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Gravitational differentiation of liquid cores of planets and natural satellites

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Abstract. Initial equations are obtained, similarity criteria are estimated and a project of simulation experiment is proposed for the gravitational differentiation of liquid cores of planets and natural satellites. It is assumed that, first, the liquid core in an adiabatic state without thermal convection and, second, the inner solid core grows during the crystallization of a heavy component from the liquid core in such a way that the buoyancy force acting on a lighter component is directed strictly along the radius. It is also assumed that the radial distribution of density in the liquid core does not change during the time interval considered. These three natural assumptions enable an analytical description of basic hydrostatic effects controlling slow growth of the solid core, gravitational stratification of the liquid core, and sources of related compositional convection. The similarity criteria of such convection are mostly the same as for thermal convection. Additional criteria are the concentration contrast ($\sim$1/10 in the Earth), the compressibility of the liquid core ($\sim$10%), and the thickness of a concentration boundary layer ($\sim 10^{-7}$) that, controlling the freezing-out of the liquid at the inner sphere, can give rise to asymmetry of the solid core. The excitation threshold of the compositional convection is much higher than a similar threshold for thermal convection, and the compositional convection itself can arise only at an intermediate stage of the gravitational differentiation of the core. Observed magnetic fields are largely due to compositional convection in the Earth's core and, probably, in deep interiors of Mercury. At the contemporary evolutionary stage of Venus' interiors, the intensity of compositional convection is most likely insufficient for the magnetic field excitation and it is undoubtedly too weak in the Mars' interiors.

1. Introduction

Convection in deep interiors of planets [*Stevenson et al.,* 1983] and their natural satellites [*Kuskov and Kronrod,* 1998] can be due to both thermal and gravitational effects [*Loper,* 1978]. Only at a certain evolutionary stage of a planet (satellite), can compositional convection, largely controlled by gravitational differentiation of its composition, arise somewhere in its deep liquid interiors. Very vigorous compositional convection should take place, for example, due to the differentiation of the Earth's core into liquid and solid phases. As was described for the first time by Braginsky [*Braginsky and Roberts,* 1995], such convection is associated with the floating-up of an excessive lighter component from the boundary with a slowly growing inner solid core, a heavier component being precipitated onto this boundary from the contracting outer liquid core.

The growth rate of the Earth's solid core can be estimated by dividing the present-day radius of the core by its age (1–3 Ga), which yields a value of $\sim 10^{-11}$ m s^{-1}. The relations governing this rate and the conditions favoring the occurrence of compositional convection have not been explored as yet. Moreover, of the best of our knowledge, even basic criteria of similarity have not been developed for compositional convection in a rapidly and almost rigidly rotating planetary spherical layer [*Starchenko,* 2000]. Only a few widely known studies (e.g. see [*Glatzmaier and Roberts,*

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1997; *Starchenko and Jones,* 2002]) were devoted to the direct numerical modeling of magnetic effects due to thermal and compositional convection. The assumptions underlying these studies are so unrealistic and their model time intervals are so limited that their results are by no means usable for developing the theory of compositional convection in a rotating spherical liquid layer.

The goal of this paper is the development of the general theory, as well as its experimental simulation basis, for gravitational differentiation and the related compositional convection in a rapidly and nearly rigidly rotating spherical liquid layer in deep interiors of planets and natural satellites.

The simplest mathematical model describing the main features inherent in the gravitational differentiation of deep interiors of planets and natural satellites is proposed in the second section of this paper. The spherical liquid layer under consideration is assumed to be an adiabatic state with no thermal convection. Moreover, for simplicity, the buoyancy acceleration that can arise in such a layer is assumed to be directed strictly along the radius.

In the third section, it is shown that, given a stationary density determined from seismic observations or evolutionary models, the initial equations for the gravitational potential, pressure, and concentration of the lighter component have a basic analytical solution. This basic solution completely defines the global growth rate of the solid inner sphere and the intensity of the possible compositional convection. Main hydrostatic effects and similarity criteria associated with this slow growth of the solid sphere modeling the inner core of a planet or a natural satellite are described theoretically.

In the fourth section, the convective instability and similarity criteria are considered for compositional convection in a rotating layer. Comparison between chemical and thermal convection patterns made it possible to utilize some results derived for thermal convection. On the other hand, unique effects inherent in compositional convection are established. The main effect is the formation of a concentration boundary layer controlling the freezing-out behavior of the liquid at the surface of the inner sphere. Specific features of this behavior can result in the asymmetry of the Earth's rigid core. The presence of the concentration layer and the effect of rapid rotation can raise significantly the excitation threshold of compositional convection as compared with thermal convection. As a result, the magnetohydrodynamic system driven by the compositional convection is likely to be in the laminar state in the Earth, near the generation threshold in Mercury and beyond the generation threshold in Venus and Mars.

In the final, fifth section, main conclusions are formulated and a project of an experimental installation for the laboratory simulation of gravitational differentiation of deep planetary interiors is discussed.

2. Formulation of the Problem

A simplified mathematical model, discussed below, is primarily developed to reconstruct main effects of the gravitational differentiation in deep liquid interiors of terrestrial planets (Mercury, Venus, the Earth, and Mars), because the dependence of their evolution on gravitational differentiation raises no doubts [*Stevenson et al.,* 1983]. Moreover, the model proposed here is applicable to large natural satellites similar in internal structure to terrestrial planets (for example, the Moon, the Jupiter's satellites Ganymede, Europa, and Callisto [*Kuskov and Kronrod,* 1998], and other similar satellites of giant planets). Finally, this model can have some implications for Neptune, Uranus, Saturn, and Jupiter, if gravitational differentiation played a substantial role in the formation of inner rigid cores in these planets.

