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Temperature distribution inside and around a lava tube

Michele Dragoni [a,*], Francesca D'Onza [b], Andrea Tallarico [c]

[a] *Dipartimento di Fisica, Università di Bologna, Bologna, Italy*
[b] *Dipartimento di Scienze della Terra, Università di Siena, Siena, Italy*
[c] *Dipartimento di Geologia e Geofisica, Università di Bari, Bari, Italy*

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Abstract

We propose a model describing the thermal effects of a lava tube. The tube is a circular cylinder, embedded in a solid medium and filled with a Newtonian liquid flowing under the gravity force. Steady-state conditions are considered. The velocity field in the tube, evaluated by the Navier–Stokes equation, is introduced into the heat equation taking into account the viscous dissipation. The temperature distribution is evaluated both inside the tube and in the surrounding solid medium. Under the assumption that the lava tube is embedded in a solid half-space, the surface heat flow due to the presence of the tube is calculated. It is shown that heat flow measurements at the Earth's surface can give information on the depth, size and temperature of the buried lava tube.

Keywords: temperature distribution; lava tube

1. Introduction

Lava tubes are a common feature of many effusive eruptions and they were described by several authors (Ponte, 1949; Peterson and Swanson, 1974; Guest et al., 1980; Greeley, 1987; Peterson et al., 1994; Sakimoto et al., 1997). The evolution of a lava channel into a tube is often a consequence of surface cooling of lava, which produces a solid crust and levees (Dragoni et al., 1995).

Thermal aspects concerning lava tubes have been investigated by several authors. Keszthelyi (1998) proposed a thermal budget of lava tubes taking into account conduction, viscous dissipation, crystallization and degassing, atmospheric convection, rain and thermal radiation and compared the heat flows due to these various mechanisms. In the present model atmospheric convection, rain and radiation are not taken into account because we consider buried lava tubes as observed on Mount Etna (Calvari and Pinkerton, 1999); moreover we evaluate not only the heat flux but also the temperature profile. Sakimoto and Zuber (1998) described the cooling of lava tubes by conduction. Our paper differs from theirs since we take into account the viscous dissipation and we introduce the solution of the Navier–Stokes equation into the heat equation: in this way we consider the velocity field of the lava and obtain a temperature profile also for the inner part of the tube. Accordingly our model depends also on gravity, lava viscosity and slope

* Corresponding author..
E-mail address: dragoni@ibogfs.df.unibo.it (M. Dragoni).

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of the tube. Lava tubes have strong implications for hazards in volcanic areas: because of the formation of tubes, lava flows may threaten areas which would not be reached by flows in open channels (Malin, 1980). In fact, tube formation may considerably increase the length of a flow, since it greatly reduces the heat loss of lava.

The formation and evolution of a lava tube is a complex phenomenon resulting from a close interplay between thermal and rheological properties of lava. In the present paper we investigate the temperature field which develops inside and around a lava tube. Lava tubes have irregular cross sections, varying from rectangular to circular and sometimes triangular (Kilburn and Guest, 1993; Peterson et al., 1994; Calvari and Pinkerton, 1999). As a first approximation, we assume that the lava tube is a sloping, right circular cylinder, filled with a high-temperature Newtonian liquid and embedded in a solid medium. Kauahikaua et al. (1998) observed that lava tubes rarely are full: however the model that we are proposing can approximately account for partially filled tubes. The liquid flows under the gravity force and heat is transferred by conduction to the surrounding medium.

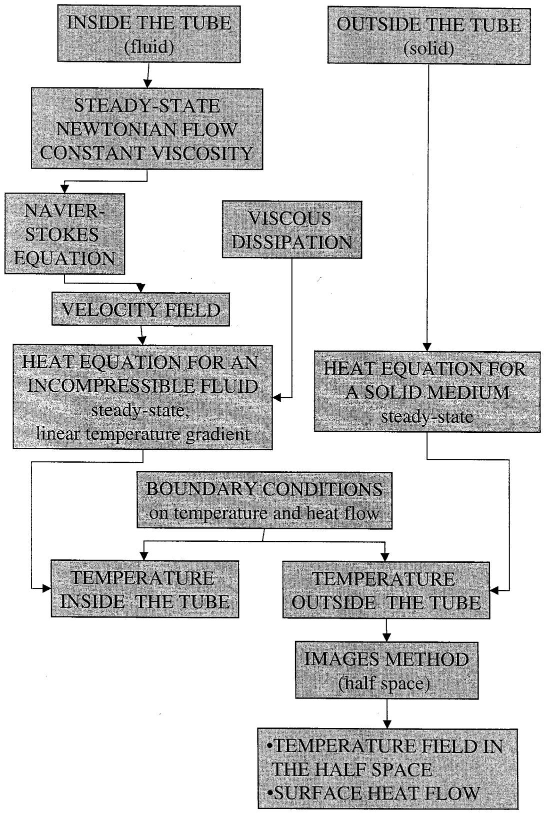

Fig. 2. Summary of the procedure.

