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Watershed modeling of rainfall excess transformation into runoff

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Abstract

In this paper an attempt is made to present a distributed physiographic conceptual model that uses the principles of flow continuity and momentum. For this purpose, the watershed under study is divided into subwatersheds keeping in view the drainage patterns and characteristics. Then the main tributaries are identified and their drainage areas are delineated to form tributary subwatersheds. The main channel subwatersheds have taken care of the remaining area in the vicinity of the main channel. The kinematic wave theory is applied for the overland runoff computations from these subwatersheds. Further, the overland flows are superimposed onto the main channel. The dynamic wave theory is used to route the flows through the main channel to compute the watershed responses at the outlet. The proposed model is tested onto a natural watershed. The computations were performed for few storm events. Comparison of the significant parameters of the computed and the observed hydrographs shows that the maximum relative error in prediction is 5.8%. Thus, the results are satisfactory. Better results can be obtained when measured rainfall-excess data are available or a more realistic loss index is adopted for rainfall-excess separation.

Keywords: Watershed modeling; Conceptual model; Surface runoff; Flow routing; Dynamic wave; Kinematic wave

1. Introduction

Recent attempts to further develop and apply physically based, distributed rainfall–runoff models are primarily in two directions. First, some attempts are aimed to improve the process representation and predictive capabilities of runoff modeling through the effective use of available experimental and observational data. Second, other analyses are applications to solve water resources engineering problems. Examples of physically based distributed models are those outlined by Freeze and Harlan (1969), Abbott et al. (1986a,b), Refsgaard and Storm (1995), and Ping and Xiaofang (1999). These models simulate the runoff process and compute the watershed response. However, the sophisticated models when applied to a comparatively large watershed requires both a great deal of time and data preparation. Therefore, it would seem fruitful to develop ways to achieve a higher degree of accuracy, without additional complexity but taking into account the physical characteristics of the watershed.

An overview of the present day hydrologic models indicates that kinematic wave (KW) theory has been widely used for simulating flow over the planes where the criteria for the KW application are satisfied. However, applications of the KW theory have been mostly restricted to urban watersheds and to some extent to the natural watersheds that have comparatively small drainage areas. On the other hand, dynamic wave (DW) models are mathematically deterministic and

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physically based process models. The DW theory is considered to be the best methodology for taking into account the prevailing flow conditions over the watershed and in channels.

Thus, the present study is aimed at developing a mathematical model utilizing the characteristics of DW theory and its KW approximation. For this purpose, the model includes components for both the overland plane and the main channel. Recognizing this structure, the mathematical formulation of the DW channel flow model is discussed first, to be followed by the KW model used to compute the lateral flows from the overland planes.

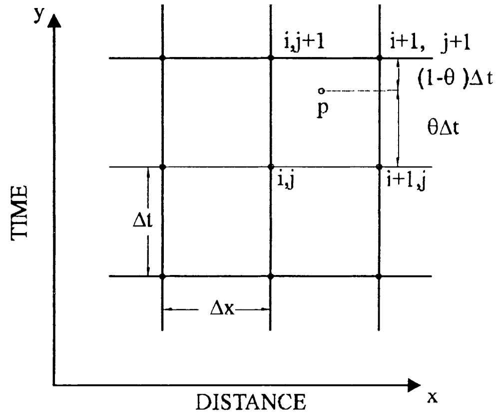

Fig. 1. Four point implicit finite difference scheme.

2. Description of mathematical model

2.1. Channel flow equations

The St Venant equations that describe one-dimensional unsteady flow in wide rectangular channels expressed per unit channel width are (Liggett and Cunge, 1975) the equation of continuity,

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = q_0 \tag{1}$$

and the momentum equation,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = g(S_o - S_f) - \frac{q_0 u}{h} \tag{2}$$

where h, depth of flow; u, velocity; S_o, the channel bed slope; x, the coordinate horizontal in flow direction; t, the time; q_0, the lateral inflow per unit time per unit channel length; g, acceleration due to gravity; and S_f, the friction slope. The friction slope is approximated by Manning's equation,

$$S_f = \frac{n^2 u|u|}{h^{4/3}} \tag{3}$$

where n is the Manning's roughness coefficient. The term u^2 appearing in Eq. (3) is replaced by $u|u|$ to account for the possibility of flow reversal.