To pinpoint main effects of gravitational differentiation, we address an idealized body in which the following conditions are valid.

(i) The spherical liquid layer under consideration is in the adiabatic state without thermal convection. This condition can be valid even in the Earth's core, where it is still not evident that the adiabatic gradient exceeds the value required for excitation of thermal convection.

(ii) Gravitational differentiation proceeds in the spherical liquid layer (the liquid core) consisting of heavy and light components. The heavy component forms a slowly growing (billions of years in the Earth) inner solid core.

(iii) The buoyancy acceleration is assumed to be strictly radial in a reference frame rotating together with the outer boundary of the liquid core, because the centrifugal acceleration is appreciably smaller than the gravitational acceleration in deep interiors of planets and satellites.

If the mass fraction of the light component (or admixture) ξ is referred to as the concentration, basic equations describing the simplified model of gravitational differentiation proposed here are (e.g. see [*Braginsky and Roberts,* 1995; *Loper,* 1978; *Starchenko,* 2001])

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{V}) = 0\ ; \tag{1a}$$

$$\begin{aligned}\nu\nabla^2\mathbf{V} &= \partial\mathbf{V}/\partial t + (\mathbf{V}\cdot\nabla)\mathbf{V}\\ &+ 2\Omega\times\mathbf{V} + (\nabla p)/\rho + \nabla U\ ;\end{aligned} \tag{1b}$$

$$\nabla^2 U = 4\pi G p\ ; \tag{1c}$$

$$\nabla\cdot(\rho\kappa\nabla\xi) = \rho(\partial\xi/\partial t + \mathbf{V}\cdot\nabla\xi)\ ; \tag{1d}$$

$$\rho = \rho(t,p,\xi)\ . \tag{1e}$$

Here the reference frame rotates at a fixed angular velocity Ω close to that of the rotating outer boundary of the liquid core at $r = r_o$, ρ is density, t is time, $\mathbf{V}$ is the velocity vector, ν is the constant kinematic viscosity coefficient, p is pressure, U is the gravitational potential, $G = 6.67\times10^{-11}$ m^3/(kg cm^2) is the gravitation constant, and κ is the kinematic diffusion coefficient of the light admixture. System (1) consists of the continuity (1a), hydrodynamic (Navier-Stokes) (1b), gravitation (1c), and diffusion (1d) equations and the equation of state (1e); the latter involves condition (i), which formally implies that the specific entropy S is uniform ($\nabla S = 0$) and depends only on time: $S = S(t)$.

The boundary conditions for the velocity in (1a) and (1b) are controlled by the structure of the outer and inner ($r = r_\mathrm{i}$) boundaries of the liquid core. At rigid boundaries in liquid cores of terrestrial planets, these conditions can be written as

$$\mathbf{V} = 0 \text{ at } r = r_\mathrm{o} \tag{2a}$$

$$\text{and } \mathbf{V} = \omega_\mathrm{i} r_\mathrm{i} \sin\theta \mathbf{1}_\varphi \text{ at } r = r_\mathrm{i}(t)\ . \tag{2b}$$

Relation (2b) and the dynamic equation for the relative angular velocity of the inner rigid boundary, derived from (1a) and (1b), make the system of equations complete. The gravitation equation (1c) is complemented by the continuity conditions imposed on the gravitational potential U and its gradients ∇U.

In the case of terrestrial planets, equations (1d) and (1e) are complemented by the outer condition of impermeability, inner condition of diffusion and phase transition condition at the inner liquid/solid interface:

$$0 = \frac{\partial \xi}{\partial r} \text{ at } r = r_\mathrm{o}\ , \tag{3a}$$

$$\frac{\partial R_\mathrm{i}}{\partial t} = -\Xi \kappa \frac{\partial \xi}{\partial r} \tag{3b}$$

$$\text{and } r_\mathrm{i}^{-1} \frac{\partial R_\mathrm{i}}{\partial t} = -\frac{F_S}{c_p}\frac{\partial S}{\partial t} - F\frac{\partial \xi}{\partial t} \text{ at } r = R_\mathrm{i}(t, \theta, \varphi)\ . \tag{3c}$$

Here, $r = R_i \approx r_i$ describes a spherically slightly asymmetric surface of the solid core; Ξ is the boundary ratio of the density of the liquid layer to the density jump at the surface of the solid sphere, providing a change in the concentration of the light admixture in the liquid; c_p is the specific heat at constant pressure; and F_S and F are positive factors controlling the thermodynamics of freezing and crystallization of the outer liquid core at the solid core surface. Taking into account the estimates presented in [*Braginsky and Roberts,* 1995; *Lister and Buffett,* 1995; *Loper,* 1978; *Starchenko and Jones,* 2002] for the Earth's core, we have

$$\Xi = 25 \pm 10\ , \tag{4a}$$

$$S_t \equiv -F_S c_p^{-1} \partial S/\partial t = (2 \pm 1.5) \cdot 10^{-17}/s \tag{4b}$$

$$\text{and } F = 50 \pm 20\ . \tag{4c}$$

The important parameter S_t, introduced in (4b), physically means a characteristic frequency of the thermogravitational differentiation of the core of a planet (or a satellite) into liquid and solid components. The numerical value of (4b) specifies, in a natural way, the age ($\sim 1/S_t$) of the Earth's rigid core that amounts to about one billion of years.

