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Journal of Hydrology 284 (2003) 77–91

Journal of Hydrology
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Stochastic hydraulic safety factor for gas containment in underground storage caverns

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Received 6 February 2002; accepted 10 July 2003

Abstract

Since the previous hydraulic safety factor for gas containment in gas storage caverns did not consider the spatial hydraulic conductivity variation, which directly affects the variation of hydraulic head and gradient, it is insufficient to fulfill the hydraulic safety from gas leakage. Therefore, based on the stochastic simulation considering the heterogeneity of hydraulic conductivity, a method for determining the hydraulic safety factor for gas containment in underground storage cavern is suggested. Instead of a single hydraulic gradient value obtained by using deterministic modeling, a possible range of a hydraulic gradient under a given probability was examined by means of stochastic simulation. The term 'stochastic safety factor' is newly defined as a head value at the water curtains, which is needed to make the critical hydraulic gradient of the cavern larger than a proposed critical gradient. By using this stochastic safety factor, the shortage of hydraulic gradient can be replenished and the risk of gas leakage due to heterogeneity of hydraulic conductivity can be reduced.

Keywords: Gas containment; Stochastic safety factor; Underground storage cavern; Heterogeneity of hydraulic conductivity

1. Introduction

Underground storage of pressurized gases in unlined rock caverns has advantages over above-ground storage in terms of safety, environment and economy, and could be commercially attractive (Homer et al., 1989). In unlined rock caverns, gas is kept from leaking by ensuring that the groundwater pressure in the surrounding rock exceeds the gas pressure in the storage caverns (Gustafson et al., 1991). High gas pressure can be achieved by locating the caverns at a sufficient depth or by installing so-called "water curtains" surrounding the caverns. Water curtains are arrays of the drilled holes, which are installed parallel over the cavern roofs as well as around sidewalls of caverns, if necessary. With a water curtain, the groundwater pressure around the caverns should be strong enough to prevent any gas

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doi:10.1016/S0022-1694(03)00275-0

leakage. When such water curtain is used, the key question then becomes 'How large of pressure difference between groundwater and gas in the cavern should be maintained? (Liang and Lindblom, 1994a). This question may be reformulated into the specific questions related to a site, such as the minimum depth of caverns below groundwater table, the minimum pressure of the water curtain, and the critical hydraulic gradient above the caverns. Åberg (1977), GEOSTOCK (1984), Goodall et al. (1988), Liang and Lindblom (1994a), and others discussed gas containment criteria for rock caverns. These criteria were decided based on the deterministic flow analysis. Therefore, the heterogeneity of hydraulic conductivity has to be considered by means of stochastic analysis to update the existing gas containment criteria.

The stochastic simulation technique has several important advantages. This approach is conceptually straightforward because it involves simply using a simulated random field for the parameters in a deterministic flow simulator. The random parameter field is generated using a simulation technique and then the flow simulator is used to solve for the head field. This process is repeated many times, and the resulting head data are analyzed statistically to determine the means, variances and covariances.

A large body of work exists which describes techniques for treating quantitatively the problems of flow and transport in heterogeneous media under conditions of data scarcity (Dagan, 1989; Gelhar, 1993). Examples for generators are the method used by Freeze (1975) in his work on flow in heterogeneous porous media, the nearest neighborhood method (Smith and Freeze, 1979; Smith and Schwartz, 1980; King and Smith, 1988), the Turning Bands Methods (Matheron, 1973; Journel, 1975; Journel and Huijbregts, 1978; Mantoglou and Wilson, 1982; Mantoglou, 1987; Tompson et al., 1987) and Fast Fourier Transform based method developed by Gutjahr (1989). A method not limited to Gaussian fields was presented by Gòmez-Hernàndez and Srivastava (1990). Griffiths et al. (1994) investigated the influence of soil variability on the expected value of 'output' quantities such as the flow rate and the exit gradient in seepage problem. Sanchez-Vila et al. (2000) performed the validation study for recent analytical formula based on extension of Thiem's equation (Sanchez-Vila et al., 1999) by numerical simulations for many heterogeneous transmissivity fields, including uncorrelated case and multigaussian fields. The numerically based techniques mentioned above have the advantages in terms of their ability to incorporate complicated boundary conditions and the influence of boundary conditions for specific site conditions (Gelhar, 1993).

This study links the hydraulics of gas storage in rocks to stochastic subsurface hydrology by introducing a new safety factor for gas containment, which is computed by an uncertain quantity generated from the spatial variability of hydraulic conductivity. A stochastic simulation method combined with finite element deterministic simulator is used to analyze the uncertainty of hydraulic heads around underground high pressurized gas storage caverns. At first, deterministic modeling analyzes the flow pattern to check if the hydraulic gradient fulfills the gas tightness condition. And then, the stochastic simulation computes the uncertainty of hydraulic heads and gradients due to the spatial variability of hydraulic conductivity. The various probability distribution functions are applied to hydraulic conductivity data to obtain the proper distribution of hydraulic conductivity at the study sites considered. The code HYDRO_GEN, a spatially distributed random field generator for correlated properties, outlined in Bellin and Rubin (1996) is used for stochastic simulation. The resultant uncertainty of hydraulic gradient is used to obtain a critical hydraulic gradient from which stochastic safety factor can be determined.

2. Hydraulic margin for gas containment

During the past decades, several gas containment criteria based on groundwater gradient or pressure were proposed. Åberg (1977) proposed that a vertical hydraulic gradient greater than 1.0 around a cavern should be maintained through the rock fractures surrounding a storage cavern during gas containment operations. This gas containment criterion is commonly used. Geostock (1984) carried out a series of tests with differently-shaped cavities by use of Hele–Shaw model. The results of these tests revealed that the pressure difference between the required groundwater depth above the cavities and the maximum tolerable pressure depends on the shape of the cavities and their environments. This relation was defined as follows

$$H > P_{max} + P + S \tag{1}$$

where

H: height from the ceilings of a cavern to groundwater level (m)
P_{max}: maximum tolerable gas pressure expressed as a head in the cavern (m)
P: shape factor (m)
S: safety factor (m)

Goodall et al. (1988) recommended that a practical design of cavern be based on the simple criterion that no gas must leak as long as the water pressure increases along all possible gas leakage paths away from the caverns. This is a generalization of Åberg's recommendation that the vertical hydraulic gradient be greater than one. On the basis of Goodall et al.'s study, Liang and Lindblom (1994b) suggested the 'critical gas pressure' defined as the maximum tolerable gas pressure for a given storage system at no gas leakage conditions.

