Geophys. J. Int. (2002) **148**, 336–339

RESEARCH NOTE

Note on scaling of peak ground acceleration and peak ground velocity with magnitude

J. Douglas

Department of Civil & Environmental Engineering, Imperial College, London, SW7 2BU, *UK. Email: john.douglas@ic.ac.uk*

Accepted 2001 August 17. Received 2001 July 27; in original form 2001 February 16

SUMMARY

The theoretical scaling of near-field peak ground acceleration and peak ground velocity with moment magnitude, M_w, is found using an L model of rupture. This scaling matches well with the magnitude scaling of recent attenuation relations.

Key words: earthquakes, fault models, strong ground motion.

1 SCALING OF PEAK GROUND ACCELERATION WITH M_W

Scholz (1982a) discusses the consequences of the L rupture model for the scaling of peak ground acceleration, PGA, with rupture length, L. This note shows that derivations of the scaling of peak ground acceleration and peak ground velocity with magnitude made using this rupture model are consistent with what has been found empirically in strong-motion attenuation relations. Therefore, suggesting that the L model of rupture is applicable to the derivation of strong-motion attenuation relations.

There are two main models of earthquake rupture: the L model and the W model (Scholz 1982b). An L model is one in which the fault is mechanically unconstrained (or loosely constrained) at the base, so that slip is determined by the length of faulting and the correlation between slip and length is explained if the stress drop is constant (Scholz 1982a). A W model is one in which the slip is constrained to be zero at the base of the fault, so that slip and stress drop are determined by the fault width (Scholz 1982b). Scholz (1994a,b) and Wang & Ou (1998) have presented results supporting the validity of the L model in describing the scaling of earthquake faults. On the other hand, Romanowicz (1992, 1994) prefers the W model of rupture.

The seismic moment of a earthquake with a vertical, rectangular rupture plane of length L, width W, rigidity μ, and slip u is

$$M_0 = \mu u L W \,. \tag{1}$$

The definition of M_w is (Kanamori 1978):

$$M_w = \frac{2}{3}\log M_0 - 6 \tag{2}$$

where M_0 is in N m.

Also, Scholz (1982b) found that the slip and the fault length for earthquakes that rupture the entire seismogenic zone are related approximately by $u = \alpha L$.

Scholz (1982a) derives eq. (3) for the estimation of near-field peak ground acceleration, $a_{\max}$, for an earthquake with rupture length L given the near-field peak ground acceleration, $a^*_{\max}$, which occurred for a unit earthquake with rupture length L^* equal to the depth of the seismogenic layer, W:

$$a_{\max} = \sqrt{\ln\frac{L}{L^*}}\, a^*_{\max} \,. \tag{3}$$

Note that eq. (3) is a simple relation which does not explicitly include many important factors that are known to influence PGA in the near-field case. These include: focal depth, focal mechanism, dip of fault, distance to the source, local soil conditions, topography and directivity. More complex models are needed to include such effects. Eq. (3) is a scaling relation which assumes that all other factors affecting near-field PGA are equal for a unit earthquake and an earthquake of rupture length L.

Eq. (3) is only valid for earthquakes that rupture the entire seismogenic layer. In fact, it only gives larger peak ground accelerations for the larger earthquake when $\sqrt{\ln L/L^*} > 1$, i.e. $L > \exp(1)L^*$. Using eq. (1) with $u = \alpha L$ to express L in terms of M_0, μ, W and α, the inequality $L > \exp(1)L^*$ with $L^* = W$, eq. (2) and converting from natural to common logarithms leads to this inequality when eq. (3) applies:

$$M_w > \frac{2}{3}\left[\log(\alpha\mu W^3) + 2\log\exp(1) - 9\right]. \tag{4}$$

Stock & Smith (2000) have recently found no evidence for a change of fault scaling from self-similarity, i.e. $M_0 \sim L^3$, to that predicted by the L model, i.e. $M_0 \sim L^2$, for normal and reverse mechanism earthquakes. However, they do find such a change for strike-slip mechanism earthquakes. This means that eq. (3) may not be useful for estimating PGA from normal or reverse earthquakes.

