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Journal of Hydrology 274 (2003) 129-144



www.elsevier.com/locate/jhydrol

# Estimation of stream flow depletion and uncertainty from discharge measurements in a small alluvial stream

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Received 18 October 2001; revised 6 November 2002; accepted 29 November 2002

# Abstract

Hydrographs were recorded at three discharge stations at a small alluvial stream during the summers of 1997 and 1998. A method of analysis was set up so that the temporal variations in discharge resulting from natural hydrological processes can be distinguished from the influence from ground water being periodically abstracted approximately 60 m from the stream. Thus, evapotranspiration from the riparian zone resulted in diurnal variations in streamflow with maximum amplitude of 3-5 l/s, whereas heavy rainfall resulted in intense short-term surface/subsurface flow from the riparian zone. The magnitude of peak-flow was of the order of 2-3 times baseflow and such events typically disturbed the hydrograph for one to three days. Baseflow increased from about 35 l/s to about 70 l/s over the studied reach, but when ground water was abstracted at rates of about 15 l/s this resulted in a reduction in discharge. Within 4-8 days the reduction stabilized at about 4 l/s and at 5-7 l/s, respectively, at discharge stations 140 and 350 m downstream of the well site. Predictions made by the analytical depletion model by [Ground Water, 37 (1999) 98] using the values of transmissivity and conductance of the streambed estimated by [Ground Water, 40 (2002) 437] by drawdown analysis compare reasonably well to the observed reduction of streamflow.

Keywords: Streamflow depletion; Hydrograph analysis; Ground water abstraction; Alluvial stream; Measurement uncertainty

# 1. Introduction

In areas in Denmark where sandy soils prevail abstraction of ground water for irrigating crops during dry summer periods is important. Abstraction from an aquifer, which is hydraulically connected to a nearby stream, will inevitably result in reduced groundwater seepage (baseflow) to the stream and eventually this may be harmful to its fauna. Depletion caused by abstraction has traditionally been predicted using the model by Theis (1941). This is based on the assumptions that a homogeneous, infinite aquifer is completely penetrated by the stream. Hantush (1965) considered pumping near a completely penetrating stream whose bed is lined with semi-pervious material while for pumping near a stream that only partially penetrates the aquifer and exchanges water with the aquifer through a semi-permeable streambed Hunt (1999) derived an analytical solution. In his model the drawdown from the pumping well can propagate in all directions in the aquifer, including areas on

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the opposite side of the stream from the abstraction point. Furthermore, the solution by Hunt (1999) includes the earlier solutions by Theis (1941) and Hantush (1965).

Several other authors have studied streamflow depletion using analytical or numerical models (e.g. Fox et al., 2002; Sophocleous et al., 1995; Spalding and Khaleel, 1991), but so far there are only a few successful field investigations (e.g. Hunt et al., 2001; Madsen, 1988) where the rate of depletion has been measured and compared to predicted values. The likely explanation is that the discharge in streams is usually large compared to the rate of pumping from a single well (Hunt, 1999). Furthermore, often the transient variations in discharge caused by variations in surface runoff and interflow exceed streamflow depletion.

Therefore a field study involving a pumping test was planned in the upper perennial reach of an alluvial stream where baseflow is low. There was no ongoing abstraction in the catchment so conditions were favourable for detecting even moderate changes in streamflow resulting from abstraction. Surface-water discharges and ground-water levels were measured during the summers of 1997 and 1998 as groundwater was abstracted near the stream for several weeks. Data were analysed by Nyholm et al. (2002) using numerical flow models, but the analysis revealed substantial problems in fitting a model to all observed data. The analysis also pointed out that it is important to have accurate measurements of streamflow depletion and estimates of the measurement uncertainty for model validation.

The purpose of this paper is to demonstrate a method that distinguishes the effects of groundwater pumping from evapotranspiration and storm events in stream discharge records. The reported methods were applied in a humid climate. The depletion of stream discharge by abstraction from an adjacent aquifer and the associated uncertainty were estimated. A second aim is to compare the measured depletion data with corresponding predictions made by the analytical model of Hunt (1999), but based on observations of hydraulic head and drawdown in the aquifer.

The paper is organised so that at first the experimental set-up is described. Then follows an analysis of the transient influences on the hydrograph resulting from the seasonal growth of weeds, interflow from the riparian zone after rainstorms, diurnal variation in evapotranspiration, and seasonal variation in baseflow. Finally, the reduction of baseflow resulting from ground water abstraction is estimated and the rate of depletion is compared to values predicted using the analytical model of Hunt (1999).

# 2. Field site and collected data

# 2.1. Hydrogeology

The catchment of the small Haller stream is part of the regional Karup catchment in central Jutland just beyond the limits of the main ice from the Weichselian glaciation (Fig. 1). In front of the ice sheet the meltwater deposited coarse sediments (mostly sand and gravel) that form a large flat outwash plain sloping gently to the NW at about 2– 3 m/km (Nyholm, 2000a). A phreatic aquifer is found below a 5–10 m deep unsaturated zone, and both units are composed of medium to coarse sand and gravel. Data from a deep observation well about 60 m from the stream indicated that at about 20–30 m depth, the aquifer is underlain by at least 50 m of clay/ silt sediments having low hydraulic conductivity (Nyholm et al., 2002).



Fig. 1. Location of the regional Karup catchment and the experimental site at the Haller å tributary.



#### 2.3. Streamflow

The narrow flood plain along the Haller stream is flat and confined to a 100 m wide valley. The valley sides are less than 2 m high at the upstream end and gradually increase to 8 m high 2 km downstream. The distribution of sediments on the flood plain is typical (e.g. Fetter, 1994; p. 321 et seq.). Thus sand is found along the valley sides, whereas peat or alluvial sand/ silt with a high organic content dominate the uppermost 1-2 m of the riparian zone. At many places, it was observed that the organic/organic-rich sediments in the riparian zone have a permeable top-layer while below this, there may be a decomposed peat or organic-rich layer having low permeability. Thus in the riparian zone ground water from the underlying aquifer must flow through the semi-permeable peat in order to reach the permeable top-layer at the surface.

The first 2 km of the stream was channelled early in the 20th century including the removal of the peat below the streambed. Thus below the stream exchange of water between the stream and the aquifer will take place through a (semi-) permeable streambed as is commonly observed for alluvial streams (e.g. Winter et al., 1998; Domenico and Schwartz, 1998, p.18).