However, these hydraulic margins are computed by using deterministic models, which do not consider the spatial variability of hydraulic conductivity. Therefore, we propose the new hydraulic safety margin obtained by use of stochastic simulation. This procedure of stochastic simulation can be described as follows:

- The hydraulic conductivity data surrounding rock caverns are collected.
- The appropriate probability distribution for the collected hydraulic conductivity data based on parameter estimation and goodness of fit tests is determined.
- The uncertainties for the head and hydraulic gradient are computed by using a stochastic simulation procedure with N times.
- The probability density function (PDF) is developed by using N outputs at the interested point of the gradient graph. In this PDF, we can set a particular value of security (e.g. 95% or 99%) and find the critical gradient value.
- The stochastic safety factor, which is needed to increase the critical hydraulic gradient up to a proposed critical hydraulic gradient (e.g. 1.0 for Åberg's condition), is defined.

3. Evaluation of gas containment by deterministic model

Rock masses are characterized by faults, joints and bedding planes. The description of groundwater flow in rock masses is complicated by these discontinuities, so it would be difficult to consider all factors in groundwater modeling. Fortunately, the host rocks for gas storage are usually hard, and massive with few fractures, so a simplified groundwater model can be used to numerically analyze the groundwater flow. A gas tightness design and hydraulic modeling are performed by continuum approach in many cases (Goodall et al., 1988; Liang and Lindblom, 1994a; Kawatani and Saito, 2000; GEOSTOCK, 2001). Although, the influence of individual fissures on groundwater flow in fractured rock masses might not be negligible, the analysis results presented in this study are based on following assumptions:

1. The groundwater flow obeys Darcy's law.
2. The medium is continuous, heterogeneous and anisotropic.
3. The groundwater flow is steady, that is, groundwater conditions are constant.
4. Groundwater is incompressible.
5. Rock around storage caverns are saturated with rock water.

The study site is a propane storage cavern in Korea. The maximum gas pressure is 8.6 bar. The ceilings of caverns are located at elevation −115 m and the water curtains are located at elevation −90 m. The head of the water curtains is artificially maintained at elevation 5.0 m, and represents the total hydraulic pressure above the cavern of 120 m in head. Since the gas pressure is equivalent to 86 m in head, the hydraulic head difference between cavern and water curtain is 34 m, which is the hydraulic safety factor determined at the design stage. The deterministic flow modeling is performed to check the hydraulic gradient around the caverns

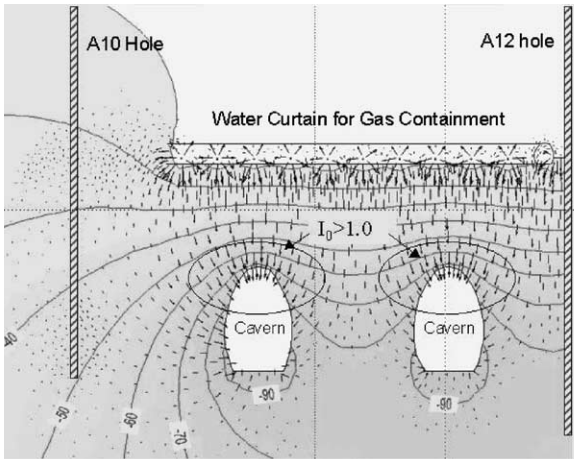

Fig. 1. The representative vertical cross section for flow analysis.

using this safety factor. The representative vertical cross section for modeling is shown in Fig. 1.

According to Goodall's criterion, at the critical gas pressure, the groundwater pressure in the vicinity of the cavern is equal to the gas pressure at some point on the boundary of cavern, i.e. $\partial p/\partial n = 0$, where p is the groundwater pressure and n is the unit normal vector at this point (see point A in Fig. 2). Hydraulic head gradient $\partial H/\partial n$ at point A determines the movement of the gas–water interface (De Marsily, 1986).

$$\frac{\partial H}{\partial n} = \frac{\partial}{\partial n}\left(\frac{p}{\rho_{\mathrm{w}} g} - n \sin\alpha\right) = \frac{1}{\rho_{\mathrm{w}} g}\frac{\partial p}{\partial n} - \sin\alpha = -\sin\alpha \qquad (2)$$

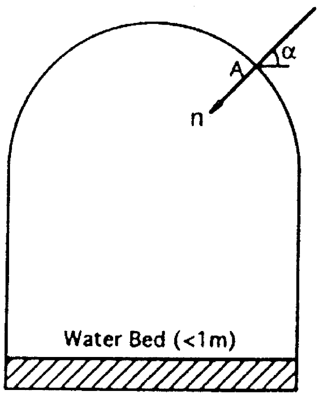

Fig. 2. Hydraulic gradient at the periphery of a cavern.

where ρ_{w} is the water density and g is the acceleration of gravity.

When point A is above the cavern bottom, $\partial H/\partial n$ is negative, and the groundwater flows towards the inside of the cavern. To prevent gas leakage, the cavern bottom is usually saturated with water (called water bed). When the water is maintained, gas–water interface does not move outward (Liang and Lindblom, 1994b).

The governing differential equation for flow analysis is the Laplace equation in two dimensions. The analyses of groundwater flow are based on a two-dimensional finite element model developed by Chung et al. (1997). This program was designed for the simulation of groundwater flow caused by the water curtain around an underground storage cavern. The symmetric global conductance matrix is solved by the Choleski decomposition method (Cook, 1981). The modeling area is 100 m wide and 100 m deep. In the modeling of groundwater flow, four-node linear rectangular elements and a linear basis function are used. The mesh size is 5 m $\times$ 5 m, thus a total of 441 nodes and 400 elements are used for the analysis of gas tightness. The mean value of hydraulic conductivity is 1.6×10^{-7} m s^{-1}, which is computed by using the data from A10 and A12 boreholes. Although, the shape of the cavern is horseshoe, the shape could be simplified to a rectangle whose height is 20 m and the width is 10 m for spatial variability simulation.

Two types of boundary condition were used. Constant head boundaries at the water curtains and the caverns are assigned because these are maintained by artificial recharge and long term gas storage schedule, respectively (Korea Petroleum Development Cooperation, 1991). No flow boundary conditions are assigned both sides of modeling area.

Fig. 3(a) and (b) show the contours of the computed hydraulic head and the computed hydraulic gradient by deterministic modeling, respectively. The hydraulic gradients are computed at the center of each element by using the computed hydraulic heads at corresponding nodes. Distinct and reasonable flow patterns as shown in Fig. 3(a) and (b) show that mesh size is appropriate for simulation.

