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Article *in* Rock Mechanics and Rock Engineering · January 2001

DOI: 10.1007/s006030170011

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Rock Mech. Rock Engng. (2001) 34 (3), 235–242

Rock Mechanics and Rock Engineering

On Study of Influences of Loading Rate on Fractal Dimensions of Fracture Surfaces in Gabbro

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1. Introduction

Studies of the effects of loading rates on rock fracture have been performed for decades (Zhang et al., 1999). However, the previous work on static or dynamic rock fracture has mainly been limited to a macro-experimental study. Although some investigations dealt with a micro-experimental study of rock fracture, they were still limited to a qualitative experimental study (Zhang et al., 2000). Therefore, a quantitative micro-experimental study of rock fracture is necessary for exploring rock fracture mechanisms. Since the 1980's fractal theory has been applied to examine the fracture surfaces of various solid materials. This was reviewed in detail by Charkaluk et al. (1998). Furthermore, the fractal study of brittle materials has indicated that the rougher the fracture surface of a brittle material is, the greater is the fractal dimension of the fracture surface (Mecholsky et al., 1989). The study by Mecholsky et al. (1989) also showed that the fractal dimensions of the fracture surfaces for several brittle materials were proportional to their fracture toughness. However, the above work, dealing with brittle materials, was performed under static loading conditions.

The present investigation measures the fractal dimensions of the fracture surfaces of the gabbro specimens fractured at various loading rates covering static and dynamic loading, and explores the relationship between the fractal dimensions and the fracture toughness of the rock.

2. Specimen Preparation

The rock specimens are similar to a short rod (SR) specimen, suggested by the International Society for Rock Mechanics (ISRM) (Ouchterlony, 1988). In the condition of low loading rates, the fracture testing was performed on a material testing system MTS 810, and such a fracture is defined as static fracture. K_{Ic} stands for static fracture toughness in mode I, and the relevant loading rate is

Table 1. Mineralogical composition (*M.C.*) and maximum grain size (*M.G.S.*) of the gabbro

M.C.	%	M.G.S. (mm)
Plagioclase	48–52	1.9
Pyroxene	35–38	1.4
Olivine	5–7	0.6
Hornblende	5	1.1
Biotite	3	0.4
Magnetite	1	0.4

$k < 10^4$ MPa m$^{1/2}$ s^{-1}. In the condition of high loading rates, the fracture testing was done with a split Hopkinson pressure bar (SHPB) system and this type of fracture is called dynamic fracture. The SHPB system is composed of a strike bar, input bar, output bar and relevant testing equipment. A wedge is tightly connected with the end of the input bar by a screw, and the rock specimen is sandwiched between the wedge and the output bar. K_{Id} stands for dynamic fracture toughness in mode I, and the relevant loading rate is $k \geq 10^4$ MPa m$^{1/2}$ s^{-1}. Here the loading rate is defined as $k = K_{Ic}/t_c$ or K_{Id}/t_c. The critical time t_c is the time interval from the start of loading to the point when the crack just becomes unstable. The experimental systems for static and dynamic fracture are described in detail by Zhang et al. (1999). In this study, all of the rock specimens or fragments (usually two halves) came from the previous fracture experiments in Zhang et al. (1999), i.e. after static or dynamic fracture testing, the fragments were preserved to measure the fractal dimensions of the rock fracture surfaces. The indirect tensile strength (as determined by the Brazilian test) of the gabbro is 17.3 MPa. Its mineralogical composition and the maximum grain size are shown in Table 1.

Immediately after fracture, the fragments of the SR specimen were carefully preserved from new damage on their fracture surfaces. To examine the vertical section of the SR specimen, the fracture surface of one of its fragments must be protected from mechanical damage induced by later machining. First, a small Bakelite plate was firmly bonded on the fracture surface of the fragment. Secondly, the fragment was cut along its axis through the notch tip. The section of the fragment was perpendicular to its fracture surface. Finally, the section was abraded flatly and polished (see Fig. 1). The polished section was provided for optical microscope observations. In order to perform SEM experiments, the polished section was sprayed with carbon. Before the SEM experiments and carbon spraying, the section surface was cleaned with alcohol.