Using eqs (2) and (1), with $u=\alpha L$, to express $\log L$ in terms of M_w, α, μ and W and then substituting the expression derived into eq. (3), it is found that

$$\log a_{\max} = -\frac{1}{2}\log(\log\exp(1)) + \frac{1}{2}\log\left[\frac{3}{4}M_w - \frac{1}{2}\log(\alpha\mu W^3) + \frac{9}{2}\right] + \log a^*_{\max}. \quad (5)$$

This paper is concerned with the comparison of this theoretical scaling of peak ground acceleration with magnitude and what has been found empirically in attenuation relations derived using strong-motion records. Therefore, eq. (5) is differentiated with respect to M_w to yield:

$$\frac{\partial\log a_{\max}}{\partial M_w} = \frac{3}{2}\log(\exp(1))\left[\frac{1}{3M_w - 2\log(\alpha\mu W^3) + 18}\right]. \quad (6)$$

Most attenuation relations for PGA have the form (Douglas 2001):

$$\log a_{\max} = b_1 + b_2 M + b_3 \log R + \cdots, \quad (7)$$

where M is the magnitude (usually M_s, M_L or M_w) and R is a distance measure (usually epicentral, hypocentral, distance to the rupture or distance to the surface projection of the rupture), which is often transformed through the addition of a constant or similar term. It has been proposed that strong ground motion does not increase without bound for increasing magnitudes and that as the magnitude increases ground motion does not increase at a constant rate. Recently many authors have allowed for this effect in the functional form of their equation through the inclusion of a negative quadratic term, b_4M^2 (Boore *et al.* 1997), although they find that b_4 is not significantly different from zero for PGA, or by including in R a magnitude-dependent term such as $R+\exp(b_5+b_6M)$ (Sadigh & Egan 1998).

Therefore, eq. (6) is related to b_2 and it would be equal to b_2 if the scaling of the logarithm of peak ground acceleration were linear with M_w.

Fig. 1 shows the magnitude scaling of log $a_{\max}$ with respect to M_w for two different α (1×10^{-5} and 10×10^{-5}), which are roughly the range of values usually found (Scholz 1982b), and three different widths of the seismogenic zone W: 15, 20 and 25 km. The curves are plotted from the smallest M_w which satisfies the condition $L>\exp(1)L^*$ derived above.

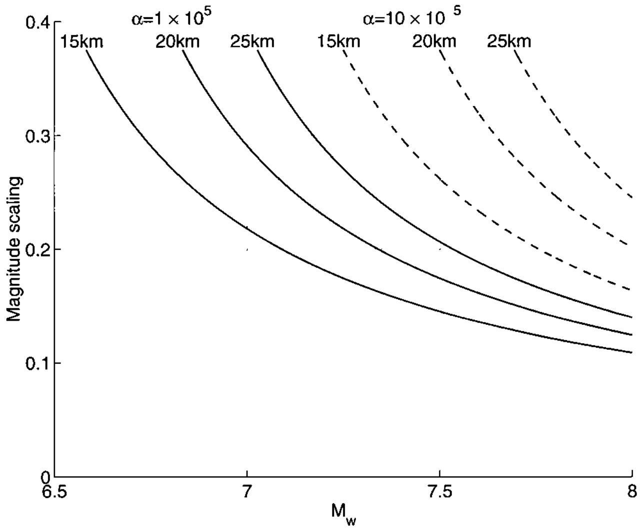

Figure 1. Theoretical magnitude scaling ($\partial \log a_{\max}/\partial \log M_w$) of $\log a_{\max}$, where $a_{\max}$ is the near-field peak ground acceleration, for the L model, i.e. eq. (6). Solid lines are for $\alpha=1\times10^{-5}$ and dashed lines are for $\alpha=10\times10^{-5}$. The scaling is given for three different widths of the seismogenic zone W: 15, 20 and 25 km.

From Fig. 1 it can be seen that the theoretical scaling of PGA with M_w is not linear and that it is not the same for all widths of the seismogenic zone or for all values of the proportionality constant α. This means that the scaling of PGA with M_w for different regions of the Earth, e.g. interplate (where α is usually low) and intraplate (where α is usually high), could be different. Also the thickness of the seismogenic layer varies regionally (Stock & Smith 2000), leading to possible differences in the scaling of PGA with M_w for different regions of the world.

The magnitude scaling (using a logarithm to base 10 and differentiating the equation with respect to magnitude) in some recent equations (which include records from earthquakes in the high-magnitude range from the near-field) are given in Table 1. It is difficult to correlate differences in the observed scaling of PGA with M_w obtained in the studies reported in Table 1 with the focal mechanisms of the earthquakes used or the seismotectonic conditions of the areas where the accelerograms used were recorded. This is because of differences in magnitude scale, distance metric and site classifications used, functional form adopted [for example, Ambraseys *et al.* (1996) only consider a linear scaling with M_s] and, probably mainly,

Table 1. Magnitude scaling in terms of a logarithm to base 10 for some recent attenuation relations for horizontal peak ground acceleration.