2. The model

We assume that the tube is a right circular cylinder, inclined by an angle α with respect to the horizontal plane and filled with a Newtonian liquid with density ρ, specific heat c_p, thermal conductivity κ and viscosity η, flowing in the tube under the gravity force.

As a consequence of the geometry, we introduce a system of cylindrical coordinates (r, θ, x), such that the x-axis coincides with the axis of the cylinder and r is the radial distance from the axis of the cylinder itself, while θ is the azimuthal coordinate (Fig. 1). Let r_1 be the radius of the cylinder.

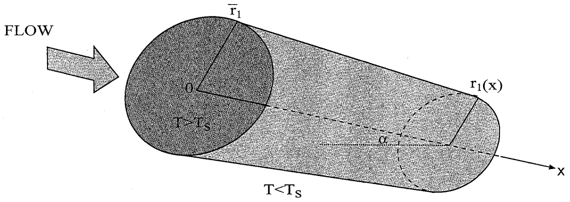

Fig. 1. Sketch of the model.

As it flows, the liquid cools down due to heat conduction in the surrounding medium. Temperature varies as a function of r and x. We assume that temperature variations are small enough to consider ρ, c_p, κ and η as uniform. We consider only the steady-state condition.

We first consider a tube embedded in an unbounded solid medium and calculate the temperature distribution inside and outside the tube. Then we consider the tube in a solid half-space, in order to evaluate the heat flow at the Earth's surface. A summary of the procedure is shown in Fig. 2.

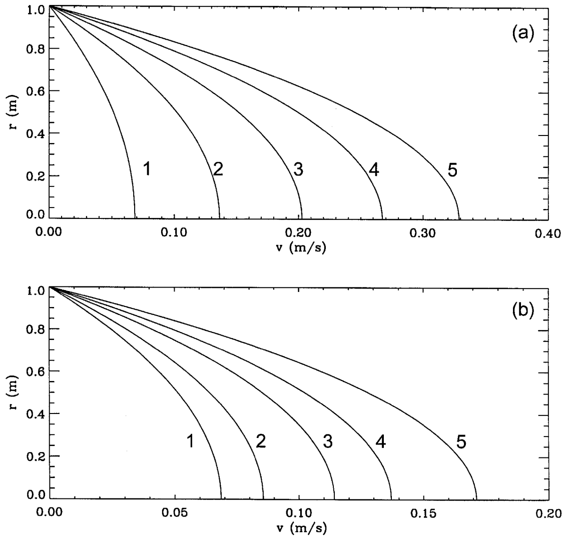

Fig. 3. Lava velocity in a cross section of the tube. Different values of the tube inclination α and the viscosity η are considered. (a) Curve 1, $\alpha=0.1$ rad, curve 2, $\alpha=0.2$ rad, curve 3, $\alpha=0.3$ rad, curve 4, $\alpha=0.4$ rad, curve 5, $\alpha=0.5$ rad; for all the curves $\eta=10\,000$ Pa s. (b) Curve 1, $\eta=10\,000$ Pa s, curve 2, $\eta=8000$ Pa s, curve 3, $\eta=6000$ Pa s, curve 4, $\eta=5000$ Pa s, curve 5, $\eta=4000$ Pa s; for all the curves $\alpha=0.1$ rad. The values of the model parameters g and ρ are fixed and shown in Table 1.

2.1. Temperature distribution inside the tube

Flow takes place in the x direction and flow velocity has the form:

$$\mathbf{v} = (0,\ 0,\ v) \tag{1}$$

The Navier–Stokes equation yields the well-known solution (e.g. Kundu, 1990; Dragoni et al., 1986):

$$v(r) = v_0\left(1-\frac{r^2}{r_1^2}\right),\ 0 \leq r \leq r_1 \tag{2}$$

where v_0 is the velocity at the center of the tube:

$$v_0 = \frac{\rho g r_1^2 \sin\alpha}{4\eta} \tag{3}$$

where g is the acceleration of gravity. Fig. 3 shows the magma velocity v inside the tube, as a function of distance r from the center, for a tube having initial radius $\bar{r}_1 = 1$ m. The typical parabolic profile of the velocity of a Newtonian fluid inside a cylindrical tube can be recognized. Due to the low thermal conductivity of rocks, the decrease of temperature T with x is very slow: we assume that it is linear, i.e.