Thus, at times significantly shorter than $1/|S_t|$, all of the model relations presented in this section can be reliably used in the study of gravitational differentiation. Moreover, with an accuracy sufficient for the model considered, we can assume below that all external parameters in (4) and (3b, 3c) are constant in time and space.

3. Basic Hydrostatic State

In order to successfully solve system (1)–(3), we should specify an initial condition as simple as possible. In what follows, this state is referred to a basic state, and all related values have the index "0".

An optimal approach is to choose such a basic state that has a given stationary radial distribution of density $\rho_0(r)$ satisfying the model of internal structure of a planet (satellite) in a certain epoch. In order to model the basic state of the Earth's liquid core in the modern epoch, it is natural to take the density distribution from the PREM seismic model [*Dziewonski and Anderson,* 1981]. Ancient epochs can be modeled, for example, with the use of the model proposed in [*Loper,* 1978], which describes the entire differentiation process of the Earth's core into its solid and liquid parts.

The basic stationary, spherically symmetric density $\rho_0(r)$ should satisfy equation of state (1e) with an adequate accuracy over the time interval of the epoch considered. Evidently, the radial density distribution in the Earth's core would change by a value on the order of 10% over about one billion years. Therefore, the characteristic time required for a significant change in the density is about ten billion years. Accordingly, if an accuracy of the order of 1% is taken for the description of the basic state, the duration of the model epoch for the Earth will not exceed a value on the order of 100 Myr.

The continuity equation (1a) will be satisfied for the basic state if no convection is present, $\mathbf{V}_0 = 0$. The remaining three equations in (1) are simplified and have the form

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial U_0}{\partial r}\right) = 4\pi G r^2 \rho_0\ , \tag{5a}$$

$$\frac{\partial p_0}{\partial r} = -\rho_0 \frac{\partial U_0}{\partial r}\ , \tag{5b}$$

$$\frac{\kappa}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho_0 \frac{\partial \xi_0}{\partial r}\right) = \rho_0 \frac{\partial \xi_0}{\partial t}\ . \tag{5c}$$

Starting from the center, equation (5a) can be easily integrated with respect to $\nabla U_0 = (\partial U_0/\partial r)\mathbf{1}_r$, because the basic density is specified everywhere throughout the epoch under consideration:

$$\partial U_0/\partial r = \left(4\pi G/r^2\right)\int_0^r r^2 \rho_0(r) dr\ . \tag{6}$$

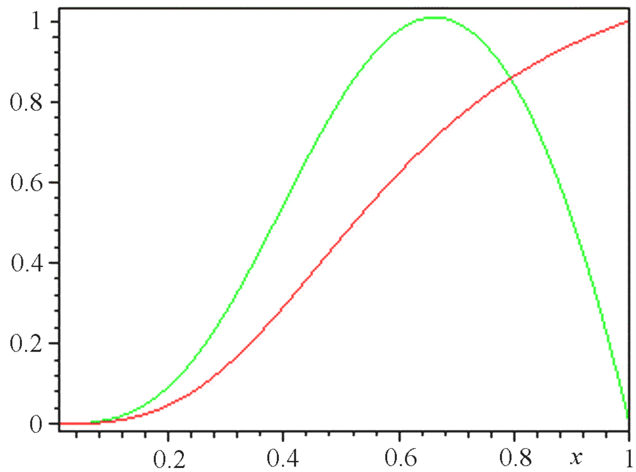

Figure 1. The curve continuously increasing from $x \equiv r_\mathrm{i}/r_\mathrm{o}$ is the normalized growth rate of the light admixture concentration (7c) in the Earth's liquid core bounded by the radii r_i and r_o. The bell-shaped curve is the gravitational differentiation energy of the core normalized to its maximum value.

The substitution of this solution into (5b) immediately provides the stationary pressure gradient.

The general solution of the diffusion equation (5c) satisfying boundary conditions (3) can be conveniently expressed through the rate of the concentration increase $\dot{\xi}$, which is constant in the epoch modeled, and the derivative ξ', which depends on the radius r alone. Using the value ξ_0^0, which is constant in the given epoch and is determined by the equation of state (1e), this general solution for the initial concentration is written in the form

$$\xi_0 = \dot{\xi} t + \int_{r_\mathrm{i}}^{r} \xi'(r) dr + \xi_0^0 \ , \tag{7a}$$

$$\xi' = -\frac{\dot{\xi}}{\kappa} \frac{\displaystyle\int_{r}^{r_\mathrm{o}} \rho_0 r^2 dr}{r^2 \rho_0} \ , \tag{7b}$$

$$\dot{\xi} = \frac{S_t r_\mathrm{i}^3 \rho_0(r_\mathrm{i})}{F r_\mathrm{i}^3 \rho_0(r_\mathrm{i}) + \Xi \displaystyle\int_{r_\mathrm{i}}^{r_\mathrm{o}} \rho_0 r^2 dr} \ . \tag{7c}$$

Hence, using (3b) and (3c), we obtain an estimate for the growth rate of the solid sphere ($\sim 10^{-11}$ m s^{-1} in the Earth):

$$\frac{\partial r_\mathrm{i}}{\partial t} \equiv \dot{r}_\mathrm{i} = r_\mathrm{i} S_t \Big/ \left[1 + r_\mathrm{i}^3 \rho_0(r_\mathrm{i}) F \Big/ \left(\Xi \int_{r_\mathrm{i}}^{r_\mathrm{o}} \rho_0 r^2 dr \right) \right] \sim r_\mathrm{i} S_t \ , \tag{8}$$

which is independent of the diffusion coefficient κ.

