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Estimating uncertainty in normalized Fry plots using a bootstrap approach

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Abstract

The normalized Fry method is a powerful and commonly used tool for measuring fabric in aggregates of packed grains. The significance of these parameters is often unclear because the associated uncertainty is unknown. Basic statistical hypotheses, such as deciding if a sample has a fabric, or if the fabrics of two samples are significantly different, requires knowledge of associated uncertainty.

For this study a bootstrap version of the normalized Fry method was developed. This program randomly selects normalized center-to-center distances, with replacement, from the population of all possible center-to-center distances. For each sample 100 bootstrap normalized Fry plots were constructed using different combinations of center-to-center distances. The variation of fabric parameters for these 100 analyses is used to estimate the uncertainty associated with the sample.

Results of bootstrap analyses of sandstones, oolitic limestones and synthetic data sets, show considerable variation in fabric parameter uncertainty, apparently related to both lithology and the degree of fabric development. Fabrics with axial ratios less than 2.0 appear only to be significant to one decimal place. The variation of fabric uncertainty makes it important to determine the uncertainty associated with individual samples. The deformation model for a field example developed based on conventional normalized Fry plots needed to be more complicated than deformation models that allowed for variation of fabric parameters.

Keywords: Normalized Fry method; Bootstrap approach; Fabric

1. Introduction

The normalized Fry method (Erslev, 1988; Erslev and Ge, 1990) is a powerful tool for measuring fabric in various types of aggregates of packed grains lacking other adequate markers. This computer-based modification of the original Fry method (Fry, 1979) allows the automated calculation of a fabric ellipse from a graphical normalized Fry plot by fitting an ellipse to the elliptical point concentration on the normalized Fry plot (Erslev and Ge, 1990). This automation removes some of the subjectivity associated with quantifying results from the method and allows fabric to be summarized with two parameters, the axial ratio (R) and orientation (ϕ) of the fabric ellipse.

Summarizing fabric with two parameters, however, has a serious downside. Important information about the quality of the data is lost because the degree of scatter of points on the Fry diagram is not taken into account. A visual examination of normalized Fry plots shows variation in the scatter of points in the maximum point concentration (Fig. 1). Some plots show very distinct elliptical concentrations of points, while on others the point distribution is very diffuse. Though ellipses can be fitted to all of these plots, clearly ellipses fitted to plots showing distinct point concentrations are more significant than ellipses fitted to diffuse point concentrations. Yet once the ellipse is fitted there is no way to recognize the quality of the original data if only the axial ratio and orientation of the fabric ellipse are specified, as is commonly done in publications to save space. Serious misinterpretation, or overinterpretation, is risked if only the fabric parameters are used.

Even if normalized Fry plots are presented, a visual examination can only produce a qualitative estimate of the quality of the data. To be able to evaluate individual samples, to compare pairs of samples, or to interpret complete data sets, it is necessary to have a way to quantify quality by determining the uncertainty associated with each fabric parameter. Basic interpretations, such as deciding if a sample has a fabric, or if the fabrics of two samples are significantly different, requires knowledge of both the fabric parameters and associated uncertainty. For example, if fabric was measured in samples close to a fault ($R = 1.5$, $\phi = 60$) and further away ($R = 1.3$, $\phi = 90$) it might be concluded that the difference was related to deformation associated with the fault. But is this difference significant? Or is the difference in fabric within the uncertainty associated with the measurements? If the uncertainty is small the

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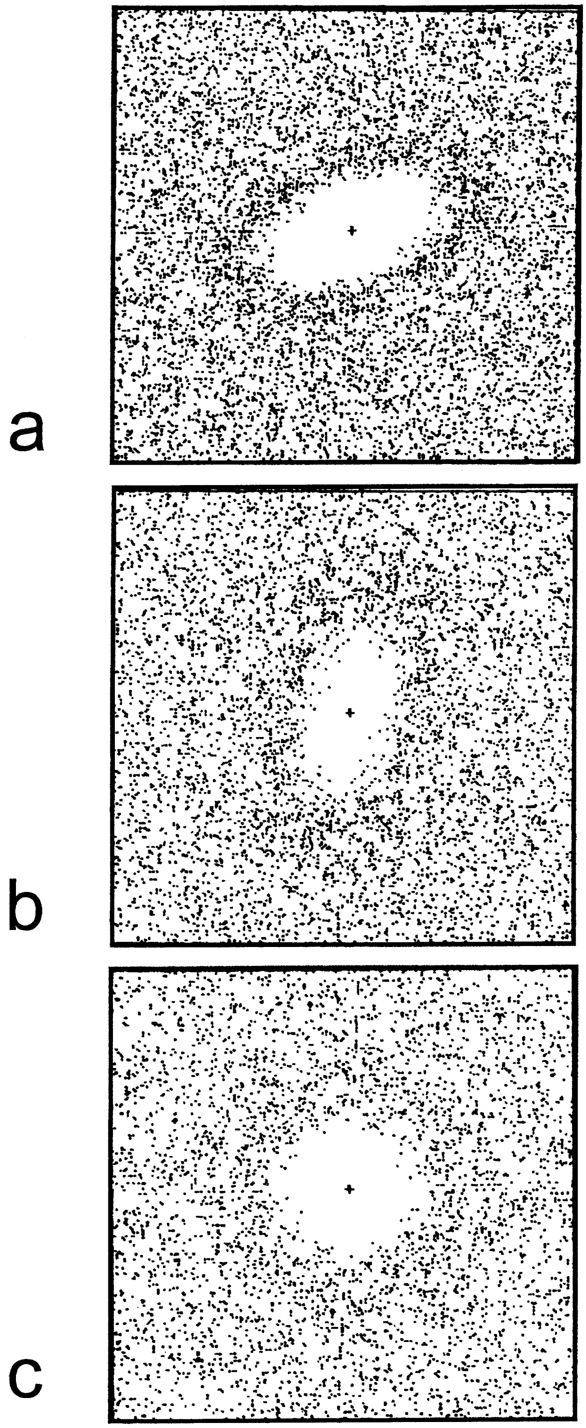


Fig. 1. Three normalized Fry plots showing different amounts of scatter. (a) Well defined maximum point density ring, (b) less well defined maximum point density ring, (c) poorly defined maximum point density ring.