As shown in Fig. 3(b), in most cells surrounding the upper part of the caverns (in dotted area), the Åberg's gas tightness condition was satisfied because

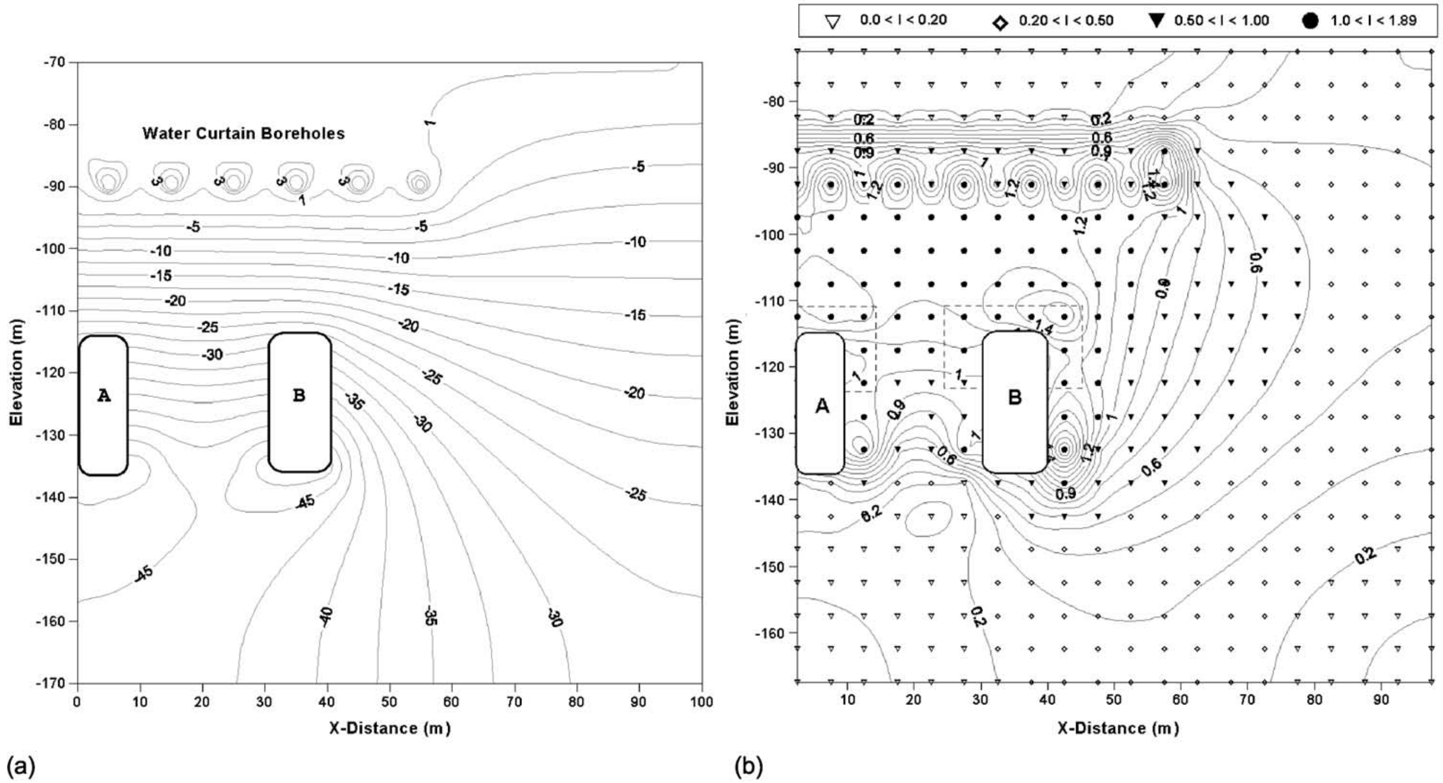

Fig. 3. The computed results by deterministic modeling. (a) The computed heads and (b) the computed hydraulic gradients.

the hydraulic gradients were greater than 1. However, among the cells surrounding the upper part of the cavern B, there is a cell ($x = 27.5$, $z = -122.5$) whose mean hydraulic gradient is less than 1, which implies that the Åberg's condition is not satisfied.

4. Evaluation of gas containment by stochastic simulation

4.1. Probability distribution of hydraulic conductivity in the modeling area

In this study, a porous medium is set up to represent a stochastic set of macroscopic elements. Within each soil type or geologic unit, the properties of these elements are assumed to come in unknown frequency distributions. Thus, an appropriate PDF for the media property has to be determined. Various probability distributions such as the two- and three-parameter gamma (three-parameter means scale, shape and location parameter, respectively), General Extreme Value (GEV), Gumbel, two- and three-parameter lognormal, log–Pearson type III are applied to determine the appropriate probability distribution model for the real field hydraulic conductivity data. Several methods such as the methods of moment, probability weighted moments (PWM), l-moments and maximum likelihood can estimate model parameters. In this study, the parameters of the selected models are estimated based on the method of PWM which gives the stable parameter estimates for a small sample size. The general form of the PWM is given by (Greenwood et al., 1979; Hosking, 1986)

$$M_{p,r,s} = E[X^p F^r(x)\{1 - F(x)\}^s] \tag{3}$$

where

p, r, s: non-negative integers
E: expectation
$F(x)$: cumulative distribution function

In general, the following two types of the population PWMs are used to estimate the parameters depending on the probability models:

$$M_{1,r,0} = E[XF^r(x)] \equiv B_r \tag{4}$$

$$M_{1,0,s} = E[X\{1 - F(x)\}^S] \equiv B'_s \tag{5}$$

And the corresponding sample PWMs are given by

$$\hat{B}_r = \frac{1}{N}\sum_{j=1}^{N} x_j \frac{(j-1)(j-2)\cdots(j-r)}{(N-1)(N-2)\cdots(N-r)}, \tag{6}$$

$r \geq 1$

$$\hat{B}'_s = \frac{1}{N}\sum_{j=1}^{N} x_j \frac{(N-j)!(N-s-1)!}{(N-j-s)!(N-1)!}, \qquad s \geq 0 \tag{7}$$

where

N: sample size
x_j: ordered statistic ($x_1 \leq x_2 \leq \cdots \leq x_N$).

Note that if $r = s = 0$, then $\hat{B}_0 = \hat{B}'_s = \bar{X}$, in which $\bar{X}$ is the sample mean.

To determine the appropriate probability distribution for the hydraulic conductivity data, the data of A10 and A12 holes in the corresponding vertical plain were used. These hydraulic conductivity data were measured by a hydraulic pressure test along the depth in these holes before the construction of the cavern. These data sets are listed in Table 1 and displayed in Fig. 4. These data were obtained from the Lugeon test by using the double packer system. Injection interval was 3 m, and a constant pressure provided the injected water during a fixed period. Hydraulic conductivity was measured from the correlation curve of injected water pressure and flow quantity of injected water (Korea Petroleum Development Cooperation, 1985).

