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DIFFERENCES BETWEEN STATIC AND DYNAMIC ELASTIC MODULI OF ROCKS: PHYSICAL CAUSES

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Elastic moduli are controlled mainly by the viscoelastic and microplastic behavior of rocks if stress and strain remain below the proportionality limit. Differences between measured static and dynamic elastic moduli are caused by different inelastic contributions to stress-strain relationships which behave as a function of strain amplitude and frequency (energy and strain rate). Static and dynamic elastic moduli can be appropriately compared at equal strain amplitudes and frequencies and at identical physical properties of solids.

Nonlinear seismics, inelastic behavior, hysteresis, strain amplitude dependence of wave velocities and attenuation, static and dynamic elastic moduli

INTRODUCTION

Discrepancy between elastic moduli obtained from static and dynamic measurements is observed at different frequencies and levels of applied strain. ("Quasistatic" moduli are hereafter called "static" for short.) Numerous experiments show that dynamic moduli are consistently greater than static moduli, up to 4–8 times for Young's moduli [1–4], at least at confining pressures below 60 MPa. However, the physical causes of this empirically known inequality remain unclear, though its understanding is essential in many theoretical and practical applications (e.g., developing techniques to predict hard-to-measure static moduli from easier-to-measure dynamic moduli, such as in hydraulic fracturing problems, etc.) [2, 5, 6].

The empirically observed differences between static and dynamic moduli (or velocities) can be explained by differences in frequency and strain amplitudes used in measurements [4]. Young's moduli obtained from ultrasonic laboratory measurements at 100 kHz–1 MHz are higher than log-derived moduli measured at 10–20 kHz, and these are higher than low-frequency (acoustic and seismic bandwidths) and static moduli: $E_{\text{ultrasonic}} > E_{\text{log}} > E_{\text{lowfreq}} > E_{\text{static}}$.

Frequency (time) dependence is related to strain rate ($\acute{\varepsilon}$) dependence. Rocks become stiffer as the frequency of the applied strain is increased, which calls for a modulus increase. Strain amplitudes applied in dynamic (wave propagation) measurements are kept small, within the porportionality limit, which corresponds to viscoelastic behavior of velocity dispersion and energy dissipation. This behavior has been explained by Biot's global fluid-flow mechanism or local mechanisms like squirt flow, stick-slip sliding, etc. [4, 7, 8].

Unlike quasistatic measurements taken at low strain rates ($\acute{\varepsilon}$), strain amplitudes in dynamic experiments can be above critical (ε_{cr}) and give rise to irreversible inelastic processes [9, 10]. In this case the effect of strain amplitude is especially well pronounced, as well as at low frequencies when viscoelastic mechanisms are restricted to effective viscosity and no local effects are observed [4, 7].

Thus the existing views of viscoelastic mechanisms responsible for the differences between static and dynamic moduli appear generally reasonable but the effect of strain amplitude requires further investigation, as it is not as simple (modulus decrease with strain) as is commonly believed [4, 11–13]. The physical mechanisms of deformation are thus to be studied with a special focus on plastic effects of rock microstructure and strain amplitude dependence of elastic moduli.

The objective of this research is to develop a better understanding of physical mechanisms responsible for the difference between static and dynamic moduli. For this, an irreversible mechanism of microplasticity, most often neglected at the mesoscopic level, is investigated in addition to viscoelastic behavior of rocks. The effect of microplasticity has been tested by quasistatic and dynamic measurements of Young's moduli which are likewise applicable to wave dynamics. Although the experiments addressed only Young's modulus, the same idea can be applied to shear strain, as shear velocity measurements reported in [4, 14] demonstrated strain amplitude dependence of S velocities and attenuation.

FORMATION OF ELASTIC MODULI

Elastic moduli represent the strain response or stiffness of material and can be found from the stress–strain diagram $\sigma(\varepsilon)$ as a ratio of the applied stress to the strain it produces. Instantaneous Young's modulus (within $\Delta\sigma$ stresses) in the stress-strain curve is given by

$$E_i = \frac{\Delta\sigma_i}{\Delta\varepsilon_i}\,. \tag{1}$$

For appropriate comparison, the static and dynamic moduli should be determined within the same range of $\Delta\sigma$, i.e., at $\Delta_{\text{static}} = \Delta\sigma_{\text{dyn}}$. However, the moduli remain different even under identical stress conditions [4]. This difference can be understood if we consider the process of deformation in more detail and analyze the constituents of the measured static and dynamic strain $\Delta\varepsilon_i$.