$$\frac{\partial T}{\partial x} = -G \tag{4}$$

with $G > 0$. If we call T_0 the temperature at the center of the tube, we can write:

$$T_0(x) = \bar{T}_0 - Gx \tag{5}$$

where $\bar{T}_0$ is the central temperature at the beginning of the tube ($x=0$ and $r=\bar{r}_1$). The temperature distribution $T(r,\ x)$ inside the tube is given by the heat equation for an incompressible fluid (e.g. Landau and Lifchitz, 1971):

$$\rho c_\mathrm{p}\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = \kappa\nabla^2 T + \frac{1}{2}\eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)^2 \tag{6}$$

where the second term in the right hand side of the equation is the heat production per unit volume and unit time due to viscous dissipation. If we consider a steady-state motion and take into account Eq. 1, the heat equation reduces to:

$$\rho c_\mathrm{p} v\frac{\partial T}{\partial x} = \kappa\nabla^2 T + \frac{1}{2}\eta\left(\frac{\partial v}{\partial r}\right)^2 \tag{7}$$

In cylindrical coordinates, the Laplacian of T has

Table 1
Numerical values for the model parameters which are kept constant in drawing the graphs

$\bar{T}_0$	1100°C
T_s	900°C
T_a	20°C
κ	3 J s^{-1} m^{-1} K^{-1}
ρ	2800 kg m^{-3}
g	9.8 m s^{-2}
c_p	837 J kg^{-1} K^{-1}
$\bar{r}_1$	1 m

The values are typical for basaltic lavas, such as Etna lavas (e.g. Murase and McBirney, 1973).

the following expression:

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial x^2} \quad (8)$$

Thanks to Eq. 4, $\partial^2 T/\partial x^2 = 0$. Moreover we have axial symmetry, i.e. no dependence on θ. Accordingly, the expression of the Laplacian can be simplified and Eq. 7 can be written as:

$$\rho c_p v \frac{\partial T}{\partial x} = \kappa\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{1}{2}\eta\left(\frac{\partial v}{\partial r}\right)^2 \quad (9)$$

or, by Eqs. 2 and 4:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = -\frac{\rho c_p v_0 G}{\kappa}\left(1 - \frac{r^2}{r_1^2}\right) + \frac{2\eta v_0^2}{\kappa}\frac{r^2}{r_1^2} \quad (10)$$

Boundary conditions on temperature and heat flow are:

$$T(0,\ x) = T_0(x) \quad (11)$$

$$\frac{\partial T}{\partial r}\Big|_{r=0} = 0 \quad (12)$$

The solution of Eq. 10 is:

$$T(r,\ x) = T_0(x) - \gamma G\left(4r^2 - \frac{r^4}{r_1^2}\right) + \beta\frac{r^4}{r_1^4},\ 0 \le r \le r_1 \quad (13)$$

where, taking into account Eq. 3:

$$\gamma = \frac{\rho c_p v_0}{16\kappa} = \frac{\rho^2 g c_p r_1^2 \sin\alpha}{64\kappa\eta} \quad (14)$$

$$\beta = \frac{v_0^2 \eta}{8\kappa \sin\alpha} = \frac{\rho^2 g^2 r_1^4 \sin\alpha}{128\kappa\eta} \quad (15)$$

Sakimoto and Zuber (1998) obtain a different solution, for which they do not assume a constant thermal gradient but assume constant temperature and heat flow at the wall of the tube. In this way they obtain G as a result of their model. It appears that G is approximately constant for reasonable distances from the vent and not too small flow rate according with our assumption in Eq. 4. We assume that the surface of the tube $r = r_1$ is the isothermal surface at the solidus temperature T_s:

$$T(r_1,\ x) = T_s \quad (16)$$

The velocity and temperature distributions within the tube control the radius of the tube itself. The liquid cools down as it flows: due to temperature decrease, the radius of the tube also decreases: therefore r_1 depends weakly on x. However, the decrease is very small and can be neglected for our purposes. From Eq. 13 calculated at $r = r_1$