3. Fractal Dimensions of Fracture Surfaces

3.1. Methods of Measuring Fractal Dimensions

The research performed by Mecholsky et al. (1989) indicated that the fracture surfaces of six aluminium materials with varied grain size, porosity, and toughness, and five glass-ceramics with different microstructures were identified to be

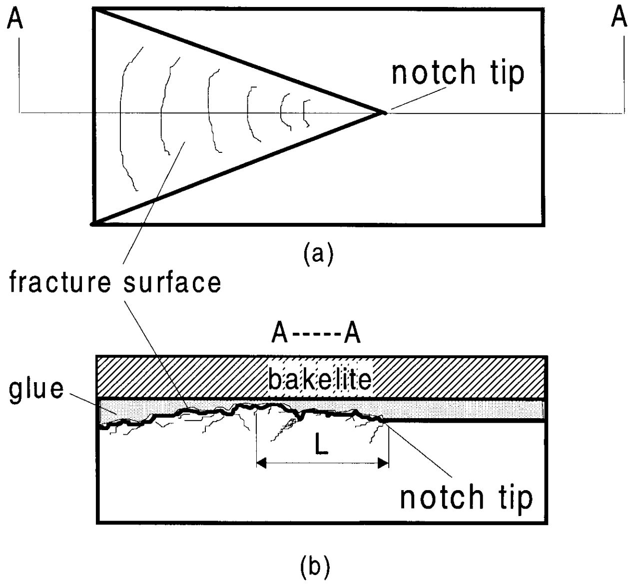

Fig. 1. Fracture surface without Bakelite (**a**) and its vertical section with Bakelite (**b**) of a short rod specimen

self-similar. Similar research has shown that the fracture surfaces of some rock materials are also self-similar or statistically self-similar (Xie, 1993). The fracture surfaces of rock materials can, therefore, be quantitatively studied by means of fractal geometry.

A study of the fracture surfaces of a solid material by fractal geometry is generally performed through measuring and analysing the fractal dimensions of the fracture surfaces. There are several methods for determining the fractal dimension of a fracture surface (Charkaluk et al., 1998; Xie, 1993; Pande et al., 1987). The present study has employed the vertical section method (Pande et al., 1987). According to this method, a fracture surface is first cut along a vertical section (in this study the section goes through the notch tip of the specimen, see Fig. 1). This section is perpendicular to the fracture surface. The section and the fracture surface intersect each other on a curved line. Such a line is used to calculate the fractal dimension of the fracture surface. For the SR specimens used in this study, the horizontal distance of the curved line was about 28 mm long from the notch tip. In fact, it is expensive to measure the fractal dimension of the fracture surface by using such a long line. However, the minimum length of the curved line used for measuring the fractal dimension should be enough to cover the region in which the main crack extends from a stable to a critical state. Thus we concluded that the horizontal length of the curved line to be measured should be further than 10 mm from the notch tip, and we therefore chose 12 mm as the horizontal length of the line for each specimen. The curve line to be measured is represented by "L" in Fig. 1.

According to fractal theory introduced by Mandelbrot (1982), it was found that certain irregular curves could be characterised by a non-integer fractal dimension, and that this value was scale-invariant. The fractal dimension, D_F, of a line can be determined by the following equation (Pande et al., 1987; George et al., 1992):

$$D_F = [(\ln L_0 - \ln L)/\ln E] + D, \tag{1}$$

where: L = length of the curved line measured;
E = scale of measurement;
L_0 = a constant;
D = topological dimension (equal to 1.0 for straight lines and 2.0 for planes).

The Cambridge Quantimet 900, set at a magnification of around 100×, was used to obtain the section profile of a fracture surface. The section profile was analysed by a computer program that measured digitally the length of the section profile by different scale lengths. Under the magnification 100, the scale of the measurement is from 2 mm to 30 mm at the increment 2 mm. The fractal dimension is then obtained from the slope of the $\ln L$ vs. $\ln E$ curves, using a least square fitting procedure. This method is simple, and several of these measurements for D_F can be made in succession by further grinding (Pande et al., 1987). Furthermore, Underwood and Banerji (1983) found that a simple serial section was equally as effective as sections taken over all possible orientations, greatly reducing the number of sections required. For this reason, we assume that the result obtained by Underwood and Banerji (1983) is valid for rock materials. Considering the above reasons, we measured and calculated one section for each specimen.

3.2. Relationship Between Fractal Dimensions and Loading Rates

According to the method described above we measured the fractal dimensions of 11 gabbro specimens. The results are shown in Table 2. From the results we find that the fractal dimension of the gabbro is correlated to the loading rate. Their

Table 2. Fractal dimensions and fracture toughness for the gabbro

Specimen	k (MPa m$^{1/2}$ s^{-1})	K_{Ic} or K_{Id} (MPa m$^{1/2}$)	D_F
C04	1.38E-1	2.75	1.027
C01	1.57E-1	2.83	1.030
No. 50	7.9E01	3.63	1.026
No. 46	1.09E02	3.28	1.027
No. 47	2.69E02	2.69	1.026
No. 12	1.56E05	6.53	1.030
No. 07	1.79E05	9.33	1.037
No. 27	3.11E05	12.43	1.043
No. 24	7.11E05	19.20	1.059
No. 23	8.77E05	20.18	1.057
No. 35	1.04E06	26.98	1.068