The organic/organic-rich sediments are close to saturation except after long dry spells, and watertolerant plants dominate the vegetation on the flood plain while grasses and shrubs are found along the valley sides. The vegetation on the outwash plain consists of farm crops, grass fields, and plantations with mixed plots of spruce or fir.

# 2.2. Climate

The Danish Meteorological Institute records daily precipitation near the field area, and the average annual rainfall over the catchment was 725 mm for the period 1961–90 (Frich et al., 1997). Annual evapotranspiration of 350 mm/year was estimated using a root zone model (Olesen and Heidmann, 1990). On the studied reach the Haller has no tributaries and yearly runoff is totally dominated by baseflow. Therefore the average recharge is estimated as precipitation less evapotranspiration, i.e. approximately 375 mm. A similar value was obtained for the average regional recharge in the regional Karup catchment (Miljøstyrelsen, 1983).

The stream is intermittent along its upper course, but only the perennial part was considered in the present investigation (Fig. 2). Three gauging stations, each with a well-defined cross section were set up at the start of the study. Station S2a was placed about 1000 m downstream of the stream source while stations S2 and S1a were placed about 425 and 800 m downstream from S2a (Fig. 2). At S1a the 1.5 m wide bed was covered with flagstones for a distance of 10 m. Vertical plating bounded the flagstones creating a rectangular stream cross section.



Fig. 2. Map of the regional piezometric surface. The positions of pumping wells, observation wells, re-infiltration site, and stream discharge stations (S1a, S2, and S2a) are marked. Bounding streamlines for S1a and S2a are also marked.



At S2 and S2a only 3 m of vertical plating was set and the flat gravel bed was left unchanged. At each station the local gradient of the stream was found as the average water surface slope over the nearest 50 m.

During summer frequent stage and discharge control measurements were made at S1a, S2 and S2a using a rule and a current meter. At each station, 5-6 vertical velocity profiles were recorded. These were fairly uniform indicating that satisfactory measurement conditions were established. In addition to the current meter data stage, velocity, and water temperature were initially monitored continuously at each station by means of a STARFLOW-6526 instrument (UNIDATA, 1994). The instrument was placed on the streambed at the centre of the stream where it continuously recorded the local depth  $(D^*)$ using a pressure transducer while the average velocity  $(V^*)$  of the vertical velocity profile above the instrument was recorded using an acoustic sensor. The  $D^*$ - and  $V^*$ -values stored every 15 min were averages based on readings every second, and by recording both velocity and depth continuously it was possible to assess the temporal change of channel roughness between discharge control measurements.

Waves on the stream surface and sediment transport (saltation) can cause small as well as unrealistically large fluctuations (spikes) in the recorded velocity (UNIDATA, 1994), which complicates the analysis. Unfortunately, in addition to this, there are major gaps in the data series because vital parts of every STARFLOW instrument corroded within the first year and despite that failing instruments were replaced successive losses were caused by new failures. However, for the periods where acceptable  $D^*$  and  $V^*$ -data were collected, the continuous discharge ( $Q^*$ ) was found as

$$Q^* = CB_{\rm st} V^* D^* \tag{1}$$

where  $B_{st}$  is the width of the stream. The value of *C* was determined by minimising

 $\Sigma (Q - CB_{st}\bar{V}^*\bar{D}^*)^2$ , where  $\bar{D}^*$  and  $\bar{V}^*$  are the averages of  $D^*$  and  $V^*$  for the intervals where the discharge (Q) was measured using a current meter. Usually the comparison comprised 5–10 current meter measurements made over a period of about 1–2 months. For S1a, for instance, C was 0.855 during days 185–233 and 0.893 during days 233–302 in 1997. In the summer of 1998, the value of C for S1a

varied even less and was 0.962 during days 225-245and 0.971 during days 245-275. The standard deviation between Q and  $Q^*$  is typically in the range 0.5-1 l/s. Similar small temporal variations in the values of C were recorded at the other stations. Finally, in order to reduce the influence of noise in the  $D^*$ - and  $V^*$ -data the  $Q^*$ -series were low-pass filtered using a 6 h moving average.

## 2.4. Ground water flow

The hydraulic head was measured in 31 wells within and near the catchment and in 16 shallow piezometers along the stream so that a piezometric map could be constructed (Fig. 2). The length of the screen and its depth varies from well to well so that the data on hydraulic head refer to different depths in the aquifer. However, none of the wells were close to the stream where the head changes most rapidly with depth. Furthermore, in an observation well with several short screens at different depths (Nyholm et al., 2002) the head at 24 m depth exceeded the head at 5 m depth (~ground water table) by only 2 cm. So this (presumably small) error was ignored.

The catchment extends approximately 4 km SE of S1a (Fig. 2). There is no information in the geological data to suggest that horizontal anisotropy is important. Therefore, Fig. 2 shows that the ground water flows from the divide in NW direction towards the stream between S2a and S1a. The information also indicates that north of the stream between S2a and S1a the ground water flows almost parallel to the stream, while near the eastern intermittent course, the ground water flows away from the stream in a northerly direction.

#### 2.5. Pumping test experiments

Two pumping wells (P1 and P2) were drilled about 60 m from the stream just upstream of S2 (Fig. 2). In P1 the screen is placed from about 8–14 m below the water table, and in P2 the screen is from the water table to about 8 m below. In 1997 during the period 25 July–26 September (days 206–269) water was abstracted from P1 at a rate which initially was  $Q_w = 16.4$  l/s. The abstracted water was pumped approximately 500 m through a pipeline into the plantation SW of S2 (Fig. 2). The optimum place to



release the water would have been in the stream downstream of S1a, but this was impossible because the concentration of ochre in the abstracted water is harmful to fish in the stream and in the hatcheries downstream of S1a. The pumping rate was observed manually on a water-meter while the drawdown was monitored in all nearby wells. After approximately 10 days of pumping ochre precipitated in the pipeline so that between day 215 and day 240 the pumping rate decreased by 2.8 l/s from its initial value (Nyholm and Rasmussen, 2000).

During the summer of 1998, two pumping tests of well P2 were made. From 24 July to 31 July (days 205–212) the abstraction rate was 14.7 l/s while from 11 August to 8 September (days 223–251) it was 14.4 l/s. No technical problems occurred this summer, but the discharge in the stream was high and fluctuating because of abnormally high precipitation.