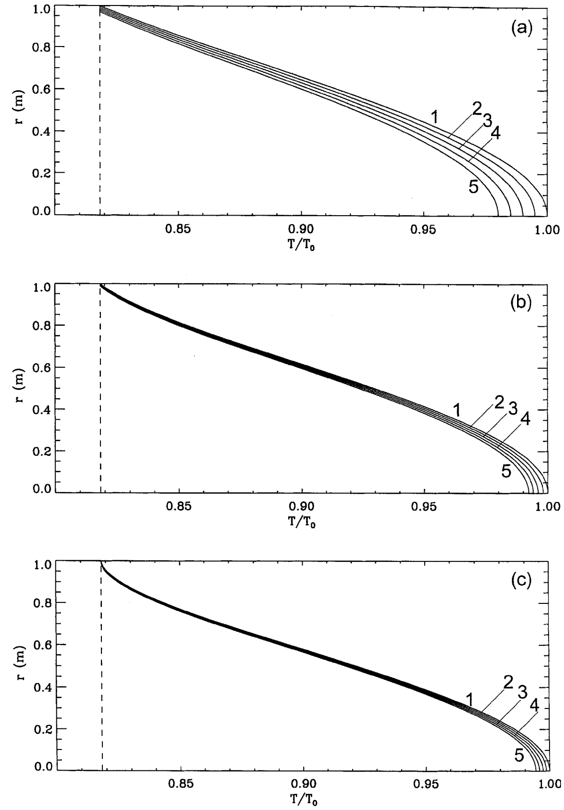

Fig. 4. Temperature inside the tube. Different values of the tube inclination α and of the distance from the beginning of the tube x are considered. (a) $\alpha = 0.1$ rad, (b) $\alpha = 0.3$ rad, (c) $\alpha = 0.5$ rad; curve 1, $x = 0$ m, curve 2, $x = 250$ m, curve 3, $x = 500$ m, curve 4, $x = 750$ m, curve 5, $x = 1000$ m. For all the curves $\eta = 10\,000$ Pa s. The values of the other model parameters are fixed and shown in Table 1.

we obtain:

$$T_s = T_0 - 3\gamma G r_1^2 + \beta \quad (17)$$

Once the initial radius $\bar{r}_1$ of the tube is given, the temperature gradient G can be obtained from Eqs. 5 and 17 as:

$$G = \frac{\bar{T}_0 - T_s + \beta' \bar{r}_1^4}{3\gamma' \bar{r}_1^4}, \bar{T}_0 \geq T_s \quad (18)$$

where:

$$\gamma' = \frac{\gamma}{r_1^2} \quad (19)$$

$$\beta' = \frac{\beta}{r_1^4} \quad (20)$$

The temperature inside the tube given by Eq. 13 is shown in Fig. 4. Three different slopes α and five different distances x from the tube origin x are considered: it appears that the lava temperature decreases as x increases. The cooling decreases for higher values of α: this is due to the greater lava velocity causing a greater supply of heat, represented by the advective term in the heat equation (Eq. 6).

At points $x > 0$, from Eq. 18 the radius of the tube is found to be:

$$r_1(x) = \sqrt[4]{\frac{T_0(x) - T_s}{3\gamma' G + \beta'}}, \; T_0 \geq T_s \quad (21)$$

The radius decreases slowly as T_0 decreases; it appears that the decrease in the radius reaches a maximum at a value of about 3%: we ignore this effect. Since dr_1/dx is very small, we neglect the decrease of the radius in the continuity equation. The temperature gradient as a function of $\bar{r}_1$ is shown in Fig. 5. It can be seen that G reaches a limit value if $\bar{r}_1$ is greater than about 2 m, such a value is:

$$\lim_{\bar{r}_1 \to \infty} G = \frac{\beta'}{3\gamma'} = \frac{g}{6C_p} \quad (22)$$

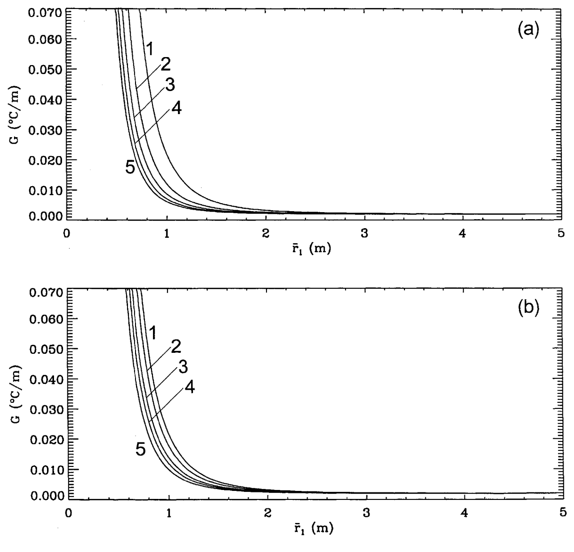

Fig. 5. Thermal gradient G as a function of the initial tube radius $\bar{r}_1$. Different values of the tube inclination α and the viscosity η are considered. (a) Curve 1, $\alpha = 0.1$ rad, curve 2, $\alpha = 0.2$ rad, curve 3, $\alpha = 0.3$ rad, curve 4, $\alpha = 0.4$ rad, curve 5, $\alpha = 0.5$ rad; for all the curves $\eta = 10\,000$ Pa s. (b) Curve 1, $\eta = 10\,000$ Pa s, curve 2, $\eta = 8000$ Pa s, curve 3, $\eta = 6000$ Pa s, curve 4, $\eta = 5000$ Pa s, curve 5, $\eta = 4000$ Pa s; for all the curves $\alpha = 0.1$ rad. The values of the other model parameters are fixed and shown in Table 1.