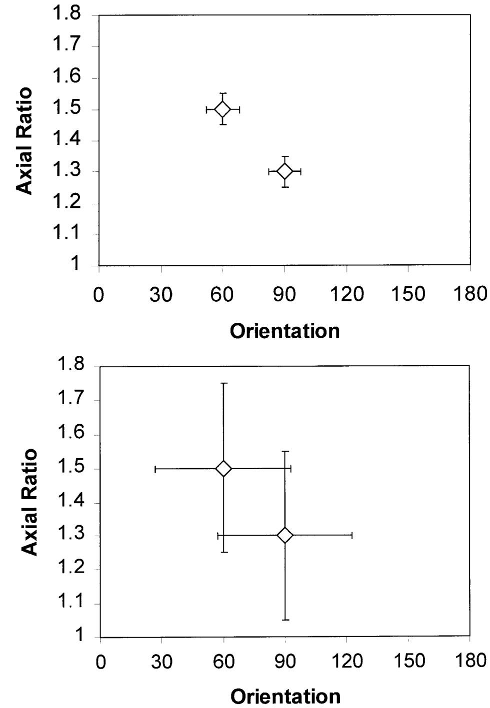


Fig. 2. Illustration of the importance of variation in fabric parameters in making interpretations. Points falling within error bars are considered insignificantly different. For the same measurements if the variability is small the samples may be significantly different (a), but if variability is large samples will not be significantly different (b).

two measurements may be different (Fig. 2a), but if the uncertainty is large the difference may not be real and conclusions drawn may be misleading (Fig. 2b). In fact if uncertainty is large enough the axial ratio of the fabric ellipse may not be significantly different from 1.0, i.e. there may not be a significant fabric in the rock.

In order to evaluate the fabric measurements, the uncertainty associated with each measurement must be determined. Ideally the uncertainty associated with a fabric measurement could be determined by making multiple fabric measurements from different parts of the same sample. If enough measurements were made, the range of fabric parameters could be used to determine the uncertainty associated with fabric in the sample. In practice this approach has serious disadvantages. Measuring fabric by the normalized Fry method is a slow process. Repetitive preparation and measurement of the same sample would be extremely time consuming, greatly reducing the amount of data that could be collected in any fabric study.

A more practical approach would be to estimate fabric variability from a single data set so that data would only have to be collected once from each sample. If a fabric measurement is a simple one, variability can be determined by standard statistical techniques. Fry plots, however, are not simple, so derivation of a direct method for calculating uncertainty would be complex and has not been done. Instead the uncertainty can be estimated using a statistical approach for resampling the same data set. Erslev and Ge (1990) made use of their own resampling procedure to test the reproducibility of the normalized Fry method, particularly with regard to the number of grains that need to be analyzed. This work was done to demonstrate the validity of the normalized Fry method but was not used as part of the method for analyzing fabric.

Table 1

Original data set	Data subsets (randomly chosen from the original data set)									
	A	B	C	D	E	F	G	H	I	J
1.1	1.4	1.4	1.4	1.2	1.4	1.2	1.2	1.4	1.4	1.4
1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.2	1.2	1.2	1.2
1.2	1.1	1.2	1.1	1.2	1.2	1.1	1.4	1.1	1.1	1.4
1.4	1.4	1.1	1.4	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Mean = 1.23	1.2	1.225	1.275	1.2	1.25	1.15	1.25	1.225	1.225	1.3
Standard error = 0.054	Bootstrap estimate of standard error = 0.042 (standard deviation of the means of each data subsets)									

For this study data resampling was carried out using a more systematic statistical technique known as the bootstrap in order to have a standard way of determining fabric uncertainties. Bootstrapping (Diaconis and Efron, 1983; Efron and Tibshirani, 1991) involves resampling the original data set to create artificial data subsets that are used to estimate the variability of fabric parameters. These artificial data subsets are created by randomly selecting elements, with replacements, from the original data set. Each subset has the same number of elements as the original set but is different because an individual element can be chosen once, more than once, or not at all.

Table 1 presents a simple example of using the bootstrap to estimate standard error. For this example the original data set contains four elements. Ten subsets are generated by randomly selecting elements from the original set (Table 1). The bootstrap approximation of the standard error is the standard deviation of the means of each data subset (Diaconis and Efron, 1983; Efron and Tibshirani, 1991).

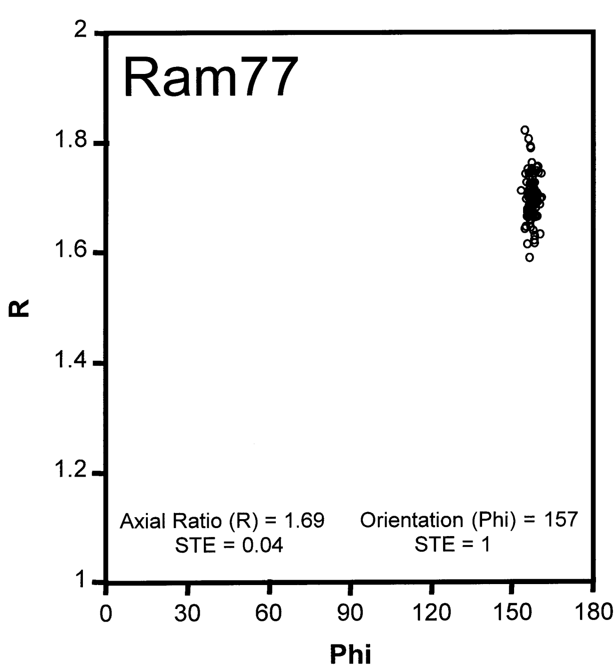


Fig. 3. Plot of fabric parameters of 100 bootstrap analysis subsets. R is the axial ratio and Phi is the orientation of the long axis of the fabric ellipse. STE is the standard error.

For this simple example the standard error of the original data set can be calculated directly and compared with the bootstrap estimate. Although in this example the bootstrap estimate of standard error is not the same as the calculated standard error (Table 1), the approximation would improve with the use of more data subsets. The power of the bootstrap is it can be used to estimate standard errors where direct calculations are not possible, such as with normalized Fry plots.

This paper applies the bootstrap to normalized Fry plots in order to study the uncertainty associated with synthetic (computer generated) and natural samples. This approach will be used to investigate the significance level at which: (i) fabric is developed in samples, (ii) fabric deviates from assumed sedimentary compaction fabrics, and (iii) fabric is different from samples in different structural positions.

2. Method

Normalized Fry plots were constructed for this study by modifying the implementation of McNaught (1994) for bootstrap analysis (for the remainder of this paper this implementation will be referred to as conventional normalized Fry analysis to distinguish it from the bootstrap approach described below). Data were collected the same way as for conventional normalized Fry analysis. Grain boundaries were traced on photomicrographs and then digitized using an image analyzer to determine the coordinates of the centroid and the area of each grain from the tracing. This list of centroid coordinates and grain areas is all the data needed to construct a normalized Fry plot using this approach. The polar coordinates of points on the normalized Fry plot are determined by calculating the direction between every possible pair of grains and the normalized center-to-center distance (D_n) between each pair (Erslev, 1988; McNaught, 1994):

$$D_n = D\pi^{1/2}/\left(A_1^{1/2} + A_2^{1/2}\right) \quad (1)$$

where D is the distance between grain centers, and A_1 and A_2 are the areas of each grain.

