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RESEARCH NOTE

Deformation of a uniform half-space due to a long inclined tensile fault

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SUMMARY

The calculation of the deformation due to shear and tensile faults in a half-space is a fundamental tool for the investigation of seismic and volcanic sources. The solution of the two-dimensional problem of a long inclined shear fault is well-known. The purpose of the present paper is to present closed-form analytical expressions for the subsurface stresses and displacements caused by a long inclined tensile fault buried in a homogeneous isotropic half-space. The expression for the Airy stress function satisfying appropriate boundary conditions at the surface of the half-space is obtained. This Airy stress function is used to derive the expressions for the displacements and stresses at an arbitrary point of the half-space. The variation of the displacement field with the horizontal distance from the fault is studied numerically. The effect of the depth of the upper edge of the fault and the dip angle on the deformation field is also examined.

Key words: deformation, dislocation, dyke injection, half-space, long tensile fault, volcanic source.

1 INTRODUCTION

Since dislocation theory was first introduced in the field of seismology by Steketee (1958), numerous theoretical formulations describing the deformation of a uniform half-space have been developed (Okada 1992). In contrast to the progress that has been made in the modelling of the deformation field due to a shear fault, the studies related to a tensile fault are scarce. However, tensile fault representation has several very important geophysical applications, such as modelling of the deformation field due to dyke injection in volcanic regions, mine collapse and fluid-driven cracks. Recent studies have shown that a large number of earthquake sources cannot be represented by the double-couple source mechanism which models a shear fault. According to Sipkin (1986), the non-double-couple mechanism might be due to tensile failure under high fluid pressure.

Maruyama (1964) obtained surface displacements due to vertical and horizontal rectangular tensile faults in a Poissonian elastic half-space. Davis (1983) modelled the crustal deformation associated with hydrofracture by a dipping rectangular tensile fault beneath the surface of an elastic half-space. Yang & Davis (1986) derived analytical expressions for the displacements, strains and stresses due to a rectangular inclined tensile fault in an elastic half-space. Bonafede & Danesi (1997) obtained an analytical solution for the displacement and stress fields produced by a long vertical tensile fault in a uniform half-space, using a Galerkin vector approach. The corresponding problem for two half-spaces in welded contact has been solved by Bonafede & Rivalta (1999).

Singh & Garg (1986) obtained integral expressions for the Airy stress function in an unbounded medium due to various 2-D sources. Beginning with these results, Rani *et al.* (1991) obtained closed-form analytical expressions for the Airy stress function, displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space due to an arbitrary line source. By integration over the width of the fault, Rani & Singh (1992) obtained the expressions for the Airy stress function, displacements and stresses in a uniform half-space due to a long dip-slip fault. The same procedure is followed in the present paper to study plane strain 2-D deformation of a uniform half-space due to a long inclined tensile fault of arbitrary depth. The 2-D solution obtained here is useful because of its considerable

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simplicity as compared to the 3-D solution given by Yang & Davis (1986). The elastic field due to a vertical tensile fault in a uniform half-space given by Bonafede & Danesi (1997) and Singh & Singh (2000) can be obtained as a particular case of the solution given in the present paper on taking the dip angle $\delta=\pi/2$.

The correspondence principle can be used for finding the viscoelastic quasi-static field from the analytical elastic solution given in the present paper (Singh & Singh 1990). The closed-form analytical solution for an inclined tensile fault in a uniform half-space may find useful applications towards extracting, from geodetic and seismic data, information about the position, depth, magma content and inclination of a buried dyke. The stresses induced by dyke opening are also thought to be responsible for the seismic activity generally observed prior to a volcanic eruption, for inducing isotropic moment tensor components and even for causing changes in the directions of the principal stresses (Bonafede & Rivalta 1999).

An arbitrary dislocation has three components: strike-slip, dip-slip and tensile (crack opening). The 2-D solutions for the strike-slip and dip-slip dislocations were given by Freund & Barnett (1976) and Rani & Singh (1992). The corresponding solution for a tensile dislocation is obtained in the present paper. Thus combining the results of Rani & Singh (1992) with the results given in the present paper one can obtain the solution for an arbitrary 2-D dislocation in a uniform half-space.

