On Bouchet's complementary hypothesis

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Abstract

Morton's proof of Bouchet's complementary hypothesis has been revisited. The proof's central assumption is that an increase in sensible heat from the surface to the air induces a similar increment in the sensible heat transfer from the air to a hypothetical potential evaporimeter. It is shown that this assumption is not necessary in the derivation. Instead, the complementary hypothesis under simplified conditions can be obtained with the help of the mass conservation equation for water vapor. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Bouchet's hypothesis (1963), as restated by Morton (1965), claims that over a large homogeneous surface, for which any heat or vapor advection is negligible, the actual areal evaporation (E [LT^{-1}]) and the potential evaporation (E_{po} [LT^{-1}]) are in a complementary relationship such as

$$E + E_{po} = \text{const.} \tag{1}$$

provided the energy balance of the surface is constant in time. Morton (1965) derived the constant to be equal to twice the regional wet environment evaporation (E_w [LT^{-1}]). Generally the potential evaporation term (E_{po}) is estimated by the Penman (1948) combination equation, while for the wet environment evaporation (E_w) term, the Priestley–Taylor (1972) approach is applied (e.g. Brutsaert and Stricker, 1979; Parlange and Katul, 1992; Kim and Entekhabi,

1997) which is based on the (assumed constant) energy balance of the surface.

A basic argument behind Eq. (1) is that, as the originally wet environment is drying out, less and less energy is consumed by evaporation and consequently this unused portion of the original energy becomes fully available to increase potential evaporation, E_{po}, by exactly the same amount as E decreased, while leaving the overall energy budget unaffected (Bouchet 1963; and more recently Morton, 1983). While the practical validity of Eq. (1) has been confirmed by Morton (1983) on a large data set from four continents, and more recently by Parlange and Katul (1992); Kim and Entekhabi (1997), no attempt has been devoted since Morton (1983) to prove Eq. (1) on a theoretical basis.

Morton (1983) derived Eq. (1) from theoretical considerations under simplifying conditions. His proof was based on Bouchet's (1963) somewhat heuristic argument above. Below we show that Eq. (1) can be derived without employing this argument under conditions described by Morton (1983).

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2. Methodology

Following Morton (1983), we are going to apply the sensible- (H) and latent-heat (LE) [W m^{-2}]) transfer equations

$$H = -\lambda \frac{\partial \Theta}{\partial z} \tag{2a}$$

$$LE = -f \frac{\partial e}{\partial z} \tag{2b}$$

over a large uniformly vegetated area with uniform soil conditions and with negligible heat/vapor advection. Here λ [W m^{-1} K^{-1}] is heat-, and f is the vapor-transfer [W m^{-1} Pa^{-1}] coefficient, Θ is potential temperature [K], e is vapor pressure [Pa] and z designates the vertical axis of the coordinate system. We further assume that a constant net available energy $R_n–G$ [W m^{-2}] (where the former is net radiation and the latter is the soil heat flux term) can be fully partitioned into sensible and latent heat. Finally, if we also stipulate that any soil-moisture change in the area would leave the overall energy balance unaffected, we can write the energy-balance of the vegetated surface as

$$R_n - G = H + LE = \text{const.} \tag{3}$$

By arbitrarily choosing two reference levels at fixed elevations over the surface, the above equations can be combined into

$$\lambda_L(\Theta_L - \Theta_U) + f_L(e_L - e_U) = \text{const.} \tag{4}$$

where the subscripts 'L' and 'U' designate the variable taken at the lower ($z = L$) or upper ($z = U$) reference level, respectively. Note that in deriving Eq. (4) it has been assumed that the vertical profiles of Θ and e are similar (see Appendix for an explanation). By introducing the difference operator δ, and assuming the temporal changes in the transfer coefficients to be negligible, Eq. (4) transforms into

$$\lambda_L(\delta\Theta_L - \delta\Theta_U) = -f_L(\delta e_L - \delta e_U) \tag{5}$$

which defines changes in the variables at the two reference levels to changes in the areal soil moisture. If the upper reference level is chosen in the top of the surface sublayer, then the second term on either side of Eq. (5) becomes negligible (Morton 1983); there-fore one obtains

$$\delta e_L = -\frac{\lambda_L}{f_L} \delta\Theta_L \tag{6}$$

which relates changes in vapor pressure and potential temperature at the lower reference level to changes in areal soil moisture.