2.2. Temperature distribution outside the tube

The temperature distribution outside the tube is given by the heat equation for the solid medium, in the absence of heat sources (e.g. Landau and Lifchitz, 1971):

$$\rho c_p \frac{\partial T}{\partial t} = \kappa \nabla^2 T \quad (23)$$

where ρ, c_p and κ are, respectively, the density, the specific heat and the thermal conductivity of the solid. Under steady-state conditions, Eq. 23 reduces to:

$$\nabla^2 T = 0 \quad (24)$$

On the basis of previous assumptions, the Laplacian of T in cylindrical coordinates, given by Eq. 8, has the last two terms equal to zero. Therefore Eq. 24 reduces to:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \quad (25)$$

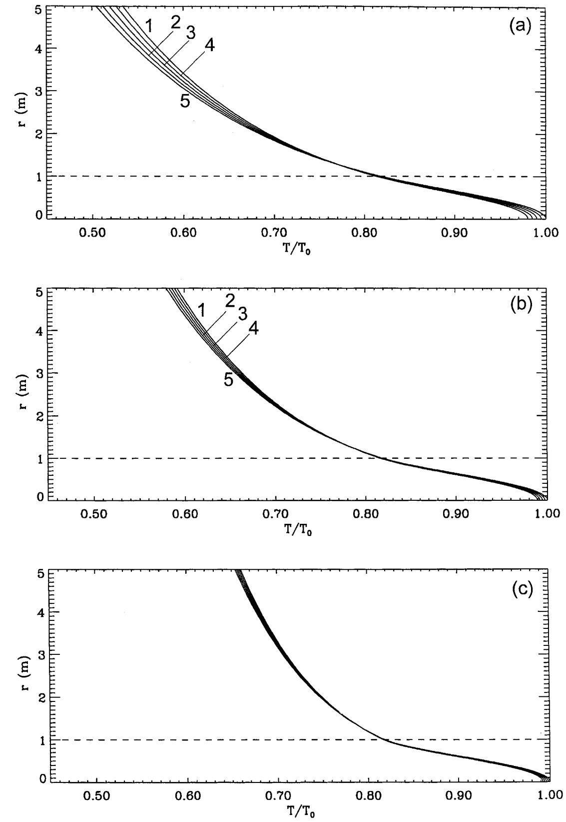

Fig. 6. Temperature inside and outside the tube. Different values of the tube inclination α and of the distance from the beginning of the tube x are considered. (a) $\alpha = 0.1$ rad, (b) $\alpha = 0.2$ rad, (c) $\alpha = 0.3$ rad; curve 1, $x = 0$ m, curve 2, $x = 250$ m, curve 3, $x = 500$ m, curve 4, $x = 750$ m, curve 5, $x = 1000$ m. For all the curves $\eta = 10\,000$ Pa s. The values of the other model parameters are fixed and shown in Table 1.

The boundary conditions are:

$$T(r_1) = T_s \tag{26}$$

$$T(r_2) = T_a \tag{27}$$

that is temperature must equate the solidus at the surface of the tube and the ambient temperature, T_a, at a distance $r_2 \gg r_1$. The value of r_2 is evaluated requiring the continuity of heat flow at the surface of the tube $r = r_1$. Of course $T_s > T_a$ is assumed. The solution of Eq. 25 is:

$$T(r, x) = \begin{cases} T_a + (T_s - T_a)\dfrac{\ln\frac{r}{r_2}}{\ln\frac{r_1(x)}{r_2}}, & r_1 \leq r \leq r_2 \\ T = T_a, & r \geq r_2 \end{cases} \tag{28}$$

where r_1 is given by Eq. 21 and r_2 will be given by Eq. 33. The temperature inside and outside the tube, given by Eqs. 13 and 28, respectively, is shown in Fig. 6 where three different slopes α are considered. The inner part of the tube cools, while the surrounding medium is heated. We consider now the heat flow density $\mathbf{q}$. Both inside and outside the tube, the radial component of $\mathbf{q}$ is given by:

$$q_r = -\kappa\frac{\partial T}{\partial r} \tag{29}$$

where κ is the thermal conductivity of the fluid or the solid, according to the case. We assume that the two conductivities are equal. From Eq. 13 we obtain:

$$q_r(r, x) = 4\kappa\left[\gamma G\left(2r - \frac{r^3}{r_1^2}\right) - \beta\frac{r^3}{r_1^4}\right] \tag{30}$$

and from Eq. 26:

$$q_r(r, x) = \kappa\frac{T_s - T_a}{r\ln\frac{r_2}{r_1}}, \quad r_1 \leq r \leq r_2 \tag{31}$$