2 THEORY

Let the Cartesian coordinates be denoted by (x_1, x_2, x_3) with the x_3-axis vertically downwards. Consider a 2-D approximation in which the displacement components u_1, u_2 and u_3 are independent of x_1 so that $\partial/\partial x_1 \equiv 0$. Under this assumption the plane strain problem $(u_1=0)$ can be solved in terms of the Airy stress function Φ such that

$$\tau_{22}=\partial^2\Phi/\partial x_3^2,\ \tau_{33}=\partial^2\Phi/\partial x_2^2,\ \tau_{23}=-\partial^2\Phi/\partial x_2\partial x_3\,, \tag{1}$$

where τ_{ij} are the stress components. Following Rani *et al.* (1991) and Singh & Singh (2000), we obtain the following expression for the Airy stress function for an inclined tensile fault of width L and infinite length (in the x_1-direction) in a uniform half-space $x_3 \geq 0$:

$$\begin{aligned}\Phi =&[\mu b/2\pi(1-\sigma)]\{(s-x_2\cos\delta-X_3\sin\delta)\log_e(S/R)\\&+2x_3[x_2(x_2\sin\delta+d\cos\delta)+x_3X_3'\sin\delta]S^{-2}\\&+2sx_3[(X_3'\sin\delta-x_2\cos\delta)\sin\delta-d]S^{-2}\}\|\,,\end{aligned} \tag{2}$$

where μ denotes the shear modulus, σ the Poisson's ratio, d the depth of the upper edge of the fault (Fig. 1), δ the dip angle, b the displacement discontinuity normal to the fault and

$$\begin{aligned}&X_3=x_3-d,\ X_3'=x_3+d\,,\\&R^2=(x_2-s\cos\delta)^2+(X_3-s\sin\delta)^2,\\&S^2=(x_2-s\cos\delta)^2+(X_3'+s\sin\delta)^2\,,\\&f(s)\|=f(L)-f(0)\,.\end{aligned} \tag{3}$$

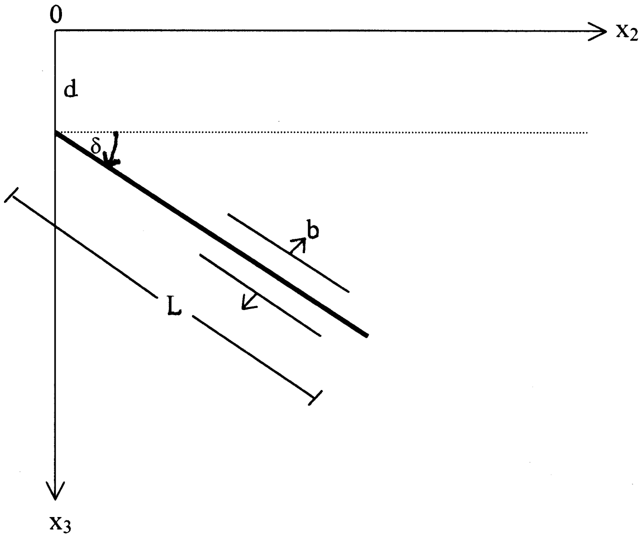

Figure 1. Geometry of a long tensile fault in a half-space $(x_3 \geq 0)$. The fault is of width L in the down-dip direction and of infinite length in the strike (x_1) direction. δ is the dip angle, d is the depth of the upper edge of the fault and b is the dislocation normal to the fault.

From eqs (1) and (2), we get the following expressions for the stresses:

$$\begin{aligned}\tau_{22}=&[\mu b/2\pi(1-\sigma)]\{[3X_3\sin\delta+x_2\cos\delta-s(1+2\sin^2\delta)]\\&\times(R^{-2}-S^{-2})-8\sin\delta(d+s\sin\delta)S^{-2}\\&+2(s-x_2\cos\delta-X_3\sin\delta)\\&\times[(X_3-s\sin\delta)^2R^{-4}-(X_3'+s\sin\delta)^2S^{-4}]\\&+4(d+s\sin\delta)[x_3(3X_3'\sin\delta-x_2\cos\delta+s+2s\sin^2\delta)\\&+2(X_3'+s\sin\delta)(X_3'\sin\delta-x_2\cos\delta+s)]S^{-4}\\&-16x_3(d+s\sin\delta)(X_3'\sin\delta-x_2\cos\delta+s)\\&\times(X_3'+s\sin\delta)^2S^{-6}\}\|\,,\end{aligned} \tag{4}$$