#### 3. Results

After a short presentation of the flow characteristics at the discharge stations during summer and a discussion of the measurement error the principal components of the hydrograph, i.e. quick responding surface/sub-surface flow, diurnal variations in streamflow, and baseflow will be considered. Finally the influence from abstraction will be discussed.

#### 3.1. Flow characteristics at the discharge stations

Daily rainfall, discharge control measurements, and continuous discharge data  $(Q^*)$  are presented for days 180-300 during 1997 in Fig. 3(a) and for days 160-280 during 1998 in Fig. 3(b). During 1998 the measurement error at S1a is higher than for the rest of the data (see below). Although the measurement error is still acceptable according to normal standards (e.g. Chow et al., 1988) the data are of little value to the depletion study and they have been omitted from the following figures. Instead selected results will be given in the text. In Fig. 3 the solid line above the labels on the x-axis indicates the period(s) during which ground-water was abstracted from either P1 or P2. The letter 'P' marks the day where pumping starts or ends. Overall, the discharge increased downstream, and during rainless periods in 1997 the discharge



Fig. 3. Precipitation and discharge at stream discharge stations during the summer periods of 1997 (a) and 1998 (b). Continuous discharge ( $Q^*$ ) is indicated by a solid line; current meter based control measurements by  $\diamond$ .

values were approximately 30 l/s at S2a, 45 l/s at S2, and 65 l/s at S1a. During 1998 the values were approximately 35, 50 and 70 l/s at the same three stations. Based on low-pass filtered discharge values the largest observed discharge during summer was estimated to near 100 l/s at S1a.

For uniform flow in a channel with a rectangular cross section of width  $B_{st}$  and depth D (so that the area  $A = D B_{st}$ ) the discharge (Q) is given by the Manning-equation (Chow et al., 1988)

$$Q = AMR^{\frac{2}{3}}I^{\frac{1}{2}} \tag{2}$$

where *M* is Manning's number ( $M = n^{-1}$  where *n* is Manning's roughness coefficient), *R* is the hydraulic radius, and *I* is the slope of the energy line. *I* was



assumed to be the same as the water surface slope and constant during summer. It is required that the flow is fully turbulent in order for Eq. (2) to be valid (Chow et al., 1988).

By re-arranging Eq. (2) the friction of the streambed and the sides can be expressed in terms of the Manning number

$$M = \frac{Q}{AR^{\frac{2}{3}}I^{\frac{1}{2}}}$$
(3)

Generally M decreases steadily during summer periods while weeds growing in the stream and along the banks increase in density and size. The changes in M are expected to be gradual and monotone since no man-made changes were made upstream of S1a. For the 1997 data (Fig. 4(a)) the value of M at S2a was  $30-35 \text{ m}^{1/3} \text{ s}^{-1}$  until the beginning of July when a seasonal decrease started. Over the next two months, M decreased to  $10-15 \text{ m}^{1/2}$  $^{3}$  s<sup>-1</sup>. At S2 the *M*-value decreased almost linearly from the initial level of about 25 m<sup>1/3</sup> s<sup>-1</sup>, to 10–  $15 \text{ m}^{1/3} \text{ s}^{-1}$  which is similar to the final value at S2a. At S1a the growth of weeds was intense downstream of the station causing backwater effects and reducing the flow velocity at the gauging station. Thus, the Manning numbers calculated from (3) will reflect the increased depth and not the small bed roughness. The initial *M*-value was about 8 m<sup>1/3</sup> s<sup>-1</sup>, but as the weeds



Fig. 4. The values of Manning's number (M) at stream discharge stations during the summer periods of 1997 (a) and 1998 (b).

downstream of the station grew it dropped to only about  $5 \text{ m}^{1/3} \text{ s}^{-1}$  at day 190. Then it gradually increased to  $7 \text{ m}^{1/3} \text{ s}^{-1}$  at day 220 and remained constant during the remaining part of the summer. The reason for the slight increase is unknown.

The spring and summer in 1998 were cold and the weeds developed poorly and late. Thus at both S2a and S2 the *M*-values were similar starting at about  $30 \text{ m}^{1/3} \text{ s}^{-1}$  at day 200 (Fig. 4(b)). During summer, *M* gradually decreased to just over  $20 \text{ m}^{1/3} \text{ s}^{-1}$  at day 280.

For each of the stations the random scatter of Mwas small indicating that the measurement uncertainty is also small. Since in the measurement of Qthere are two unknowns (depth and velocity) as well as a temporal change in the channel roughness (M) it is not straightforward to assess the uncertainty on the discharge. Thus it was estimated using a statistical model (Appendix A) which is based on several simplifying assumptions. At the gauging stations, for instance, the width of the stream  $(B_{st})$  was 5–10 times larger than the depth (D) so the hydraulic radius R-D. Furthermore, the variation of Q was small so it is assumed that the slope of the water surface (I) is independent of Q. Finally, the variation of M was approximated by either one or two linear segments during each summer period in order to estimate the measurement errors on the discharge values  $(\pm 2\nu)$ ; Appendix A).

Results from the analysis are given in Table A1. Generally the standard deviation of observation errors of the logarithmic depth ( $\tau$ ) is small except at S2a during 1997 and at S1a during 1998. At S2a the level of the bed was influenced by intermittent sediment transport during 1997. Occasionally sand deposited along the south side of the stream and influenced the measurement profile. This is reflected in the relatively large variation of M (Fig. 4(a)) and in the fairly high values of  $\tau$  and probably also in the fairly high values of the standard deviation of logarithmic depth ( $\omega$ ). At S1a there was no backwater during 1998. Combined with the smooth bed this resulted in a shallow flow and complicated the assessment of the velocity profile. The measurement uncertainties of the discharge values at the three stations are (Table A1):

1997—days 186–302: 
$$\pm 2\nu_{1a} = \pm 1.9 \text{ l/}$$
  
s;  $\pm 2\nu_2 = \pm 1.3 \text{ l/s}$ ;  $\pm 2\nu_{2a} = \pm 2.5 \text{ l/s}$ ;



1998—days 167–273: 
$$\pm 2\nu_{1a} = \pm 3.5$$
 l/s  
 $\pm 2\nu_2 = \pm 1.6$  l/s;  $\pm 2\nu_{2a} = \pm 1.1$  l/s.

Typically  $\pm 2\nu = \pm 1-2$  l/s which is a fairly small measurement error. This indicates that the effort of creating flume like conditions at the gauging stations was valuable except at S1a during the exceptionally wet year 1998.