2.2. Method of solution

The four-point implicit scheme (Preissmann, 1961) is employed for the solution of channel flow equations. As shown in Fig. 1, the flow domain is divided into a number of flow reaches. Similarly, the time variable is discretized and the solutions at discrete time intervals are sought. Accordingly, for a point like 'p' located in a rectangular grid, the average values of the functions and derivatives are given by,

$$f = \frac{\theta}{2}(f_{i+1}^{j+1} - f_i^{j+1}) + \frac{1-\theta}{2}(f_{i+1}^{j} + f_i^{j}) \tag{4}$$

$$\frac{\partial f}{\partial x} = \theta\left(\frac{f_{i+1}^{j+1} - f_i^{j+1}}{\Delta x}\right) + (1-\theta)\left(\frac{f_{i+1}^{j} - f_i^{j}}{\Delta x}\right) \tag{5}$$

$$\frac{\partial f}{\partial t} = \frac{1}{2\Delta t}\left(f_{i+1}^{j+1} - f_{i+1}^{j} + f_i^{j+1} - f_i^{j}\right) \tag{6}$$

where θ is a time-weighting coefficient. In Eqs. (4)–(6), f may represent u, h, q_0, S_f,… so that u, h, q_0, $\partial u/\partial x$, $\partial u/\partial t$, $\partial h/\partial x$, and $\partial h/\partial t$ at point 'p' are approximated in terms of the corresponding values at the four corner grid points. On substituting the average values of the Eqs. (4)–(6) for the appropriate terms in Eqs. (1) and (2), the following equations are obtained:

$$\begin{aligned} F_{2i} = {} & x_{2i+1} + x_{2i-1} + \frac{2\Delta t}{\Delta x}[\theta^2\{x_{2i+2}x_{2i+1} - x_{2i}x_{2i-1}\} \\ & + \theta(1-\theta)\{x_{2i+2}h_{i+1}^{j} - x_{2i}h_i^{j} + u_{i+1}^{j}x_{2i+1} - u_i^{j}x_{2i-1}\} \\ & + (1-\theta)^2\{u_{i+1}^{j}h_{i+1}^{j} - u_i^{j}h_i^{j}\}] \\ & - h_{i+1}^{j} - h_i^{j} - 2\Delta t Q_0 = 0, \quad i = 1,2,\ldots,N-1 \end{aligned} \tag{7}$$

where

$$Q_0 = \frac{\theta}{2}[q_{i+1}^{j+1} + q_i^{j+1}] + \frac{1-\theta}{2}[q_{i+1}^j + q_i^j]$$

and

$$\begin{aligned} F_{2i+1} = {} & x_{2i+2} + x_{2i} + \frac{\Delta t}{\Delta x}[\theta^2(x_{2i+2}^2 - x_{2i}^2) \\ & + 2\theta(1-\theta)\{x_{2i+2}u_{i+1}^j - u_i^j x_{2i}\} \\ & + (1-\theta)^2\{(u_{i+1}^j)^2 - (u_i^j)^2\}] \\ & + \theta\Bigg[\frac{2g\Delta t}{\Delta x}(x_{2i+1} - x_{2i-1}) \\ & + gn^2\Delta t\left\{\frac{x_{2i+2}|x_{2i+2}|}{x_{2i+1}^{4/3}} + \frac{x_{2i}|x_{2i}|}{x_{2i-1}^{4/3}}\right\} \\ & + \Delta t\left\{\frac{q_{i+1}^{j+1}x_{2i+2}}{x_{2i+1}} + \frac{q_i^{j+1}x_{2i}}{x_{2i-1}}\right\}\Bigg] \\ & + (1-\theta)\Bigg[\frac{2g\Delta t}{\Delta x}(h_{i+1}^j - h_i^j) \\ & + gn^2\Delta t\left\{\frac{u_{i+1}^j|u_{i+1}^j|}{(h_{i+1}^j)^{4/3}} + \frac{u_i^{j+1}|u_i^j|}{(h_i^j)^{4/3}}\right\} \\ & + \Delta t\left\{\frac{q_{i+1}^j u_{i+1}^j}{h_{i+1}^j} + \frac{q_i^j u_i^j}{h_i^j}\right\}\Bigg] \\ & - (u_{i+1}^j + u_i^j + 2gS_0\Delta t) = 0, \quad i = 1,2,\dots,N-1 \end{aligned} \tag{8}$$