$$\begin{aligned}\tau_{33}=&[\mu b/2\pi(1-\sigma)]\{(x_2\cos\delta-X_3\sin\delta-s\cos2\delta)(R^{-2}-S^{-2})\\&+2(x_2\cos\delta+X_3\sin\delta-s)\\&\times[(X_3-s\sin\delta)^2R^{-4}-(X_3'+s\sin\delta)^2S^{-4}]\\&+4x_3(d+s\sin\delta)[x_2\cos\delta-3X_3'\sin\delta-s(1+2\sin^2\delta)]S^{-4}\\&+16x_3(d+s\sin\delta)(X_3'\sin\delta-x_2\cos\delta+s)\\&\times(X_3'+s\sin\delta)^2S^{-6}\}\|\,,\end{aligned} \tag{5}$$

$$\begin{aligned}\tau_{23}=&[\mu b/2\pi(1-\sigma)]\{(X_3\cos\delta-x_2\sin\delta)(R^{-2}-S^{-2})\\&+2(x_2\sin\delta-X_3\cos\delta)\\&\times[(X_3-s\sin\delta)^2R^{-4}-(X_3'+s\sin\delta)^2S^{-4}]\\&-4x_3(d+s\sin\delta)(x_2\sin\delta+3X_3'\cos\delta+s\sin2\delta)S^{-4}\\&+16x_3(d+s\sin\delta)(x_2\sin\delta+X_3'\cos\delta)\\&\times(X_3'+s\sin\delta)^2S^{-6}\}\|\,,\end{aligned} \tag{6}$$

$$\tau_{11} = \sigma(\tau_{22} + \tau_{33}) = [\sigma/(1+\sigma)](\tau_{11} + \tau_{22} + \tau_{33})$$
$$= [\sigma\mu b/\pi(1-\sigma)]\{(X_3 \sin\delta + x_2 \cos\delta - s)(R^{-2} - S^{-2})$$
$$- 4\sin\delta(d + s\sin\delta)S^{-2} + 4(d + s\sin\delta)(X_3' + s\sin\delta)$$
$$\times (X_3' \sin\delta - x_2\cos\delta + s)S^{-4}\}\|\,, \quad (7)$$

$$\tau_{12} = \tau_{13} = 0\,. \quad (8)$$

The strains can be calculated through the relations

$$2\mu e_{22} = (1-\sigma)\tau_{22} - \sigma\tau_{33}\,,$$
$$2\mu e_{33} = (1-\sigma)\tau_{33} - \sigma\tau_{22}\,,$$
$$2\mu e_{23} = \tau_{23},\ e_{11} = e_{12} = e_{13} = 0\,. \quad (9)$$

Corresponding to the stresses given by eqs (4)–(8), the displacements are given by

$$u_1 = 0,\ 2\mu u_i = -\partial\Phi/\partial x_i + (1-\sigma)\int(\tau_{22} + \tau_{33})dx_i\ (i = 2, 3)\,. \quad (10)$$

We find

$$u_2 = [b/4\pi(1-\sigma)]\{(3-2\sigma)\cos\delta \log_e(R/S)$$
$$+ 2(1-\sigma)\sin\delta[\tan^{-1}(x_2 - s\cos\delta - s\sin\delta)$$
$$- \tan^{-1}(x_2 - s\cos\delta)/(X_3' - s\sin\delta)]X_3\cos\delta - x_2\sin\delta)$$
$$\times [X_3 + s\sin\delta R^{-2} - (X_3' + s\sin\delta)S^{-2}] + 2(d + s\sin\delta)$$
$$\times [(1-2\sigma)(x_2\sin\delta + d\cos\delta) + 2(1-\sigma)x_3\cos\delta]S^{-2}$$
$$- 4x_3(d + s\sin\delta)(X_3' + s\sin\delta)(x_2\sin\delta + X_3'\cos\delta)S^{-4}\}\|\,, \quad (11)$$

$$u_3 = [b/4\pi(1-\sigma)]\{(1-2\sigma)\sin\delta\log_e(R/S)$$
$$+ 2(1-\sigma)\cos\delta[\tan^{-1}(X_3 - s\sin\delta)/(x_2 - s\cos\delta)$$
$$- \tan^{-1}(X_3' + s\sin\delta)/(x_2 - s\cos\delta)] + (X_3 - s\sin\delta)$$
$$\times (s - x_2\cos\delta - X_3\sin\delta)(R^{-2} - S^{-2}) + 2(d + s\sin\delta)$$
$$\times [2(1-\sigma)(x_2\cos\delta - X_3'\sin\delta - s) + 3x_3\sin\delta]S^{-2}$$
$$- 4x_3(d + s\sin\delta)(X_3' + s\sin\delta)$$
$$\times (X_3'\sin\delta - x_2\cos\delta + s)S^{-4}\}\|\,. \quad (12)$$