# 3.2. Surface/sub-surface flow $(Q_R)$

Inspection of the rainfall and discharge series shows that when isolated rainfall events of less than about five millimetres follow a dry period of a week or more, the discharge remains unaffected by the rain. In such cases precipitation is intercepted by the vegetation or stored in the root zone so that rainfall must exceed a threshold value before the excess rain (the effective precipitation; Shaw, 1994, p. 321) generates quick surface/subsurface runoff,  $Q_{\rm R}$ .

During intense summer storms, or a series of smaller storms, precipitation exceeds the threshold value and creates sudden peaks in the discharge. This is illustrated using  $Q^*$ -data from rain periods for which the  $D^*$  and  $V^*$  records are of good quality at one station at least. The first event includes days 208-209 at the start of the pumping test in 1997 (Fig. 3(a)) when the data from S2a and S2 were of good quality. One major rainfall of 29.6 mm fell on day 208 while on each of the three previous days and on day 209 the precipitation was between four and six mm (Fig. 5(a)). However, only the heavy rain on day 208 had a noticeable influence on  $Q^*$ . Because ground water abstraction started on day 206 a linear trend was fitted to the five control measurements between day 206 and day 215 in order to estimate the slight decrease in baseflow. Finally, in order to assess  $Q_{\rm R}$  the trend was subtracted from the discharge series. The duration of the  $Q_{\rm R}$ -peak (Fig. 5(b)) was less than one day, and the maximum value was 2-3 times larger than the baseflow component, but field inspection gave no indication of surface runoff outside the flood plain. Combined with the short response time this indicates that  $Q_{\rm R}$  is most likely a rapid surface/sub-surface contribution from the topographically low meadows adjacent to the stream as observed in other watersheds in humid climates (Dunne and Black, 1970; Hewlett and Nutter, 1970) and in agreement with the partial

area contribution concept (Ragan, 1968). The hypothesis is supported by the following simple calculations. Let the area  $(A_R)$  of the zone contributing to  $Q_R$  be  $A_R = L_R \times W_R$  where  $L_R$  is the distance along the stream from the source to the discharge station where  $Q_R$  was found and  $W_R$  is the constant width of the zone. During a rainfall of size  $P_e$  the volume  $(V_e)$  of water on the zone is  $V_e$  ( $= P_e \times A_R$ ). This corresponds to the volume of the rapid discharge  $(V_R = \int Q_R dt)$  if influences from evapotranspiration and changes in storage in the riparian zone are neglected. For the rain on day 208 a value of  $W_R \sim 30$  m was calculated for the two stations (Table 1).

On rainy days evapotranspiration usually will be one to two mm at most so it is fair to neglect



Fig. 5. Rainfall-runoff relationships for a large rainfall during 1997: (a) hydrographs for S2 and S2a; (b) rapid surface/subsurface component at S2 and S2a. Please note differences in scaling of axis.

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Table 1 Parameters derived for the rapid surface/surface component ( $Q_R$ ) for three (large) rainfall events

Period (days)	Station	Duration(s)	Precipitation, $P_{e}$ (mm)	Volume, $V_{\rm R}$ (m <sup>3</sup> )	Length, $L_{\rm R}$ (m)	Width, $W_{\rm R}$ (m)
208-211, 1997	S2			$1.2 \times 10^{3}$	1425	29
	S2a	$9.9 \times 10^4$	29	$9.5 \times 10^2$	1000	33
245-252, 1997	S2a	$4.3 \times 10^{5}$	28	$1.4 \times 10^{3}$	1000	50
233–238, 1998	S2	$3.5 \times 10^{5}$	34	$2.6 \times 10^{3}$	1425	54

influences from evapotranspiration. The maximum thickness of the unsaturated zone along the stream is approximately two decimetres, but at most places it is only few cm thick. Since the specific yield of the peat was estimated to  $S_y = 5\%$  (Nyholm, 2000b) only a small fraction of  $P_e$  was involved in the change in storage in the unsaturated zone. Furthermore, for rainfalls similar to the one discussed here, piezometer observation showed a considerable change (decimetres) in the piezometric level in the peat during rainfall, but within one day after rainfall ceased the level deviated less than 0.10 m from its value prior to rainfall. The change in level corresponds to a change of storage of about 5 mm, i.e. only a small fraction of  $P_e$ .

A similar analysis was made using data from two other periods with large rainfalls. During the period between day 244 and 254, for instance, several precipitation events occurred. Some events were short, less than an hour, while others lasted some hours. Thus the maximum precipitation of 15.3 mm fell on day 246 and resulted in a peak on the hydrograph lasting about one day at S2a. Following the same principles as sketched above the conversion of data gave  $W_{\rm R} \sim 50$  m for this event (Table 1). The last event comprises the discharge at S2 between days 231 and 240 during the extremely wet summer of 1998. After a couple of weeks with frequent small rainfalls, a large storm on days 233 and 234 resulted in a total rainfall of 32 mm. The discharge increased quickly on day 233, but on day 238 it had returned to the level prior to the storm. For this case  $W_{\rm R} = 54$  m (Table 1), which is the largest of the calculated  $W_{\rm R}$ values. Thus the data from the flat Haller catchment indicate an expansion of the saturated area with rainfall characteristics similar to observations made in a small, hilly watershed in Vermont (Dunne and Leopold, 1978). Thus for short intense rainfalls during the summer period it is safe to assume that their

influence on the hydrograph is negligible within one to two days. When several days of rainfall occur it may take up to almost a week before baseflow is undisturbed by quick responding surface/sub-surface flow.

#### 3.3. Diurnal variations in the discharge

In the low-pass filtered  $Q^*$ -series diurnal variations in the discharge can be observed during rainless summer periods (Fig. 6). The discharge is minimum in the afternoon (2–4 pm) and maximum during the night. Typically, the diurnal variations are of the order of 1–2 l/s, but at S2a and S1a they can amount to about 3 and 5 l/s, respectively. The small variations occur on overcast or rainy days while the large variations occur on days with maximum incoming solar radiation. Therefore it seems likely that the variation can be ascribed to the uptake of water by plants from a waterlogged riparian zone as observed at headwater stations in a larger Danish stream (Erup, 1981). Thus discharge values recorded for instance near 8 am or 6 pm when discharge is close to its daily



Fig. 6. Diurnal variations of streamflow during summer 1998 (days 200–214). The first abstraction period starts day 205.

average are relatively unaffected by the diurnal variation of evapotranspiration from the riparian zone.