The PWM parameter estimates for each probability model are displayed in Table 2. Among the probability models, the three-parameter lognormal (lognormal-3) and log–Pearson type III models show the NG result, which indicates that the estimated parameters for these two models do not meet the validity conditions for each distribution type. Also, the goodness- of- fit tests for five probability models are performed using the χ^2-test, Kolmogorov–Smirnov test and Cramer von Mises test. The results of the goodness- of- fit tests are shown in Table 3. The three-parameter gamma (gamma-3) distribution is rejected based on three goodness- of- fit tests, while others are accepted at 5% significance level.

Table 1
Hydraulic conductivity data from the storage cavern (adapted from Korea Petroleum Development Cooperation (1985))

Hole No.	Depth (el., m)	K ($\times 10^{-7}$ m s^{-1})	Hole No.	Depth (el., m)	K ($\times 10^{-7}$ m s^{-1})
A10	−71 ~ −74	2.595	A12	−75 ~ −78	1.830
	−74 ~ −77	2.664		−79 ~ −82	0.498
	−77 ~ −80	3.139		−83 ~ −86	1.036
	−82 ~ −85	2.020		−87 ~ −90	1.235
	−87 ~ −90	2.651		−91 ~ −94	1.356
	−92 ~ −95	2.273		−95 ~ −98	1.333
	−98 ~ −101	1.894		−101 ~ −104	1.201
	−103 ~ −106	2.273		−104 ~ −107	1.263
	−106 ~ −109	1.894		−108 ~ −111	1.544
	−111 ~ −114	2.146		−111 ~ −114	2.092
	−116 ~ −119	2.273		−115 ~ −118	1.449
	−119 ~ −122	2.020		−118 ~ −121	2.183
	−124 ~ −127	1.998		−123 ~ −126	1.731
	−127 ~ −130	2.188		−127 ~ −130	0.429
	−130 ~ −133	2.399		−135 ~ −138	1.115
	−133 ~ −136	2.399		−140 ~ −143	1.709
	−136 ~ −139	1.832		−146 ~ −149	0.857
	−140 ~ −143	1.768		−152 ~ −155	0.943
	−143 ~ −146	1.427		−156 ~ −159	0.772
	−148 ~ −151	1.768		−159 ~ −162	1.544
	−155 ~ −158	1.768		−162 ~ −165	1.614
	−163 ~ −166	2.378		−165 ~ −168	1.499
				−172 ~ −175	0.858

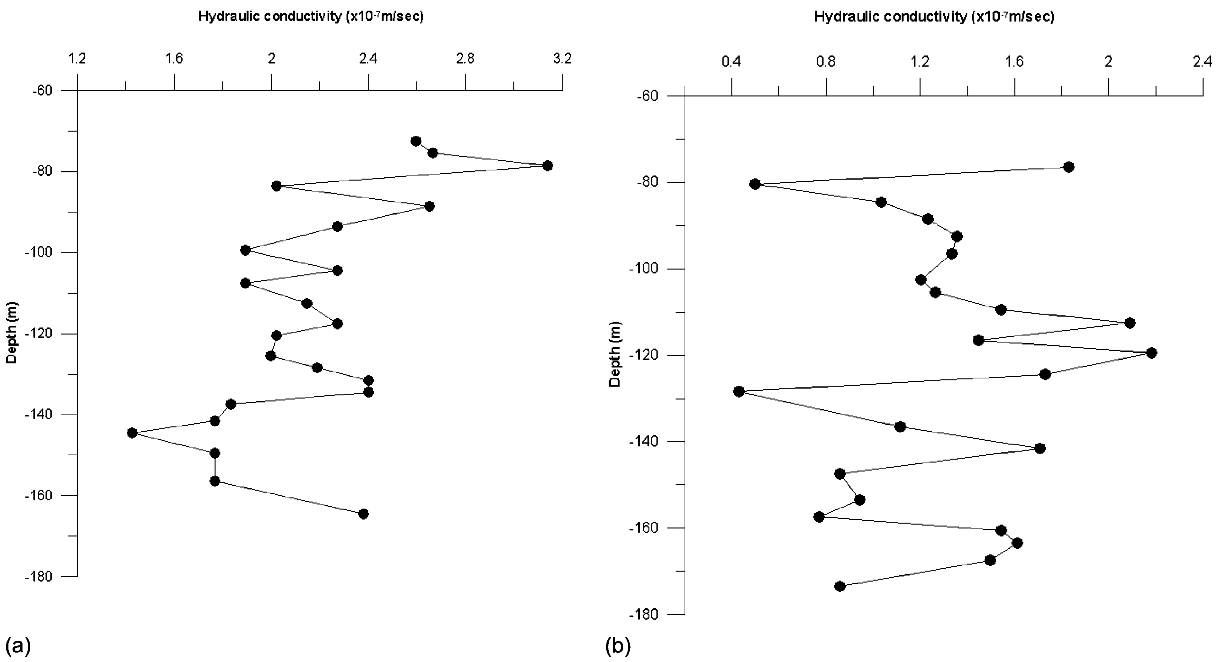

Fig. 4. Vertical distribution of hydraulic conductivity data. (a) A10 Hole and (b) A12 Hole.

Table 2
The estimated parameters for each probability distribution

Distribution	Location parameter	Scale parameter	Shape parameter	Result
Gamma-2	0.000	0.429	0.227	OK
Gamma-3	−67.851	0.005	46.540	OK
GEV	1.535	0.635	0.355	OK
Gumbel	1.441	0.502	–	OK
Lognormal-2	0.000	0.562	0.350	OK
Lognormal-3	−5.952	2.042	0.080	NG
Log–Pearson type III	1.089	−0.306	2.000	NG

4.2. Stochastic simulation considering spatial variability of hydraulic conductivity

In this study, among the 4 models in Table 3, the two-parameter lognormal distribution is selected as an appropriate model for the stochastic simulation because this model is accepted by goodness- of- fit tests and this model is generally assumed as appropriate for the hydraulic conductivity data, as shown in many studies (McMillan, 1966; Freeze, 1975; Woodbury and Sudicky, 1991; Massmann and Hagley, 1995). In other words, the logarithm of hydraulic conductivity values follows a normal distribution. Usually, we generate a lognormal field by generating a $N(\mu, \sigma)$ field and then taking the exponential of it. Thus, a lognormal field $Y(x) = \ln K(x)$ is believed to simulate realistic hydraulic conductivity values.

