. For each rule R_r, the corresponding degree of applicability can be determined according to the t-norm (Zimmermann, 1985; Kruse et al., 1994), i.e.

$$\alpha_r = \min\{m_r^{(1)}(x_1),\ m_r^{(2)}(x_2), \ldots, m_r^{(p)}(x_p). \tag{A8}$$

Normally, different formulae are used to determine the degree of applicability α for each IF–THEN rule. To demonstrate how to determine the degree of applicability α, a simple example is presented here.

Assume that there are two rivers, River 1 and River 2, contributing to the outlet discharge of a watershed and that we want to establish the IF–THEN rules for flood forecasting at the outlet of the watershed concerned, according to the discharges of the two upstream rivers. The discharges in the two rivers, denoted by $Q^{(1)}$ and $Q^{(2)}$, are considered to be the input variables, and the discharge at the outlet of the watershed, denoted by $Q^{(0)}$, is the control variable (output). For simplicity, we partition both $Q^{(1)}$ and $Q^{(2)}$ into two domains, i.e. low flow (LF), and medium flow (MF). So, altogether, there are four fuzzy sets involved whose membership functions are defined as

$$m_{(\mathrm{LF})}^{(1)}(Q^{(1)}) = \begin{cases} \dfrac{Q^{(1)}}{100}, & 0 < Q^{(1)} < 100, \\ 2 - \dfrac{Q^{(1)}}{100}, & 100 \le Q^{(1)} < 200, \end{cases} \tag{A9}$$

$$m_{(\mathrm{MF})}^{(1)}(Q^{(1)}) = \begin{cases} \dfrac{Q^{(1)}}{100} - 1, & 100 \le Q^{(1)} < 200, \\ 3 - \dfrac{Q^{(1)}}{100}, & 200 \le Q^{(1)} < 300, \end{cases} \tag{A10}$$

$$m_{(\mathrm{LF})}^{(2)}(Q^{(2)}) = \begin{cases} \dfrac{Q^{(2)}}{400}, & 0 < Q^{(2)} < 400, \\ 2 - \dfrac{Q^{(2)}}{400}, & 400 < Q^{(2)} < 800, \end{cases} \tag{A11}$$

$$m_{(\mathrm{MF})}^{(2)}(Q^{(2)}) = \begin{cases} \dfrac{Q^{(2)}}{400} - 1, & 400 \le Q^{(2)} < 800, \\ 3 - \dfrac{Q^{(2)}}{400}, & 800 \le Q^{(2)} < 1600, \end{cases} \tag{A12}$$

where $m_{(\mathrm{LF})}^{(1)}(Q^{(1)})$and $m_{(\mathrm{MF})}^{(1)}(Q^{(1)})$represents the fuzzy sets of the low flow and medium flow on the River 1, respectively, and $m_{(\mathrm{LF})}^{(2)}(Q^{(2)})$ and $m_{(\mathrm{MF})}^{(2)}(Q^{(2)})$ represents the fuzzy sets of the low flow and medium flow on the River 2, respectively.

Assume that there are four IF–THEN rules to determine the value of $Q^{(0)}$, of the form

$$R_1: \text{ IF } (Q^{(1)} \text{ is “low flow”},\ Q^{(2)} \text{ is “low flow”}) \quad \text{THEN } Q^{(0)} \text{ is}\ldots, \tag{A13.1}$$

$$R_2: \text{ IF } (Q^{(1)} \text{ is “low flow”},\ Q^{(2)} \text{ is “medium flow”}) \quad \text{THEN } Q^{(0)} \text{ is}\ldots, \tag{A13.2}$$

$$R_3: \text{ IF } (Q^{(1)} \text{ is “medium flow},\ Q^{(2)} \text{ is “low flow”}) \quad \text{THEN } Q^{(0)} \text{ is}\ldots, \tag{A13.3}$$

R_4 : IF ($Q^{(1)}$ is "medium flow",

$$Q^{(2)} \text{ is "medium flow")} \quad \text{(A13.4)}$$

THEN $Q^{(0)}$ is...

Given that $Q^{(1)} = 110$ m^3 s^{-1} and $Q^{(2)} = 490$ m^3 s^{-1}, then we can calculate that $m^{(1)}_{(LF)}(Q^{(1)} = 110) = 0.9$, $m^{(1)}_{(MF)}(Q^{(1)} = 110) = 0.1$, $m^{(2)}_{(LF)}(Q^{(2)} = 490) = 0.8775$, and $m^{(2)}_{(MF)}(Q^{(2)} = 490) = 0.1225$, according to the corresponding membership functions (A9) to (A12).

According to Eq. (A8), for the IF–THEN rule R_1, the degree of applicability is calculated as

$$\alpha_1 = \min\{m^{(1)}_{(LF)}(Q^{(1)} = 110),\ m^{(2)}_{(LF)}(Q^{(2)} = 490)\}$$
$$= \min\{0.9,\ 0.8775\} = 0.8775. \quad \text{(A14)}$$

For the other three rules,R_2, R_3, and R_4, we determine that $\alpha_2 = 0.1225$, $\alpha_3 = 0.1$, and $\alpha_4 = 0.1$, respectively. Among these four rules, the rule R_1 has the largest degree of applicability, with $\alpha_1 = 0.8775$, which means that the given information of the input variables has the largest possibility to fulfil the premises of the rule R_1.

The results for this simple example considered above are considered in Section 3 of this paper, in an elaboration of this example in the context of the Takagi–Sugeno fuzzy system.

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