where

$$\left.\begin{array}{ll} x_1 = h_1^{j+1} & x_2 = u_1^{j+1} \\ \vdots & \vdots \\ x_{2i-1} = h_i^{j+1} & x_{2i} = u_i^{j+1} \\ x_{2i+1} = h_{i+1}^{j+1} & x_{2i+2} = u_{i+1}^{j+1} \\ \vdots & \vdots \\ x_{2N-1} = h_N^{j+1} & x_{2N} = u_N^{j+1} \end{array}\right., \quad i = 1,2,\dots,N-1$$

Since N nodal points are positioned on $(j+1)$th row, there would be $(N-1)$ rectangular grid and $(N-1)$ interior nodal points. Therefore, the total number of equations are $2(N-1)$ with $2N$ unknowns. To make the system determinate two more equations are obtained by defining the upstream and the downstream boundary conditions. The upstream boundary condition as a function of time might be of the following form:

$$F_1(x_1) = x_1 - h_{up}(t) = 0 \tag{9}$$

where $h_{up}(t)$ is the ordinate of stage hydrograph at upstream section at time $j+1$. If at the downstream boundary, the stage-velocity relationship is known from the property of the control section, then a relationship like the following form becomes available as the second supplementary equation provided by the downstream boundary.

$$\begin{aligned} F_{2N} &= x_{2N} - (A_0 + A_1x_{2N-1} + A_2x_{2N-1}^2 \\ &\quad + A_3x_{2N-1}^3) \\ &= 0 \end{aligned} \tag{10}$$

where A_0, A_1, A_2, and A_3 are constant coefficients. The Eqs. (7)–(10) lead to a determinate system of nonlinear equations. Following the Newton–Raphson's algorithm, the solution of this system of equations is obtained.

3. The Kinematic wave model

A unit width of the rectangular plane is considered for the runoff generation. The KW equation is written as,

$$\frac{\partial h_0}{\partial t} + \alpha m h_0^{m-1}\frac{\partial h_0}{\partial x} = q_i \tag{11}$$

where $\alpha = \sqrt{S_0/n_0}$, $m = 5/3$, n_0, overland Manning's roughness coefficient; q_i, lateral inflows (rainfall excess intensity); and h_0, overland flow depth.

The second-order Lax–Wendroff explicit scheme reported by Lax and Wendroff (1960) is used for solving Eq. (11). The flow domain is represented by x-t plane for the spatial and time coordinates as shown in Fig. 2. Following the notations in Fig. 2, Eq. (11) is

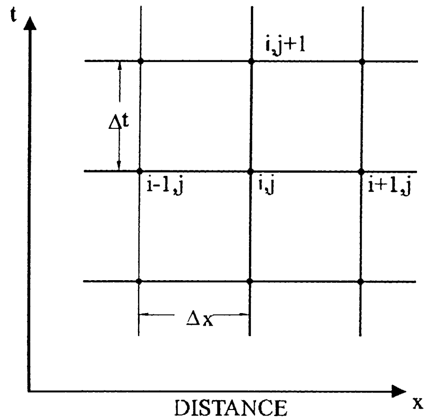

Fig. 2. Grid definition for explicit finite difference scheme.