The radial heat flow is positive and directed outward from the tube. Eq. 28 satisfies the boundary condition of vanishing flow at $r = 0$. The continuity of flow at $r = r_1$ requires that:

$$\ln\frac{r_2}{r_1} = \frac{T_s - T_a}{4(\gamma G r_1^2 - \beta)} \tag{32}$$

from which the appropriate value of r_2 can be derived:

$$r_2 = r_1 \exp\left(\frac{T_s - T_a}{4(\gamma G r_1^2 - \beta)}\right) \tag{33}$$

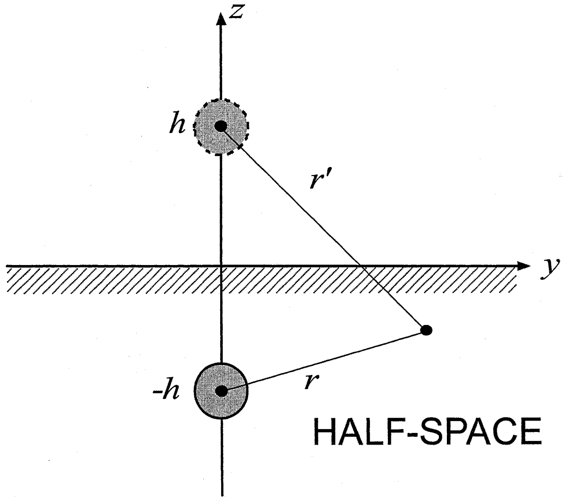

Fig. 7. Sketch of the method of images. The dashed line represents the image tube.

We neglect the longitudinal component q_x of heat flow, since $\partial T/\partial x$ is small.

2.3. Lava tube in a half-space

In order to evaluate the temperature field and the surface heat flow due to a buried lava tube, we introduce a Cartesian coordinate system (x, y, z) and assume that the tube is embedded in the solid half-space $z>0$. The axis of the cylinder is the right line parallel to the x-axis at $z=-h$. This means that the tube is parallel to the Earth's surface: this is assumed for the sake of simplicity, considering that the free surface is not horizontal on the flanks of a volcano. In order to find the solution in the half-space, we apply the method of images. We consider an image lava tube, the axis of which is the right line at $z=h$: its surface is at temperature $T=-T_s$ (Fig. 7). By superposition of the temperature fields generated by the two tubes, the boundary conditions at the Earth's surface $z=0$ are satisfied. We consider a plane $x=$constant, where the radius of the tube is $r_1(x)$, and r and r' are the distances of a generic point on this plane from the axes of the real and image lava tubes, respectively. Taking into account only the zone outside the tube $r \geq r_1$, by Eq. 33 we obtain:

$$T(r,\ r') = (T_s - T_a)\frac{\ln\frac{r}{r'}}{\ln\frac{r_1}{r_2}},\quad r_1 \leq r \leq r_2 \tag{34}$$

The temperature field on the plane yz is shown in Fig. 8 for three different depths of the tube. In each case the temperature vanishes at the Earth's surface. The radial heat flow density is obtained in the same way by Eq. 31:

$$q_r(y,\ z) = \kappa\frac{T_s - T_a}{\ln\frac{r_2}{r_1}}\left(\frac{1}{r} + \frac{1}{r'}\right),\quad r_1 \leq r \leq r_2 \tag{35}$$

