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Low-field variation of magnetic susceptibility and its effect on the anisotropy of magnetic susceptibility of rocks

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SUMMARY

The theory of the low-field anisotropy of magnetic susceptibility (AMS) assumes a linear relationship between magnetization and the magnetizing field. This assumption is precisely valid in diamagnetic and paramagnetic minerals by definition, while in ferrimagnetic and antiferromagnetic minerals this relationship is in general non-linear (represented by a hysteresis loop), being linear only with very weak fields in which the initial susceptibility is measured. Recently, it has been shown that, in using common measuring fields, the field-independent susceptibility is measured in magnetite, while in pyrrhotite, haematite and titanomagnetite it may often be outside the initial susceptibility range. The problem can be solved in three ways. The simplest way is using very weak measuring fields (less than 10 A m^{-1}), but this can result in significant lowering of sensitivity and precision. The second way is to respect the non-linearity and measure the susceptibility in so many directions that contour diagram of directional susceptibilities can be presented instead of a susceptibility ellipsoid. The third way is to measure the AMS within the Rayleigh law range and calculate the initial directional susceptibilities from which the AMS can be correctly determined using linear theory.

Key words: anisotropy, field variation, magnetic susceptibility.

INTRODUCTION

The theory of the low-field anisotropy of magnetic susceptibility (AMS) of rocks is based on the assumption of a linear relationship between magnetization and the magnetizing field. This assumption is precisely valid in diamagnetic and paramagnetic minerals that show a linear relationship by definition in common measuring fields, the susceptibility then being field independent. In ferromagnetic minerals *sensu lato* (comprising ferrimagnetic, antiferromagnetic and ferromagnetic *sensu stricto*), the relationship is in general non-linear, represented by a hysteresis loop. However, in very weak magnetic fields this relationship is also linear even in ferromagnetic minerals *sensu lato*, the corresponding field-independent susceptibility being called the initial susceptibility. The instruments for measuring the AMS of rocks use relatively weak fields (see Table 1) assuming that they are weak enough for the initial susceptibility to be measured.

Chlupáčová (1984) noticed that susceptibilities of pyrrhotite ores measured by the KLY-2 Kappabridge differed substantially from those measured by the LAM-24 Astatic Magnetometer despite both instruments being calibrated in the same way (by a calibrating coil) and the susceptibilities of magnetite-bearing rocks measured by these two instruments were effectively the same. As the magnetizing field of the KLY-2 Kappabridge is 300 A m^{-1} and that in the LAM-24 Astatic Magnetometer is about 40 A m^{-1}, the differences in susceptibility were accounted for the differences in the magnetizing fields and, therefore, the field dependence of the magnetic susceptibility of pyrrhotite was hypothesized. Her work was confirmed and extended by Worm (1991) and Zapletal (1992) who found a very conspicuous field variation in the susceptibility in some pyrrhotite specimens.

Worm *et al.* (1993) also investigated, among other parameters, the grain-size dependence of the field variation of the susceptibility. They found that while the field variation of susceptibility was very conspicuous in large grains (typically hundreds of micrometers in size), it was hardly observable in small grains (less than 30 μm).

Markert & Lehmann (1996) treated the field variation of the AMS theoretically for the Rayleigh region of the magnetization curve. They generalized the originally scalar Rayleigh law to three dimensions, introducing the initial susceptibility tensor and the Rayleigh tensor, and developed a technique for simultaneous measurement of both tensors using a vibrating sample magnetometer.

Hrouda & Quade (1997) and Hrouda *et al.* (1998) investigated the AMS of single crystals of haematite from the Sao Juliao quarry, Minas Gerais, Brazil, in the fields ranging from 20 to 400 A m^{-1}. They found virtually no field variation of the susceptibility parallel to the *c*-axis, but strong variation along the basal plane. The minimum susceptibility direction was precisely parallel to the *c*-axis and the susceptibility ellipsoid was virtually rotational oblate. The calculation of the degree of AMS using the theory and program of

Table 1. Field intensity in various AMS meters.

Instrument	Field intensity	Reference
Low-field torque meter	800–6400 A m^{-1}	King & Rees (1962)
Molspin anisotropy delineator	400 A m^{-1}	Collinson (1993)
SI Sapphiro instrument	80 A m^{-1}	Borradaile *et al.* (1999)
KLY-1 Kappabridge	150 A m^{-1}	Jelínek (1973)
KLY-2 Kappabridge	300 A m^{-1}	Jelínek (1980)
KLY-3 Kappabridge	300 A m^{-1}	Jelínek & Pokorný (1997)
KLY-3S Kappabridge	300 A m^{-1}	Jelínek & Pokorný (1997)

Jelínek (1977) failed, because the calculated minimum susceptibilities were strongly negative, having evidently nothing to do with diamagnetism as revealed through direct measurement of the susceptibility along the *c*-axis.

Jackson *et al.* (1998) investigated the field variation of susceptibility of titanomagnetites ranging in composition from magnetite to strongly titaniferous titanomagnetite in the fields ranging from 0.1 to 2000 A m^{-1}. They found no field variation of susceptibility in pure magnetite, but strong variation in titanomagnetites; the variation become stronger with more titaniferous titanomagnetite.

De Wall (2000) investigated the AMS of titanomagnetite-bearing dyke rocks from the Hegau volcanics, SW Germany, in fields of 30 and 300 A m^{-1}. No differences were found in the orientations of the principal susceptibilities and in the shape of the susceptibility ellipsoid, while in the degree of AMS the differences were revealed, becoming larger the more different the titanomagnetite composition was from magnetite.

All of the above investigations have shown that, using common AMS instruments, the field-independent susceptibility is measured in the case of magnetite, while in the case of titanomagnetite, coarse-grained pyrrhotite and haematite the fields may be too strong so that the measured susceptibility is outside the range of the initial susceptibility. The AMS calculated using the linear theory can then give rise to imprecise results. The present paper analyses this problem theoretically and discusses possible solutions to the problem.

FIELD VARIATION OF BULK SUSCEPTIBILITY—LITERATURE AND NEW EXPERIMENTAL DATA

Magnetization of multidomain materials in low fields (less than the coercivity) is often described by the empirical Rayleigh law

$$M = kH + \alpha H^2, \tag{1}$$

where M is the magnetization, H is the intensity of the magnetizing field, k is the initial susceptibility and α is the Rayleigh coefficient. The measured direct field susceptibility, $\kappa = M/H$, can be obtained through dividing eq. (1) by H,

$$\kappa = k + \alpha H. \tag{2}$$

As deduced by Neél (1942) on the basis of the Preisach diagram, the relationship between the initial susceptibility and the Rayleigh coefficient is as follows:

$$\alpha = ck^2, \tag{3}$$

where c is a proportionality constant. Combining eqs (2) and (3) yields

$$\kappa = k + ck^2 H. \tag{4}$$

Figs 1–4 show the data of the field variation of susceptibility in various minerals. The data were partially obtained through scanning and subsequent digitizing the literary data and partially measured by the present author. It is obvious from Fig. 1 that there is virtually no field variation of susceptibility in magnetite and magnetite-bearing rocks measured in the fields used in common AMS meters (*cf.* Table 1). Linear theory in calculating the AMS of magnetite-bearing rocks is fully legitimate.