transformed to a finite difference form:

$$h_i^{j+1} = h_i^j - \Delta t\left\{\alpha\left[\frac{h_{i+1}^{jm} - h_{i-1}^{jm}}{2\Delta x}\right] - q_i^j\right\} + \frac{m\alpha}{4}\frac{\Delta t^2}{\Delta x}\left[\left[h_{i+1}^{jm-1} + h_i^{jm-1}\right] \times\left\{\alpha\left[\frac{h_{i+1}^{jm} - h_i^{jm}}{\Delta x}\right] - \left[\frac{q_{i+1}^j - q_i^j}{2}\right]\right\} - \left[h_i^{jm-1} + h_{i-1}^{jm-1}\right]\left\{\alpha\left[\frac{h_i^{jm} - h_{i-1}^{jm}}{\Delta x}\right] - \left[\frac{q_i^j + q_{i-1}^j}{2}\right]\right\} + \frac{2\Delta x}{m\alpha\Delta t}\left\{q_i^{j+1} - q_i^j\right\}\right] \quad (7)$$

It should be noted here that to determine h_i^{j+1} requires knowledge of h_i^j, h_{i+1}^j and h_{i-1}^j. Thus, Eq. (12) can compute the overland flow profile at points $i = 2, 3, \ldots, N-1$ on a time line. The solution at $i = 1$ is provided by the upstream boundary condition:

$$h(0,t) = 0 \quad (13)$$

The depth of flow at the outlet ($i = N$) can be obtained using the first order scheme,

$$h_N^{j+1} = h_N^j + \Delta t\left[\alpha\left\{\frac{(h_{N-1}^j)^m - (h_N^j)^m}{\Delta x}\right\} + q_N^j\right] \quad (14)$$

Eqs. (12)–(14) along with the initial conditions can simulate the overland flow over a plane. It is assumed that the surface of the plane is initially dry, then,

$$h(i,0) = 0, \quad i = 1, 2, \ldots, N \quad (15)$$

Finally, the flow depth is converted into a discharge using a rating curve in the following form:

$$q_0 = \alpha h_0^m \quad (16)$$

The stability and convergence criteria for this scheme is well documented by Price (1974), Liggett and Cunge (1975), and Liu et al. (1992).

4. Proposed physiographic model

The open book type physiographic models have been used by various researchers (Singh, 1976) for the application of KW models for distributed hydrologic systems. Since within a natural watershed, the soil type, landuse, channel configurations, slopes, etc. may vary, the open book type models are likely to distort the actual physiographic effects on the runoff process. Therefore, for a comprehensive coverage of these distributed aspects, a physiographic model was developed.

The proposed model consists of tributary subwatersheds and the distributed main channel subwatersheds. As shown in Fig. 3(a) the main tributaries (T_j) are identified. The corresponding subwatersheds have the areas (A_j). Therefore, the remaining portions marked ΔA_j form the main channel subwaterseds (j standing for the number).

Modeling for each of the tributary subwatersheds is accomplished as an open book type model. These are folded onto the main channel. For this purpose, the rectangular configurations of the tributary subwatersheds are placed parallel to the main channel. The length of each plane is equal to the total length of the tributaries and the placement is set from the point of confluence to the upstream end as shown in Fig. 3(b). Modeling for the main channel subwatersheds is accomplished on similar lines as the tributary subwatersheds. These are then folded onto the main channel. The KW theory is used to compute the overland surface runoffs, q_{Tj} from tributary subwatersheds, and q_{Mj} (from the main channel subwatersheds). In the overlapping portions of the main channel, the lateral flows (q_{Mj} and q_{Tj}) are superimposed to form reaches (R_j) as