If the liquid spherical layer cools, its entropy decreases with time and we have $S_t > 0$, as in (4b). In this case, the growth rate of the inner solid sphere (8) is positive and the concentration gradient is negative, i.e. $\xi' < 0$ in (7b). The corresponding basic hydrostatic state described by the stationary quantities $\rho_0, \nabla U_0, \nabla p_0, \xi'(r)$ and $\dot{\xi} = \mathrm{const}$ is generally unstable. Therefore, given a positive growth rate of the inner sphere, even very small deviations from such a basic state with cooling can excite convection (for details, see below). Vice versa, if the liquid in the layer is heated, $S_t < 0$, the inner radius of the layer decreases and the basic state (5)–(8) is stable with respect to any arbitrarily small perturbations.

Basic state (7) and its energy characteristics [*Lister and Buffett,* 1995; *Starchenko and Jones,* 2002] are fully determined by the time derivative of concentration (7c). Considering that the density in liquid cores of terrestrial planets varies insignificantly ($\Delta\rho_0/\rho_0 \le 10\%$), this derivative can be approximated, within a reasonable accuracy, by the value

$$\dot{\xi} \approx (S_t/F)/[1 + (\Xi/F)(x^{-3} - 1)/3] \ ,$$

where $x \equiv r_\mathrm{i}/r_\mathrm{o}$ is the ratio of the inner radius to the outer radius. Figure 1 plots the function $1/[1 + (\Xi/F)(x^{-3} - 1)/3]$ for the Earth, and very similar behavior of this function should be expected for the other terrestrial planets. Thus, the specific energy density of gravitational differentiation, which is directly proportional to (7c), is negligibly small for $x \le 0.1$ and subsequently starts rising. The total energy of gravitational differentiation is found through multiplying (7c) by the volume of the liquid layer and vanishes at $x = 1$, as is seen from Figure 1.

The stationary difference of concentration across the liquid layer $\Delta\xi$ or the relative concentration difference of density is determined by the integration of (7b):

$$\Delta\xi \equiv -\int_{r_\mathrm{i}}^{r_\mathrm{o}} \xi' dr = \frac{\kappa^{-1} S_t r_\mathrm{i}^3 \rho_0(r_\mathrm{i})}{F r_\mathrm{i}^3 \rho_0(r_\mathrm{i}) + \Xi \displaystyle\int_{r_\mathrm{i}}^{r_\mathrm{o}} \rho_0 r^2 dr} \times \int_{r_\mathrm{i}}^{r_\mathrm{o}} \left(r^{-2} \rho_0^{-1} \int_{r}^{r_\mathrm{o}} \rho_0 r^2 dr \right) dr \ . \tag{9}$$

Given the molecular diffusion coefficient $\kappa \sim 10^{-5}$ m^2 s^{-1} generally accepted for the Earth's liquid core, the value (9) is on the order of 10^{-1} in the modern epoch.

As seen from (9) and (7a), the quantity $\Delta\xi \sim \xi_0$ is actually a characteristic of the light admixture concentration. In the modern epoch, the same estimate of the order of 10^{-1} was independently obtained for this concentration in the Earth in [*Braginsky and Roberts,* 1995; *Lister and Buffett,* 1995; *Loper,* 1978; *Starchenko and Jones,* 2002]. Therefore, all of the values and estimates used in this paper are self-consistent.

4. Description of the Gravitational Convection

Let the density, gravitational potential, pressure and concentration be represented as sums of basic (see above) and relatively small convective components: $\rho_0+\rho$, U_0+U, p_0+p and $\xi_0+\xi$, respectively. Then, taking into account equations (5)–(7), describing the initial state, can be rewritten in the following, "convective" form:

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{V}) + \nabla\cdot(\rho_0\mathbf{V}) = 0\ ; \tag{10a}$$

$$\begin{aligned}\nu\nabla^2\mathbf{V} &= \partial\mathbf{V}/\partial t + (\mathbf{V}\cdot\nabla)\mathbf{V} + 2\Omega\times\mathbf{V}\\ &+ (\rho_0+\rho)^{-1}\nabla(p_0+p) - \rho_0^{-1}\nabla p_0 + \nabla U\ ;\end{aligned} \tag{10b}$$

$$\nabla^2 U = 4\pi G\rho\ ; \tag{10c}$$

$$\begin{aligned}&\nabla\cdot\left[\left(\rho_0+\rho\right)\kappa\nabla\xi\right]+\nabla\cdot\left(\rho\kappa\xi'\mathbf{1}_\mathrm{r}\right)\\ =\ &\rho_0\partial\xi/\partial t + \rho\dot{\xi} + \left(\rho_0+\rho\right)\mathbf{V}\cdot\left(\xi'\mathbf{1}_\mathrm{r}+\nabla\xi\right)\ ;\end{aligned} \tag{10d}$$

$$\begin{gathered}\rho = \rho'_p p + \rho'_\xi\xi\ ,\\ \text{where } \rho'_p = \frac{\rho'_0}{\partial p_0/\partial r} \text{ and } \rho'_\xi = \frac{\rho'_0}{\partial\xi_0/\partial r}\\ \text{determine } \rho'_0 \equiv \frac{\partial\rho_0}{\partial r}\ .\end{gathered} \tag{10e}$$