Here the bootstrap approach differs from the conventional normalized Fry analysis. Instead of plotting points representing all possible pairs of normalized center-to-center

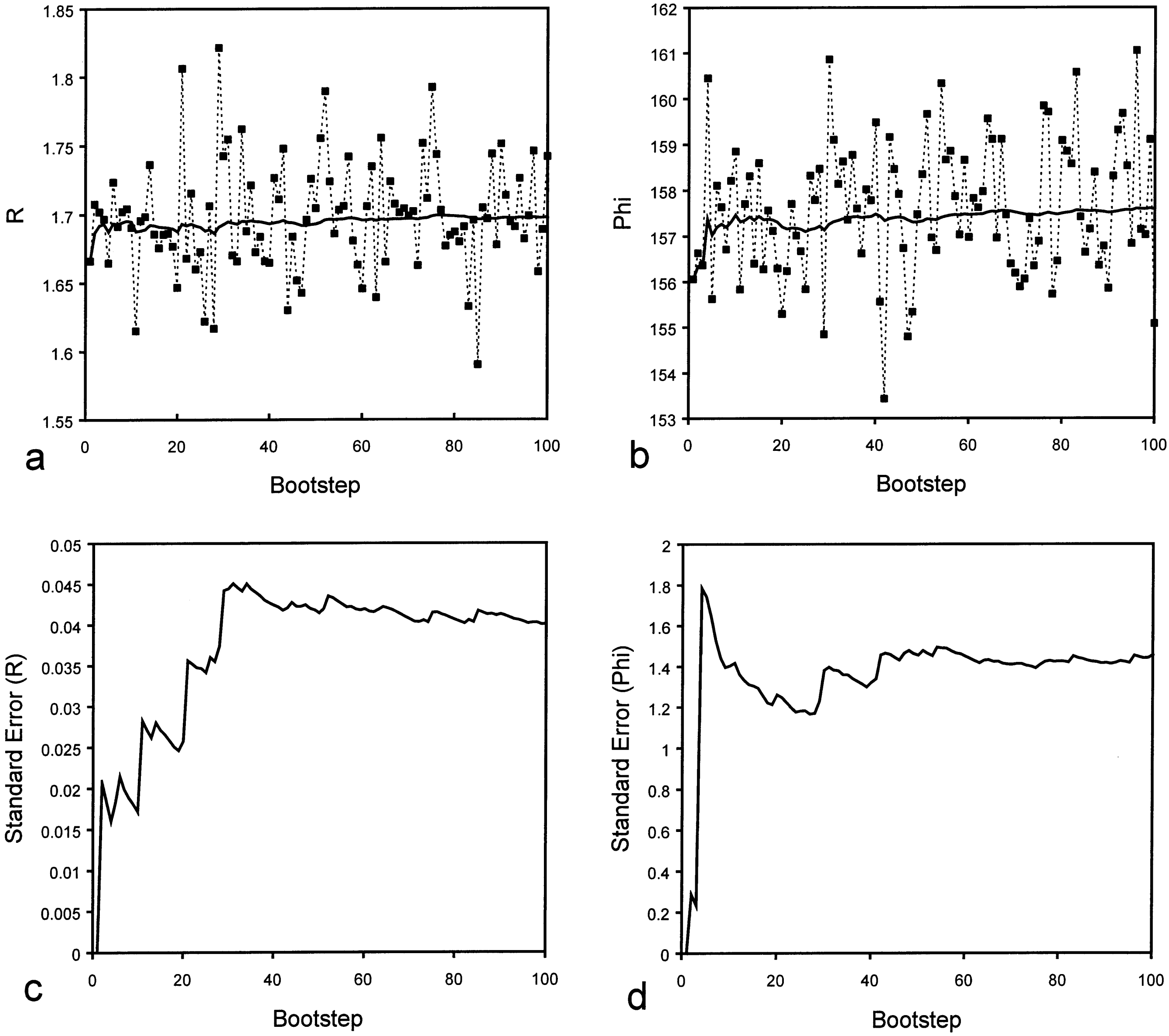


Fig. 4. Effects of number of increments used for bootstrap analysis on fabric determination. Estimated axial ratio for each bootstep (a) and estimated orientation of fabric ellipse for each 100 bootsteps (b). Solid lines in (a) and (b) indicate the running mean of axial ratio and orientation, respectively. (c) and (d) show the running calculation of standard error of axial ratio and orientation with each additional bootstep. Note all running calculations converge on values well before the 100 increment.

distances, for the bootstrap approach normalized center-to-center distances are selected randomly, with replacements, from the population of all possible center-to-center distances. Each iteration of the bootstrap process will use a different randomly selected data subset. Note that this differs slightly from the earlier bootstrap approach of McNaught (1994) where individual grains, rather than center–center distances were randomly selected.

Once the first bootstrap data subset has been chosen, it is treated as a conventional Fry data set using the approach of McNaught (1994). A first estimate of the fabric ellipse is determined from the normalized Fry plot by imposing a virtual circular grid over plot to locate the region of maximum point density in each radial sector. The normalized center-to-center distances in that region are averaged to determine the radial value of a point on the fabric ellipse. The length and width of each grid region exceeds the spacing between grid regions so points on the Fry plot lie in several regions. This prevents splitting of regions of maximum point density. Because the grid used to determine the fabric ellipse is circular it will tend to underestimate the fabric of elliptical maximum point density rings. This problem is avoided by taking an iterative approach that incrementally removes fabric until the axial ratio value of the remaining fabric ellipse falls below an arbitrary threshold (for this study 1.005). Combining the amount of fabric