In Figs 2–4, it is clear that the susceptibility of pyrrhotite, haematite and titanomagnetite measured in fields below 10 A m^{-1} is more or less field independent and can therefore be regarded as representing the initial susceptibility. In higher fields, the measured susceptibilities are in general strongly field dependent, clearly not representing the initial susceptibility. An attempt was made to fit the measured data to the Rayleigh law for various field intervals

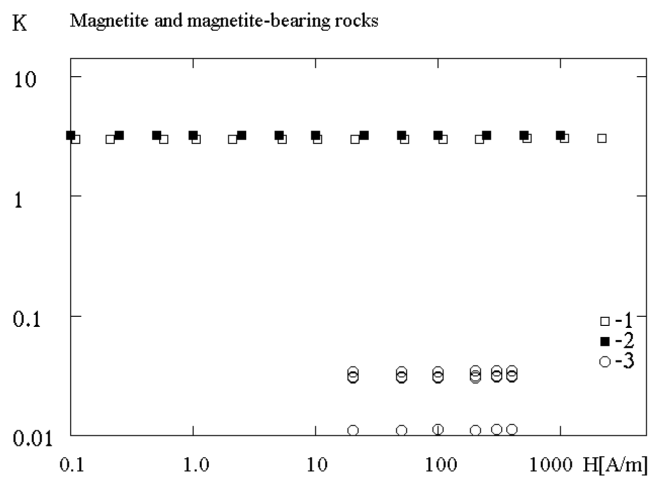

Figure 1. Field variation of magnetic susceptibility in magnetite and magnetite-bearing rocks. Legend: (1) synthetic magnetite, Jackson *et al.* (1998), (2) synthetic magnetite, Worm *et al.* (1993), (3) various magnetite-bearing rocks measured by the present author using a specially adapted KLF-3 mini-Kappa instrument.

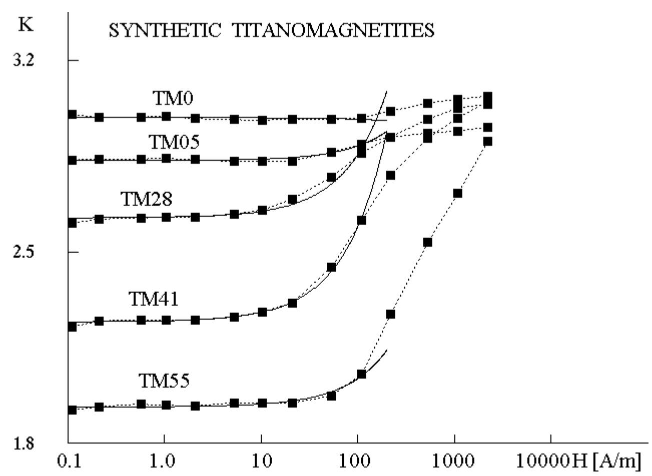

Figure 2. Example of the field variation of the magnetic susceptibility in synthetic titanomagnetites. The individual plots, connected by dotted straight lines, are adopted from Jackson *et al.* (1998). The solid line represents the Rayleigh law curve in the field interval 0.1–300 A m^{-1} constructed through least-squares fitting using eq. (2).

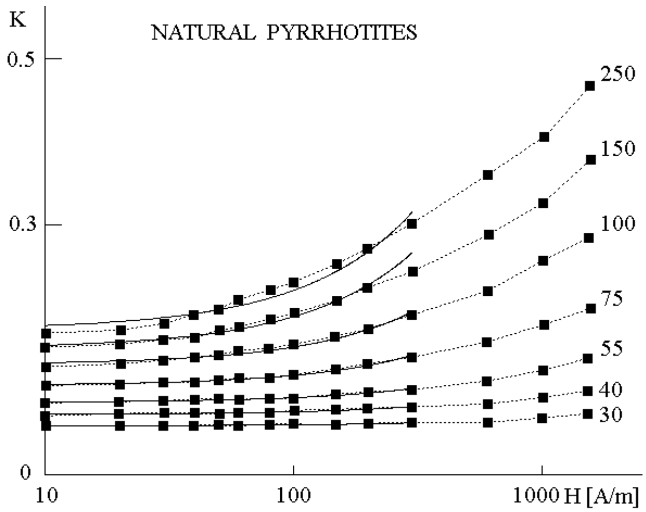

Figure 3. Example of the field variation of the magnetic susceptibility in pyrrhotites of variable grain size (denoted in microns) from the locality of the Ortano mine on Elba Island, Italy. The individual plots, connected by dotted straight lines, are adopted from Worm *et al.* (1993). The solid line represents the Rayleigh law curve in the field interval 0.1–300 A m^{-1} constructed through least-squares fitting using eq. (2).

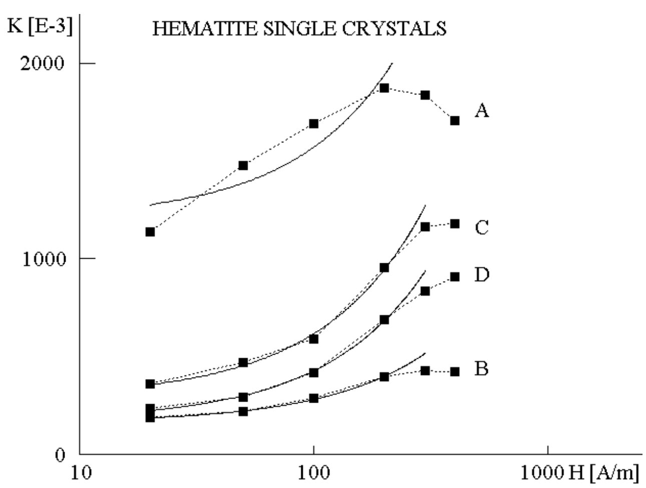

Figure 4. Example of the field variation of the magnetic susceptibility in haematite single crystals from the Sao Juliao quarry, Minas Gerais, Brazil. The individual plots, connected by dotted straight lines, are adopted from Hrouda & Quade (1997) and Hrouda *et al.* (1998). The solid line represents the Rayleigh law curve in the field interval 0.1–300 A m^{-1}, constructed through least-squares fitting using eq. (2).

using the least-squares method and eq. (2). It was revealed that the empirical data follow the Rayleigh law relatively closely up to the fields of about 300 A m^{-1} (solid line in Figs 2–4), and in higher fields deflect from it significantly. The results are summarized in Table 2 showing for each curve (indicated by the same symbol as in Figs 2–4) the initial susceptibility (k), Rayleigh coefficient (α), Rayleigh coefficient to initial susceptibility ratio (α/k), constant c, ratio of the initial susceptibility to the susceptibility measured at the field intensity of 300 A m^{-1} (k/κ_{300}), and the ratio of the αH member to the susceptibility measured at the field intensity of 300 A m^{-1} ($\alpha H/\kappa_{300}$). The field intensity of 300 A m^{-1} was selected as a reference value, because it is used in the commercially most extended AMS meters, namely the KLY-2 and KLY-3S/KLY-3 Kappabridges.