Bulk strain includes a reversible and irreversible components:

$$\varepsilon_i = \varepsilon_e + \varepsilon_r\,. \tag{2}$$

Reversible strain, in turn, consists of ideally elastic (ε_{i-e}) and viscoelastic ($\varepsilon_{\upsilon-e}$) components

$$\varepsilon_e = \varepsilon_{i-e} + \varepsilon_{\upsilon-e}\,. \tag{3}$$

Strictly speaking, irreversible strain includes a viscoplastic ($\varepsilon_{\upsilon-p}$) and a microplastic (ε_{μ}) components and strain associated with frictional sliding of rock particles during closure of voids and macrocracks (ε_{sl}). However, the components responsible for plastic and viscoplastic processes may be neglected in our case of small and medium strain (low-amplitude waves), when stress and strain remain below the yield point. The ε_{sl} strain is also neglected as we deal with consolidated rocks under overburden pressure having microcracks but no large open cracks [2]. Then bulk strain includes three main components:

$$\varepsilon_i = \varepsilon_{i-e} + \varepsilon_{\upsilon-e} + \varepsilon_{\mu}\,. \tag{4}$$

The component $\varepsilon_{\upsilon-e}$ represents an ideally elastic Young's modulus $E_{i-e} = \Delta\sigma_i/\Delta\varepsilon_{\upsilon-e}$ corresponding to strain response of monocrystalline grains and rock skeleton as a whole. Two latter terms in (4) represent the inelastic component responsible for nonlinear departure from the Hooke's law and from $E_{i\text{–}e}$, respectively. The strain $\varepsilon_{\upsilon-e}$ corresponds to viscoelastic behavior of rocks dependent on the magnitude and time of stress [1]. The presence of the viscoelastic component $\varepsilon_{\varepsilon-e}$ leads to hysteresis, i.e., incoincidence of loading and unloading arms of stress–strain, $\sigma(\varepsilon)$, diagrams and the appearance of quasistatic and dynamic residual strain. Once a deformation cycle is completed, residual strain relaxes with time and decreases to zero. As a result, the system returns to the initial stress-strain state.

In a rheological model for a Maxwell body, viscoelastic strain at $\sigma = \text{const}$ is controlled by time (t_σ) and effective viscosity (η_{ef}) related with Young's modulus and relaxation time $T_{\text{rel}} = \eta_{\text{ef}}/E_i$ [3] as

$$\varepsilon_{\upsilon-e} = \frac{\sigma t_\sigma}{\eta_{\text{ef}}} = \frac{\varepsilon_i E_i t_\sigma}{\eta_{\text{ef}}} = \frac{\varepsilon_i t_\sigma}{T_{\text{rel}}}\,. \tag{5}$$

It follows from (5) that the strain $\varepsilon_{\upsilon-e}$ of rocks with invariable viscosity, Young's modulus (T_{rel} = const), and applied stress depends on time which must differ in quasistatic and dynamic measurements. If the above conditions are satisfied, viscoelastic strain in the quasistatic state is higher than dynamic strain ($\varepsilon^{\text{st}}_{\upsilon-e} > \varepsilon^{\text{dyn}}_{\upsilon-e}$) at the account of a longer time t_σ. Shorter time thus corresponds to a greater strain rate $\acute{\varepsilon}$ (or frequency) and lower $\varepsilon_{\upsilon-e}$. Then, the effect of frequency on the difference between static and dynamic moduli manifests itself in the changes of viscoelastic strain leading to changes in Young's modulus. Normally the time of quasistatic measurements is 1–10 s, and the half-periods of compression-extension cycles in dynamic measurements are orders of magnitude shorter.

On the contrary, dynamic stress is much greater than static stress. Therefore, the static and dynamic moduli will differ because of different contributions of the viscoelastic component ($E_{i-\mathrm{dyn}} > E_{i-\mathrm{st}}$) even if quasistatic and quasidynamic stresses are assumed equal.

Instantaneous (local) Young's modulus with regard to (1) and (4) is given by

$$E_i = \frac{\Delta\sigma_i}{\Delta\varepsilon_{i-e} + \Delta\varepsilon_{\upsilon-e}(t_\sigma) + \Delta\varepsilon_\mu(|\varepsilon|)}\,, \tag{6}$$

where $\Delta\varepsilon_{\upsilon-e}(t_\sigma)$ is the time-dependent viscoelastic component and $\Delta\varepsilon_\mu(|\varepsilon|)$ is the strain amplitude dependent microplastic component (with variable sign) of bulk strain.

Thus, the viscoelastic effect is related to time and the microplastic effect to strain amplitude, or applied energy.