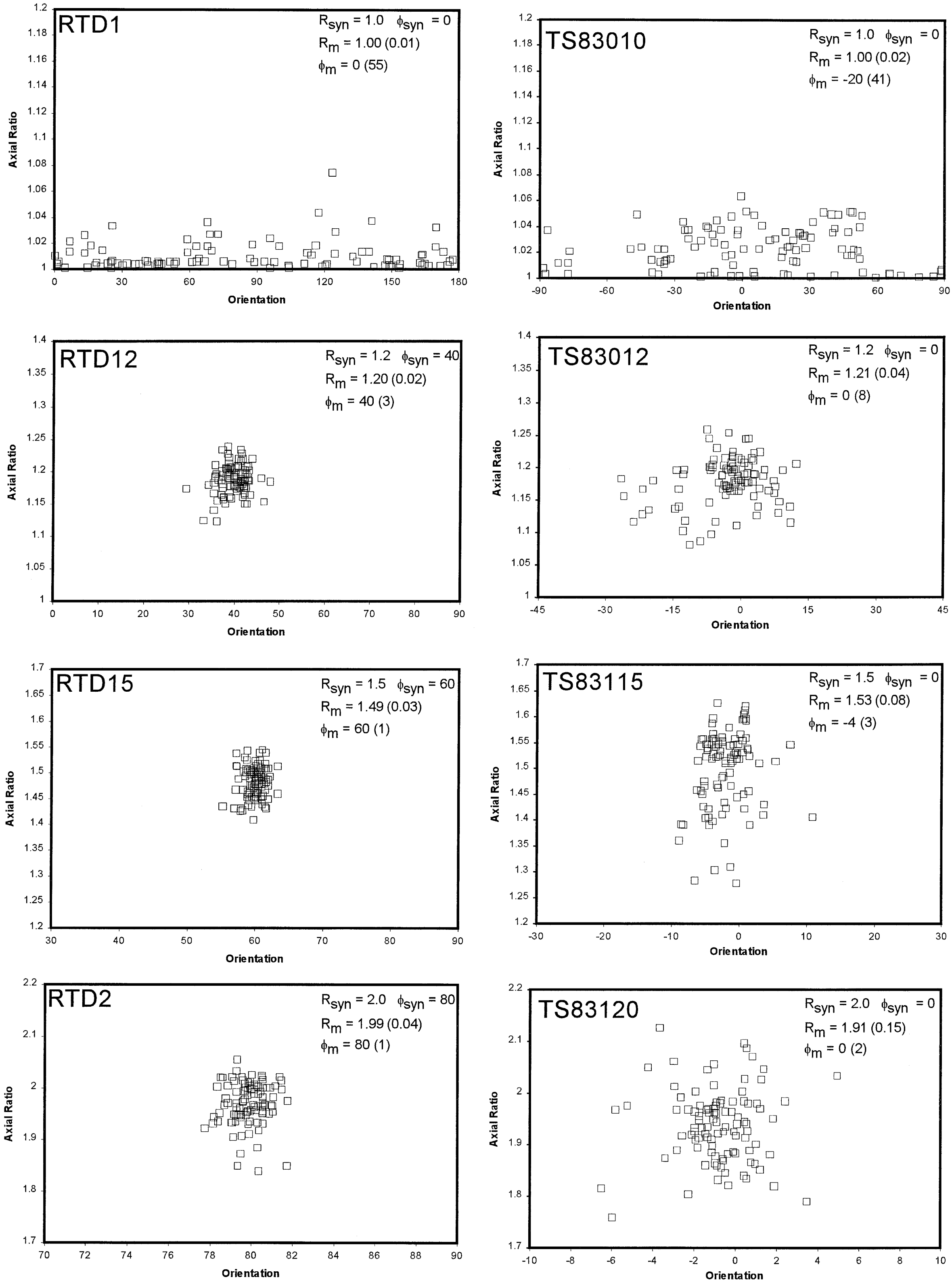
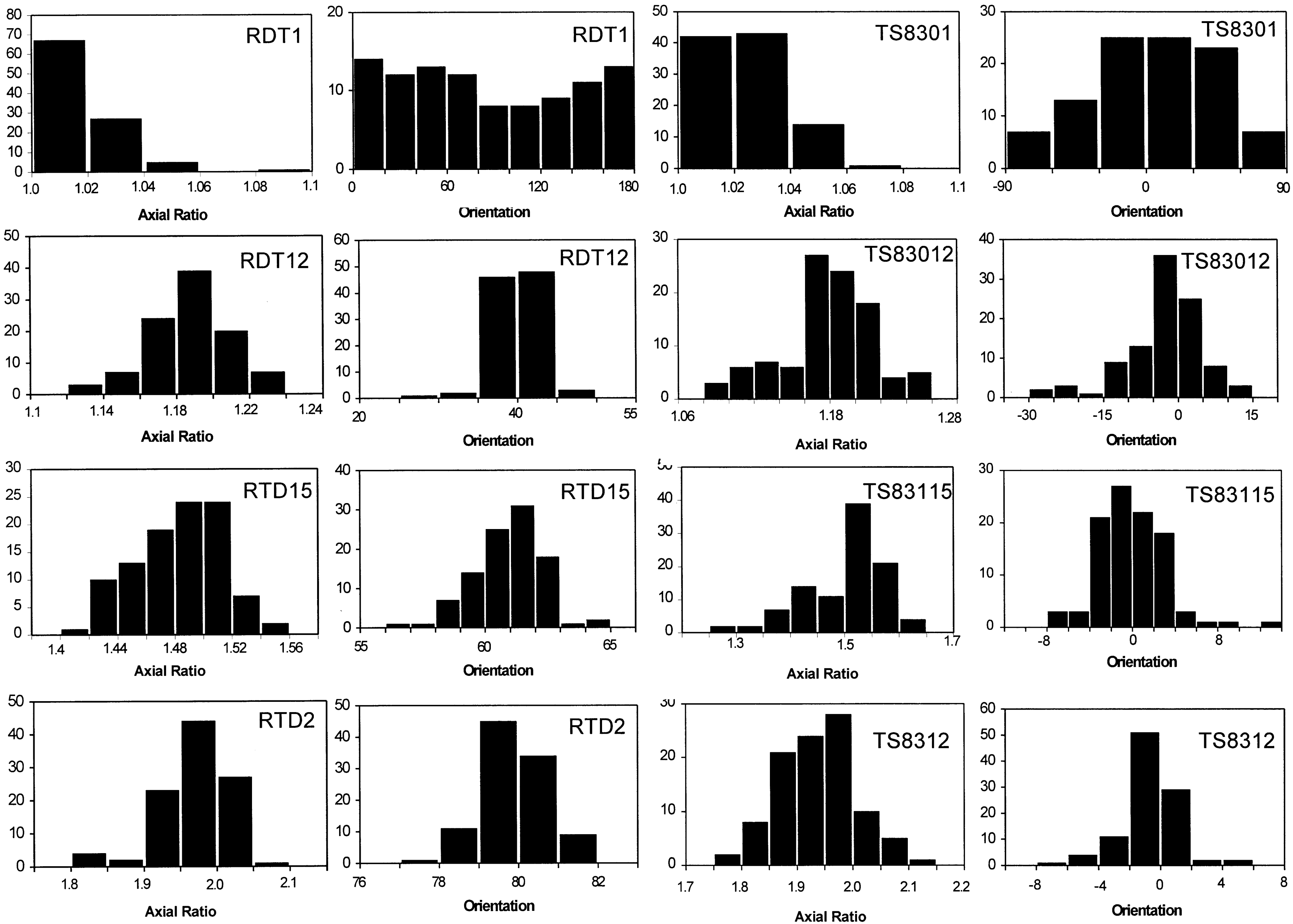


Fig. 5. Bootstrap results from analysis of synthetic data sets. Data set RDT consists of elliptical objects from figure 11b of Erslev and Ge (1990). Data set TS consists of polygonal objects from figure 9 of McNaught (1994). Objects in each data set were rotated and copied to insure no initial fabric and then subjected to synthetic deformation using a strain ellipse with axial ratios of 1.2, 1.5, 2.0. For data set RDT the orientation of the long axis of the strain ellipse was varied for synthetic deformation (40, 60, and 80°). R_{syn} and ϕ_{syn} are the parameters of the imposed synthetic fabric. R_m and ϕ_m are the measured fabric parameters. The standard error for each measured fabric parameter is given in parentheses after the measured value.