Table 2. Resolution of susceptibility versus field curves.

Curve	Fig.	k	α (A m^{-1})	α/k (A m^{-1})	c (A m^{-1})	k/κ_{300}	$\alpha H/\kappa_{300}$
Titanomagnetite							
TM0	2	2.988	0.000 08	0.000 03	0.000 01	0.99	0.01
TM05	2	2.835	0.000 42	0.000 15	0.000 05	0.96	0.04
TM28	2	2.635	0.001 52	0.000 58	0.000 22	0.85	0.15
TM41	2	2.255	0.002 64	0.001 17	0.000 52	0.74	0.26
TM55	2	1.926	0.001 48	0.000 77	0.000 40	0.81	0.19
Pyrrhotite							
250	3	0.175	0.000 47	0.002 69	0.015 39	0.55	0.45
150	3	0.152	0.000 38	0.002 53	0.016 65	0.57	0.43
100	3	0.132	0.000 21	0.001 59	0.012 02	0.68	0.32
75	3	0.107	0.000 12	0.001 12	0.010 43	0.75	0.25
55	3	0.087	0.000 06	0.000 64	0.007 39	0.84	0.16
40	3	0.073	0.000 03	0.000 43	0.005 99	0.88	0.12
30	3	0.059	0.000 01	0.000 20	0.003 33	0.94	0.06
Hematite							
B	4	0.173	0.002 55	0.014 82	0.085 87	0.18	0.82
C	4	0.290	0.003 28	0.011 30	0.038 93	0.23	0.77
D	4	0.163	0.001 19	0.007 27	0.044 61	0.31	0.69

Table 2 shows that in titanomagnetite the contribution of the member αH is relatively weak, being stronger the larger the Ti component is. In pyrrhotite, the contribution of αH is much stronger, sometimes constituting one-half of the susceptibility measured; the proportion of αH increases with increasing grain size. In the basal plane of haematite, αH may even dominate the measured susceptibility. In order to gain an idea of the effect of αH on the measured susceptibility according to the field intensity, Fig. 5 was constructed showing the contribution of αH to the measured susceptibility ($K_q = \alpha H/\kappa$) against the field intensity (H) for several α/k ratios derived from the data presented in Table 2. It can be seen in the figure that the contribution of αH to the measured susceptibility is very low in weak fields and can be neglected in practice, while in stronger fields, comparable to those used in the most AMS meters, this contribution may be significant, being stronger the higher the α/k ratio is.

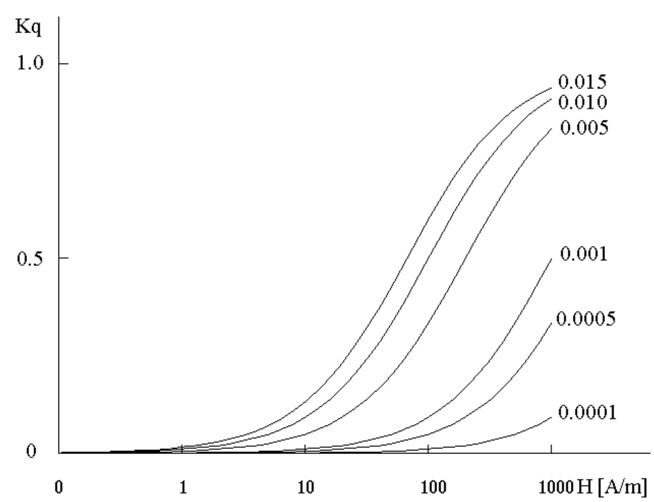

Figure 5. Plot of the contribution of the αH member in eq. (2) to the measured susceptibility ($K_q = \alpha H/\kappa$) against the field intensity (H) for several α/k ratios (denoted on the right-hand sides of individual curves).

FIELD VARIATION OF AMS

The theory of the low-field AMS is based on the assumption of a linear relationship between magnetization and the magnetizing field, traditionally described as follows:

$$\mathbf{M} = \boldsymbol{\kappa}\mathbf{H}, \tag{5}$$

where **M** is the magnetization vector, **H** is the field intensity vector and $\boldsymbol{\kappa}$ is the symmetric second-rank tensor of magnetic susceptibility. In the Rayleigh law range, the relationship between magnetization and the magnetizing field is described by the Rayleigh law generalized to three dimensions (Markert & Lehmann 1996):

$$\mathbf{M} = \boldsymbol{k}\mathbf{H} + \boldsymbol{\alpha}H\mathbf{H}, \tag{6}$$

where H is the modulus of the vector **H**, $\boldsymbol{k}$ is the second-rank initial susceptibility tensor and $\boldsymbol{\alpha}$ is the second-rank Rayleigh tensor. The components of the initial susceptibility tensor and those of the Rayleigh tensor are related as follows (Markert & Lehmann 1996):

$$\alpha_{ij} = ck_{ij}^2, \tag{7}$$

where c is a constant of proportionality.

It is obvious from eq. (6) that the assumption of linearity is no longer valid in the Rayleigh region and the AMS theory based on eq. (5) is in principle incorrect. However, the linear AMS theory is so simple, elegant and beautiful that the anisotropists would advocate using it even though it is not fully correct provided that the errors introduced by using it are not too large. Let us investigate these errors in practical examples.

The principle of the AMS determination lies in measuring the susceptibility of a specimen in at least six independent directions, the measured susceptibilities being called the directional susceptibilities (Janák 1965), and subsequent fitting the susceptibility tensor to these data using the least-squares method. In some instruments, the directional susceptibilities are measured directly (e.g. the KLY-1, KLY-2 and KLY-3 Kappabridges use the 15-directions design, SI-2 Sapphiro uses a 24-directions design). The other instruments (e.g. Low Field Torque Magnetometer, Molspin Anisotropy Delineator, KLY-3S Kappabridge) measure a slowly rotating specimen; consequently, a large set of data derived from directional susceptibilities (e.g. susceptibility differences or susceptibility components perpendicular to the magnetizing field) is obtained. Measuring more than six directional susceptibilities enables a test for anisotropy to be made which serves to verify whether the differences between the principal susceptibilities are great enough compared with measurement errors for the specimen to be regarded as being anisotropic. In the Jelínek (1977) method used in this paper, the F-test is used for this testing. In addition, the error in fitting the tensor as well as the errors in determining the principal susceptibilities and principal directions can be evaluated. The former error is evaluated as follows. After fitting the susceptibility tensor to the measured data, the difference between the measured and fitted data is calculated for each of the 15 directions and normalized using the measured value. The fitting error is then characterized as the average value of the absolute values of these 15 normalized differences. The precision in determining the directions of the principal susceptibilities is characterized by so-called error angles. For each principal direction, two error angles are defined parallel to the principal planes of the susceptibility ellipsoid. These angles delimit a region within which the true principal direction lies with a probability of 95 per cent (for details see Jelínek 1977).