Reference	Scaling	Notes
Ambraseys *et al.* (1996)	0.266	Assuming M_s and M_w are equivalent for $M_w \geq 6.5$
Ambraseys & Douglas (unpublished data 2001)	0.202	Assuming M_s and M_w are equivalent for $M_w \geq 6.5$
Boore *et al.* (1997)	0.229	
Campbell (1997)	0.206–0.066	Strike-slip, $R=10$ km, $M_w=6.5$–8
Campbell (1997)	0.164–0.025	Reverse, $R=10$ km, $M_w=6.5$–8
Cousins *et al.* (1999)	0.296	
Sadigh & Egan (1998)	0.167–0.094	Rock sites, $R=10$ km, $M_w=6.5$–8
Sadigh & Egan (1998)	0.139–0.079	Soil sites, $R=10$ km, $M_w=6.5$–8

Table 2. Magnitude scaling in terms of logarithm to base 10 for some recent attenuation relations for horizontal peak ground velocity.

Reference	Scaling	Notes
Bommer *et al.* (2000)	0.390	Assuming M_s and M_w are equivalent for $M_w \geq 6.5$
Joyner & Boore (1981)	0.489	
Sadigh & Egan (1998)	0.369–0.191	Rock sites, $R=10$ km, $M_w=6.5$–8
Sadigh & Egan (1998)	0.319–0.148	Soil sites, $R=10$ km, $M_w=6.5$–8

because of the distribution of the data used in terms of magnitude and distance. For example, the data sets of Ambraseys *et al.* (1996) and Cousins *et al.* (1999) contain many records from the intermediate and far-field regimes and so the magnitude scalings found probably do not reflect the true near-field behaviour, whereas the data sets of Campbell (1997) and Ambraseys & Douglas (unpublished data 2001) feature only records from the near-source regime and so probably better reflect the true near-field scaling of PGA with M_w.

Fig. 1 suggests that the near-field non-linear scaling of PGA with M_w modelled by, for example, Campbell (1997) and Sadigh & Egan (1998) is theoretically justified. Also, the magnitude scaling they obtained (see Table 1) matches reasonably well with the theoretical scaling for $\alpha = 1 \times 10^{-5}$ and $W = 15$ km, which are reasonable estimates (Scholz 1982b; Stock & Smith 2000) for western North America where most of the accelerograms they used were recorded.

Comparing Fig. 1 and Table 1 shows that the magnitude scaling found empirically matches approximately the theoretical scaling found above. Therefore, Scholz's L model can be applied to derive strong-motion attenuation relations for PGA.

2 SCALING OF PEAK GROUND VELOCITY WITH M_W

Scholz (1982a) also derives eq. (8) for the prediction of the near-field peak ground velocity (PGV) near the ends of the fault, $v_{\max}$, for an earthquake with rupture length L given the near-field peak ground velocity, $v^*_{\max}$, which occurred for a unit earthquake with rupture length, L^*, equal to the depth of the seismogenic layer, W:

$$v_{\max} = \sqrt{\frac{L}{L^*}}\, v^*_{\max} \qquad (8)$$

Similarly to eq. (3), eq. (8) is a simple relation which does not explicitly include many factors known to influence PGV (see Section 1).

Unlike eq. (3) this equation gives a larger peak ground velocity for all $L > L^*$. Using the same steps as for the peak ground acceleration case this equation predicts that the logarithm of the near-field peak ground velocity near the ends of the fault should scale linearly with M_w. The actual equation is

$$\log v_{\max} = -\frac{1}{4}\log \mu\alpha W^3 + \frac{9}{4} + \frac{3}{8} M_w + \log v^*_{\max}\,. \qquad (9)$$

Thus

$$\frac{\partial \log v_{\max}}{\partial M_w} = \frac{3}{8} = 0.375\,. \qquad (10)$$

The magnitude scaling (using a logarithm to base 10 and differentiating the equation with respect to magnitude) in some recent equations (which include records from earthquakes in the high-magnitude range from the near-field) are given in Table 2. There are large differences between the magnitude scaling of PGV found empirically in the different studies reported in Table 2. As for PGA, these differences probably reflect variations in the independent variables, functional form and data sets used by the different authors rather than differences in seismotectonic conditions or focal mechanisms. Table 2 shows that the magnitude scaling found empirically approximately matches the theoretical scaling found above. Therefore, Scholz's L model can be applied to derive strong-motion attenuation relations for PGV.