RDT1
Axial Ratio
RDT1
Orientation
TS8301
Axial Ratio
TS8301
Orientation
RDT12
Axial Ratio
RDT12
Orientation
TS83012
Axial Ratio
TS83012
Orientation
RTD15
Axial Ratio
RTD15
Orientation
TS83115
Axial Ratio
TS83115
Orientation
RTD2
Axial Ratio
RTD2
Orientation
TS8312
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TS8312
Orientation

removed for each increment, the axial ratio and orientation of the fabric ellipse can be calculated. This is the fabric estimate for the first bootstrap subset.

The above procedure is repeated for each of the subsets of randomly selected combinations of normalized center-to-center distances, until 100 values of axial ratio and orientation have been found. The standard deviations of the axial ratio and orientation values approximate the standard error of the axial ratio and orientation of the overall fabric ellipse.

3. Results

3.1. RAM77

The deformed oolite illustrated in figure 7.7 of Ramsay and Huber (1983, p. 112) has become a defacto standard for testing strain analysis methods, and serves as an excellent illustration for interpreting bootstrap results. Bootstrap results for normalized Fry plots are easiest to view by plotting axial ratio versus orientation of each of the 100 subset analyses (Fig. 3). The scatter of these points represents the uncertainty associated with the measurement of fabric in this sample. Quantitatively the standard errors for axial ratio and orientation are 0.04 and 1, respectively. A previously calculated axial ratio (1.69) and orientation (157) for this sample (McNaught, 1994) was plotted in the center of the cluster of points on this plot. Axial ratios determined by Erslev and Ge (1990) using normalized Fry analysis (1.64) and by Ramsay and Huber (1983) using conventional Fry analysis (1.7) also lie within this cluster of points representing uncertainty in this sample.

Crucial in this discussion is the decision to use 100 iterations in the bootstrap process. The choice to use 100 data subsets is arbitrary, but seems adequate because estimates of standard errors stabilize before all 100 data subsets are considered (Fig. 4). Additional subsets would probably improve results, but computational time, which is already considerable, becomes unreasonable.

3.2. Synthetic data

The bootstrap approach was tested on two groups of synthetic data sets with known imposed strain. Data sets in the first group (RDT) are made up of computer-generated elliptical objects (figure 11b of Erslev and Ge (1990)). Data sets in the second group are made up of computer-generated polygonal objects (figure 9 of McNaught (1994)). This second data set was developed to approximate aggregates of nonelliptical grains. The original objects in each data set were rotated and duplicated to ensure no initial fabric. Then each set was subject to the indicated synthetic plane strains ($R_{syn} = 1.2$, 1.5, 2.0) by adjusting coordinates of the centers of each grain. In addition, for the RDT data sets, the orientation of the principal axes of synthetic strain was changed in each case.

Conventional normalized Fry results (R_m, ϕ_m) closely agree with the imposed axial ratio and orientation of the synthetic deformation (R_{syn}, ϕ_{syn}) (Fig. 5). Bootstrap results cluster around the imposed synthetic deformation for each set. The standard error for axial ratio increases and the standard error for the orientation decreases with increasing imposed synthetic deformation (Fig. 5). Histograms of bootstrap estimates of axial ratio and orientation show a generally symmetric distribution (Fig. 6), except for data sets with no imposed synthetic fabric.

3.3. Sheeprock Thrust sheet

The bootstrap approach to normalized Fry analysis can be illustrated by a field example from the Sheeprock Thrust sheet in Utah. The Sheeprock Thrust is a major thrust in the Provo salient of the Sevier Belt of the North America Cordillera. The thrust carries a thick section of Proterozoic and Early Cambrian quartzites in its hanging wall (Mukul and Mitra, 1998). The uniform lithology of the hanging wall strata provides an excellent area for studying deformation associated related to thrust emplacement without the complications of lithology variations. The strain in these quartzites has been studied in detail (Mukul, 1998; Mukul and Mitra, 1998).

For this study six samples were selected from the much larger data set of Mukul (1998) and were analyzed using the bootstrap normalized Fry approach. Three of these samples serve as a good illustration of the method because, when they are viewed down plunge, they represent the fabric that exists in a vertical profile above the horizontal thrust. Qualitatively, the fabric is weakest away from the fault (Fig. 7a) and strongest in the sample closest to the fault (Fig. 7c).

To compare the interpretations of the fabric in these samples, each was analyzed using both the conventional- and the bootstrap-normalized Fry methods. Quantitative results from the conventional normalized Fry method (Fig. 8) show an increase in axial ratio from $R = 1.06$ away from the fault to $R = 1.80$ near the thrust. The long axis of each fabric ellipse verges in the transport direction (to the east).

The basic interpretation made from this data is that fabric develops because of increasing shear closer to the fault. Developing a deformation model to explain this data requires more than fault-parallel simple shear because the measured orientations and axial ratios of the fabric ellipses cannot be produced by simple shear of an initial circular marker (Fig. 9). The three data points from the Sheeprock Thrust sheet lie below the $\alpha = 1$ curve. Instead the deformation model requires additional deformation increments.

Fig. 6. Histograms of bootstrap estimates of axial ratio and orientation for synthetic data sets RDT and TS. Vertical axis for each is the number of bootstrap estimates in the given range. Total number of measurements for each histogram is 100.