In order to better understand its field variation, the AMS of some magnetite- and pyrrhotite-bearing rocks and ores and of four single crystals of haematite was measured in the fields of 20, 50, 100, 200, 300 and 400 A m^{-1} using the specially adapted KLF-3 mini-Kappa instrument (Sapík 1988) and the Jelínek (1977) rotatable design using 15 measuring directions (Fig. 6). Unfortunately, this instrument was not originally designed for measuring the AMS, its sensitivity being 1×10^{-6} [SI] and precision of measurement being 2 per cent. For this reason, the investigated specimens were selected in such a way as to be strongly magnetic and/or anisotropic in order to keep reasonable precision in the determination of the AMS. Namely, our long years of experience with AMS have shown that the maximum minus minimum susceptibility difference should be at least one order higher than the measurement error.

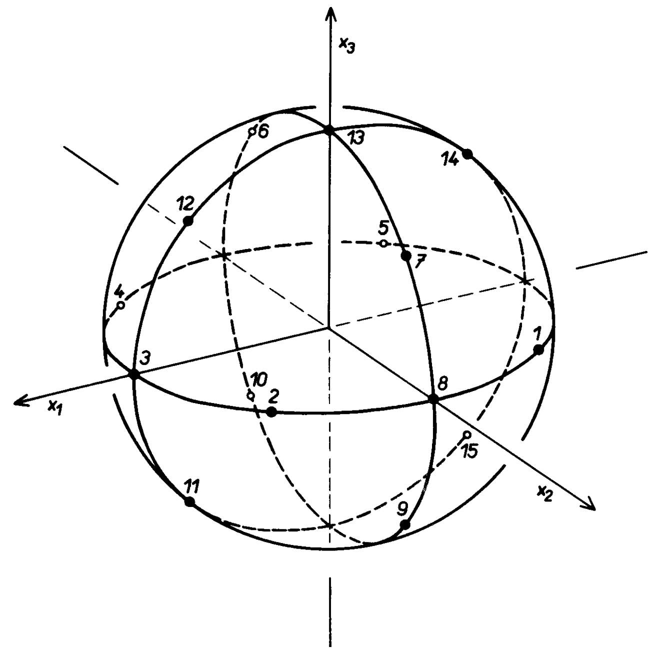

Figure 6. Rotatable design of measuring directions used in the Jelínek (1977) method for the AMS determination. Adopted from Jelínek (1977).

The results are presented in Tables 3–5 showing for each specimen investigated the measuring field (H), mean susceptibility (K_m), degree of AMS (h) and shape parameter (U) defined as follows:

$$K_m = (k_1 + k_2 + k_3)/3$$
$$h = 100(k_1 - k_3)/K_m \tag{8}$$
$$U = (2k_2 - k_1 - k_3)/(k_1 - k_3)$$

where $k_1 \geq k_2 \geq k_3$ are the principal susceptibilities. The parameters h and U are preferred for more frequently used P and T parameters, because they are based on susceptibility differences and hence are not affected by negative values of the minimum susceptibility sometimes obtained through linear fit to non-linear data. In addition, the $k_1 - k_3$ difference, F-statistics of the test for anisotropy ($F > 3.4817$ to indicate a statistically anisotropic specimen on a likelihood level of 0.05, Jelínek 1977), fitting error (E) and error angles of the principal directions ($E12$, $E23$, $E13$) are presented.

Magnetite

The values of the F-statistics are high in all specimens and in all fields (Table 3)—much higher than the critical value ($F_{crit} = 3.4817$). This, together with the $k_1 - k_3$ differences being about 2×10^{-3}, while the measurement error varies from 2 to 6×10^{-4}, means that the

Table 3. AMS of magnetite-bearing rocks measured in various fields.

H (A m^{-1})	K_m [E-6]	h	U	F	$k_1 - k_3$ [E-6]	E	$E12$	$E23$	$E31$
1MM									
20	11 047	15.4	0.10	111.9	1698	0.7	7.8	6.4	3.5
50	11 053	15.1	0.03	234.1	1672	0.5	5.0	4.7	2.4
100	11 093	15.4	0.11	202.4	1706	0.5	5.9	4.7	2.6
200	11 080	15.3	0.05	151.9	1693	0.6	6.4	5.7	3.0
300	11 100	15.6	0.04	120.6	1736	0.7	7.1	6.5	3.4
400	11 140	16.5	0.14	382.5	1838	0.4	4.5	3.4	1.9
22M									
20	30 893	7.1	−0.26	183.4	2200	0.7	4.4	7.5	2.8
50	30 887	7.0	−0.13	519.4	2174	0.4	2.9	3.8	1.6
100	31 087	7.8	−0.16	291.0	2419	0.6	3.8	5.3	2.2
200	31 207	7.1	−0.25	489.5	2219	0.5	2.7	4.5	1.7
300	31 333	7.5	−0.21	402.4	2347	0.5	3.1	4.8	1.9
400	31 540	7.3	−0.30	417.6	2302	0.5	2.9	5.2	1.9
23M									
20	34 207	7.1	0.48	610.1	2415	0.5	6.0	2.1	1.6
50	34 200	6.6	0.56	804.5	2247	0.4	6.3	1.8	1.4
100	34 227	6.8	0.49	565.4	2317	0.5	6.5	2.2	1.6
200	34 367	6.8	0.58	881.3	2344	0.4	6.3	1.7	1.3
300	34 453	6.9	0.55	987.1	2395	0.4	5.5	1.6	1.2
400	34 553	7.0	0.52	2841.2	2405	0.2	3.0	1.0	0.7
24M									
20	30 333	8.6	0.52	669	2609	0.5	6.3	2.0	1.5
50	30 373	8.3	0.45	2029	2515	0.3	3.1	1.2	0.9
100	30 427	8.2	0.46	1099	2504	0.3	4.3	1.6	1.2
200	30 480	8.3	0.53	764.5	2533	0.4	6.0	1.8	1.4
300	30 613	8.7	0.48	1542	2676	0.3	3.8	1.3	1.0
400	30 727	8.8	0.53	771.7	2695	0.5	5.9	1.8	1.4

Table 4. AMS of pyrrhotite-bearing specimens measured in various fields.