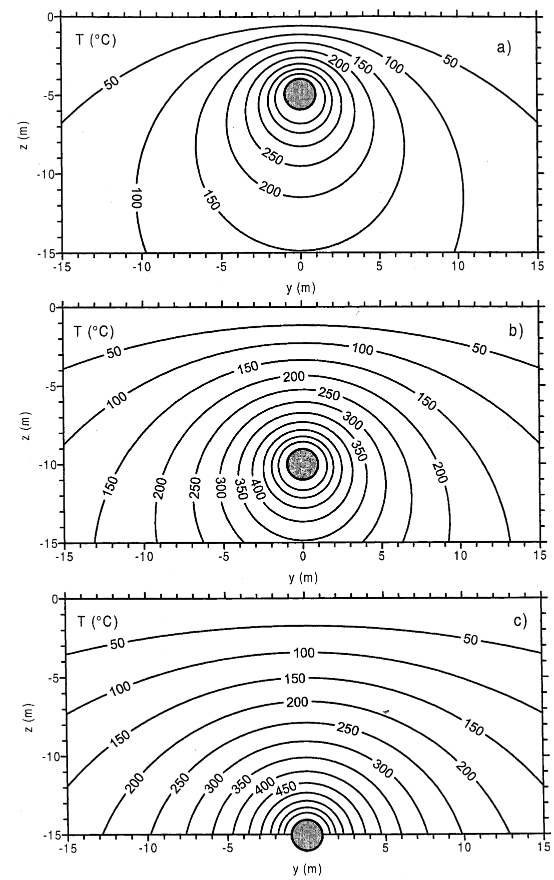

Fig. 8. Temperature field outside the tube on the plane yz. Different values of the tube depth h are considered: (a) $h=5$ m, (b) $h=10$ m, (c) $h=15$ m;$x=0$ m, $\eta=10\,000$ Pa s. The values of the other model parameters are fixed and shown in Table 1.

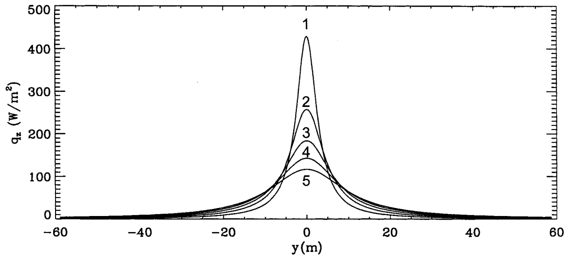

Fig. 9. Vertical component q_z of the heat flow at the Earth's surface. Different values of the tube depth h are considered: curve 1, $h=3$ m, curve 2, $h=5$ m, curve 3, $h=7$ m, curve 4, $h=9$ m, curve 5, $h=11$ m; for all the curves $\eta=10\,000$ Pa s, $x=0$ m, $\alpha=0.1$. The values of the other model parameters are fixed and shown in Table 1.

where:

$$r=\sqrt{(h-z)^2+y^2} \tag{36}$$

and:

$$r'=\sqrt{(h+z)^2+y^2} \tag{37}$$

At the free surface $z=0$, Eq. 35 becomes:

$$q_r(y)=\kappa\frac{T_s-T_a}{\ln\frac{r_2}{r_1}}\frac{2}{\sqrt{h^2+y^2}} \tag{38}$$

We are interested in the heat flow component q_z, which is normal to the free surface. It is given by:

$$q_z(y)=q_r(y)\frac{h}{\sqrt{h^2+y^2}} \tag{39}$$

A graph of $q_z(y)$ is shown in Fig. 9, where five different depths of the tube are considered. We can integrate Eq. 39, obtaining the total flow Q across the Earth's surface (per unit length of the tube):

$$Q=\int_{-\infty}^{\infty}q_z(y)\mathrm{d}y \tag{40}$$

From Eqs. 38 and 39, one obtains:

$$Q=\frac{2\pi\kappa(T_s-T_a)}{\ln\frac{r_2}{r_1}} \tag{41}$$

which is independent of h. Eq. 41 is also the heat flow (per unit length of the tube) across a cylindrical surface of arbitrary radius r surrounding the tube itself. Hence we can write:

$$q_z(y)=\frac{Q}{\pi}\frac{h}{y^2+h^2} \tag{42}$$

At some distance from the tube, the heat flow must be fairly independent of the shape of the tube and of the flow cross section. The energy equation (Eq. 7) shows that the heat flow is approximately proportional to the lava flow rate in the tube (neglecting viscous dissipation). The flow rate F of a completely filled cylindrical tube with circular cross section can be calculated by integrating the flow velocity Eq. 2, giving:

$$F=\frac{\pi\rho^2 g\sin\alpha}{8\eta}r_1^4 \tag{43}$$

Therefore we shall preferably associate a measured heat flow value Q with a lava flow rate F, rather than with a tube radius r_1. From Eqs. 41 and 43, we obtain:

$$F=\frac{\pi\rho^2 g r_2^4\sin\alpha}{8\eta}e^{-8\pi\kappa(T_s-T_a)/Q} \tag{44}$$