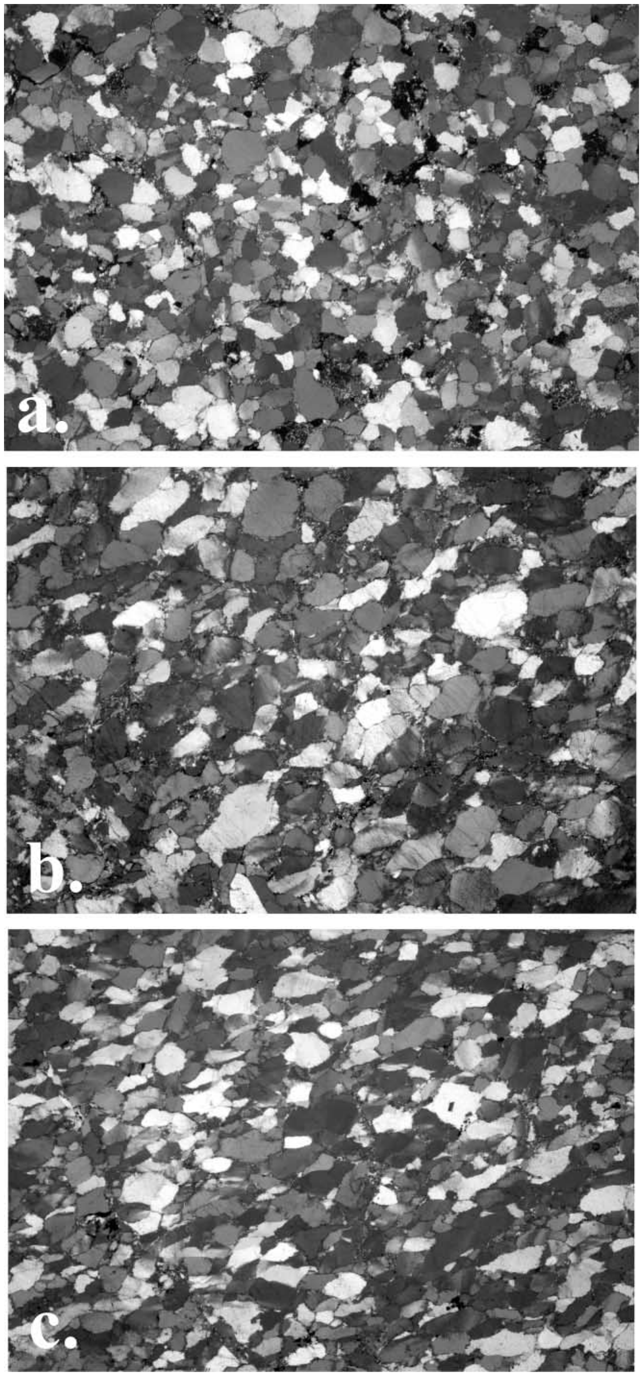

Fig. 7. Photomicrographs of thin section cut in the transport plane from samples taken above the Sheeprock Thrust. (a) Sample 33S, furthest from thrust, (b) Sample 37S, (c) Sample 47S, closest to thrust. For the relative positions of samples see Fig. 8.

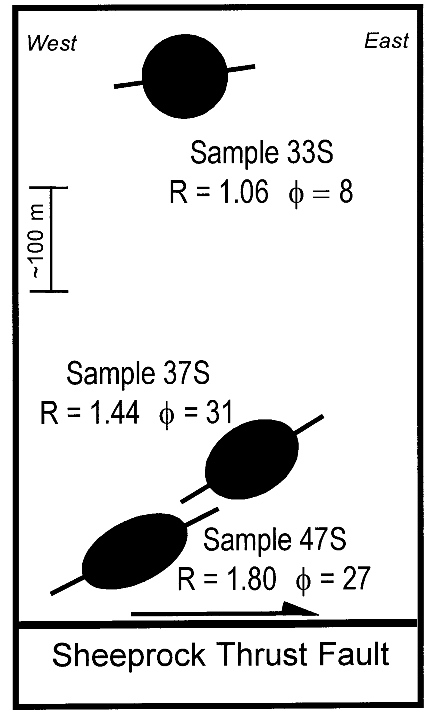

Fig. 8. Summary of conventional normalized analysis of samples above the Sheeprock Thrust. Positions of fabric ellipses represent the position of samples relative to the thrust as determined from down plunge projections. R is the axial ratio of fabric ellipse and ϕ is the orientation of the long axis of the fabric ellipse relative to the fault.

A number of additional deformation increments are possible, including dilation (non-plane strain), stretching in the transport direction, and vertical shortening (sedimentary compaction). One possible simple model calls for early sedimentary compaction, increasing with depth, followed by simple shear above the fault, with increasing amounts of shear closer to the fault. This model is by no means a unique solution, but it does explain the measured deformation using deformation increments that can be reasonable considered geologically possible for this area.

The deformation model above, however, is based on the result of conventional normalized Fry method and is only as good as the original data. But is the variation between fabric measurements significant, and if so, how significant is it? Are all the steps in the deformation model significant, or are we adding steps to explain noise in the data rather than actual deformation in the rock? Bootstrap analysis allows us to address these important questions.

Bootstrap normalized Fry analysis was performed on the same raw data used for the conventional normalized Fry analysis above (Fig. 10). The three samples show minimum overlap in axial ratio (Fig. 9). Sample 33S, the sample furthest from the fault, does not appear to have a significant fabric (its axial ratio is not significantly different than 1.0) using an arbitrarily selected 90% confidence interval, and appears to be significantly different from the other two samples (Fig. 9). Samples 37S and 47S both have a significant fabric. A simple statistical z-test of the two sample's axial ratios suggests that the axial ratios are significantly different at the 90% significance level. There is no significant difference in fabric ellipse orientation between the two samples. This small data set from the Sheeprock Thrust sheet indicates a significant fabric is only developed in the lower part of the sheet. There is a significant increase in

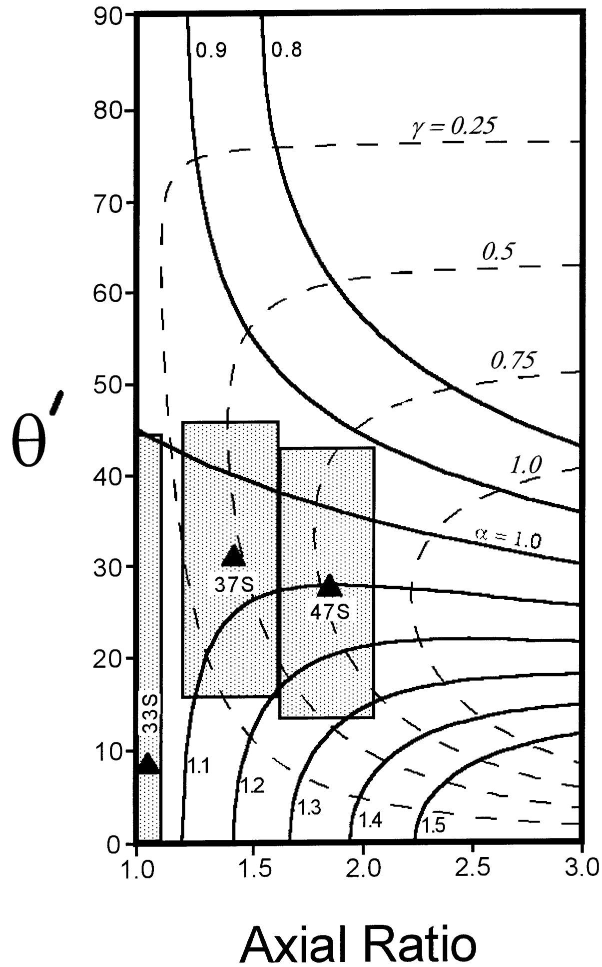

Fig. 9. Graph of axial ratio and orientation of a strain ellipse produced by combinations of simple shear (dashed curves, $\gamma = 0.25, 0.5, 0.75, 1.0$) and stretch parallel to the shear direction (solid curves, $\alpha = 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5$) (Sanderson, 1982). Because we are only dealing with axial ratios for this study, positive stretches are equivalent to two-dimensional compaction perpendicular to the shear plane. Data for the Sheeprock Thrust shown. Triangles represent results for conventional normalized Fry analysis. Boxes represent results of bootstrap normalized Fry. The mean values are located at the center of each box. The sides of each box = ±1.65 standard error for axial ratio and orientation.

fabric intensity approaching the fault but there is no significant change in fabric ellipse orientation, though the lower two fabric ellipses are inclined to bedding by a significant amount.