H (A m^{-1})	K_m [E-6]	h	U	F	$k_1 - k_3$ [E-6]	E	$E12$	$E23$	$E31$
P39									
20	318	203.5	0.35	7.2	648	110.0	37.6	20.2	14.0
50	591	161.4	0.95	71.0	954	57.8	74.4	5.2	5.0
100	864	160.2	0.96	150.1	1384	5.8	71.6	3.6	3.5
200	1082	160.6	0.88	624.5	1739	3.5	25.7	1.8	1.7
300	1224	160.4	0.85	648.1	1963	3.8	21.1	1.8	1.6
400	1300	158.9	0.85	848.1	2065	3.5	17.9	1.5	1.4
P48									
20	2916	180.1	0.51	29.0	5250	45.4	27.1	9.5	7.2
50	4544	189.8	0.31	104.1	8622	38.4	10.7	5.7	3.7
100	5630	184.0	0.36	469.3	10 361	21.9	5.5	2.6	1.8
200	6435	178.9	0.41	836.4	11 510	1.8	4.5	1.9	1.3
300	7061	179.2	0.40	2411.9	12 653	1.2	2.6	1.1	0.8
400	7359	178.8	0.40	2832.0	13 156	1.1	2.4	1.0	0.7
P48A									
20	96	233.5	−0.05	9.3	224	32.9	22.1	24.2	12.1
50	136	224.1	0.07	6.5	304	53.2	28.8	25.5	14.3
100	206	221.2	0.09	46.5	455	29.9	11.9	10.0	5.5
200	232	212.6	0.05	115.0	493	20.6	7.3	6.6	3.5
300	256	206.6	0.16	210.4	529	16.4	6.2	4.4	2.6
400	279	201.4	0.35	90.4	561	26.9	12.1	5.9	4.0
80P									
20	209	197.9	0.54	6.6	414	75.5	49.0	19.1	14.9
50	356	185.1	0.57	43.5	659	47.1	25.8	7.6	5.9
100	462	174.6	0.68	120.9	806	35.2	21.9	4.3	3.6
200	549	162.5	0.87	233.8	892	29.2	36.9	2.9	2.7
300	600	160.3	0.84	210.3	962	33.0	32.7	3.1	2.9
400	641	161.1	0.89	359.4	1033	27.4	36.3	2.3	2.2

specimens are anisotropic enough with respect to the measurement error in all fields. The E error characterizing the fit of the tensor to the measured data is very low, less than 1 per cent in all specimens and all fields. The error angles characterizing the precision in determining the orientations of the principal directions are also very low, being of the order of a few degrees. Consequently, the error in fitting the tensor is therefore lower than the measurement error and the linear theory of the AMS is fully legitimate in this case.

The mean susceptibility in individual specimens shows virtually no field dependence. Only in the two strongest fields is the mean susceptibility slightly higher than those measured in the other fields. The degree of AMS is relatively low and obviously field independent. The shape parameter indicates predominantly planar magnetic fabric in specimens 23M and 24M, the magnetic fabric on transition between linear and planar in specimen 1MM and predominantly linear magnetic fabric in specimen 22M. It is relatively variable within a specimen, but evidently showing no systematic field variation. The directions of the principal susceptibilities are virtually coaxial in all measurement fields in each specimen (Fig. 7), the angular differences in orientations of the principal susceptibilities in various fields being comparable to the error angles.

Pyrrhotite

The values of the F-statistics are higher than the critical value ($F_{crit} = 3.4817$) in all specimens in all fields (Table 4). However, these values clearly increase with field in all specimens. The mean susceptibility also increases with the field, reaching values three times higher in a field of 400 A m^{-1} than in a field of 20 A m^{-1}. The $k_1 - k_3$

Table 5. AMS of hematite single crystals measured in various fields.

H (A m^{-1})	K_m [E-6]	h	U	F	$k_1 - k_3$ [E-6]	E	$E12$	$E23$	$E31$
A									
20	736 993	178.5	0.38	204.2	1315 680	14.8	8.5	3.9	2.7
50	916 686	175.5	0.38	34.4	1 608 967	44.0	20.0	9.4	6.5
100	1 156 024	178.9	0.44	156.5	2 068 589	26.8	10.8	4.3	3.1
200	1 288 419	167.8	0.61	1120	2 162 354	10.8	6.1	1.5	1.2
300	1 261 561	158.4	0.72	900.5	1 997 935	11.3	9.5	1.6	1.3
400	1 193 083	149.5	0.76	440	1 783 778	14.6	15.7	2.2	1.9
B									
20	151 318	169.0	0.63	264.7	255 803	12.1	13.1	3.0	2.4
50	200 745	174.8	0.54	158.7	350 963	21.2	13.3	4.0	3.1
100	283 233	184.7	0.39	116.2	523 216	36.0	11.5	5.1	3.5
200	453 891	188.0	0.38	62.1	853 133	8.0	15.3	7.0	4.8
300	554 580	186.1	0.36	163.9	1 032 240	6.0	9.2	4.4	3.0
400	617 243	178.6	0.46	273.6	1 102 149	5.0	8.7	3.2	2.3
C									
20	243 064	176.7	0.46	408.5	429 470	17.0	7.1	2.6	1.9
50	344 168	180.5	0.44	85.1	621 223	5.4	14.7	5.8	4.2
100	449 994	177.3	0.50	191.1	797 705	4.6	11.2	3.7	2.8
200	668 153	166.9	0.81	104	1 115 482	9.3	36.3	4.5	4.0
300	797 524	164.8	0.77	302	1 314 558	6.4	19.8	2.7	2.4
400	789 788	155.1	0.89	453	1 225 120	5.0	32.3	2.1	2.0
D									
20	124 307	192.2	0.14	109.7	238 968	24.9	8.3	6.3	3.6
50	150 789	190.6	0.21	218.2	287 359	21.3	6.4	4.2	2.5
100	194 328	186.1	0.34	82.3	361 606	44.1	12.5	6.2	4.2
200	266 822	178.4	0.44	219.2	476 010	3.6	9.2	3.6	2.6
300	275 246	159.6	0.69	3259	439 237	0.9	4.5	0.8	0.7
400	274 889	153.1	0.74	587.6	420 745	2.1	12.7	1.9	1.7