The quantities ρ'_p, and $\rho'_\xi(r)$ in (10e) are functions defined through the basic density $\rho_0(r)$. In addition, we used relations derived for planets from numerical and laboratory simulations: $\rho_0 \gg |\rho|$, $|p_0| \gg |p|$ and $\xi_0 \gg |\xi|$. Based on these relations and the estimate $|\partial/\partial t| \sim |\mathbf{V}\cdot\nabla|$, typical of convection, the exact equations (10a)–(10c) can be reduced to a simplified system:

$$\nabla\cdot\left(\rho_0\mathbf{V}\right) = 0\ ; \tag{11a}$$

$$\begin{aligned}\nu\nabla^2\mathbf{V} &= \partial\mathbf{V}/\partial t + \left(\mathbf{V}\cdot\nabla\right)\mathbf{V}\\ &+ 2\Omega\times\mathbf{V} + \left[\rho_0^{-1}\nabla p - \rho_0^{-2}\rho\nabla p_0 + \nabla U\right]\ ;\end{aligned} \tag{11b}$$

$$\begin{aligned}&\nabla\cdot\left(\rho_0\kappa\nabla\xi\right)+\left[\nabla\cdot\left(\rho\kappa\xi'\mathbf{1}_r\right)-\rho\dot{\xi}\right]\\ =\ &\rho_0\left(\partial\xi/\partial t + \mathbf{V}\cdot\nabla\xi\right)+\rho_0 V_r\xi'\ .\end{aligned} \tag{11c}$$

Using (10e) and (5)–(7), the bracketed terms in (11b) and (11c) can be transformed into a form advantageous for their further effective application. First, we write separately the buoyancy acceleration from (11b):

$$\begin{aligned}&\left[\rho_0^{-1}\nabla p - \rho_0^{-2}\rho\nabla p_0 + \nabla U\right]\\ =\ &\rho_0^{-1}\nabla p - \rho_0^{-2}\left(\rho'_p p + \rho'_\xi\xi\right)\nabla p_0 + \nabla U\\ =\ &\nabla\left(\rho_0^{-1}p + U\right) - \rho_0^{-2}\rho'_\xi\xi\nabla p_0\\ =\ &\nabla\left(\frac{p}{\rho_0} + U\right) + \frac{\rho'_\xi}{\rho_0}\frac{\partial U_0}{\partial r}\xi\mathbf{1}_r\ .\end{aligned} \tag{12a}$$

The bracketed term in (11c) can then be transformed to the form

$$\begin{aligned}&\left[\nabla\cdot\left(\rho\kappa\xi'1_\mathrm{r}\right)-\rho\dot{\xi}\right]\\ =\ &\kappa\xi'\left(\mathbf{1}_r\cdot\nabla\rho\right)+\rho\left[\nabla\cdot\left(\kappa\xi'\mathbf{1}_r\right)-\dot{\xi}\right]\\ =\ &\left(\kappa\xi'/\rho_0\right)\left[\rho_0\partial\rho/\partial r - \rho\partial\rho_0/\partial r\right]\\ =\ &\kappa\xi'\rho_0\partial\left[\left(\rho'_p p + \rho'_\xi\xi\right)/\rho_0\right]/\partial r\ .\end{aligned} \tag{12b}$$

It is evident from this relation and from (9) that the value (12b) is $\Delta\xi$ times smaller than the first term on the left-hand side of (11c), which is usually minimal in a larger part of the liquid layer. Hence, since $\Delta\xi \ll 1$ in the case studied, the contribution of (12b) to (11c) can be neglected in the models considered here.

Finally, introducing the effective pressure $P \equiv \rho_0^{-1}p + U$ from (12a), we obtain the simplest convective system of equations

$$\nabla\cdot\mathbf{V} = -\rho_0^{-1}\rho'_0 V_r\ ; \tag{13a}$$

$$\begin{aligned}\nu\nabla^2\mathbf{V} &= \partial\mathbf{V}/\partial t + \left(\mathbf{V}\cdot\nabla\right)\mathbf{V}\\ &+ 2\Omega\times\mathbf{V} - g_0\xi\mathbf{1}_r + \nabla P\ ;\end{aligned} \tag{13b}$$

$$\rho_0^{-1}\nabla\cdot\left(\rho_0\kappa\nabla\xi\right) = \partial\xi/\partial t + \mathbf{V}\cdot\nabla\xi + V_r\xi'\ . \tag{13c}$$

Here, $g_0(r) \equiv -(\rho'_\xi/\rho_0)\partial U_0/\partial r$ is the gravitational acceleration determined from (6) and (10e), which is responsible for the concentration buoyancy. This acceleration is directly proportional to ordinary gravitational acceleration $\partial U_0/\partial r$, with the proportionality factor ρ'_ξ/ρ_0 amounting to ~0.6 in the Earth [*Braginsky and Roberts,* 1995]. The convection intensity is controlled by the product of g_0 and the stationary concentration gradient $-\xi'$ given by (7).

The system of 8th order (13) is complemented by six boundary conditions (2) for the vector $\mathbf{V}$ and by the following two conditions imposed on the convective part of the concentration ξ, below referred to simply as concentration:

$$\frac{\partial\xi}{\partial r} = 0 \text{ at } r = r_\mathrm{o}\ , \tag{14a}$$

$$\frac{\partial\xi}{\partial r} = \frac{Fr_\mathrm{i}}{\Xi\kappa}\frac{\partial\xi}{\partial t} \text{ at } r = r_\mathrm{i}(t) - F\int_0^t r_\mathrm{i}\frac{\partial\xi}{\partial t}dt\ . \tag{14b}$$

Here, $r_\mathrm{i} = r_{\mathrm{i}0} + \dot{r}_\mathrm{i}t$ is the radius of the solid sphere, linearly varying with time; $\dot{r}_\mathrm{i} = \mathrm{const}$ is determined in (8); and $r_{\mathrm{i}0} = \mathrm{const}$ is the initial radius at the time moment $t = 0$, when the system under consideration is adequately described by the spherically symmetric basic state (see the preceding section). The remaining parameters in (14b) are determined in (3b, 3c) and (4a, 4c).