Now let's consider a hypothetical evaporimeter that measures potential evaporation (E_p) in the area. Using the elevation ('P') of the evaporating surface of our evaporimeter as a third reference level ($z = P$), Eq. (5) can be reformulated using this new reference level in place of the upper one as

$$\lambda_L(\delta\Theta_P - \delta\Theta_L) = -f_L(\delta e_P - \delta e_L) \tag{7}$$

The only physically meaningful solution of Eqs. (6) and (7) combined is when $\delta e_P = \delta\Theta_P = 0$. Note that the other algebraic solution, $-\lambda_L/f_L = \delta e_P/\delta\Theta_P$, is physically invalid, since $e_P = e_{sat}$ at the surface of the evaporimeter, which is a monotonic function of T_P ($= \Theta_P$), the temperature of the evaporating water surface.

The solution $\delta e_P = \delta\Theta_P = 0$ for the evaporimeter shows that the surface temperature of the hypothetical evaporimeter cannot change in response to changes in areal evaporation (Morton, 1983). Note that when combining Eqs. (6) and (7), it was implicitly assumed that the evaporimeter due to its hypothetical zero order size would not change the e and Θ values being measured at the lower reference level.

Assuming that the energy that is being available by decreasing areal evaporation as a response of depleted soil moisture is fully used by the evaporimeter, Morton (1983) showed that Bouchet's complementary hypothesis (Eq. (1)) must be true. In other words, evaporation from a hypothetical evaporimeter would increase exactly by that amount as areal evaporation decreased. [Note that this may not be strictly valid for an evaporation pan, as a substitute for a hypothetical evaporimeter, where the pan geometry becomes an additional factor (Brutsaert and Parlange, 1998)]. Below we show that the areal and potential evaporation are indeed complementary, without resorting to assumptions about the energy transfer between the drying environment and a hypothetical evaporimeter. Instead we make use of the mass conservation equation.
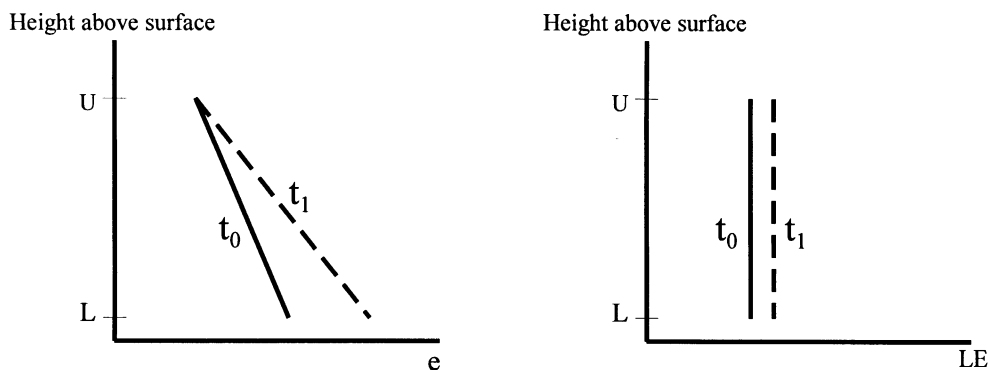
Fig. 1. Schematics of linear vapor pressure profiles with height above the surface and the corresponding latent heat profiles at two different times, t_0 and t_1. L designates the lower, U the upper reference level.

The change in latent heat at the lower reference level is

$$\delta LE_L = \frac{f_L(\delta e_L - \delta e_U)}{C_1} \cong \frac{f_L \delta e_L}{C_1} \qquad (8)$$

where C_1 is a constant, a function of the height of the lower and upper reference levels and of the water-vapor vertical profile. For example, for a logarithmic vertical vapor profile C_1 is equal to $L\ln(L\,U^{-1})$, where 'ln' is the natural logarithm (see Appendix). The change in latent heat at the surface of the evaporimeter is

$$\delta LE_P = \frac{f_P(\delta e_P - \delta e_L)}{C_2} = \frac{-f_P \delta e_L}{C_2} \qquad (9)$$

where we made use of $\delta e_P = 0$. The complementary hypothesis results if $\delta LE_L = -\delta LE_P$, which requires the equality of $f_L C_2$ and $f_P C_1$. By choosing a suitable lower reference level, the equality of these two terms can be achieved as is shown below.