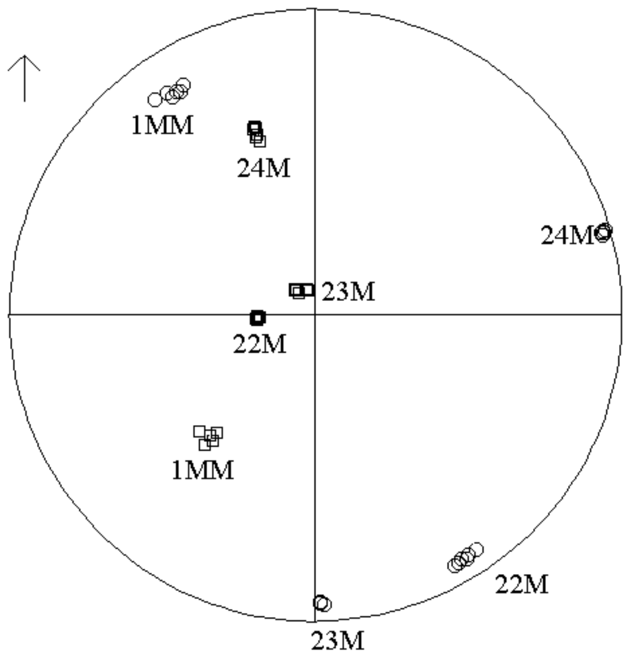

Figure 7. Variations of the orientations of the maximum (square) and minimum (circle) susceptibility directions with field in magnetite-bearing rocks. Specimen names correspond to those in Table 3. Specimen coordinate system, equal-area projection on lower hemisphere.

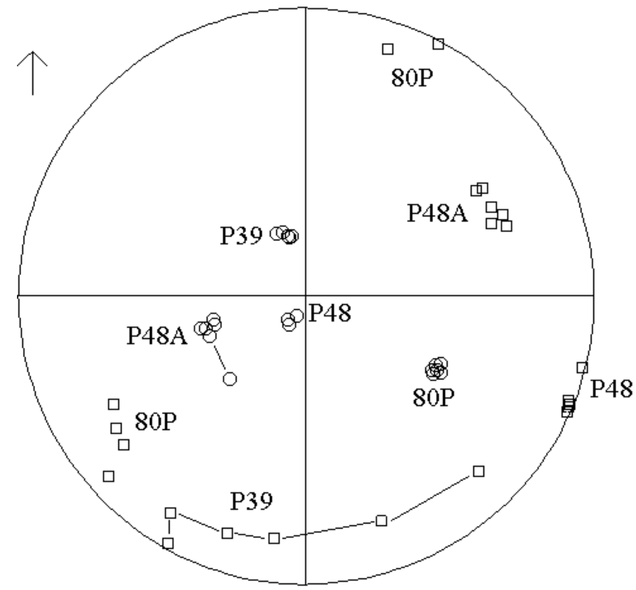

Figure 8. Variations of the orientations of the maximum (square) and minimum (circle) susceptibility directions with field in pyrrhotite-bearing rocks. Specimen names correspond to those in Table 4. Specimen coordinate system, equal-area projection on lower hemisphere.

differences are very large, also showing a clear increase with the field. The degree of AMS is extremely high, showing either no field variation or a very weak tendency to decrease with the field. The magnetic fabric, as indicated by the shape parameter, ranges from that on transition between linear and planar to clearly planar. It is relatively variable within individual specimens, but shows no clear variation with field.

The E errors characterizing the fit of the tensor to the measured data are mostly very high, much higher than those in magnetite. Except for fields of 100–400 A m^{-1} in specimen P39 and fields of 200–400 A m^{-1} in specimen P48, where the E values are of the order of per cent, the E values are of the order of tens of per cent. These high values exist despite $k_1 - k_3$ differences being mostly two orders higher than the measurement error. Consequently, the high values of the E error indicate a very bad fit of the tensor to the measured data. A possible explanation of this phenomenon is that there is no longer a linear relationship between the magnetization and the field.

The error angles $E23$ and $E31$ characterizing the precision in determination of the minimum susceptibility direction are mostly low, of the order of degrees, only exceptionally are they higher in the weakest measuring fields. The error angle $E12$, one of the error angles characterizing the precision in determination of the maximum susceptibility direction, is in contrast mostly relatively high, of the order of tens of degrees. In specimens P39 and 80P, this angle is high in all measurement fields, indicating, in agreement with the shape parameter value, that the specimens have rotational susceptibility ellipsoids. In specimens P48 and P48A, this angle is high in low fields and low in higher fields, indicating that the difference between the maximum and intermediate susceptibilities are relatively low and these susceptibilities can be distinguished only with difficulty using the KLF-3 instrument.

The minimum susceptibility directions are virtually coaxial in all measurement fields within each specimen (Fig. 8). The maximum susceptibility directions are virtually coaxial in all measurement fields in the specimens P48 and P48A. In the specimens P39 and 80P, they are more scattered in the $k_1 - k_2$ plane in accordance with the above-mentioned fact that these specimens have almost rotational susceptibility directions.

Haematite

The values of the F-statistics are higher than the critical value ($F_{crit} = 3.4817$) in all specimens in all fields (Table 5). The mean susceptibility increases with field reaching three to four times higher values in a field of 400 A m^{-1} than in a field of 20 A m^{-1}. The $k_1 - k_3$ differences are very large, also showing a clear increase with the field. The degree of AMS is extremely high, showing either no field variation or a weak tendency to decrease with field. The magnetic fabric, as indicated by the shape parameter, is clearly planar in all specimens in all fields. It is relatively variable within individual specimens, but shows no clear variation with field.

The E errors characterizing the fit of the tensor to the measurement data are relatively high, in almost all cases higher than the measuring error, despite $k_1 - k_3$ differences being very high. Consequently, the high values of the E error indicate a very bad fit of the tensor to the measured data.

The error angles $E23$ and $E31$ characterizing the precision in determination of the minimum susceptibility direction are mostly low, of the order of degrees or even less. The error angle $E12$ characterizing the precision in determination of the maximum susceptibility direction in the $k_1 - k_2$ plane is slightly higher.

The minimum susceptibility directions are virtually coaxial in all measurement fields within each specimen. The maximum susceptibility directions are more scattered in the $k_1 - k_2$ plane. Even though they show no clear systematic change with field, the directions measured in weaker fields (20, 50, 100 A m^{-1}) are separated from those measured in stronger fields (200, 300, 400 A m^{-1}, see Fig. 9).

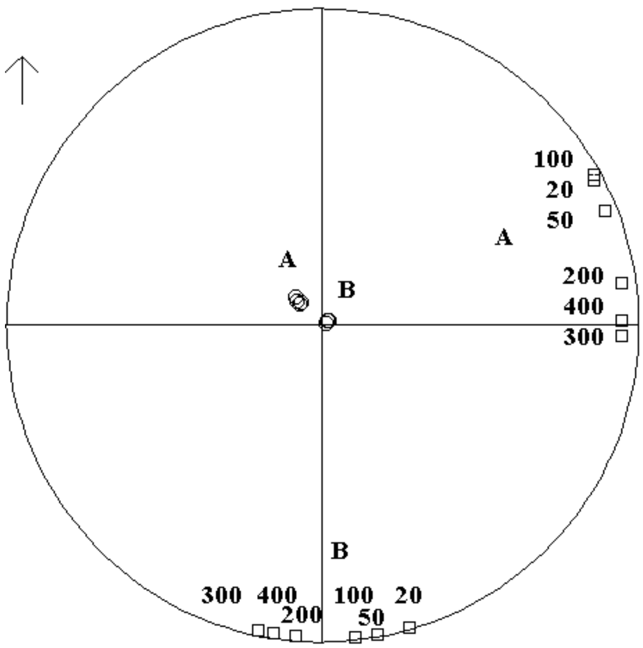

Figure 9. Variations of the orientations of the maximum (square) and minimum (circle) susceptibility directions with field in haematite single crystals A and B. Intensity of field (in A m^{-1}) is indicated with symbols for the maximum susceptibility. Specimen coordinate system, equal-area projection on lower hemisphere.