In this way, the model can account in an approximate fashion for the cases in which the tube is not a circular cylinder or is only partially filled with lava. For instance, a flow rate F may correspond to a completely filled tube with radius r_1 or to a half-filled tube with radius:

$$r'_1=\sqrt[4]{2}r_1\approx 1.2r_1 \tag{45}$$

3. Conclusions

The importance of lava tubes in the evolution of effusive eruptions is widely recognized. The thermal effects of a lava tube are related to the size and geometry of the tube as well as to the thermal, rheological and dynamical parameters of flowing lava. The model proposed in this paper describes the cooling of a cylindrical tube and the heating of the surrounding medium. In order to obtain an analytical solution, the model has been simplified in several respects: in particular, steady-state conditions are considered, the tube has a regular shape and a constant slope, the downslope temperature gradient is assumed to be constant, and the change of viscosity with temperature is neglected. Moreover the tube is completely filled with lava, while observations indicate that filling is usually only partial. These approximations are considered acceptable, in view of the aim of the present model, that is to provide a first quantitative insight into the relationships existing between the many physical quantities involved in the problem. The model illustrates how the temperature field around a lava tube depends on the tube radius and slope, on the lava viscosity and on the distance from the beginning of the tube, other quantities like the thermal conductivity of rocks and the solidus temperature being considered as fixed. Moreover it has been shown that the heat flow at the Earth's surface is a function of the depth, size and temperature of a buried lava tube. Therefore measuring surface heat flow during an eruption may be an efficient method to discover the existence of active lava tubes at depth and to investigate their characteristics. Of course in complex lava fields, where several active flows are present, the thermal anomaly connected with a lava tube may be perturbed by other thermal sources.

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References

Calvari, S., Pinkerton, H., 1999. Lava tube morphology on Etna and evidence for lava flow emplacement mechanisms. J. Volcanol. Geotherm. Res. 90, 263–280.

Dragoni, M., Bonafede, M., Boschi, E., 1986. Downslope flow model of a Bingham liquid: implications for lava flows. J. Volcanol. Geotherm. Res. 30, 305–325.

Dragoni, M., Piombo, A., Tallarico, A., 1995. A model for the formation of lava tubes by roofing over a channel. J. Geophys. Res. 100, 8435–8447.

Greeley, R., 1987. The Role of Lava Tubes in Hawaiian Volcanoes. U.S. Geol. Surv. Prof. Paper 1350, pp. 1589–1602.

Guest, J.E., Underwood, J.E., Greeley, R., 1980. Role of lava tubes in flows from the Observatory Vent, 1971 eruption on Mount Etna. Geol. Mag. 117, 601–606.

Kauahikaua, J.P., Cashman, K.V., Mattox, T.N., Heliker, C.C., Hon, K.A., Mangan, M.T., Thornber, C.R., 1998. Observations of the basaltic lava streams in tubes from Kilauea Volcano, island of Hawaii. J. Geophys. Res. 103, 27303–27323.

Keszthelyi, L.P., 1998. A preliminary thermal budget for lava tubes on the Earth's and planets. J. Geophys. Res. 100, 20411–20420.

Kilburn, C.R.J., Guest, J.E., 1993. Aa lavas of Mount Etna, Sicily. In: Kilburn, C.R.J., Luongo, G. (Eds.), Active Lavas, Chap. 3. University College London, London.

Kundu, P.K., 1990. Fluid Mechanics, Chap. 9. Academic Press, San Diego, CA.

Landau, L., Lifchitz, E., 1971. Méchanique des Fluides, Chap. 2. MIR, Moscow.

Malin, M.C., 1980. Lengths of Hawaiian lava flows. Geology 8, 306–308.

Murase, T., McBirney, A.R., 1973. Properties of some common igneous rocks and their melts at high temperatures. Geol. Soc. Am. Bull. 84, 3563–3592.

Peterson, D.W., Holcomb, R.T., Tilling, R.I., Christiansen, R.L., 1994. Development of lava tubes in the light of observations at Mauna Ulu, Kilauea Volcano, Hawaii. Bull. Volcanol. 56, 343–360.

Peterson, D.W., Swanson, D.A., 1974. Observed formation of lava tubes during 1970–1971 at Kilauea volcano, Hawaii. Stud. Speleol. 2, 209–222.

Ponte, G., 1949. Riassunto delle principali osservazioni e ricerche fatte sull'Etna. Bull. Volcanol. 9, 65–80.

Sakimoto, S.E.H., Crisp, J., Baloga, S.M., 1997. Eruption constraints on tube-fed planetary lava flows. J. Geophys. Res. 102, 6567–6613.

Sakimoto, S.E.H., Zuber, M.T., 1998. Flow and convective cooling in lava tubes. J. Geophys. Res. 103, 27465–27487.