The effect of the microplastic component of bulk strain is indicated by residual strain which does not disappear with time and is observed below critical strain (proportionality limit). This effect is due to irreversible reorganization of rock microstructure during deformation. Microplasticicity differs from viscoelasticity as it is associated with the critical strain ε_{cr} ($\sim 10^{-6}$, for the used measurement accuracy) and the effect is jump-like. At strain below ε_{cr}, microplasticity is absent and viscoplastic strain alone influences the elastic moduli.

Microplastic effects generally increase bulk strain and decrease Young's modulus but are often more complex being dependent on strain amplitude and rock physics, which is reflected in the behavior of elastic moduli. The $\sigma(\varepsilon)$ relationships for microplastic rocks may be nonmonotone or even jump-like, which is impossible for the viscoelastic response.

It is also essential that microplasticity is independent of time (at least for metallic and other polycrystals below ~1 MHz) [15], but strong energy dependence leads to strain amplitude dependence of elastic moduli [10]. This can be illustrated by static Young's modulus plots for some rocks from deposits in West Siberia (Fig. 1) where the strain amplitude dependence is clearly nonlinear. The modulus may either increase or decrease, which causes ambiguity in its static estimates if the strain range is not specified.

In addition to energy, elastic moduli are affected by the physical state of rocks associated with overburden pressure and fluid saturation. Microplastic effects are stronger in fluid-saturated than in dry rocks. Young's modulus decreases on transition from dry to saturated state and shows oscillatory changes with increasing strain (Fig. 2). Therefore, comparison of moduli, even in quasistatic measurements, is incorrect if the conditions are not specified.

Static Young's modulus may either decrease or increase with strain, as follows from positive and negative curvatures of $\sigma(\varepsilon)$ plots (Fig. 3) [9, 10, 14]. Positive curvature is typical of less consolidated or highly porous

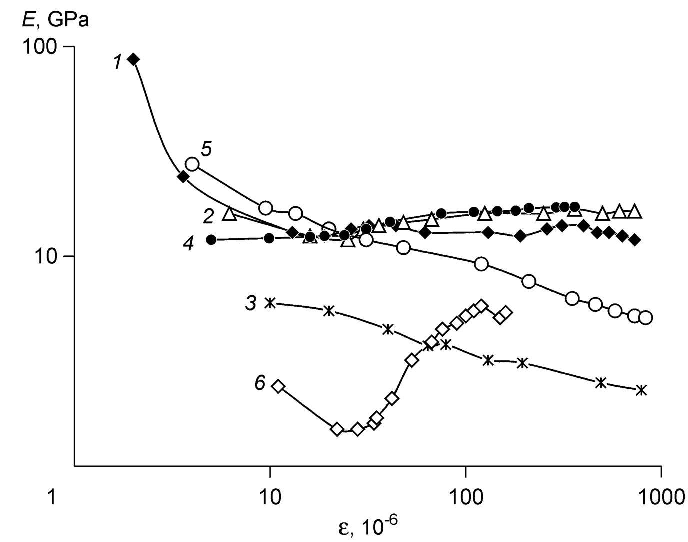

Fig. 1. Behavior of Young's modulus as a function of strain amplitude. ***1*** **— marl;** ***2*** **— bituminous mudstone;** ***3*** **— fine-grained sandstone;** ***4*** **— dolomite;** ***5*** **— coarse-grained sandstone;** ***6*** **— brittle sandstone. Depth 2100–2800 m.**

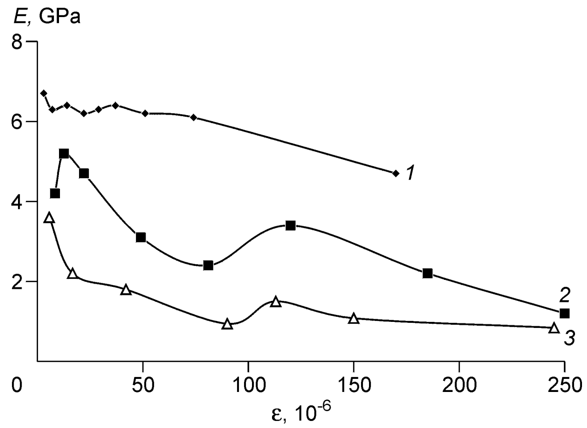

Fig. 2. Behavior of Young's modulus as a function of strain amplitude in Omba sandstone (2400 m). ***1*** **— dry;** ***2*** **— partially saturated;** ***3*** **— fully saturated.**