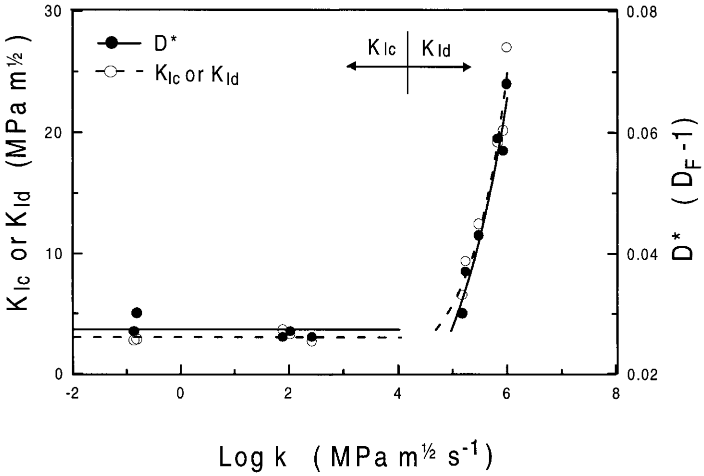

Fig. 2. Relationship between fractal dimensions and loading rates for gabbro. For dynamic fracture: $\log D^* = 0.37 \log k - 3.41$; $\log K_{Id} = 0.63 \log k - 2.4$. Here "log" represents the base 10 logarithm

relationship is shown in Fig. 2 and may be expressed by the following equations:

$$D^* = 0.027, \quad \text{when } k < 1 \times 10^4 \text{ MPa m}^{1/2}\text{s}^{-1} \tag{2a}$$

$$\log D^* = 0.37 \log k - 3.41, \quad \text{when } k \geq 1 \times 10^4 \text{ MPa m}^{1/2}\text{s}^{-1}, \tag{2b}$$

where: $D^* = D_F - 1$ (D^* is also called fractal increment);
log = base 10 logarithm.

For Eq. (2a) the standard deviation of D^* is equal to 0.0016, and for Eq. (2b) the coefficient of determination is 0.96. This shows that the scatter of the experimental data is much smaller than the level of the fractal increment D^*.

4. Relationship Between Fracture Toughness and Fractal Dimension

The results in Fig. 2 or Eq. (2) indicate that the fractal dimensions of the gabbro specimens fractured under static loading do not change too much, i.e. they can be approximately taken as a constant. However, the D_F of the gabbro fractured under dynamic loading increases with increasing loading rates. It is very interesting that this relationship is consistent with that between the fracture toughness of the gabbro and the loading rates (Zhang et al., 1999). According to the data in Table 2 we can find that the value of D_F for a gabbro specimen is correlated with its K_{Ic} or K_{Id}. Their relationship is shown in Fig. 3 and can be described by Eq. (3):

$$\left.\begin{array}{l} K_{Ic} \\ K_{Id} \end{array}\right\} = A(D^*)^B, \tag{3}$$

where A and B are constants obtained from the regression analysis of the experimental data. For Eq. (3) the coefficient of determination is 0.93. The values of A

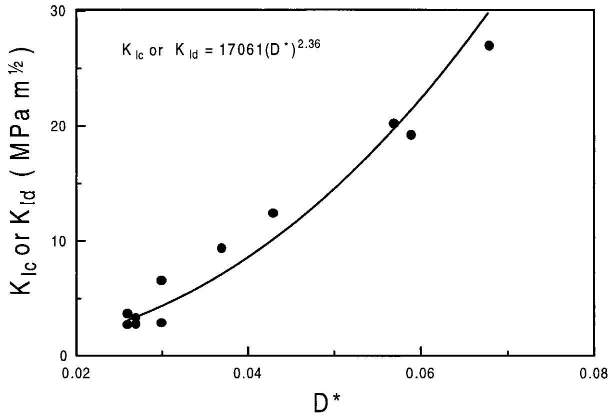

Fig. 3. Relationship between fracture toughness and fractal dimensions for gabbro

and B obtained from the data for the gabbro in Table 2 are 1.71×10^4 and 2.36, respectively. Equation (3) indicates that the fracture toughness of the gabbro increases with increasing fractal increment of its fracture surface. In other words, the fracture toughness increases with increasing roughness of the fracture surfaces. This result is similar to that obtained by Mecholsky et al. (1989). This implies that the gabbro probably has similar mechanical properties to those brittle materials investigated by Mecholsky et al. (1989).

5. Discussion

According to the experimental results for the energy partitioning of the same gabbro in the process of dynamic fracture (Zhang et al., 2000), we have:

$$\log W_L = 0.94 \log k - 5.09, \tag{4}$$

where: W_L is the energy absorbed by rock specimen during fracturing.