Boundary condition (14b) differs essentially from all boundary conditions known studies of thermal convection, which is usually examined in the Boussinesq approximation (e.g. see [*Braginsky and Roberts,* 1995; *Starchenko,* 2000]). Due to (14b), compositional convection inevitably involves the presence of a specific inner boundary layer that controls the freezing dynamics of liquid at the boundary of the inner sphere. Since the convection inevitably becomes asymmetric after a sufficiently long time interval, the shape of the inner core should eventually deviate, on a significant level, from a spherically symmetric shape. This provides a fairly simple explanation to the asymmetry of the Earth's inner core established from seismological data.

The thickness of the unique concentration layer $\Delta r = \kappa \Xi r_\mathrm{o}/(Fr_\mathrm{i} V_*)$ is estimated by balancing typical values and using as a characteristic time the ratio of the outer size r_o of the system to the characteristic velocity V_*. The corresponding similarity criterion is described by the number

$$\delta \equiv \Delta r/r_\mathrm{o} = \kappa \Xi/(Fr_\mathrm{i} V_*) \ll 1 \ , \tag{15}$$

which amounts to $\sim 10^{-7}$ for the value $V_* = 10^{-4}$ m s^{-1}, molecular diffusion and the parameters considered above.

As distinct from the widely known Boussinesq convection, the compositional convection under study involves a similarity criterion for (13a) characterizing the stratification of density:

$$d \equiv \max_r |r_\mathrm{o} \rho_0'/\rho_0| \ . \tag{16}$$

This effect, albeit small in the Earth's core ($d \sim 10^{-1}$) [*Dziewonski and Anderson,* 1981], can be significant for convection and magnetism [*Braginsky and Roberts,* 1995; *Starchenko,* 2001].

Unlike (15) and (16), many other numbers characterizing similarity criteria of compositional convection are either analogous to or coincide with well-known numbers of thermal convection. Thus, rapid and nearly rigid rotation of liquid interiors of a planet (or a satellite) is characterized, respectively, by the Ekman number E and the Rossby number ε :

$$E \equiv \nu r_\mathrm{o}^{-2}/\Omega \ll 1 \ , \tag{17a}$$

$$\varepsilon \equiv V_* r_\mathrm{o}^{-1}/\Omega \ll 1 \ . \tag{17b}$$

It is evident from this definition that these numbers are the same for both thermal and compositional convection. The lower bound $\varepsilon \geq 10^{-6}$ for the Rossby number is determined reasonably well from long-term geomagnetic and present-day seismic observations [*Glatzmaier and Roberts,* 1997].

The lower bound $E \geq 10^{-15}$ for the Ekman number is determined from the iron value of the molecular viscosity $\nu \geq 10^{-6}$ m^2 s^{-1}, obtained under conditions typical of the Earth's liquid core [*Wijs et al.,* 1998]. If the actual value of viscosity is close to this lower bound, the relative thickness of the Ekman viscous boundary layer $E^{1/2} \sim 10^{-7}$ virtually coincides with the thickness of the concentration layer δ from (15). As demonstrated below, such a coincidence is unlikely to be accidental. Then, setting $E^{1/2}$ equal to δ, we obtain a simple estimate for the concentration velocity:

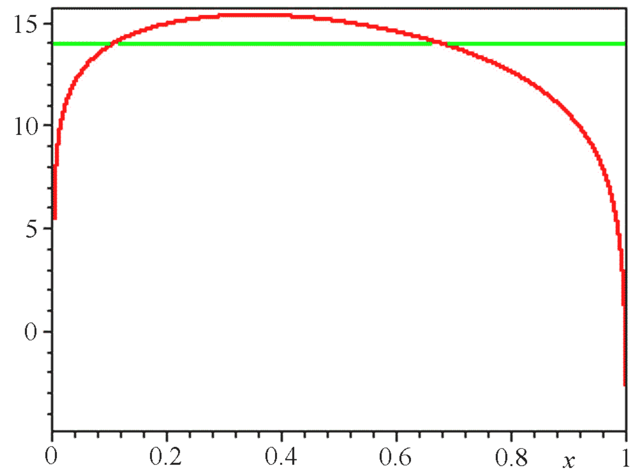

Figure 2. The straight line is the plot of $\log(C_\mathrm{cr})$ (C_cr is the critical value from (20)). The curved line is the plot of $\log(C)$ (C is the concentration number from (19)) as a function of x (see the capture to Figure 1). The compositional convection is excited at $C > C_\mathrm{cr}$.

$$V_* = (\Xi/F) r_\mathrm{o} \sqrt{\Omega/\nu} (\kappa/r_\mathrm{i}) \sim E^{-1/2} \kappa/r_\mathrm{i} \ . \tag{18}$$