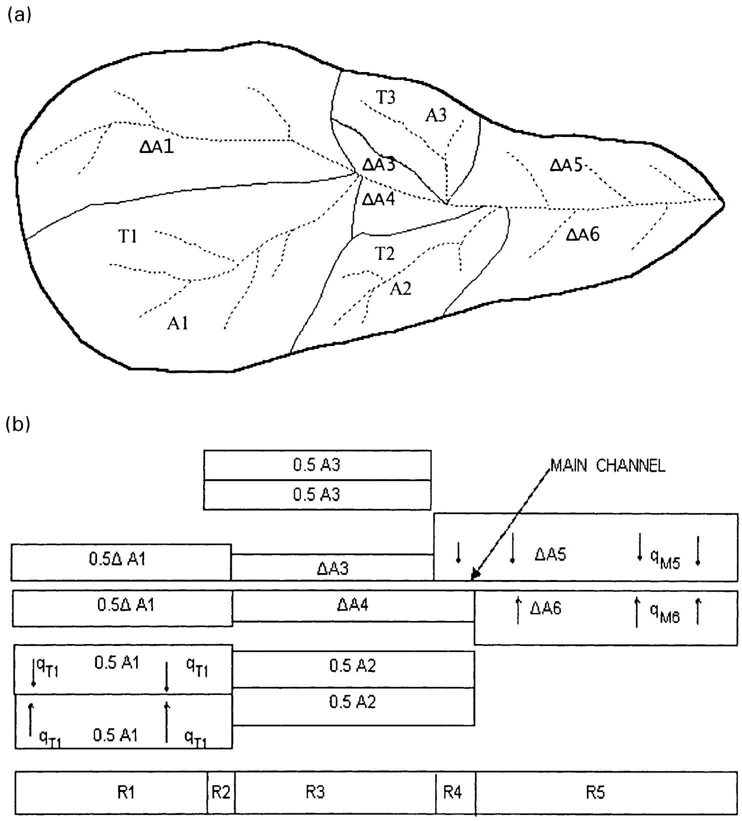

Fig. 3. Watershed Physiographic Model. (a) Delineation of subwaatersheds and (b) conceptual representation.

shown in Fig. 3(b). Thus, each reach (R_j) has a uniform lateral inflow (q_{0j}). The distributed lateral flows (q_{0j}) are routed through the main channel using the DW theory for each unit width of the channel cross-section.

5. Model application

The validity of the proposed physiographic model was applied to the Kolar watershed. River Kolar has a length of 92.5 km and drains an area of 870.85 km^2, where it forms a confluence with Narmada River at Satrana, India. Four raingauges are installed on the watershed indicated as 1–4 in Fig. 4(a).

Twelve main tributaries were identified, and these are marked as T_1–T_{12}. The remaining portions (total watershed area minus the tributary subwatershed areas) form the main channel subwatersheds. Following the water divide, 14 spatially distributed main channel subwatersheds are identified. The physiographic parameters of these subwatersheds were measured from a topographical map (Table 1).

Modeling of the tributary subwatersheds is accomplished as an open book type and conceptually represented by two rectangular planes that have the length equal to that of the main channel and a width such that the total area equals the watershed area. The 14 main channel subwatersheds are conceptually represented through rectangular planes that have areas equal to the area of the subwatersheds. The length of each plane is set equal to the stretch of the subwatershed along the main channel. The width of the plane is calculated accordingly. Following the same procedure, all of the 14 subwatersheds are modeled and placed along the length of the main channel. Therefore, the 12 tributary subwatersheds and the 14 main channel subwatersheds are folded onto the main channel to arrive at a conceptual