Table 3
The results of goodness of fit tests

Distribution	Test	Computed value	Tabulated value	Result
Gamma-2	χ^2	0.89	7.81	OK
	K–S	0.09	0.18	OK
	CVM	0.09	0.46	OK
Gamma-3	χ^2	74.08	5.99	NG
	K–S	0.58	0.18	NG
	CVM	2.09	0.46	NG
GEV	χ^2	1.27	5.99	OK
	K–S	0.06	0.18	OK
	CVM	0.01	0.46	OK
Gumbel	χ^2	3.93	7.81	OK
	K–S	0.11	0.18	OK
	CVM	0.15	0.46	OK
Lognormal-2	χ^2	5.27	7.81	OK
	K–S	0.10	0.18	OK
	CVM	0.09	0.46	OK

The usual method of treating uncertainties in hydraulic conductivity values is by fitting a covariance function (usually exponential) and finding the estimated variance and mean values from the small number of data available and then generating several realizations of the field using a random field generator. Each of these fields is then used in a Monte Carlo simulation to compute the groundwater head and gradient.

In this study, the computer code HYDRO_GEN, a spatially distributed random field generator for correlated properties outlined in Bellin and Rubin (1996), is used for generating two-dimensional space random functions with an assigned covariance structure. The code is based on heteroscedasticity of the interpolation coefficients of the multinormal space random function. The generated fields can be made conditional to field measurements. When local data are not available, HYDRO_GEN will generate unconditional realizations, honoring the prescribed spatial statistics. In most situations, field data are quite scarce and expensive to obtain, hence, the ability to use the data for model inference and for conditioning is important. In this study, among various covariance functions, exponential covariance function (C_Y) is selected.

$$C_Y(x, z) = \sigma_Y^2 \exp\left\{-\left[\left(\frac{x}{\lambda_x}\right)^2 + \left(\frac{y}{\lambda_y}\right)^2\right]^{1/2}\right\} \quad (8)$$

where λ_x, λ_y are the directional $\ln K(x)$ correlation length scales and σ_Y^2 is the variance of $\ln K(x)$.

The mean and standard deviation of the log-transformed hydraulic conductivity data in Table 1 (A10 and A12 holes) are given as $\mu_Y = -4.79$ ($Y = \ln(K)$, K in cm s^{-1}) and $\sigma_Y = 0.19$.

The properties of a sedimentary unit, a formation, or a larger scale of a set of formations, can be characterized by the covariance between two points separated by Δx (Appelo and Postma, 1993):

$$\mathrm{cov}[\ln K_{x+\Delta x}, \ln K_x] = \frac{1}{n}\sum_{x=1}^{n}(\ln K_{x+\Delta x} - \overline{\ln K})(\ln K_x - \overline{\ln K}) \quad (9)$$

where $\overline{\ln K}$ is the mean of a log-transformed set of permeability measurements. With the covariance known, an estimated autocorrelation function Γ_c can be found if the distribution is lognormal

$$\Gamma_c = \frac{\mathrm{cov}}{\sigma^2_{\ln K}} \tag{10}$$

where $\sigma^2_{\ln K}$ is the variance of the permeability distribution. The correlation length can be obtained from spatially distributed data and be independently stated for vertical and horizontal permeabilities, and also for porosities or any other parameter, which influences flow. Anisotropy is therefore easily introduced. Fig. 5 shows the autocorrelation estimates calculated from Eq. (10) for boreholes A10 and A12 in Table 1. Fluctuations of estimated autocorrelation functions around zero are found.

Although, vertical correlation length (λ_v) could be found from the estimated covariance function by using e^{-1} correlation distance, it is not easy to determine the value because of fluctuation and abrupt decrease. Therefore, more general information for estimating correlation length is introduced. Fig. 6 shows data up to an overall scale of 1 km and emphasizes the significant anisotropy of the spatial correlation structure in many sedimetary materials by using the data from previous study (Gelhar, 1993). The horizontal and vertical correlation scales determined at a given site are connected by lines.

Since overall scale of measument is about 70 m in this study, correlation lengths for a similar overall scale could be referred from in Fig. 6. Refering to the results ($\sigma_Y = 0.4$, $\lambda_h = 8$, $\lambda_v = 3$) from Goggin et al. (1988) where the vertical scale is 60 m, we can estimate that the value of the vertical correlation scale (λ_v) is 5 m, and that of the horizontal correlation scale (λ_h) is 15 m. In general, if the covariance function corresponds to a first-order autoregressive process equivalent to the negative exponential covariance function, then the variance of the mean is:

$$\mathrm{var}[\bar{X}] = \frac{\sigma^2}{N}\frac{1+a}{1-a}; \qquad a = e^{-(\Delta x/\lambda)} \tag{11}$$

This result shows that, because the process X is autocorrelated, the variance of the mean is larger than that for independent samples. Accordingly, the estimated variance can be approximately two times larger than the independent sample variance when $a = e^{-1}$. Then, the estimated standard deviation σ_Y would be 0.27. Using these properties, HYDRO_GEN generates the 1000 randomly heterogeneous hydraulic conductivity fields. The smoothed log conductivity field is shown in Fig. 7.

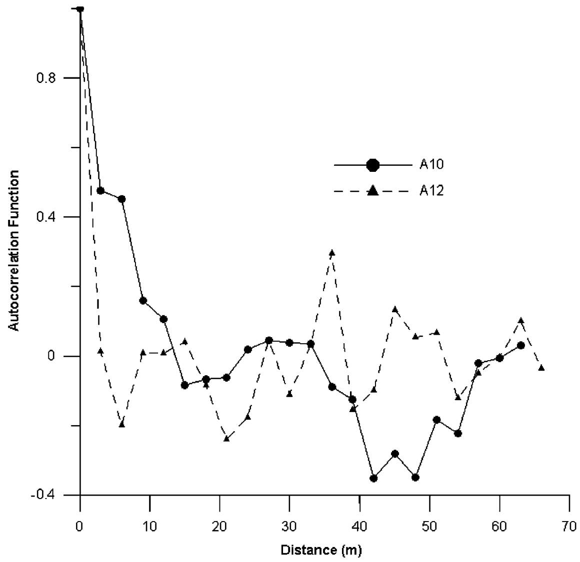

Fig. 5. Estimated autocorrelation function of two data sets.

To estimate the effect of spatial variability statistically, the Monte Carlo method is used. The hydraulic conductivity values, K_i can be generated by using the following equation

$$K_i = 10^{Y_i}, \qquad i = 1,\dots,p \tag{12}$$

where p = number of finite elements.