It has been verified that on taking $d=0$, $\delta=90°$ in eqs. (4)–(12) the results of Singh & Singh (2000) for a vertical tensile fault are obtained as a particular case. Similarly, the results of Bonafede & Danesi (1997) for a vertical tensile fault are obtained as a particular case of the corresponding results for an inclined tensile fault given here if we replace b by $-b$ due to the difference in the sign convention of the two studies.

3 Numerical results and discussion

We wish to study the 2-D displacement field due to a long inclined tensile fault of width L in a uniform half-space. For numerical calculations, we take $\sigma=0.25$ and define the following dimensionless quantities:

$$Y = x_2/L,\ Z = x_3/L,\ D = d/L,\ U_i = u_i/b\,,$$

where b is the displacement discontinuity normal to the fault. Thus, Y is the dimensionless horizontal distance from the upper edge of the fault, Z the dimensionless depth, U_2 the dimensionless horizontal displacement, and U_3 the dimensionless vertical displacement (uplift is $-U_3$).

The expressions for the surface displacements are obtained on putting $x_3=0$ in eqs (11) and (12). We find

$$u_2 = (b/\pi)[-\sin\delta\tan^{-1}(x_2 - s\cos\delta)/(d + s\sin\delta)$$
$$+ (x_2\sin\delta + d\cos\delta)(d + s\sin\delta)R_0^{-2}]\|, \quad (13)$$

$$u_3 = (b/\pi)[\cos\delta\tan^{-1}(x_2 - s\cos\delta)/(d + s\sin\delta)$$
$$+ (x_2\cos\delta - d\sin\delta - s)(d + s\sin\delta)R_0^{-2}]\|, \quad (14)$$

where

$$R_0^2 = (x_2 - s\cos\delta)^2 + (d + s\sin\delta)^2\,. \quad (15)$$

Fig. 2 shows the variation of the horizontal surface displacement with the horizontal distance from the upper edge of the fault for four values of the dip angle $\delta=0°, 15°, 60°, 90°$. Fig. 2(a) is for a horizontal tensile fault with dislocation in the vertical direction. In this case, the horizontal displacement is antisymmetric about the surface point vertically above the mid-point of the fault. As the depth of the fault increases, the maximum horizontal displacement at the surface decreases. Figs 2(b, c) are for $\delta=15°$ and $60°$, respectively. For a surface-breaking fault ($D=0$), the horizontal surface displacement is discontinuous at the origin (upper edge of the fault), the magnitude of discontinuity being b sin δ. Fig. 2(d) is for a vertical tensile fault with dislocation in the horizontal direction. For a vertical tensile fault the horizontal displacement is antisymmetric about the origin. For a surface-breaking vertical fault, the horizontal displacement has a discontinuity of magnitude b at the origin.

Fig. 3 shows the variation of the surface uplift ($-U_3$) with the horizontal distance from the upper edge of the fault for four values of the dip angle $\delta=0°, 15°, 60°, 90°$. Fig. 3(a) is for a horizontal tensile fault with dislocation in the vertical direction. In this case, the vertical displacement is symmetric about the surface point vertically above the mid-point of the fault. As the depth increases, the maximum uplift decreases. Figs 3(b and c) are for $\delta=15°$ and $60°$, respectively. For a surface-breaking fault ($D=0$), the uplift is discontinuous at the origin (upper edge of the fault), the magnitude of the discontinuity being b cos δ. For a surface-breaking vertical fault (Fig. 3d), the uplift is symmetric about the origin and has a non-zero value at that point. However, for a buried vertical fault, the uplift is symmetric about the origin and vanishes at that point.