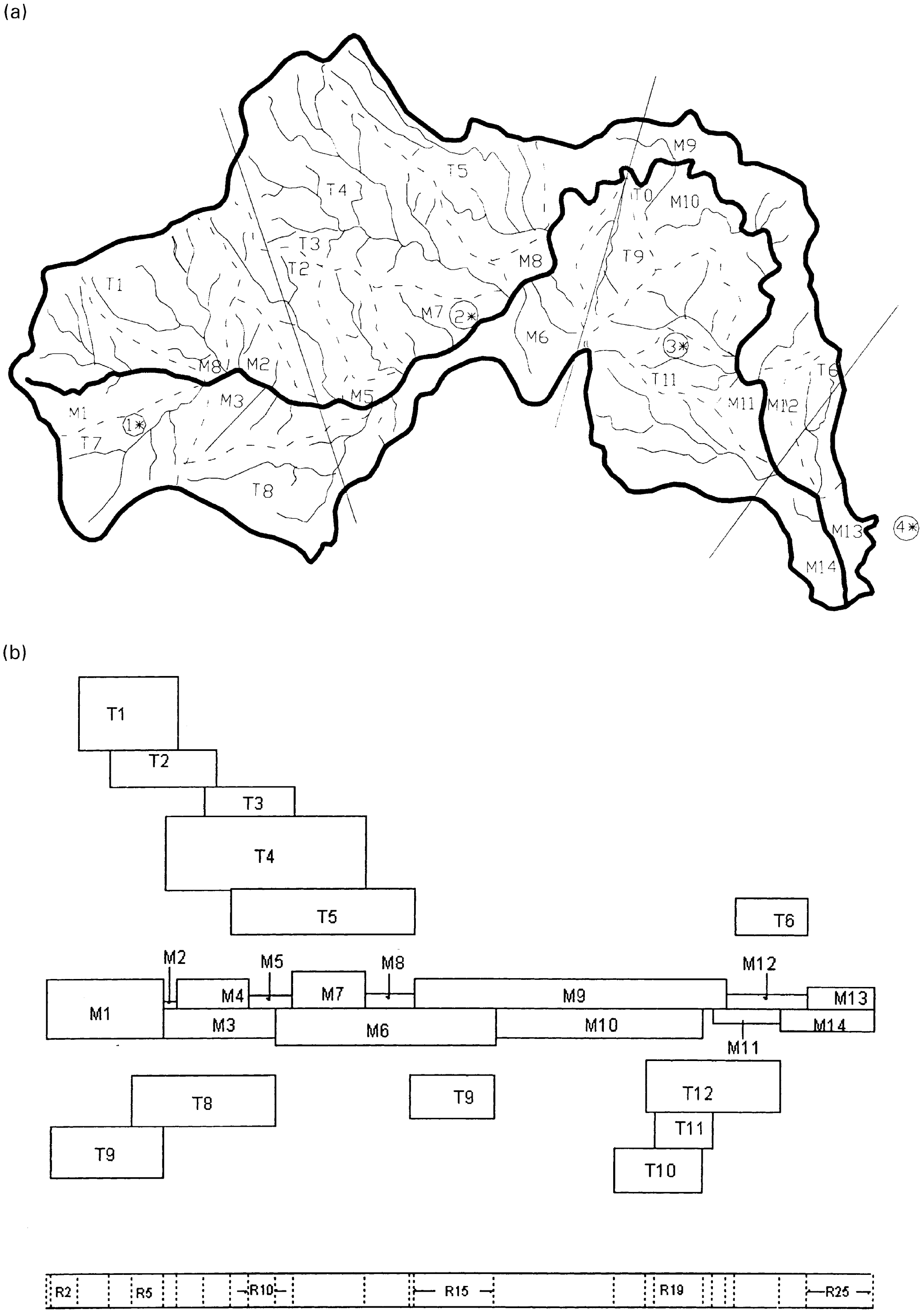

Fig. 4. Physiographic Model of Kolar River Watershed. (a) Delineation of subwatersheds and (b) conceptual representation.

Table 1
Physiographic parameters of the tributary and the main channel subwatersheds

Sl No.	Sub-watershed	Length of longest stream (km)	Overland slope	Overland width (0.5 W) (m)	α	Δx (m)
1	*T*1	11.0	0.006364	2491.0	1.330	249.10
2	*T*2	15.5	0.005161	1274.0	1.197	212.33
3	*T*3	10.0	0.004000	925.0	1.054	18.00
4	*T*4	22.5	0.044400	2511.0	3.512	251.10
5	*T*5	20.5	0.005854	1433.0	1.275	204.71
6	*T*6	8.0	0.003125	1338.0	0.932	223.00
7	*T*7	12.5	0.003600	1718.0	1.000	245.43
8	*T*8	16.0	0.002938	1697.0	0.903	212.33
9	*T*9	9.5	0.006316	1178.0	1.325	235.60
10	*T*10	10.0	0.016000	1613.0	2.108	201.63
11	*T*11	6.5	0.012310	1288.0	1.849	214.66
12	*T*12	15.0	0.011330	1446.0	1.774	205.75
13	*M*1	13.0	0.003500	2067.0[a]	0.981	206.70
14	*M*2	1.5	0.050000	400.0[a]	3.727	133.33
15	*M*3	12.5	0.009400	2136.0[a]	1.612	213.60
16	*M*4	8.0	0.010200	1969.0[a]	1.679	218.77
17	*M*5	5.0	0.020200	990.0[a]	2.368	247.50
18	*M*6	24.5	0.008900	2236.7[a]	1.576	223.67
19	*M*7	8.0	0.011470	2718.8[a]	2.021	247.16
20	*M*8	5.5	0.035900	1113.6[a]	3.159	222.72
21	*M*9	35.0	0.047700	2095.0[a]	3.640	209.50
22	*M*10	23.5	0.043900	1822.3[a]	3.492	202.48
23	*M*11	7.5	0.016500	1213.3[a]	2.140	242.66
24	*M*12	9.0	0.008500	1172.2[a]	1.539	234.44
25	*M*13	7.5	0.015800	1273.3[a]	2.094	212.21
26	*M*14	10.0	0.012900	1630.9[a]	1.892	203.75