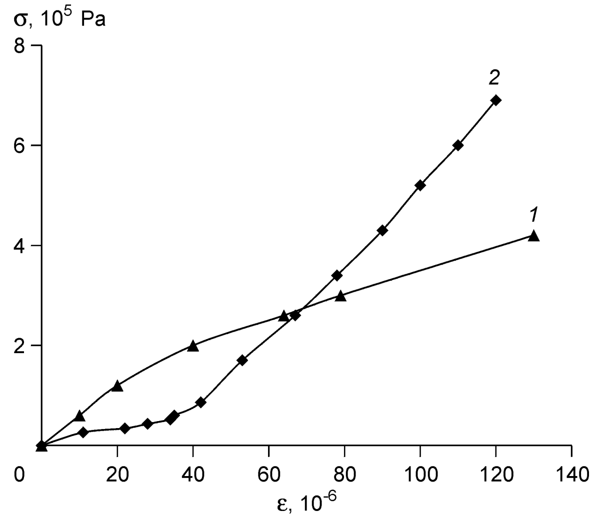

Fig. 3. Stress–strain diagrams. ***1*** **— decreasing Young's modulus (positive curvature);** ***2*** **— increasing Young's modulus (negative curvature).**

rocks, such as sandstones, and negative curvature is more often encountered in higher consolidated low-porosity rocks, such as dolomites. The latter (Fig. 4) is of special interest for our study as the respective strain amplitude dependence of moduli causes *P* velocity increase with amplitude.

The above considerations are illustrated by laboratory measurements of static and dynamic moduli in low-porous dolomites (Yurubchen and Madra fields) which show a hysteretic behavior. Figure 5 demonstrates the behavior of static (E_{static}) and dynamic (E_{dyn} and G_{dyn}) moduli as a function of overburden pressure. All moduli increase with pressure. The dynamic moduli increase by 20% (E_{dyn}) and 18% (G_{dyn}) from 2 to 60 MPa and make very narrow hysteresis loops in the $2 \to 60 \to 2$ MPa cycle which record a vanishing inelastic contribution. The static modulus E_{static} becomes times as high and approaches E_{dyn} at 60 MPa. The increase in static moduli may be due to higher strength and stiffness typical of rocks that show negative curvature of the σ(ε) diagram (Fig. 4). On unloading the modulus increases abruptly three-fold on the end point of the hysteresis loop and decreases along the unloading arm which is above the loading arm (Fig. 4). A similar jump of Young's modulus on the end point of hysteresis loops was predicted [16] and experimentally observed [4, 17] earlier. Note that in our experiment, the static modulus E_{static} is higher than the dynamic modulus E_{dyn} at a certain range of overburden pressures. The

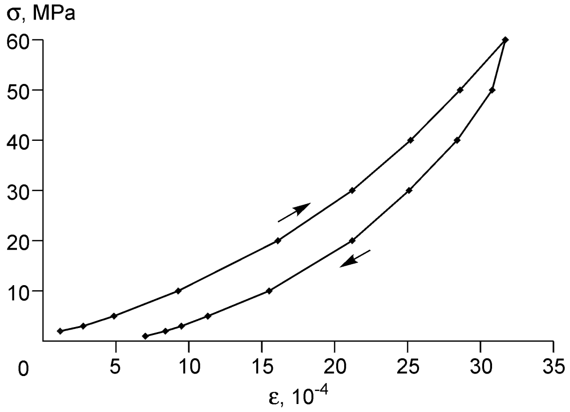

Fig. 4. Stress-strain diagram for dolomite (Madra field). H = 2480 m, K_p = 2.4%.

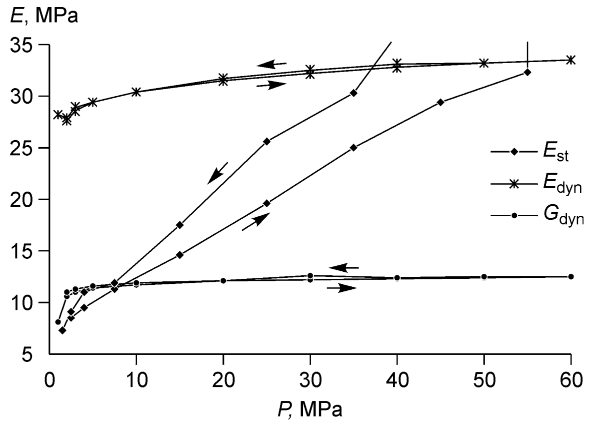

Fig. 5. Behavior of static (E_{st}) and dynamic (E_{dyn}, G_{dyn}) elastic moduli as a function of overburden pressure.

dynamic moduli follow the static behavior at all pressures: E_{dyn} and G_{dyn} increase with the amplitude of acoustic signals like the static modulus that increases with strain.