#### 3.4. The baseflow component

Comparison of the values during rainless periods reveals that baseflow increases downstream by about  $0.04 \text{ ls}^{-1} \text{ m}^{-1}$ . The ratio between baseflow at S2 and S2a is 1.4 while for S1a and S2a it is 2. Likewise, the ratio between the distances from the head of the stream to S2 and from the head of the stream to S2a is 1.4, while the ratio between the distances from the head to S1a and from the head to S2a is 1.8. The similarity of the baseflow and length ratios indicates that the seepage of ground water to the stream is uniform upstream of S1a.

During a period with constant (or zero) recharge, baseflow is usually described by an exponential-type process  $(Q = Q_0 \exp[-t/c])$  where c is the recession coefficient (Tallaksen, 1995). At S2a 350 m upstream from the abstraction point where (delayed) influence from pumping hardly can be detected (Fig. 3(a)), the recession curve is smooth and approximately follows a log-linear trend. This indicates that apart from influences from rapid sub-surface flow and diurnal variations caused by evapotranspiration from the riparian zone there are probably no other short-term variations that significantly influences baseflow during summer. Therefore, the discharge data from S2a were used to estimate the seasonal decrease of baseflow during summer periods. In 1997 the total decrease of baseflow was 9 l/s from about day 215 (35 l/s) to day 270 (26 l/s). This corresponded to a recession coefficient  $c = 1.6 \times 10^7$  s.

During the summer of 1998 large amounts of precipitation fell between days 177 and 200 after an otherwise cold and wet spring (Fig. 3(b)). Control measurements were few in early summer, and the start of the seasonal decrease (day 200) was estimated from the behaviour of the  $Q^*$ -values at S1a and S2. On day 200 the discharge was 39 l/s at S2a. Likewise, the data from S2a indicated that the decrease of baseflow ceased at about day 270 when the discharge was 29 l/s. Thus, during 1998 baseflow was slightly higher than during 1997, primarily because of expansion of the area contributing to baseflow upstream of S2a. Thus the recession coefficient at S2a was  $c = 2.0.10^7$  s which is only slightly higher than the 1997-value, and

it seems fair to assume that the temporal change of baseflow between day 200 and day 270 can be removed by assuming a simple exponential decrease.

#### 3.5. The influence from abstraction

The discharge at S1a and S2 was reduced by the abstraction of ground water from the wells P1 and P2 near the stream (Fig. 2). Thus, the control measurements indicate that relative to the undisturbed baseflow level prior to abstraction there was a detectable decrease in the discharge (i.e. larger than the estimated uncertainties of the discharge) within one to two days after pumping started while a slight increase occurred when pumping ended (Fig. 3). At S2a no similar change in the discharge was observed. Therefore the seasonal drift of baseflow was removed from the discharge data using the S2a-recession curve found in Section 3.4.

The drift corrected control measurements and the continuous measurements at S2a (Fig. 7(a) and (b)) scatter around a horizontal line with a variation of about  $\pm 2-3$  l/s except for a few higher discharge values influenced by rainfall. On rainless days prior to abstraction the corrected measurements at S1a and S2 fluctuate around constant levels ( $Q_{ini}$ ). When abstraction started there was a decrease in discharge that can be detected for approximately one week. During the abstraction period in 1997 and the second period in 1998 the discharge at each station then seemed to fluctuate about an almost constant level  $(Q_{low})$  for approximately 2-3 rainless weeks before rainfall influenced the discharge. Finally, during the last week of the pumping period the discharge values at S1a and S2 once more approached  $Q_{low}$ . When pumping stopped the increase in the discharge was sudden, i.e. within 1-2 days, but after approximately one week the drift-corrected control measurements again fluctuated around  $Q_{ini}$ .

The baseflow level that corresponds to the maximum influence from abstraction  $(Q_{low})$  was found as the average of the trend-corrected control measurements that were observed within the abstraction period after more than 5 days of pumping (Table 2). Only data that were recorded on days with less than 5 mm of precipitation were included in the calculations. Likewise, the undisturbed value of baseflow prior to abstraction  $(Q_{ini})$ 



Fig. 7. Discharge corrected for the seasonal reduction of baseflow during the summer periods of 1997 (a) and 1998 (b). Values are given from just before pumping starts to 10 days after pumping stops.

was calculated as the average value of the trendcorrected control measurements recorded either before abstraction started or more than five days after it ended (Table 2). The standard errors ( $\sigma_{ini}$ and  $\sigma_{low}$ ) are also listed in Table 2.

The maximum value of the reduction of streamflow  $(\Delta Q)$ , was found as  $\Delta Q = Q_{\text{ini}} - Q_{\text{low}}$ . When  $\sigma_{\text{t}}$  denote the standard error of  $\Delta Q$  then  $\sigma_{\text{t}} = (\sigma_{\text{ini}} 2 + \sigma_{\text{low}} 2)^{1/2}$ ,

and the uncertainty of  $\Delta Q$  is  $\pm 2\sigma_t$  (Table 2). The calculations indicate that at S1a the reduction after more than 5 days of pumping is 46% of the average abstraction rate while the reduction at S2 is only 27%.

Two pumping tests were performed during 1998, but as explained earlier the quality of the data for S1a is less good than during 1997. The first abstraction period was from day 205 to day 212, but precipitation started a few days within the abstraction period and disturbed the discharge values. The second test was performed between days 223 and 251. Here the discharge was influenced by precipitation on days 227-228, but otherwise the relative influence was estimated as explained above (Table 2). Again heavy precipitation disturbed the period after abstraction ended. The maximum values of the reduction ( $\Delta Q$ ) are similar to the values found for 1997, but the large uncertainty at S1a makes this estimate less useful than the 1997 value.

A constant streamflow depletion could occur if the stream becomes perched (e.g. Rushton, 1999), but observations in piezometers along the stream show that the hydraulic head in the aquifer never dropped below the bottom of the stream (Nyholm et al., 2002). However, the influence from abstraction increases logarithmically with time. Therefore, beyond the first week the change in discharge due to abstraction was comparable to the measurement error and thus could not be detected. The analysis also indicates that the influence from abstraction is somewhat smaller at S2 than at S1a. This is expected since the wells P1 and P2 are placed in the central part of the ground water flow to the stream (Fig. 2) so that both ground water flowing towards the stream between S2 and S2a and between S2 and S1a is diverted towards the pumping well.