[a] Total width.

configuration of the proposed model, which is shown in Fig. 4(b).

The rainfall-excess function has been computed individually for four raingauge stations located in the watershed. The time distribution of rainfall excess for each storm event is obtained rain-gauge-wise using the runoff factor for the storm and a ϕ-index. In case a tributary or a main channel subwatershed gets divided between the two polygons, the weighted average rainfall excess is computed. The computed rainfall-excess functions are fed to the overland flow KW model.

The overland surface runoff from different tributary subwatersheds as well as from the main channel subwatersheds have been computed using Eqs. (12)–(16) for all the planes. The time step is taken as 300 s throughout the overland flow computations. The space step Δx, adopted for overland planes are given in Table 1. Accordingly, the lateral flows to channel (q_{0j}) from all of the 26 planes are computed. In regions where the overland planes have overlapped, the subroutine developed for this purpose superimposes the outflows. Thus, at a given time, all along the length of the main channel, in its different parts, different lateral flows are received. Twenty-five reaches varying from 0.5 to 13.5 km were identified. The function q_{0j} forms the distributed input to the channel.

The DW theory is used to route flows through the main channel. The channel slope was taken as 0.003 and $n = 0.026$. The time weighting coefficient θ is assigned a value of 0.67. The time steps Δt is taken as 1 h. A thin layer of uniform flow was assumed as initial condition. For the downstream boundary condition, the stage-velocity curve is developed. The values of the regression coefficients (Eq. (10)) are as follows: $A_0 = 0.4027035$, $A_1 = 1.829082$, $A_2 = -0.2362494$, and $A_3 = 0.01625036$. The channel flow computations