To estimate numerically the intensity of compositional convection sources, we determine rotational ($E \ll 1$) concentration number C. To do this, we utilize the rotational Rayleigh number R known in thermal convection (e.g. see [*Starchenko,* 2000]). Replacing in the R definition the thermal density drop $\alpha \Delta T$ by the analogous value $\Delta \xi$ from (9), we obtain

$$C \equiv g_* \Delta \xi (r_\mathrm{o} - r_\mathrm{i})/(\Omega \kappa) \ , \tag{19}$$

where g_* is the characteristic gravitational acceleration and the layer thickness $r_\mathrm{o} - r_\mathrm{i}$ is used as the characteristic size. Concentration number (19) is fairly large ($C \approx 2 \cdot 10^{15}$) if the following values, typical of the contemporary liquid core of the Earth are accepted: $\Delta \xi \approx 10^{-1}$, $\kappa \approx 10^{-5}$ m^2 s^{-1}, $\Omega = 7 \cdot 10^{-5}$ s^{-1}, $r_\mathrm{o} - r_\mathrm{i} = 2 \cdot 10^6$ m and $g_* \approx 7$ m s^{-2}. The C value is plotted in Figure 2 as a function of the relative radius of the inner solid core $x \equiv r_\mathrm{i}/r_\mathrm{o}$.

To initiate convection, it is necessary that C exceed a certain critical value C_cr. In the case of thermal convection, if R exceeds the critical Rayleigh number $R_\mathrm{cr} \sim E^{-1/3} \leq 10^5$, sharply asymmetric convection with periods $t_* \sim E^{2/3} r_\mathrm{o}^2/\nu$ arises [*Busse,* 1970; *Jones et al.,* 2000]. In the Earth's core, this corresponds to the typical velocity $V_* = r_\mathrm{o}/t_* \sim 10^{-3}$ m s^{-1}. If the preferable excitation of convection of the same type were possible in the concentration system considered here, this would lead to the formation of an overly narrow concentration boundary layer ($\delta \sim 10^{-8}$) in accordance with (15). We show that the existence of even much wider layer would require an incomparably larger value of the critical number and an essentially different convection pattern.

The system is at the convection excitation threshold if the time derivatives of the relative concentration and velocity are close to zero in (13b, 13c): $|\xi^{-1}\partial\xi/\partial t| \sim 0$ and $|V^{-1}\partial V/\partial t| \sim 0$. In this case on the strength of (2b), the radial velocity in the concentration boundary layer (15) has a characteristic value of about δV_*. The characteristic values $|V_r\xi'| \sim \delta V_* \Delta\xi / r_\mathrm{o}$ and $g_* \xi_*$ of the "generating" terms in (13b, 13c) should be close to the respective values $\kappa\xi_*/(r_\mathrm{o}\delta)^2$ and $\nu(\delta V_*)/(r_\mathrm{o}\delta)^2$ for the diffusive and viscous terms hindering the generation. According to definition (19), this yields at $r_\mathrm{o} \sim (r_\mathrm{o} - r_\mathrm{i})$ an estimate for the largest possible value of the critical concentration number, expressed through the thickness of the concentration boundary layer:

$$C_\mathrm{cr} \sim E/\delta^4 \; . \tag{20}$$

Given the relative thickness $\delta \sim 10^{-7}$ typical of the Earth and $E = \delta^2$, we obtain $C_\mathrm{cr} \sim 10^{14}$, which is only a little smaller than the contemporary concentration number C according to Figure 2. Therefore, compositional convection can be laminar. This is an entirely new fact, because previously, by analogy with asymmetric thermal convection [*Braginsky and Roberts,* 1995; *Lister and Buffett,* 1995; *Starchenko,* 2000], such chemical (or gravitational) convection was always supposed to have an essentially nonlinear, turbulent and complicated pattern. The structure of compositional convection should also be simpler than that of thermal convection, because it is controlled by a more symmetric concentration boundary layer, which should be related to well-studied viscous boundary layers.

Presently, the intensity of compositional convection is close to its maximum, as is evident from Figure 2. On the contrary, in the distant past, when the Earth's solid core was small enough, compositional convection was not so intense. There even existed a critical radius of the solid core starting from which the convection was excited. The convection will start attenuating beginning from a certain time moment in future and will stop at the second critical radius, when the thickness of the liquid layer becomes too small, as is seen from Figure 2. It is quite probable that presently the radius of the radius of the solid core exceeds the second critical value in Mars and has not attained the first critical value in Venus. This is a likely reason why these planets do not have own significant magnetic fields. Even if compositional convection exists in the interiors of Mercury, its intensity is nearly critical because the own magnetic field of the planet is very weak and irregular.

5. Project of an Experiment and Conclusions

Our project of an experimental installation is largely similar to that described in [*Sumita and Olson,* 1999] and already used for modeling thermal convection. Therefore, below we do not go into technical detail but focus on significant distinctions from the model proposed in [*Sumita and Olson,* 1999] that are beneficial to the effective use of the installation proposed here primarily for modeling compositional convection and the related differential rotation. Evidently, this installation is also applicable to the modeling of thermal and combined convection in deep interiors of planets.

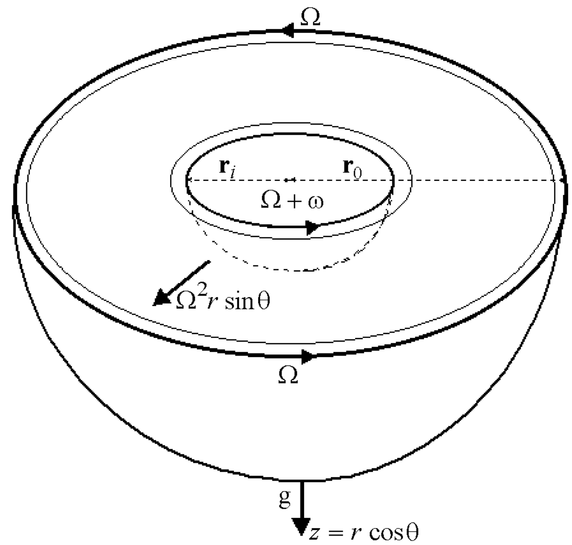

Figure 3. Layout of an experimental installation for the laboratory simulation of gravitational differentiation and differential rotation of deep interiors of planets and natural satellites.