Then the FEM model is used to solve the head values on a set of nodal points within the flow domain. By repeating the analysis 1000 times, the frequency

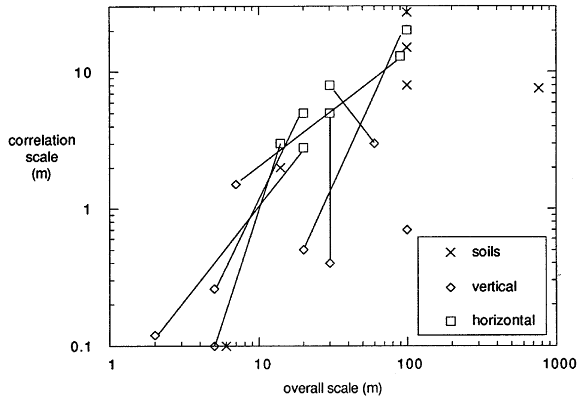

Fig. 6. Correlation scale with overall scale less than 1 km (After Gelhar, 1993).

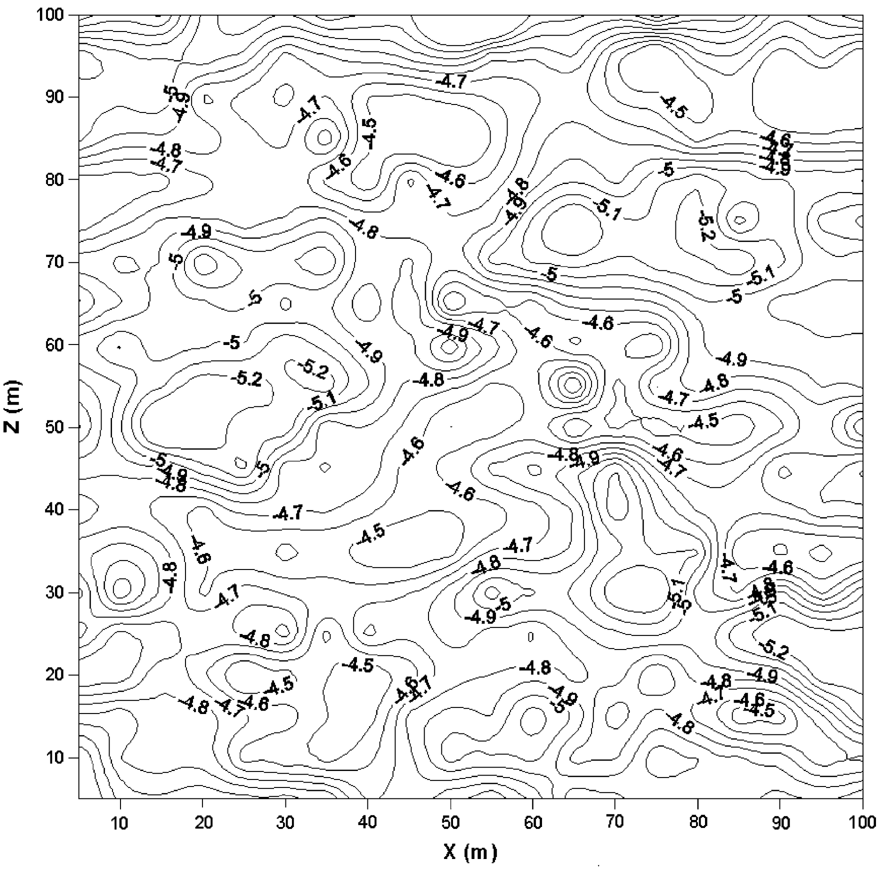

Fig. 7. Smoothed log hydraulic conductivity contours (Y = ln K(x), K in cm/s).

distributions of hydraulic head and gradient at the nodal points in the finite element grid can be analyzed to estimate the statistical properties representing the uncertainty of the hydraulic head and gradient.

4.3. Uncertainty in hydraulic head and gradient

The groundwater flow equation was solved for each realization and the effects of heterogeneity were investigated. The results are shown in Fig. 8. The standard deviations of hydraulic head (σ_ϕ) for the correlated case ranged from 0 m to 3.1 m, and these values are three times larger than those of the uncorrelated case which ranged from 0 to 0.9 m. The larger standard deviation of hydraulic head was due to the joint probability densities in correlation structure which is not considered in uncorrelated case. A complete statistical description of the hydraulic conductivity continuum requires a joint probability density function of K at various locations (Gelhar et al., 1977).

The uncertainty in the predicted hydraulic heads is large between caverns and water curtains, where the mean hydraulic gradients are relatively large. However, this large uncertain region is located some distance away from the known constant head boundaries, such as cavern boundaries and water curtains, in which the uncertainty goes to zero (Chung et al., 2000). The water curtains have the constant head boundaries in which hydraulic heads are maintained as elevation 5.0 m by artificial management, and the caverns also have the known boundaries in which the gas pressure has been maintained constant for a long time. Especially, σ_ϕ is higher in

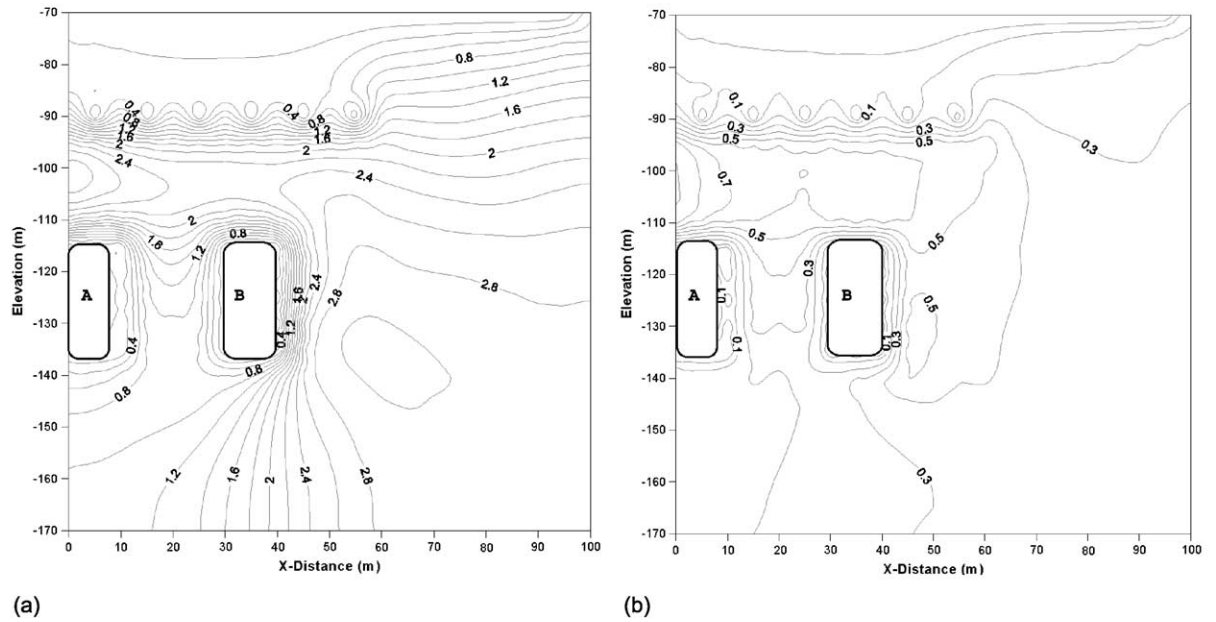

Fig. 8. The distribution of standard deviation of hydraulic head (σ_ϕ). (a) correlated case ($\lambda_x = 15$ m, $\lambda_z = 5$ m) and (b) uncorrelated case.