The W_L mainly consists of two parts: (1) the fracture and damage energy W_{FD} used for producing fracture surfaces, macro-branching cracks, and inner micro-cracking damage in the specimen, and (2) the kinetic energy W_K of the flying specimen or its fragments. From both Eqs. (2b) and (4) we can get:

$$W_L = A_1 (D^*)^{B_1}, \tag{5}$$

where: $A_1 = 3.72 \times 10^3$; $B_1 = 2.54$.

According to the energy analysis by Zhang et al. (2000), W_K is a small proportion of the W_L, and most of the W_L is used to produce fracture surfaces, macro-branching cracks, and inner micro-cracking damage. For example, at $k \leq 1 \times 10^6$ MPa m$^{1/2}$ s^{-1}, the $W_K \leq 0.14\ W_L$ (Zhang et al., 2000). Equation (5) indicates that the energy used for producing the fracture surfaces, the branching cracks, and the inner cracking damage increases with increasing roughness of the fracture surfaces.

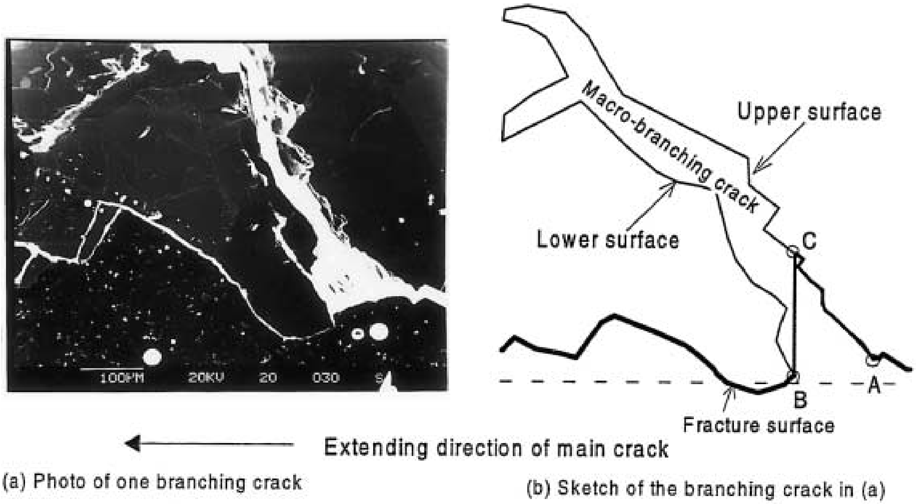

Fig. 4. A macro-branching crack on the section of gabbro specimen No. 07 ($k = 1.79 \times 10^5$ MPa m$^{1/2}$ s^{-1}, $K_{Id} = 9.33$ MPa m$^{1/2}$)

Figure 4 shows a typical branching crack of the specimen No. 07 listed in Table 2. It is certain that the crack branching influences the D_F of a fracture surface. In this study, for the purpose of simplicity, the initial part of the branching crack was only taken into consideration. In other words, from the initial point of the lower surface of the branching crack a straight line, which is roughly perpendicular to the plane (dash line in Fig. 4) of the fracture surface of the specimen, was drawn. Thus the straight line and the upper surface of the branching crack met each other at the point "C" shown in Fig. 4, and the curve "A-C-B" in the figure was taken as the part of the fracture surface representing the branching crack. This approximate treatment is based on two experimental facts: (a) Crack branching is related to loading rate, and it increases with increasing loading rate. (b) The angle between a branching crack and fracture surface is also related to loading rate. Particularly, the angle has an increasing tendency with increasing loading rate. Obviously, this treatment possibly makes the calculated D_F to be smaller than the true D_F of the fracture surface more or less, because the length of branching crack is not fully considered in determination of the D_F. In addition, in some cases of dynamic rock fracture, one branching crack produces new branching cracks during its propagation (Zhang et al., 2000). In such cases, how much length of the branching crack should we use to determine the D_F so as to get a reasonable D_F? To answer this question, a further study is needed.

6. Conclusion

The fractal dimensions of the static fracture surfaces of gabbro were approximately constant. However, the fractal dimensions of the dynamic fracture surfaces increased with the loading rates. In addition, the fractal dimensions of the fracture surfaces of the gabbro increased with increasing its fracture toughness.

In order to consider macro-crack branching in determining the fractal dimension of a rock specimen as fully as possible, it is necessary to further study both dynamic rock fracture characteristics and the relevant fractal theory.

Acknowledgements

The financial support from the Ministry of the Metallurgical Industry of China and SKB in Sweden was greatly appreciated. The authors would like to thank Dr. Finn Ouchterlony for his valuable comments for the manuscript.

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