Table 2

Values of baseflow before abstraction starts ( $Q_{ini}$ ), its standard error ( $\sigma_{ini}$ ), and the standard error for the discharge measurements made on rainless days from five days within the pumping period ( $\sigma_{low}$ ). Values of the long-term reductions of baseflow ( $\Delta Q$ ) and the assessed experimental uncertainty ( $\pm 2\sigma_i$ ) are also given

	S1a-1997	S2-1997	S1a-1998	S2-1998
$Q_{\rm ini}$ (l/s)	69.4	48.3	70.5	48.5
$\sigma_{\rm ini}$ (l/s)	0.7	0.7	3.29	1.86
$\sigma_{\rm low}$ (l/s)	0.5	0.51	0.39	0.38
$\sigma_{\rm t} = (\sigma_{\rm ini}2 + \sigma_{\rm low}2)^{1/2}$	0.9	0.9	3.3	1.9
$\Delta Q \pm 2\sigma_{\rm t}  ({\rm l/s})$	$6.6 \pm 1.8$	$4.0 \pm 1.8$	$7.8\pm 6.6$	$4.3 \pm 3.8$



The transient change in baseflow at the two stations that takes place during the first 8-10 days of an abstraction period is most clearly depicted by the trend-corrected  $Q^*$ -values that are of best quality. Thus, values  $(Q_{1a,p}^* \text{ and } Q_{2,p}^*)$  were selected the first 8-10 days after abstraction started, but only at 8 am and 6 pm in order to avoid the influence from diurnal variations in baseflow. For days that were undisturbed by precipitation the relative temporal change of baseflow at a station, e.g. at S1a, was found as  $\Delta Q_{1a}^*/Q_w = (Q_{1a,p}^* - Q_w)^*$  $Q_{1a,ini})/Q_w$  where  $Q_w$  was the rate of pumping. The temporal change of baseflow during the first 8 days after abstraction ended was also computed using values  $(Q_{1a,r}^* \text{ and } Q_{2,r}^*)$  recorded at 8 am and 6 pm at the two stations. Here the relative temporal change of baseflow at a station, e.g. at S1a, was found as  $\Delta Q_{1a}^*/Q_w = (Q_{1a,r}^* - Q_{1a,end}^*)/Q_w$  where  $Q_{1a,end}^*$  was the continuous baseflow value just before abstraction ended. However, due to missing or bad data or intense rainfall some periods were discarded. Thus for both stations data recorded during and after abstraction in 1997 were accepted, while for 1998 only data recorded during the second abstraction period were accepted. At both stations there was considerable data scatter so the three data series were merged so that only the average stream depletion  $(\Delta Q/Q_w)$  is presented (Fig. 8(a) and (b)) together with error bars for the 95% confidence limits. Values are plotted versus a dimensionless time  $(4Tt/a^2S_y)$ , where  $S_y$  is the specific yield of the water table aquifer, T is transmissivity, and a is the shortest distance between the well and the stream.

# 4. Comparison of streamflow depletion data and analytical model predictions

Hunt (1999) derived an analytical solution for the influences from pumping from a fully penetrating well near a stream that has a semi-permeable streambed and only partially penetrates the isotropic and homogeneous aquifer. The stream is linear, placed at x = 0, and extends from  $y = -\infty$  (infinite upstream) to  $y = \infty$  (infinite downstream). When water is abstracted at a constant rate  $Q_w$  from time t = 0 to t,



Fig. 8. Measured streamflow depletion  $(\Delta Q/Q_w)$  and 95% confidence limits plotted as function of dimensionless time  $(4Tt/(S_ya^2))$ . Data obtained at S1a (a) and S2 (b) are average values (for the same dimensionless time) recorded during abstraction in 1997 and 1998 and after abstraction ended in 1997. Broken lines show predicted streamflow depletion at discharge station S1a, S2, S2a, and for an infinitely long stream using the model by Hunt (1999).

the relative reduction of streamflow is

$$\frac{\Delta Q}{Q_{\rm w}} = \operatorname{erfc} \sqrt{\frac{S_{\rm y} a^2}{4Tt}} - \exp\left(\frac{\lambda^2 t}{4S_{\rm y}T} + \frac{\lambda a}{2T}\right)$$
$$\operatorname{erfc} \left(\sqrt{\frac{\lambda_2 t}{4S_{\rm y}T}} + \sqrt{\frac{S_{\rm y} a^2}{4Tt}}\right)$$
(4)

where  $\lambda = K'b/m'$  is the leakage factor of the streambed which has hydraulic conductivity (K'),

width (b) and thickness (m') and the remaining parameters are as defined earlier. The position on the stream where a is found defines y = 0.

For a finite stream reach with a discharge station placed at  $y = y_L$  the stream depletion at this station is obtained by integrating leakage q(y, t) along the y-axis from  $-\infty$  to  $y_L$ ,

$$\frac{\Delta Q}{Q_{\rm w}} = \frac{1}{Q_{\rm w}} \int_{-\infty}^{y_{\rm L}} q(y,t) \mathrm{d}y = \frac{\lambda}{Q_{\rm w}} \int_{-\infty}^{y_{\rm L}} \varphi(0,y,t) \mathrm{d}y \quad (5)$$

where  $\varphi(x, y, t)$  is the drawdown as function of time, *t*, and the co-ordinates *x* and *y* (Hunt, 1999; Eq. (30)):

$$\varphi(x,y,t) = \frac{Q_{w}}{4\pi T} \left\{ E_{I} \left[ \frac{(a-x)^{2}+y^{2}}{4\pi T/S} \right] - \int_{0}^{\infty} e^{-\theta} E_{I} \left[ \frac{(a+|x|+2T\theta/\lambda)^{2}+y^{2}}{4\pi T/S} \right] d\theta \right\},$$

$$(-\infty < x < \infty, -\infty < y < \infty, 0 < t < \infty)$$
(6)

In order to compare measured depletion to predictions using Eqs. (4) or (5) the dimensionless time  $4Tt/S_{\rm v}a^2$  and the leakage coefficient  $\lambda$  must be evaluated. As mentioned in the introduction numerical models were set up as part of the study. Values of aquifer transmissivity  $(T = 0.01 \text{ m}^2 \text{ s}^{-1})$  and streambed leakage factor ( $\lambda = 1.6 \times 10^{-4} \text{ ms}^{-1}$ ) were obtained from calibrating a steady-state numerical ground water model to observations of the hydraulic head in the aquifer and baseflow at 4 stations along the Haller stream (Nyholm et al., 2002). The specific yield was estimated to  $S_y = 10\%$  from drawdown analysis (Nyholm et al., 2002), but the value is unreasonably low for Danish outwash plains. Miljøstyrelsen (1983), for instance, reports values of  $S_{\rm v} = 25\%$  or more as typical for the region, and laboratory drainage experiments on undisturbed core samples from near the ground water table at P1 and P2 give values of  $S_v = 25\% - 30\%$  (S. Christensen, unpublished data). Therefore it was decided to use  $S_{\rm v} = 25\%$  in the calculations of the predicted depletion at the discharge stations as well as for a station infinitely downstream from the abstraction site (Fig. 8).