MODELLING THE EFFECT OF THE SUSCEPTIBILITY FIELD DEPENDENCE ON AMS

The effect of the susceptibility field variation on the AMS was investigated using a simple model based on eqs (6) and (7). A certain initial susceptibility tensor was considered specified by the mean susceptibility, degree of AMS and shape parameter. Then, 15 directional initial susceptibilities following the Jelínek (1977) design (Fig. 6) were calculated using the Janák (1965) formula

$$K_d = \mathbf{n}_t \boldsymbol{k} \mathbf{n}, \tag{9}$$

where K_d is the directional susceptibility, $\boldsymbol{k}$ is the initial susceptibility tensor, $\mathbf{n}$ is the column matrix of direction cosines specifying the direction under consideration and $\mathbf{n}_t$ is the transpose to $\mathbf{n}$.

Respecting eq. (7), 15 directional 'measured' susceptibilities were calculated using eqs (3) and (4) and rounded to four digits. These data served as input data for the ANISO-20 program based on the Jelínek (1977) linear theory of AMS calculation. The results are presented in Table 6 showing the model status (initial susceptibility, susceptibilities 'measured' at specified fields), the h and U parameters as well as the error (E) in fitting the susceptibility tensor.

Table 6. Modelling the effect of susceptibility field dependence on AMS.

Status	K_m	[E-6]	h (per cent)	U	E
initial	500		38.5	0.60	0.1
1	513 A m^{-1}	[E-6]	39.0	0.60	0.1
3	523 A m^{-1}	[E-6]	39.4	0.61	0.4
10	560 A m^{-1}	[E-6]	41.9	0.59	0.8
30	663 A m^{-1}	[E-6]	47.1	0.59	2.4
100	1029 A m^{-1}	[E-6]	57.1	0.59	8.2
300	2072 A m^{-1}	[E-6]	66.3	0.59	24.8

It is clear from the table that the E error is very low for the initial susceptibility, in fact reflecting only the rounding errors. In the fields of 1 and 3 A m^{-1}, all the parameters are relatively near those for the initial susceptibility. In stronger fields, mainly those of 100 and 300 A m^{-1}, both the mean susceptibility and the degree of AMS are much higher than those of the initial susceptibility. The shape parameter changes with field only gently. The fitting error increases with field strongly, reflecting the situation that the susceptibility is no longer represented by a second-rank tensor in stronger fields. Even though the model used is very simple, its results are in good agreement with the data measured on haematite- and pyrrhotite-bearing rocks and ores.

DISCUSSION

The fields used in the common AMS meters enable the field-independent susceptibility in diamagnetic and paramagnetic minerals as well as in magnetite to be measured. The linear theory of the AMS is then fully legitimate. In pyrrhotite, haematite and titanomagnetite, the fields are an order of magnitude stronger than the fields in which the initial susceptibility is reliably measured (Table 1). Consequently, the calculation of AMS using linear theory (eq. 5) is in principle incorrect in this case. However, the field variation of the low-field susceptibility is strongly grain size dependent in these minerals (for example, see Fig. 3), being in fact significant only in relatively coarse-grained (hundreds of micrometres) mineral grains. In fine grains, this dependence is very weak and the use of the linear theory is therefore also legitimate. In addition, there is no necessity to investigate the grain size of the magnetic mineral under consideration, because if the fit of the susceptibility tensor to the measured data is excellent, it is evident that the field dependence of AMS is insignificant in such a specimen.

The field variation of the AMS may be a significant problem in coarse-grained pyrrhotite, haematite and titanomagnetite. Let us discuss how this problem can be solved.

Using very weak measuring fields (less than 10 A m^{-1}) would theoretically solve the problem in the best and purest way. However, it would be impractical, because using a very weak field would no doubt result in significantly lowering the sensitivity and the precision of the AMS measurement. It is obvious that this solution is applicable only to relatively strongly magnetic and anisotropic rocks.

Some instruments measuring the AC susceptibility (for example, the LakeShore Model 7130 AC susceptometer, Jackson *et al.* 1998) have a capability of separate measurement of the in-phase and out-of-phase (quadrature) susceptibility components. According to Worm *et al.* (1993), in the Rayleigh law range the in-phase (κ') and out-of-phase (κ'') susceptibility components are

$$\kappa' = k + \alpha H \tag{10}$$

$$\kappa'' = 4\alpha H/3\pi. \tag{11}$$

Then, these two components can be combined to determine the initial susceptibility

$$k = \kappa' - 3\pi\kappa''/4. \tag{12}$$

Directional initial susceptibilities can then be used to construct the tensor of initial susceptibility. As far as the present author knows, no commercial AMS instrument has this capability and this approach is therefore confined to relatively strongly magnetic and anisotropic rocks.

The problem can also be solved in a pure way using the approach of Markert & Lehmann (1996), i.e. direct measurement of the initial

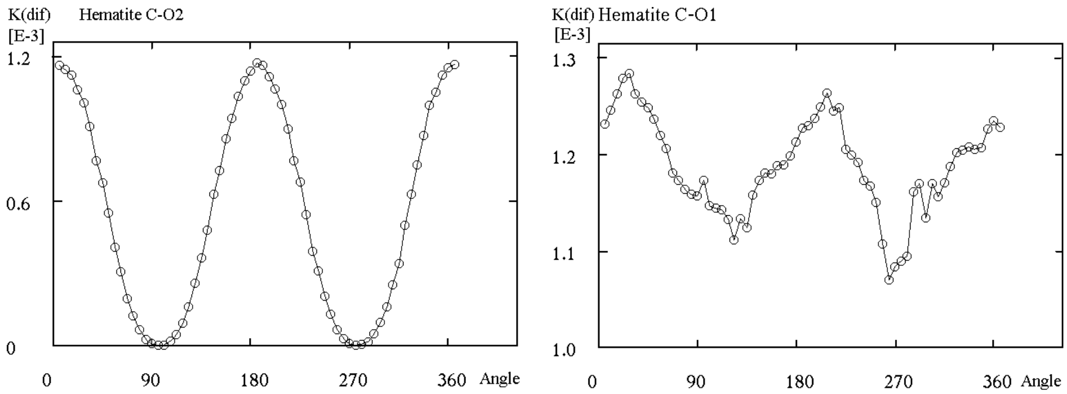

Figure 10. Plane AMS in the haematite single crystal C (a) measured plane perpendicular to the basal plane (b) measured plane parallel to the basal plane.

susceptibility tensor and the Rayleigh tensor (eq. 6) using a vibrating sample magnetometer. Unfortunately, this instrument is not primarily destined for measuring the AMS, the measurement of the whole Rayleigh loops being rather time consuming and the sensitivity and accuracy in measuring weakly magnetic rocks being probably insufficient. Realizing the problems of this instrument in measuring the AMS of standard-sized specimens because of the inhomogeneous picking up field (e.g. Kelso *et al.* 2002), one cannot consider this solution to be ideal.