From eq. (13) we find that, for a surface-breaking ($d=0$) long tensile fault, the horizontal surface displacement (u_2) on the hanging wall side is maximum for $x_2=L$ cos δ and

$$(u_2)_{max} = b((\pi/2)\sin\delta + \cos\delta)/\pi\,.$$

For a fault which does not break the surface,

$$u_2 = u_3 = 0 \text{ at } x_2 = -d\cot\delta\,.$$

This is the point at which the fault when extended meets the surface.

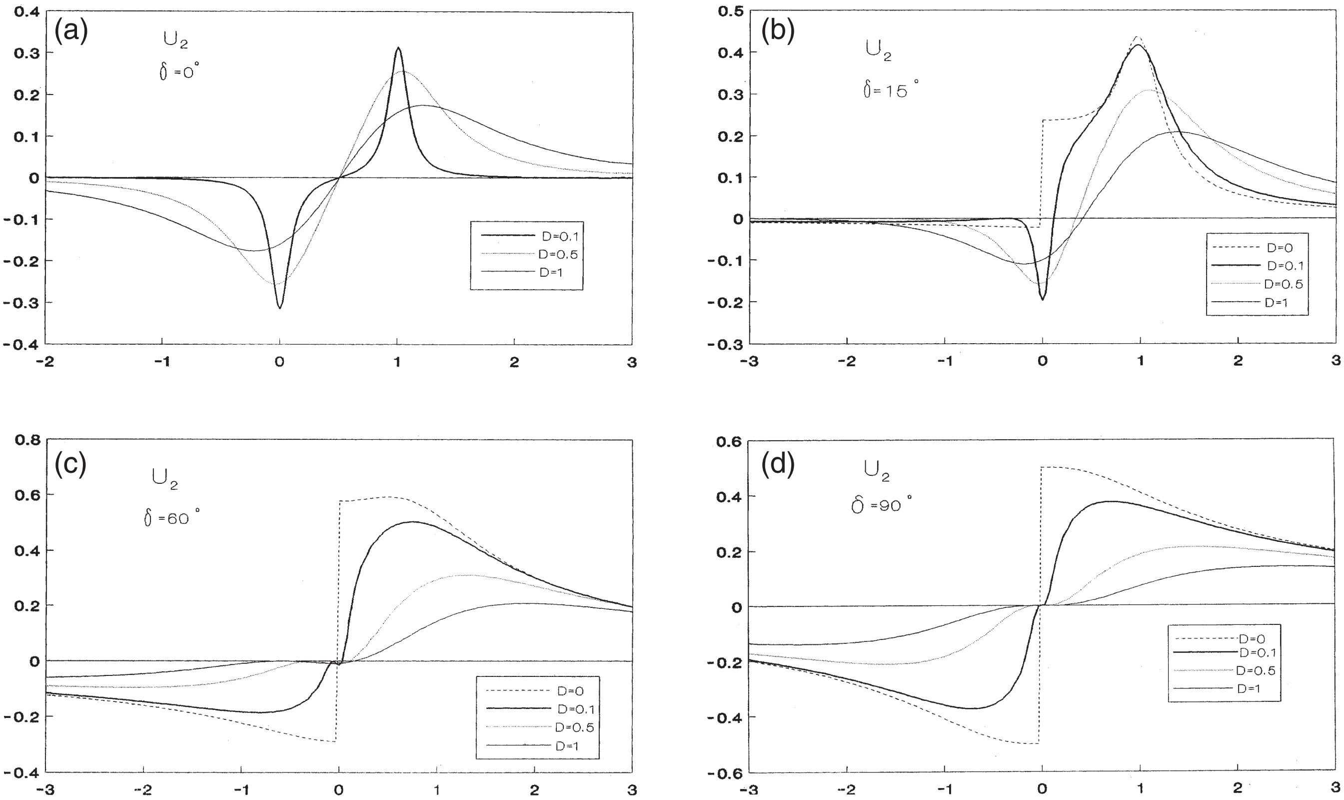

Figure 2. (a) variation of the dimensionless horizontal surface displacement $U_2=u_2/b$ with dimensionless horizontal distance $Y=x_2/L$ from the upper edge of the fault for various values of the dimensionless depth $D=d/L$ of the upper edge of the fault for $\delta=0°$. (b) variation of U_2 with Y for $\delta=15°$. (c) variation of U_2 with Y for $\delta=60°$. (d) variation of U_2 with Y for $\delta=90°$.