Inspection of Fig. 8(a) shows that the measured depletions at S1a falls below the predicted values during the first 3–4 days of the abstraction period

(Fig. 8(a)); but the difference is not significant. During the last approximately 3 days the predicted depletion lies closer to the observed data. At S2 measured data and predicted values coincide during the initial and last part of the period (Fig. 8(b)), but during the middle part observed depletion is systematically below predicted. Also here the differences between measured and predicted values are not significant except for a few values. Therefore the observed discrepancies may be mostly caused by measurement error. The fact that measured depletion predominantly falls below the predicted values during the early stages of the abstraction period might indicate that violation of some of the assumptions that Eq. (4) is based on may be another cause. Thus, some vertical anisotropy of the aquifer is observed (Nyholm et al., 2002) and neither the screen in P1 or P2 fully penetrates the aquifer. In particular when pumping is close to the stream and relatively deep in the aquifer this may produce a lag in the observed influence on discharge relative to the analytical prediction.

# 5. Discussion

The sub-catchment that contributes baseflow to the stream between S2a and S1a is almost entirely located SE of the reach (Fig. 2). The bounding streamlines for the sub-catchment are indicated on Fig. 2 and its area estimated to  $\sim 2.8 \times 10^6$  m<sup>2</sup>. If evenly distributed over the year and the area, the recharge rate  $R_{\rm ch} = 375$  mm/y will produce an average increase in baseflow between S2a and S1a of 33 l/s. The increase in baseflow as measured between S1a and S2 is 35 l/s, i.e. almost the same value as found from the simple mass balance. There is no indication in the hydraulic head data that underflow is important so the close agreement between estimated and measured increase in baseflow indicates that the average recharge rate of the aquifer has been assessed fairly accurately.

In the analysis of the influence from pumping it was assumed that streamflow at S2a was uninfluenced by abstraction because of the distance from the pumping wells. The station is placed at y = -310 m and integrating (6) from  $-\infty$  to -310 m with the parameters given above indicates that during most part of the pumping period the influence is negligible (Fig. 8(b)). Only near the end of the period is

depletion similar to the measurement uncertainty (1-2 l/s). Therefore the calculation indicates that S2a is placed far enough upstream from P1 and P2 that influences from pumping were insignificant beyond the reach of the stream that was considered here.

Likewise it was assumed that streamflow depletion at S1a was close to the maximum value obtained infinitely downstream. As can be seen from Fig. 8(a) the values obtained from Eq. (4) during the pumping period are only slightly larger than the values for S1a indicating that this assumption was also justified.

When abstracted water is used for irrigation in humid regions, the irrigation rate is normally controlled at a rate, which (almost) prevents reinfiltration, and thus has no influence on baseflow. At the Haller site the streambed is permeable because of man made influences. Nevertheless, constant abstraction for periods of 1-2 months resulted in depletion of streamflow of only approximately 5-7 l/s, i.e. about 40% of the rate of abstraction. The hydrology of the field site is simple with only three components contributing to streamflow during summer, i.e. variations due to the diurnal variation of evapotranspiration, quick responding runoff from the riparian zone due to intense rainfall, and the reduction of baseflow during summer. However, the natural components of streamflow produce variations similar to or larger than the influence from abstraction. This makes it difficult to detect precisely the influence from abstraction despite the fact that the study site is situated near the source of the stream where the discharge is small. Because the stream was artificially channelled in last century the leakage coefficient of the streambed is expected to be fairly high. Its value  $(\lambda = 1.6 \times 10^{-4} \text{ ms}^{-1})$  is not far from the value Dovleston obtained the for Drain  $(\lambda = 0.752 \times 10^{-4} \text{ ms}^{-1})$ , which was excavated in the 1860's in New Zealand (Hunt et al., 2001). Despite this presumably fair connection between stream and aquifer, when using Eq. (4) with the given values of T,  $S_v$ , and  $\lambda$  to predict the influence from pumping, this must continue at a constant rate for approximately 1000 days before the calculated rate of depletion exceeds 90% of the pumping rate. Thus, for a less permeable streambed measurements must be made over a very long period with stable baseflow in order to estimate  $\lambda$  from depletion data,

which will be difficult in practice. This finding is in agreement with Christensen (2000).

#### 6. Conclusions

The alluvial stream that was studied in the present investigation flows in a small valley on an outwash plain. During summer the two major components of the discharge are baseflow and quick responding runoff from the riparian zone. Mass balance analysis indicates that the measured  $0.04 \, \mathrm{l \, s^{-1}/m^{-1}}$  increase in baseflow between the upstream and downstream discharge stations is equivalent to the average recharge of the aquifer. Between July and September, baseflow decreases by about the same amount  $(\sim 10 \text{ l/s})$  at three stations along the reach. This corresponds to about 20% for the furthest upstream station and to 11% for the furthest downstream station. On warm sunny days, the discharge at the most downstream station approximately 1800 m downstream from the source has small diurnal variations of 5 l/s or less because of evapotranspiration from the riparian zone.

Runoff from the riparian zone has large temporal variations. A sudden increase in the discharge will occur if rainfall exceeds about 5 mm. After a rainless period of a week or longer, even intense rainfall will only influence the hydrograph for 1-2 days, but following rainy periods the influence lasts longer, typically 3-4 days. Mass balance studies suggest that during summer the rapid response from rainfall is generated in the riparian zone along the stream.

Abstraction of ground water 60 m from the stream causes a reduction in the discharge to begin within approximately one day. During two long pumping tests in 1997 and 1998, it was noted that within about a week the relative depletion of streamflow stabilizes at about 40% of the pumping rate. However, given the experimental error this may be caused by the fact that the logarithmic diminishing rate of depletion makes changes beyond the initial phase of pumping difficult to determine.