The mass conservation equation for water vapor over a homogeneous surface, with only the x-component (u) of the velocity field retained for sake of simplicity, can be written (e.g. Sutton, 1934; Brutsaert, 1982) as

$$u\frac{\partial q}{\partial x} = \frac{\partial}{\partial z}\left(K_v \frac{\partial q}{\partial z}\right) \qquad (10)$$

where q is specific humidity, and K_v is the eddy diffusivity for water vapor [$m^2\ s^{-1}$]. Over a large homogeneous field the horizontal differences in q vanish, which results in the $K_v\ \partial q/\partial z$ term be constant in the vertical direction. This term is the latent-heat

transfer Eq. (2b) divided by the latent heat of vaporization for water and the specific humidity ($\approx 0.622e/P_a$, where P_a is atmospheric pressure) being substituted for e. Eq. (10) shows that the vertical latent-heat transfer is constant in the vertical direction for the surface sublayer over a large homogeneous surface (Gotz and Rakoczi, 1981) (see Fig. 1 for illustration). Using this property of the latent-heat transfer we can write

$$LE_L = LE_P \qquad (11a)$$

for all times outside the location of the evaporimeter, from which it follows that

$$\delta LE_L = \delta LE_P \qquad (11b)$$

must also be true for all times. Eq. (11b) can be rewritten using the reference-level vapor pressure values as

$$C_2 f_L \delta e_L = C_1 f_P(\delta e_P - \delta e_L) \qquad (11c)$$

where we assumed again that the δe_U term is negligible (Fig. 1). By choosing the lower reference level at a height where originally the vapor pressure is half of that at the evaporimeter level, the two terms involving the constants (C_1 and C_2) and the vapor-transfer coefficients (f_L and f_P) become equal. This proves Bouchet's complementary hypothesis, which says that δLE_L indeed equals $-\delta LE_P$.

3. Summary and conclusions

The complementary hypothesis, first proposed by Bouchet (1963), has been derived under simplifying conditions by Morton (1965, 1983). The present proof

follows Morton's (1983) approach, but utilizes the mass conservation equation in the final steps of the proof instead of relying on additional assumption of energy transfer between the environment and a hypothetical evaporimeter. The proof is based on the following assumptions: (a) the study area is sufficiently large, having homogeneous surface and soil-moisture properties; (b) heat and vapor advection is negligible to the area; (c) temporal changes in vapor pressure and potential temperature at the top of the surface layer are negligible compared to their change close to the surface; (d) the potential temperature and vapor pressure vertical profiles are similar; and (e) the net available energy is constant.

In practical applications one cannot expect all the above requirements to be met. The net energy term, $R_n - G$, may easily change with drying conditions of the environment, still, recent tests of Bouchet's complementary hypothesis based on physical (Parlange and Katul, 1992) and model simulation experiments (Kim and Entekhabi, 1997) confirm its validity under typical environmental conditions. For example, Parlange and Katul (1992) suggested a simple adjustment of Eq. (1) in the presence of significant heat advection and diurnal variability of the energy terms.

The purpose of the present study was to show that the complementary hypothesis can be derived with a smaller number of presumptions than was presented by Morton (1965, 1983), and by doing so, the basic and somewhat heuristic argument of Bouchet and Morton about energy transfer can be avoided.

Acknowledgements

Appendix A

Let us assume that the following equation describes the vapor pressure profile with elevation (z)

$$-\frac{\mathrm{d}e}{\mathrm{d}z} = \frac{c}{z} \tag{A1}$$

where c is a constant, to be obtained from measure-

ments. Integrating Eq. (A1) one obtains

$$z = z_0 \exp\left(\frac{e_0 - e}{c}\right) \tag{A2}$$

or equally

$$e = e_0 + c\ln\left(\frac{z_0}{z}\right) \tag{A3}$$

Applying Eq. (A3) over the two reference levels $z = U$ and $z_0 = L$, the constant, c, becomes

$$c = \frac{e_U - e_L}{\ln\left(\dfrac{L}{U}\right)} \tag{A4}$$

from which one can write

$$-\frac{\mathrm{d}e}{\mathrm{d}z}\bigg|_{z=L} = \frac{c}{L} = \frac{e_U - e_L}{L\ln\left(\dfrac{L}{U}\right)} = \frac{e_U - e_L}{C} \tag{A5}$$

where C is a constant, a function of the two reference level heights and the vapor pressure profile.

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