If we were able to measure the AMS of a specimen in several fields outside the initial susceptibility range, but still within the Rayleigh law range, we would be able to determine the initial directional susceptibilities indirectly from eq. (2). Then, using these directional susceptibilities as input data, we could calculate the tensor of the initial susceptibility. The problem with this approach is the relatively narrow and variable Rayleigh law range for different minerals and the lack of sufficiently sensitive and precise instruments to measure the AMS in variable fields.

The problem can also be solved through extending the standard measurement of the AMS made using a sensitive and precise instrument by measuring one directional susceptibility in several fields using less precise, but more available, instrument. By fitting the straight line to the directional susceptibility versus field intensity data one can calculate the initial susceptibility and the Rayleigh coefficient in eq. (2) and consequently also the c constant in eq. (3); this way of determining the c constant is possible because of eq. (7). Knowing the c constant, one can calculate a set of directional initial susceptibilities to be used as input data for the AMS calculation, through solving eq. (4). Eq. (4) is quadratic and one obtains two roots, one positive and one negative, the latter being evidently unreal. The susceptibility tensor calculated in this way retains the precision in determination of the orientations of the principal directions and corrects the overestimated principal values. The problem of this approach is that both the AMS and field variation of the directional susceptibility must be measured within the Rayleigh law range, which is relatively narrow and variable for different minerals. In addition, it should be emphasized that all of our measurements were made using a rotatable design for the measurement directions (Fig. 6); this design does not favour any particular direction. If no rotatable design were used, it is not certain whether the precision in determination of the principal directions would be retained.

The problem can also be solved in such a way that instead of determining the c constant by measurement, it can be estimated from data from the literature. Through calculating variable sets of directional susceptibilities for the c constant varying in the vicinity of the estimated value, one can find, in an iterative way, such a set of directional susceptibilities that gives rise to the minimum fitting error.

The other way of solving the problem is respecting the non-linearity and measuring the susceptibility in so many directions that a contour diagram of the directional susceptibility can be presented instead of the susceptibility tensor. Unfortunately, this is laborious and time consuming. For instance, if we adopted the design of measurement directions used in elastic anisotropy to produce contour diagrams, and measured the directional susceptibility of a specimen in 132 directions (e.g. Pros & Babuška 1967), the laboriousness of this the technique would be obvious compared with the 15 directions used in measuring standard AMS. In addition, the beauty and elegance would be lost of the AMS presentation in simple terms of principal susceptibilities and parameters derived from them and orientations of magnetic foliation and magnetic lineation. On the other hand, this approach would be efficient if more detailed results were obtained than those arising from the simple susceptibility tensor. The potential of this approach is shown below.

The AMS of the above-mentioned haematite single crystals was also measured by the KLY-3S Kappabridge (Jelínek & Pokorný 1997) modified in such a way that 64 directional susceptibilities may have been measured in each of three perpendicular planes and recorded on disk. Fig. 10(a) shows these susceptibilities in a haematite crystal measured in the plane perpendicular to the basal plane. It is clear that the susceptibility curve follows the sinusoidal curve very well, which means that the AMS in this plane is well represented by an ellipse. Fig. 10(b) shows these susceptibilities measured parallel to the basal plane. It is clear that the susceptibility curve no longer follows the sinusoidal curve, which means that the AMS in the basal plane is represented neither by an ellipse nor by a circle, which would be expected, following Neumann's principle (*cf.* Nye 1957; Hrouda 1973), if the relationship between the magnetization and field were linear. High-field magnetic anisotropy of haematite crystals, as summarized by Stacey & Benerjee (1974), indicates that the magnetization within the basal plane is not isotropic, showing more complex pattern (threefold or sixfold) controlled by the crystal lattice. Similar reasons may apply to the separation of the lower-field maximum susceptibilities from the stronger-field ones (Fig. 9).

CONCLUSIONS

The investigation of the field variation of the low-field magnetic susceptibility and its anisotropy in various different magnetic minerals have led to the following conclusions.

(1) The theory of the low-field AMS of rocks is based on the assumption of a linear relationship between magnetization and the magnetizing field, resulting in a field-independent susceptibility. This relationship is valid in diamagnetic and paramagnetic minerals by definition and in magnetite where no field variation of susceptibility and its anisotropy has been observed in the low fields used in common AMS meters. In addition, in pyrrhotite-, haematite- and titanomagnetite-bearing rocks, in which these minerals are very fine-grained, the field variation of susceptibility is insignificant. Using linear theory in calculating the AMS is fully legitimate in all of these cases.

(2) In pyrrhotite-, haematite- and titanomagnetite-bearing rocks, in which these minerals are relatively coarse-grained (typically hundreds of micrometers), a clear field variation of the magnetic susceptibility may exist even in the low fields used in common AMS meters, often resulting in a poor fit of the susceptibility tensor to the measured data. Strictly speaking, linear theory in calculating the AMS is in general incorrect in this case. Fortunately, the linear theory gives rise to an overestimate of the AMS magnitudes on one hand, but retains the precision in determining the ellipsoid shape and orientations of the principal susceptibilities on the other.

(3) The problem (2) can be solved through using very weak measuring fields (less than 10 A m^{-1}). However, it would be impractical, because of it significantly lowering the sensitivity and the precision of the AMS measurement. It is obvious that this solution is applicable only to relatively strongly magnetic and anisotropic rocks.

(4) The other way of solving this problem would be measuring the AMS in several fields outside the initial susceptibility range, but still within the Rayleigh law range, and determining the initial directional susceptibilities indirectly. The problem with this approach is the narrow and variable Rayleigh law range in various minerals.

(5) The next way of solving the problem is respecting the non-linearity and measuring the susceptibility in so many directions that a contour diagram of the directional susceptibility can be presented instead of the susceptibility tensor. Unfortunately, this is laborious and time consuming. In addition, the beauty and elegance would be lost of the AMS presentation in simple terms of principal susceptibilities and parameters derived from them and orientations of magnetic foliation and magnetic lineation.

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