There is some indication of small systematic deviations between model and data when observed short term influences of pumping are compared to the predicted influences from the analytical model by Hunt (1999), but data scatter prevents definitive

conclusions. The studied stream is small and it was possible to create and maintain almost flume-like measurement conditions at the discharge stations. Therefore, in most cases the uncertainty of the discharge measurements is unusually low ( $< \pm 2\%$ ). Nevertheless, had the stream been less well connected to the aquifer, or had the discharge been somewhat higher then field recognition of the influence from abstraction from a single well would have been impossible to resolve from the transient influences on discharge caused by variation in precipitation and evapotranspiration.

# Acknowledgements

The authors are grateful to Jens Ledet Jensen who assisted with the analysis presented in Appendix A and grateful to Jens Ove Nielsen for valuable discussions. Iben Nilsson and Søren Andresen collected discharge data during 1998. The County of Viborg, Denmark, partly funded the investigation.

# Appendix A

For a channel with cross sectional area A, hydraulic radius R, and slope I the discharge Q is given by the Manning equation

$$Q = AMR^{\frac{2}{3}}I^{\frac{1}{2}}$$

where *M* is the Manning number. Let the wetted perimeter be *P* at a gauging station so that the hydraulic radius is R = A/P. Since at the stations the cross section is rectangular and has width  $B \ge D$  then the following approximation can be made

$$R = \frac{A}{P} = \frac{BD}{B+2D} \approx D \tag{A1}$$

so that Eq. (2) can be simplified to

$$Q = AMI^{\frac{1}{2}}R^{\frac{2}{3}} \approx MD^{\frac{5}{3}}BI^{\frac{1}{2}} = MD^{\frac{5}{3}}C_1$$
(A2)

During summer periods with small variations of Q the slope (I) is virtually independent of Q so that it is reasonable to assume that  $C_1$  is the same for all measurements. Taking the logarithm gives

$$\log(Q_i) = \frac{5}{3} \log D_i + \log M_i + \log C_1$$
 (A3)

where the subscript *i* indicates values at a specific time. For an observation,  $Y_i$ , of log  $(Q_i)$  assume

$$Y_i = \log(Q^*_i) = \log(Q_i) + V_i \tag{A4}$$

where the observation error  $V_i \sim N(0,\sigma^2)$  i.e. normally distributed with variance  $\sigma^2$ . Likewise, for an observation,  $Z_i$ , of log  $(D_i)$  assume

$$Z_i = \log(D^*_i) = \log(D_i) + U_i \tag{A5}$$

where the observation error  $U_i \sim N(0, \tau^2)$ . Finally it is assumed that  $\log (D_i) \sim N(Z_m, \omega^2)$ . Because  $Q_i$  and  $D_i$  are measured by different methods it is reasonable to assume that  $V_i$  and  $U_i$  are independent. In order to simplify the calculations we assume that the errors on the depth readings and on the discharge values are multiplicative.

Using Eqs. (A4) and (A5) we rewrite (A3) to

$$Y_i - V_i = \frac{5}{3}Z_i - \frac{5}{3}U_i + f_{t_i}$$
(A6)

where the term  $f_t$  describes the temporal variation of M ( $f_{t_i} = \log M_{t_i} + \log C_1$ ) with  $t_i$  denoting the day number of observation 'i'. Thus

$$Y_i - \frac{5}{3}Z_i = f_{t_i} + V_i - \frac{5}{3}U_i$$
  
  $\sim N\left(f_{t_i}, \sigma^2 + \left(\frac{5}{3}\right)^2 \tau^2\right)$  (A7)

Plots of  $Y_i - 5/3Z_i$  versus day number  $t_i$  indicate that during the summer period  $f_t$  can be approximated mostly by one, but in a couple of cases two linear segments.

We also have

$$Z_i \sim N(Z_m, \omega^2 + \tau^2) \tag{A8}$$

and

$$\operatorname{Cov}\left(Y_{i} - \frac{5}{3}Z_{i}, Z_{i}\right)$$
$$= E\left\{\left(Y_{i} - \frac{5}{3}Z_{i} - f_{t_{i}}\right)(Z_{i} - Z_{m})\right\} = -\frac{5}{3}\tau^{2} \qquad (A9)$$

Let  $\hat{f}_{t_i}$  be the estimated values obtained by using either one or two linear segments, and let the average logarithmic depth be denoted  $\hat{Z}_m$ . Then estimates of

Table A1

Parameters obtained from the analysis of the measurement errors in the discharge. The analysis is based on values of discharge  $(m^3 s^{-1})$  and depth (m) recorded at S1a, S2, and S2a

Station and year	Standard deviation of observation error of logarithmic depth, $\tau$	Standard deviation of logarithmic depth, $\omega$	Standard deviation of observation error of logarithmic discharge, $\sigma$
S1a97 S1a98 S297 S298 S297	0 0.026641 0.006385 0 0.028046	0.073504 0.077869 0.053949 0.020360 0.112538	0.015360 0.028536 0.029471 0.017207 0.038734
S2a97 S2a98	0.028040	0.026187	0.016581

the parameters  $\tau$ ,  $\omega$ , and  $\sigma$  are obtained as

$$\tau^{2} = \begin{cases} \frac{3}{5} \frac{1}{n-k} \sum \left( Y_{i} - \frac{5}{3} Z_{i} - \hat{f}_{t_{i}} \right) (Z_{i} - \hat{Z}_{m}) & \text{if } > 0 \\ 0 & \text{if } \leq 0 \\ (A10) \end{cases}$$

$$\omega^{2} = \frac{1}{n-1} \sum_{i} (Z_{i} - \hat{Z}_{m})^{2} - \tau^{2}$$
(A11)

$$\sigma^2 = \frac{1}{n-k} \sum \left( Y_i - \frac{5}{3} Z_i - \hat{f}_{t_i} \right)^2 - \frac{25}{9} \tau^2$$
(A12)

where *n* is the number of measurements and k = 2when one linear segment is used for  $\hat{f}_t$  and k = 3 if two linear segments are used for  $\hat{f}_t$ . The results are given in Table A1.

Let  $E_Q$  denote the mean discharge. Since  $Y = \log Q$  the standard deviation ( $\nu$ ) of Q is given by (Hald, 1971)

$$\nu = E_{Q}\sqrt{\exp(2\sigma^{2}) - \exp(\sigma^{2})}$$
(A13)

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