DISCUSSION

Analysis of mechanisms associated with Young's modulus shows that the empirically observed differences between static and dynamic moduli are due to different contributions of inelastic processes in static and dynamic deformation and different strain amplitudes. In the conditions we assume, the moduli are influenced by the viscoelastic and microplastic behavior of rocks which is rather complicated and depends on rocks' physical properties and applied energy.

The observed decrease in Young's modulus at lower frequencies ($E_{ultrasonic} > E_{log} > E_{lowfreq} > E_{static}$) [4, 12, 17] is consistent with the idea of a higher viscoelastic contribution at lower strain rates $\acute{\varepsilon}$. Transition from ultrasonic to acoustic and lower frequencies (under identical stress conditions) is accompanied by a decrease in strain rate $\acute{\varepsilon}$ and effective viscosity (η_{ef}), which increases viscoelastic deformation and decreases the Young's modulus of rocks. Effective viscosity is meant as measured viscosity that includes global viscosity, responsible for the rock macrostructure, and local viscosity. Effective viscosity is small at low frequencies and at low strain rates when local and global (Biot's) flow mechanisms are weak [7]. Therefore, global viscosity provides relaxed state at low frequencies and unrelaxed behavior and higher stiffness of rocks at higher frequencies.

The effect of strain amplitude is related to energy. The strain amplitude dependence of elastic moduli at

$\varepsilon_i > \varepsilon_{cr}$ can be attributed to microplasticity. However, strain amplitude dependence of velocities (moduli) was observed also at $\varepsilon_i < \varepsilon_{cr}$ at frequencies below 1 Hz when frequency dependence is vanishing [4]. Therefore, there may exist critical strains $\varepsilon_{cr\,(1-n)}$ much below the known values.

The strain amplitude dependence of elastic moduli at $\varepsilon_i > \varepsilon_{cr}$, i.e., in the presence of microplastic behavior, manifests itself in diverse ways. Strain amplitude is obviously the key parameter responsible for the difference between static and dynamic moduli, as this difference remains even after correcting for frequency effects. However, the effect of strain amplitude is more complex and is not restricted to modulus decrease with strain. Elastic moduli behave in different ways in different stress-strain diagrams: The static Young's modulus decreases with strain in positive-curvature $d\sigma/d\varepsilon$ derivative (curve 1 in Fig. 3) but increases in negative-curvature plots with increasing derivative $d\sigma/d\varepsilon$ (curve 2 in Fig. 3). In S-shaped $\sigma(\varepsilon)$ diagrams Young modulus may both increase and decrease at certain strain ranges. Therefore, the range of strain amplitudes at which elastic moduli are measured is essential, at least in quasistatic experiments or at weak oscillations.

In this respect it is pertinent to note the limitations of the theoretical explanation for the differences between static and dynamic moduli based on inappropriate initial strain amplitudes and frequencies. For instance, the Preisach-Mayergoyz (P-M) model of hysteretic systems is based on the initial $\sigma(\varepsilon)$ relationship obtained near the source [16, 18]. The time-dependent stress function was defined so that pressure was raised slowly from zero to maximum and then lowered back to zero imitating a quasistatic behavior, and small rapid pressure excursions corresponded to dynamic conditions. The reported stress-strain diagram was a large hysteresis loop for quasistatic pressure enclosing small but steeper loops for dynamic stress (wave propagation), and it was concluded that a modulus measured dynamically was larger than that measured quasistatically [16]. However, the difference between static and dynamic moduli was due not only to the choice of the P-M space but also to different loading rates in the quasistatic and dynamic measurements $d\sigma_{dyn}/dt > d\sigma_{st}/dt$, which is equivalent to frequency changes (though frequency effects were declared negligible).

On the other hand, the P-M modelling of rocks by an effective set of hysteretic mesoscopic elastic units (HMEU) [16] predicts stress-strain relationships with both positive and negative curvatures (as in Fig. 3) corresponding to moduli decreasing and increasing with strain amplitude, which is consistent with our results.

CONCLUSIONS

1. Static and dynamic Young's moduli are mostly controlled by viscoelastic and microplastic behavior of rocks at stresses and strains below critical values (within the conventional proportionality limit).
2. The differences between static and dynamic moduli observed in experiments are caused by different contributions of inelastic mechanisms at quasistatic and dynamic loading as a function of frequency and amplitude (strain rate and energy).
3. Appropriate comparison of static and dynamic moduli is possible only under identical strain rate and energy conditions.

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