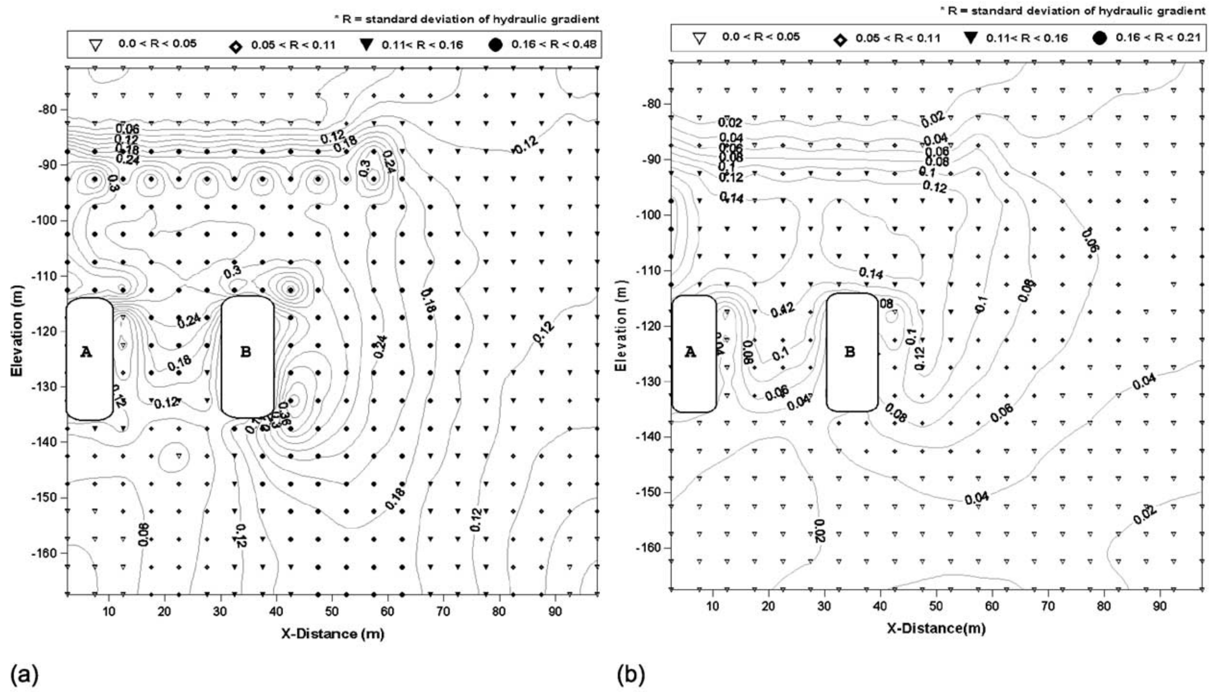

Fig. 9. The distribution of standard deviation of hydraulic gradient (σ_g). (a) correlated case ($\lambda_x = 15$ m, $\lambda_z = 5$ m) and (b) uncorrelated case.

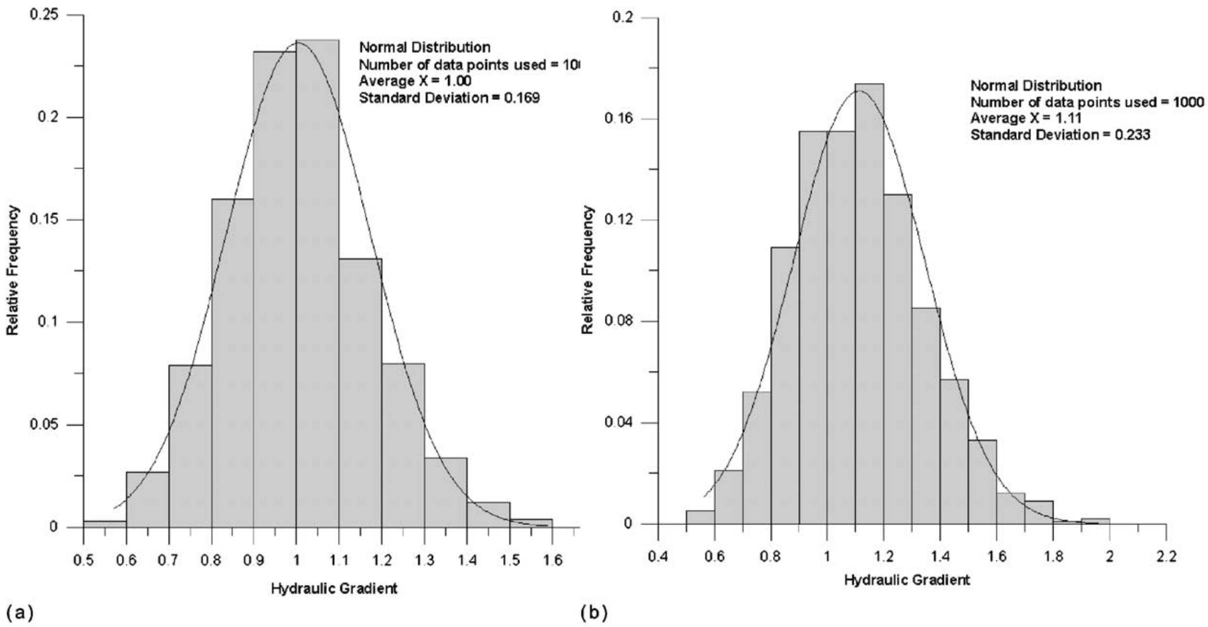

Fig. 10. The probability distribution functions of hydraulic gradient. (a) Cell No. 1 ($x = 27.5$ m, $z = -122.5$ m); (b) Cell No. 2 ($x = 27.5$ m, $z = -117.5$ m).

the upper region of the caverns than in any other region.

A second integrated measure of the uncertainty in the model prediction is defined by the hydraulic gradient variability. The standard deviations of hydraulic gradient (σ_g) are plotted in Fig. 10. The correlation of hydraulic conductivity has a significant effect on the uncertainty of the hydraulic gradient. When hydraulic conductivities are correlated, the standard deviations of the hydraulic gradient, ranged from 0 to 0.48 (Fig. 9(a)). These values are two times greater than the standard deviations of the uncorrelated case, which ranged from 0 to 0.21 as shown in Fig. 9(b).