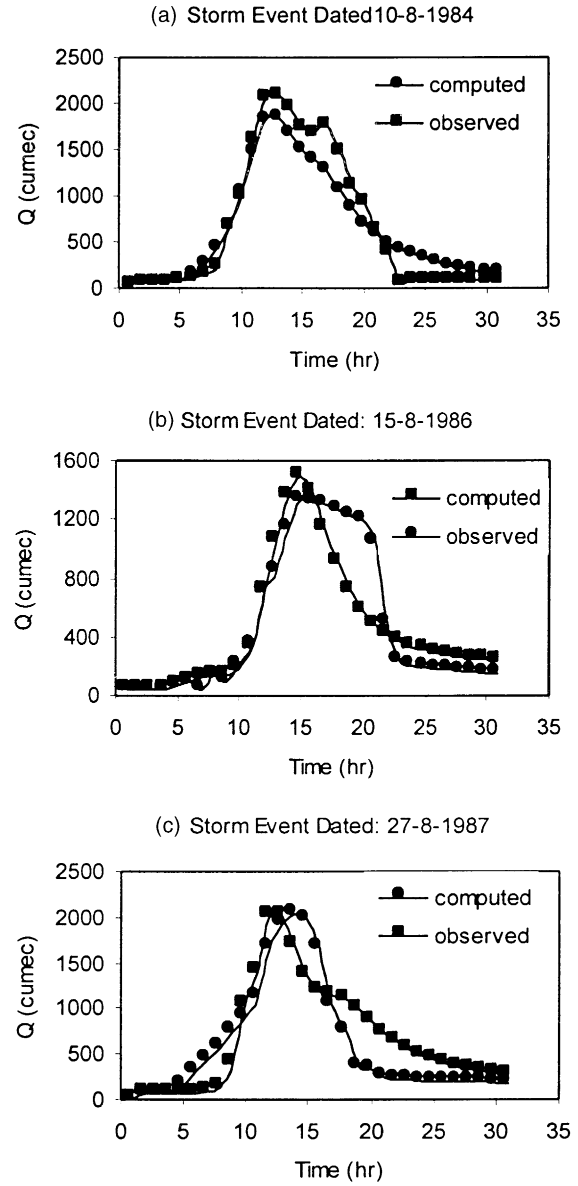

Fig. 5. Comparison of the computed and the observed hydrographs.

for differential lateral flows (q_{0j}) are made for five storm events, with the computed hydrographs compared with the observed hydrograph. Three of them are reproduced in Fig. 5(a)–(c). Comparison of the significant parameters of the computed and observed hydrographs are given in Table 2.

6. Summary and conclusions

The overall aim of this study was to develop a suitable surface hydrologic model capable of taking into account the distributed aspects of the physiography and landuse of the natural watersheds. The watersheds may vary in size ranging from a few square kilometers to a few thousands square kilometers. The model should have the capabilities of accounting for landuse changes that might be introduced due to watershed management practices which in turn may influence the runoff process.

The KW theory and the DW theory based hydrologic models currently being used for solving the St Venant's equations, have the capabilities of taking into account the distributed nature of watershed physiography provided that suitable physiographic models are adopted for use. In the present study, the KW theory is applied for routing the overland flows. The theory has its limitations in application that the appropriate conditions must be satisfied. The criteria adopted for its application is $F_r^2 k > 5$, where F_r, Froud number and k, kinematic wave number (Moramarco and Singh, 2000).

The DW theory along with the four-point implicit finite difference scheme has been preferred for routing the flows through the channel. This may have the advantages of routing the flows where the KW theory may not be applicable.

Table 2
Comparison of parameters of the computed hydrographs with the observed hydrographs of Kolar River

Sl No.	Storm dated	Parameters of observed hydrograph			Parameters of computed hydrograph			Error in prediction	
		DRH volume MCM	DRH peak (cumec)	Tp (h)	DRH volume MCM	DRH peak (cumec)	Tp (h)	Absolute	Relative (%)
1	10.8.84	66.864	2027	12	65.849	1776	12	1.015	1.518
2	15.8.86	40.866	1242	14	39.567	1396	14	1.299	3.2
3	27.8.87	50.725	1898	13	53.68	1915	11	2.955	5.8

The mathematical theories, developed in the study, have been incorporated in the proposed physiographic model. In this model, the watershed under consideration is split up into tributary and main channel subwatersheds. Drainage characteristics of the area happen to be the criteria adopted for the demarcation of the subwatersheds. In order to obtain the conceptual configuration, the surface runoffs coming from each of these subwatersheds are folded onto the main channel. The final physiographic pattern so arrived at will remain unique for the watershed under consideration. For the proposed configuration, each of the subwatersheds becomes the elementary unit from which the runoff responses are to be computed. The surface runoffs from the overlapping overland planes are superimposed to compute the lateral flows coming to the main channel. Flows are routed through the main channel to compute the outflow hydrographs at the outlet.

Erroneous initial conditions generate numerical oscillations in the results, which may take several time steps to get damped. However, to overcome the problems related to initial condition steady flow was allowed to continue for few time steps so as to recompute the initial conditions and damp the oscillations.

However, inaccuracies (if any) in the first few time steps of the computed hydrographs are due to inadequate initial conditions posed in the computations which are unavoidable. In the proposed approach, computation of rainfall-excess distribution is another important step, which influences the results significantly. The inaccuracies in the remaining parts of the computed hydrographs are partly due to the application of the constant ϕ-index in separating rainfall-excess-functions where, field measurements or a more realistic approach could definitely improve the results.

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