Figure 3 schematically illustrates the layout of the experimental installation modeling the heat-and-mass transfer under conditions typical of planetary deep interiors. We replaced spheres by hemispheres for the following reasons. First, the combined centrifugal and gravitational field is capable of reproducing the equatorial symmetry of the planetary gravitational field, whereas spheres virtually cannot ensure such a symmetry of this buoyancy field under laboratory conditions. Second, the central hemisphere can actually float, like a solid planetary core at the center of the spherical liquid layer. Finally, the use of hemispheres instead of spheres considerably facilitates effective monitoring and needed measurements.

An important element of the experimental installation is a light and transparent cap that has the shape of a thin disk and bounds the liquid layer surface from above (see Figure 3). The cap both retains the inner hemisphere at the center and prevents the free surface from buckling due to the rapid rotation. The cap material should make its viscous coupling with the layer liquid as small as possible. At the cap boundaries with the hemispheres, it is advantageous to place light bearings rigidly connected with the hemispheres. Then, the angular rotation velocity of the cap can be used for estimating the average angular velocity of the spherical layer.

Note that the laboratory field of the centrifugal and gravitational buoyancy is opposite in direction to the planetary field modeled. Therefore, in order to reproduce real effects, the laboratory gradients of temperature and concentration should be opposite to counterparts in planets. Thus, the radius of the inner hemisphere, colder than the outer one, should decrease in the laboratory experiment in order to adequately model hydrodynamic effects associated with growing hotter solid cores of planets.

Now we estimate the angular velocities of the hemispheres providing the best fit of the spherically symmetric (radial) field of gravitational buoyancy and the slightly differential rotation of a liquid core.

The angular velocity of the outer hemisphere is fixed in the laboratory experiment (i.e. $\Omega = \text{const}$ in Figure 3) in order to model the rotation of the outer planetary mantle, whose moment of inertia is much larger than that of the liquid layer. This value of Ω and the gravitational acceleration $g = 9.8$ m s^{-2} determine the equipotential lines of the acceleration field

$$\left(\Omega \sin\theta\right)^2/2 + gr\cos\theta = \text{const} \tag{21}$$

modeling the gravitation in the interiors of planets and natural satellites in the rotating reference frame (r, θ, φ) shown in Figure 3. The closer the equipotential lines (21) to the contours $r = \text{const}$, the better the reproduction of the radial gravitational field of a planet. An adequate reproduction of this field is particularly important, because all of the main processes controlling the compositional convection occur tight here. In this respect, the main line of family (21) that is tangent to the line $r = r_\mathrm{i}$ at $z = r\cos\theta = r_\mathrm{i}$ and ensures such a reproduction is determined by the criterion of closeness to $r = \text{const}$ reducing to the minimization of the area bounded by these lines and the axis $s = r\sin\theta$. It is easy to prove that the absolute minimum of this area is attained at an angular velocity close to the value

$$\Omega = 1.2\sqrt{g/r_\mathrm{i}}\ . \tag{22}$$

Hence, we obtain that, for example, at $r_\mathrm{i} = 0.35$ m the outer hemisphere should make one revolution per second for the best possible reproduction of the radial field of concentration buoyancy near the solid core of a planet.

The angular velocity of the inner hemisphere ($\Omega + w_\mathrm{i}$ in Figure 3) should be slightly different from that of the outer hemisphere in order to model a slightly differential rotation in planetary cores. Thus, the reproduction of the differential rotation in the Earth's core requires that the condition $0 \le w_\mathrm{i}/\Omega \le 10^{-5}$ be valid [*Glatzmaier and Roberts,* 1997]. The measurement of such small differences between angular velocities will require high-precision instrumentation such as layer-scanning laser illumination and interferometers.

To sum up, we present the main results of this work.

(1) Under assumptions natural for terrestrial planets, a fully analytical description is obtained for basic hydrostatic effects that control slow growth of an inner solid core in a planet, gravitational stratification of the liquid core and the associated sources of compositional convection.

(2) A system of equations governing the virtually unexplored compositional convection is derived. Main similarity criteria the observance of which is a prerequisite for successful laboratory simulation of such planetary convection are substantiated.

(3) New similarity criteria, discovered in this study, characterize the initial concentration contrast ($\Delta\xi \sim 0.1$ for the Earth), the compressibility of the liquid core ($d \approx 0.1$ for the Earth) and the relative thickness of a concentration boundary layer ($\delta \sim 10^{-7}$ for the Earth) that, controlling the liquid freezing process, can also determine the asymmetry of the planetary solid core.

(4) The study showed for the first time that the excitation threshold of compositional convection should be considerably higher than that of thermal convection. Therefore, convection in deep interiors of planets and natural satellites can be nearly laminar in spite of very large values of the concentration number $C(\sim 10^{15}$ in the Earth), which is similar to the rotational Rayleigh number R.

(5) The observed planetary magnetic fields yield evidence that a system driven by compositional convection can be in a laminar regime in the Earth, near the excitation threshold in Mercury and beyond the excitation threshold in Venus and Mars.

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