3 CONCLUSIONS

The equations derived by Scholz (1982a) for the estimation of near-field peak ground acceleration and velocity from large earthquakes are expressed in terms of the moment magnitude, M_w. The magnitude scaling from these theoretical equations is similar to that found in a number of recent strong-motion attenuation relations. This suggests that the L model of rupture can be applied for the derivation of near-field strong-motion attenuation relations.

ACKNOWLEDGMENTS

I thank Prof. N.N. Ambraseys for his help and enthusiasm towards this work, Drs S.K. Sarma and P.M. Smit for reviewing this paper before submission and two anonymous reviewers for their useful comments and suggestions.

REFERENCES

Ambraseys, N.N.,Simpson, K.A. & Bommer, J.J., 1996. Prediction of horizontal response spectra in Europe, *Earthquake eng. struct. dyn.*, **25,** 371–400.

Bommer, J.J.,Elnashai, A.S. & Weir, A.G., 2000. Compatible acceleration and displacement spectra for seismic design codes, *Proc. 12th World Conf. Earthquake Eng.*.

Boore, D.M.,Joyner, W.B. & Fumal, T.E., 1997. Equations for estimating horizontal response spectra and peak acceleration from western North American earthquakes: a summary of recent work, *Seism. Res. Lett.*, **68,** 128–153.

Campbell, K.W., 1997. Empirical near-source attenuation relationships for horizontal and vertical components of peak ground acceleration, peak ground velocity, and pseudo-absolute acceleration response spectra, *Seism. Res. Lett.*, **68,** 154–179.

Cousins, W.J., Zhao, J.X. & Perrin, N.D., 1999. A model for the attenuation of peak ground acceleration in New Zealand earthquakes based on seismograph and accelerograph data, *Bull. New Zealand Soc. Earthquake Eng.*, **32,** 193–220.

Douglas, J., 2001. A comprehensive worldwide summary of strong-motion attenuation relationships for peak ground acceleration and spectral ordinates (1969 to 2000), *ESEE Report 01-1,* Department of Civil & Environmental Engineering, Imperial College, London. Also available on internet at: http://www.esee.cv.ic.ac.uk/reports.htm.

Joyner, W.B. & Boore, D.M., 1981. Peak horizontal acceleration and velocity from strong-motion records including records from the 1979 Imperial Valley, California, earthquake, *Bull. seism. Soc. Am.,* **71,** 2011–2038.

Kanamori, H., 1978. Quantification of earthquakes, *Nature,* **271,** 411–414.

Romanowicz, B., 1992. Strike-slip earthquakes on quasi-vertical transcurrent faults: inferences for general scaling relations, *Geophys. Res. Lett.,* **19,** 481–484.

Romanowicz, B., 1994. Comment on 'A reappraisal of large earthquake scaling' by C. Scholz, *Bull. seism. Soc. Am.,* **84,** 1675–1676.

Sadigh, R.K. & Egan, J.A., 1998. Updated relationships for horizontal peak ground velocity and peak ground displacement for shallow crustal earthquakes, *Proc. 6th US Nat. Conf. Earthquake Eng.*

Scholz, C.H., 1982a. Scaling relations for strong ground motion in large earthquakes, *Bull. seism. Soc. Am.,* **72,** 1903–1909.

Scholz, C.H., 1982b. Scaling laws for large earthquakes: consequences for physical models, *Bull. seism. Soc. Am.,* **72,** 1–14.

Scholz, C.H., 1994a. A reappraisal of large earthquake scaling, *Bull. seism. Soc. Am.,* **84,** 215–218.

Scholz, C.H., 1994b. Reply to Comments on 'A reappraisal of large earthquake scaling' by C. Scholz, *Bull. seism. Soc. Am.,* **84,** 1667–1678.

Stock, C. & Smith, E.G.C., 2000. Evidence for different scaling of earthquake source parameters for large earthquakes depending on fault mechanism, *Geophys. J. Int.,* **143,** 157–162.

Wang, J.-H. & Ou, S.-S., 1998. On scaling of earthquake faults, *Bull. seism. Soc. Am.,* **88,** 758–766.