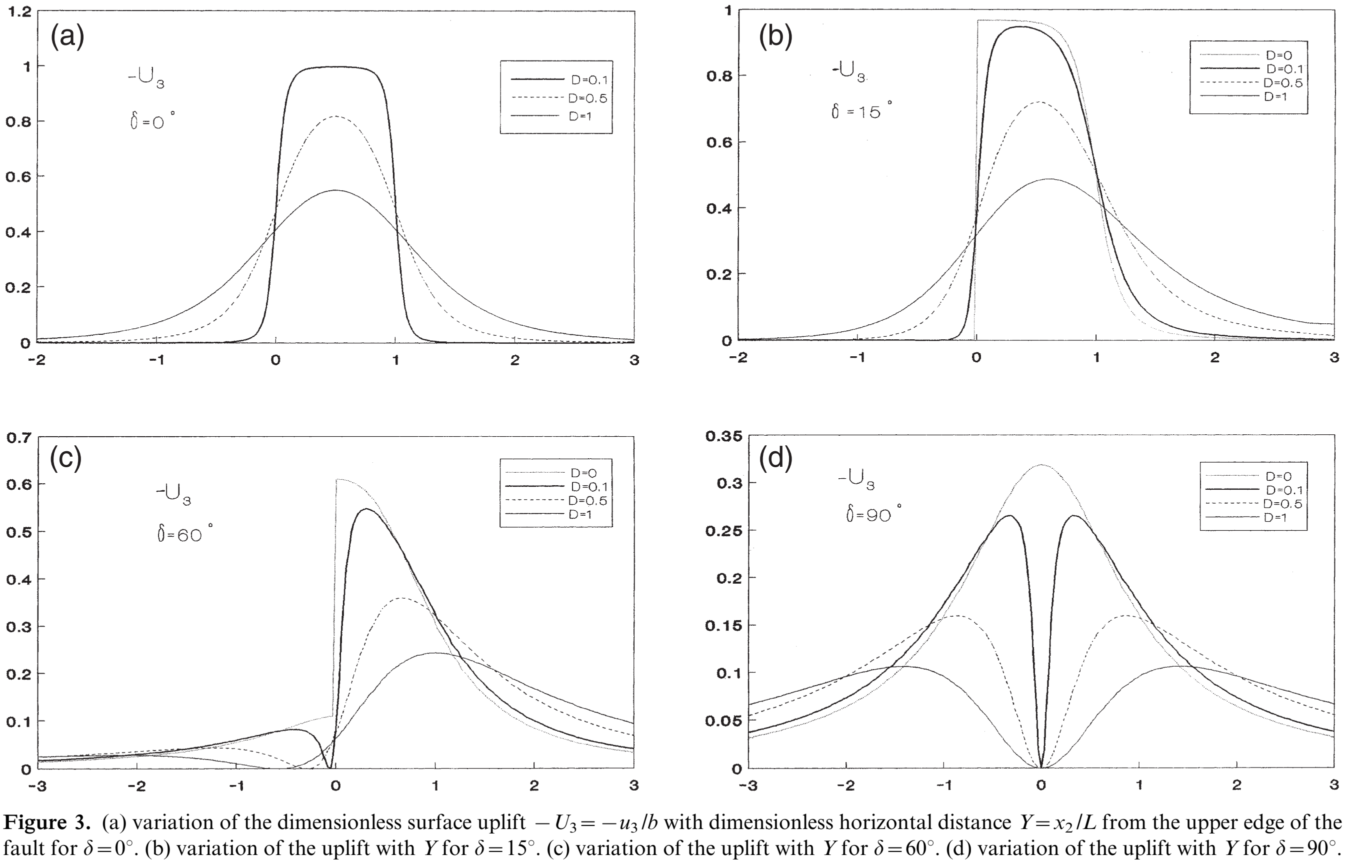

Figure 3. (a) variation of the dimensionless surface uplift $-U_3=-u_3/b$ with dimensionless horizontal distance $Y=x_2/L$ from the upper edge of the fault for $\delta=0°$. (b) variation of the uplift with Y for $\delta=15°$. (c) variation of the uplift with Y for $\delta=60°$. (d) Variation of the uplift with Y for $\delta=90°$.

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