Modeling the fabric determined by the bootstrap approach is relatively straightforward. Because the fabric ellipses are significantly inclined to bedding and verge in the transport direction it seems likely that fault parallel shear is responsible. None of the fabric ellipses are significantly different (at the 90% confidence level) from the curve that represents simple shear, so all three fabric measurements could be explained by different amounts of fault parallel simple shear. This is not the only deformation model that is possible; the data is also consistent with a two-stage history involving different amounts of vertical compaction followed by fault parallel shear. The exact combination of these two components can vary, but the data restricts possible combinations. More complex deformation models are also possible, but the bootstrap approach to normalized Fry analysis has placed constraints on which models are allowed. Further, this approach makes possible the distinction between steps in the deformation model that are required to explain the data, and steps in the deformation model that are allowed by the data.

4. Discussion

The bootstrap approach presented here provides a systematic way for evaluating the variability associated with normalized Fry plots. Because the method makes use of the same data files used by the conventional normalized Fry method it does not require additional time collecting and entering data, though it does require additional computer time. Using this approach gives a standard way to evaluate and compare fabric data.

Standard errors for both axial ratio and orientation of the fabric ellipse depend on both the axial ratio of the fabric ellipse and the lithology of the sample (Fig. 11). Patterns for synthetic data (data sets RDT and TS) indicate that the standard error for axial ratios increase (Fig. 11a) and the standard error for orientation decreases (Fig. 11b) with increasing axial ratio. This pattern is also followed by the Sheeprock Thrust sheet data sets, with the exception of one sample. This sample, 37S, the intermediate fabric sample, appears to have a bimodal distribution of grain shapes (Fig. 7b). Some of the grains appear to have developed a preferred orientation while others remain more or less equal dimensional, as if deformation was further along in some grains than others. If this occurred it would complicate the distribution of center-to-center distances, leading to higher standard errors for axial ratio.

Care needs to be taken in interpreting data sets with low axial ratios. The distribution of bootstrap estimates of axial ratio in these data sets (RTD1 and TS8301, Fig. 6) is asymmetric since values of axial ratios cannot be less than 1.0. This can lead to an underestimate of standard error, though the standard error is much larger than the difference between the calculated axial ratio and 1.0, which is an indication that the sample lacks a fabric.

Lithology appears to be the other important factor in determining standard errors (Fig. 11). Analysis of synthetic elliptical objects of data set RTD produces the lowest standard errors. The deformed oolite from figure 7.7 of Ramsay and Huber (1983), which is made of natural elliptical objects, also falls along the trend for elliptical objects. Analysis of synthetic data sets of polygonal objects (data set TS) produces higher standard errors, especially for axial ratios. Four of the samples from the Sheeprock Thrust sheet follow this trend, suggesting the synthetic data is not a bad approximation for some natural polygonal aggregates.

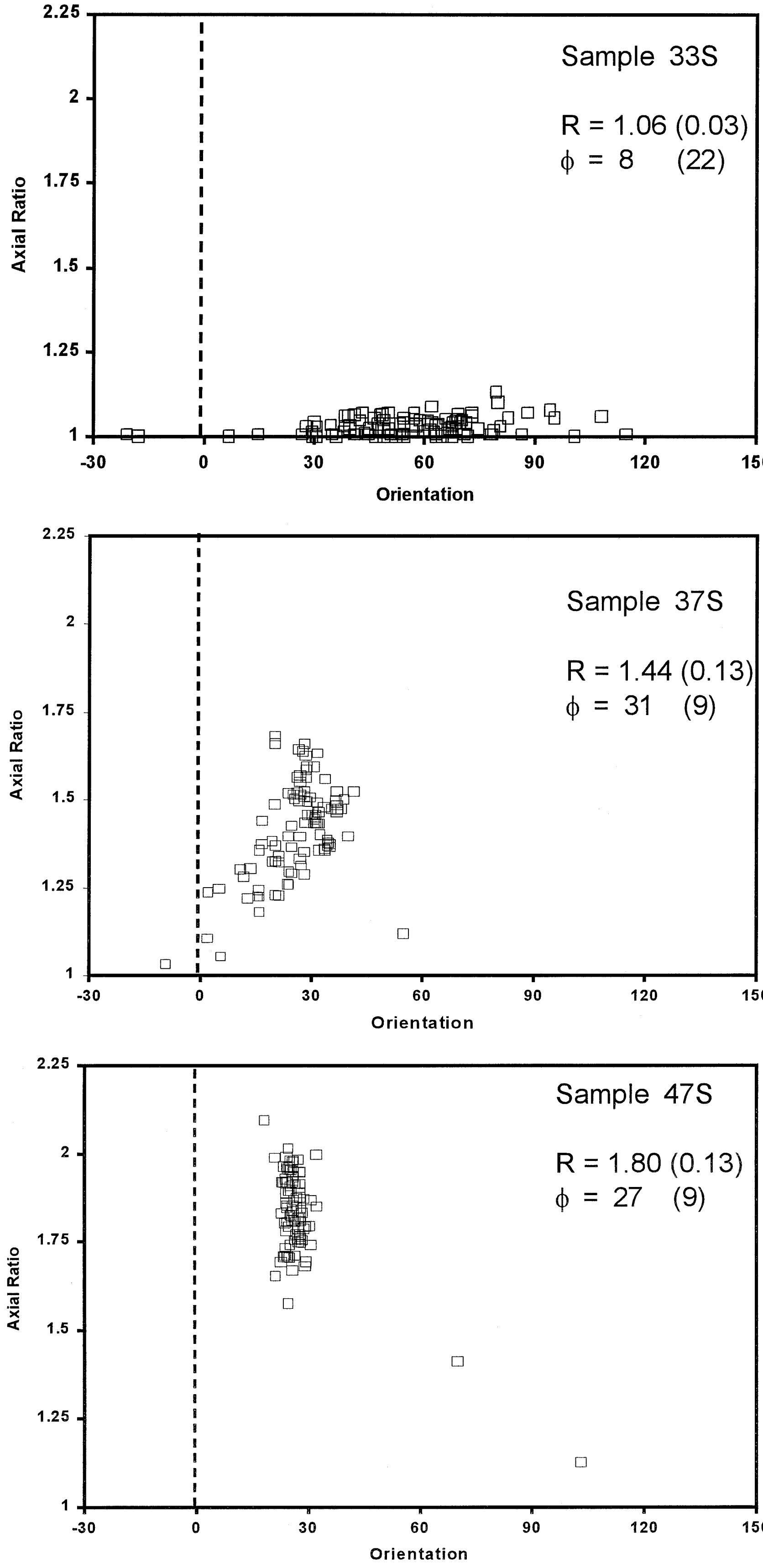

Fig. 10. Results of bootstrap analysis of three samples from the Sheeprock Thrust. Zero degree orientation represents the position of bedding and the shear plane. Conventional normalized Fry axial ratio (R) and orientation (ϕ) are shown for each sample along with standard errors determined by bootstrap analysis in parenthesis.