As mentioned in Section 4.2, the Åberg's gas tightness can be maintained by steady mean hydraulic gradient (I_0) which is greater than 1 in the periphery of a cavern. However, due to the heterogeneity of ln K, uncertainty in hydraulic gradient could affect the gas tightness of some cells in the upper part of cavern.

To find the critical value of the hydraulic gradient, PDFs using 1000 outputs at two interested cells were developed as shown in Fig. 10. In these PDFs, we could set a particular value for the security, i.e. 95% or 99% and find the critical gradient value. According to these PDFs, the possible ranges of hydraulic gradients at two cells in the upper part of cavern B are examined with a given probability as shown in Table 4.

With a given probability of 95%, critical hydraulic gradient can be obtained as 0.644 at cell No. 2 as shown in Table 4. When the proposed critical

Table 4
Critical ranges of hydraulic gradient with given probabilities

Cell No.	Location	Critical range of hydraulic gradient with given probabilities	
		95%	99%
1	$x = 27.5$ m, $z = -122.5$ m	$0.662 < I < 1.338$	$0.493 < I < 1.507$
2	$x = 27.5$ m, $z = -117.5$ m	$0.644 < I < 1.576$	$0.411 < I < 1.809$

hydraulic gradient (e.g. 1.0 for Åberg's criteria) is greater than this critical hydraulic gradient, gas tightness could not be satisfied at that point. Therefore, the hydraulic gradient should be increased as much as for the proposed value (e.g. 0.356 for Åberg's condition) to prevent gas leakage. With a 99% security, the critical hydraulic gradient was 0.411 at cell No. 2. As the probability of security increases, the critical hydraulic gradient decreases in relation to the standard deviation of the hydraulic gradient (σ_g).

4.4. Stochastic hydraulic safety factor

The results of Section 4.3 show that the uncertainty of the hydraulic head and gradient due to the heterogeneity of hydraulic conductivity directly affects gas tightness. With a given probability, the critical hydraulic gradient can be defined as follows

$$I_C = \min[\mu_x - \alpha\sigma_x] \tag{13}$$

where

α : coefficient representing a given probability
μ_x : mean hydraulic gradient of the upper part of a cavern for each Monte Carlo realizations
σ_x : uncertainty of μ_x due to the hydraulic conductivity variation.

To fulfill the gas tightness condition under a given probability, I_C should be increased to a proposed critical hydraulic gradient. A proposed critical hydraulic gradient can be determined as an appropriate value for a specific cavern condition. Several parameters, such as the possibility of raising the water curtain head and seepage water etc., should be taken into account for a realistic operation of a gas storage cavern.

Hence, the head values at the water curtains should be increased up to a certain degree when the boundary condition of a cavern remains unchanged. Here, we define this value as the stochastic safety factor (S_S) for preventing gas leakage and added this value to the existing hydraulic margin.

$$H > P_{max} + P + S + S_S \tag{14}$$

where S_S is a stochastic hydraulic safety factor (m) under a given probability.

To obtain this value, we must find the relationship between hydraulic head at the water curtains and critical hydraulic gradient at a specific point. Because the relationship between critical hydraulic gradient and hydraulic head is linear, a linear regression equation can be obtained by plotting the variation of the critical hydraulic gradient according to the variation of the hydraulic head at the water curtains. For example, at the point where the critical hydraulic gradient with 95% security occurs, the linear relation between head at the water curtains and critical hydraulic gradient could be obtained as shown in Fig. 11. This result was computed by increasing head values at the water curtains from 5 to 20.0 m. The linear equation is given by

$$Y = 0.0085X - 0.603 \text{ (at Cell No. 2)} \tag{15}$$

where

X : head increment at the water curtains (m)
Y : critical hydraulic gradient with a 95% security.

If this stochastic safety factor at the water curtains is applied, hydraulic gradients in all cells surrounding upper part of cavern could be greater than the proposed value. For example, by using Eq. (14), stochastic

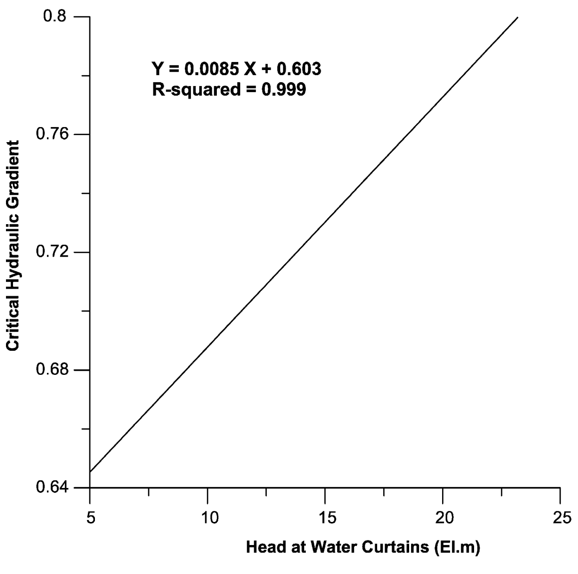

Fig. 11. Linear relationship between head at the water curtain and hydraulic gradient.

safety factor with a 95% security could be determined as 16.6 m which increases the critical hydraulic gradient from 0.644 to 0.744 at the cell No. 2. Accordingly, the risk of gas leakage can be reduced as much.

5. Conclusions

The existing hydraulic safety factor for gas containment around gas storage caverns has been computed by using traditional deterministic flow modeling in which spatial hydraulic conductivity variation was not considered. However, it is found that this safety factor is insufficient in meeting hydraulic safety from gas leakage, because the spatial hydraulic conductivity variation directly affects the variation of hydraulic head and gradient. Therefore, this study suggests the method of determining hydraulic safety factor for gas containment of underground storage cavern by stochastic simulation, considering the heterogeneity of hydraulic conductivity. The possible effects of hydraulic conditions were investigated by examining the effects of heterogeneity.

The results of this study can be summarized as follows:

1. In evaluating the hydraulic safety of an underground LPG storage cavern, both the flow system operating within the flow domain and the nature of the spatial heterogeneities in hydraulic conductivity must be considered.
2. The critical ranges of hydraulic gradient with given probabilities of security could be computed by using stochastic simulation.
3. This study suggests the new term 'stochastic safety factor' which can be defined as the increment of the head value at the water curtains. This factor should be added to a degree which makes the critical hydraulic gradient of the upper part of the cavern larger than the proposed hydraulic gradient value.
4. By using this stochastic safety factor, the shortage of hydraulic gradient can be replenished and the risk of gas leakage due to heterogeneity of hydraulic conductivity can be reduced.

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