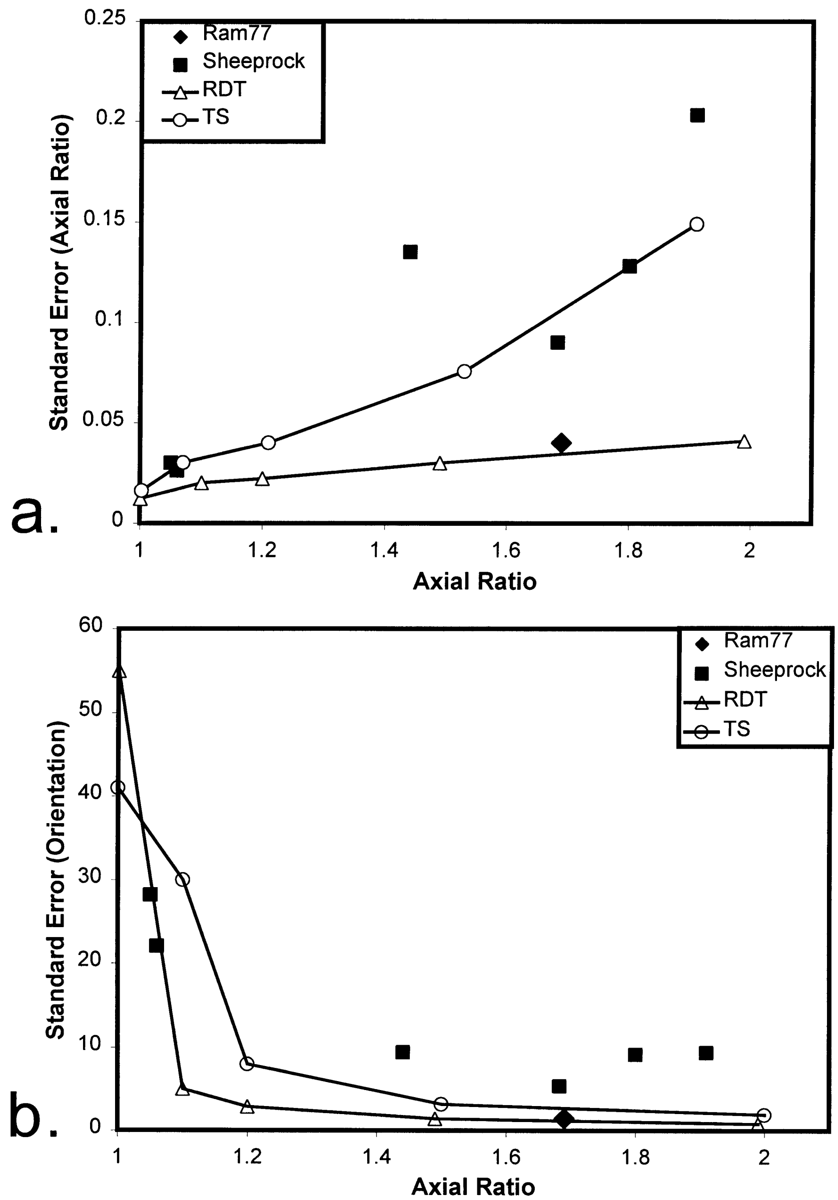

Fig. 11. Relationship of standard errors for axial ratios and for orientation to the measured axial ratio. Lines connect data points for synthetic data. RTD is the elliptical data set, TS is the polygonal data set. Ram77 is the deformed oolite in figure 7.7 of Ramsay and Huber (1983), Sheeprock are the samples from the Sheeprock Thrust sheet, including three samples from elsewhere in the Sheeprock sheet.

In general caution is needed in interpreting any individual normalized Fry plot. From the limited data in this study it is doubtful that axial ratios determined by normalized Fry measurements of natural aggregates are significant to more than one decimal place, and for aggregates of irregularly-shaped grains the significance may be much less.

The importance of taking into account the variability associated with fabric measurements made by the normalized Fry method when developing deformation models was illustrated by the Sheeprock Thrust sheet field example. A more elaborate deformation history was proposed based on the conventional normalized Fry analysis rather than on the bootstrap normalized Fry approach. The variability associated with the fabric parameters allows for simpler explanations of fabric development. While unique deformation histories could not be worked out from the limited set of available fabric data, constraints on deformation history could be developed. This may be the most important aspect of use using the bootstrap approach; it allows recognition of which part of a deformation model is required by the data, and what part of the deformation model is permitted by the data. For the samples from the Sheeprock Thrust sheet all of the fabric can be explained by different amounts of fault parallel simple shear, but the evidence does not exclude the possibility of other components of deformation, such as preshear vertical compaction or stretching in the transport direction.

Though it is convenient to discuss variability in terms of confidence intervals because it allows for discrimination between 'significant and insignificant' results, it is important to recognize that the selection of these cutoffs is arbitrary. If a lower confidence level was chosen more of the results could be considered 'significant', likewise if a higher confidence level was chosen more of the results could be considered 'insignificant'. The main goal of the bootstrap approach should not be determining whether results meet some arbitrary level, but to provide a way to evaluate study results. More important than the confidence interval of a single study is comparison of confidence intervals between studies. Results from studies using higher confidence intervals should be given more weight.

5. Conclusions

Results of bootstrap analyses show considerable variation in fabric parameter uncertainty, apparently related to both lithology and the degree of fabric development. For the natural samples measured in this study the standard errors for the axial ratio of the fabric ellipse varied between 0.05 and 0.2. The standard errors for the orientation of the fabric ellipse was generally less than 10°, except in cases of very weak fabric. Axial ratio measurements appear to be significant to only one decimal place.

Bootstrap analysis of normalized Fry plots provides a method for estimating the uncertainty associated with fabric measurements. This information is important for evaluating the quality of data, and can be an important guide in making structural interpretations. For the field examples presented in this study, allowing for variability in fabric measurements permitted adoption of a simpler deformation model.

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