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Morphometry of Quartz Aggregates in Granites: Fractal Images Referring to Nucleation and Growth Processes[1]

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Knowledge of mineral aggregate morphologies is of importance to analyze characteristic differences in rock-forming features. For quantifying these differences, the fractal geometry of quartz aggregate cuts digitized from polish sections of different types of granites has been studied. As an approach to measure fractal dimension (D), a power-law dependence of square of aggregate cuts on their linear size has been used. The D values thus calculated mainly increase from 1.48–1.62 for amazonite granites to 1.63–1.70 for alaskite granites and 1.75–1.81 for standard granites. To account for the data of morphometry, the model of nucleation and growth as applied to silicate melt freezing has been considered. For comparison between the nature and model textures, the fractal properties of cluster cuts in the system of overlapping spheres randomly distributed with random radii have been investigated through computer simulation. It has been demonstrated that the distributions of quartz aggregates in granites may be simulated by homogeneous or heterogeneous Poisson models, and both order of crystallization and metamorphic recrystallization should be taken for explaining textural variability. The results of the simulation have enabled the granitic texture to be discussed with respect to the random configuration of the spatial percolation cluster.

KEY WORDS: morphometry, fractal, granite, texture, nucleation and growth, poisson model, percolation cluster.

INTRODUCTION

The interpretation of igneous processes requires quantitative description of reaction pathways, kinetics, and thermodynamic conditions under which crystallization happened. These features of magmatic evolution in turn can be comprehended through the analysis of texture of plutonic rocks provided that the morphological properties of their component parts (such as mineral aggregates and mineral grains) are adequately specified and the links between morphology and genesis are known. Actually, defined conformities are considered in terms of genetically valuable

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models concerning the nature of origin of rocks recorded by their morphologies like dihedral angles, grain shape, or grain size distribution. The pertinent models, for example, are available for volcanic rocks (Cashman and Marsh, 1988; Marsh, 1988) and rocks exposed to recrystallization (Ehrlich and others, 1972; Kretz, 1994; Vernon, 1968).

One added approach to quantifying texture involves the examination of intergranular relations of mineral aggregates in order to identify nonrandom distribution of grains in a rock matrix. The similar model for granitoid rocks, introduced by Vistelius (1966, 1987) under the name of the concept of ideal granite, states that succession of feldspar and quartz grains on line traverses through many granites have the property of Markov chain with transition probabilities that may be predicted from Tuttle and Bowen's experimental studies in the system $NaAlSi_3O_8$–$KALSi_3O_8$–SiO_2–H_2O (Tuttle and Bowen, 1958). In this case, an ideal granite is considered as eutectic composition consisting only three mineral phases (albite, orthoclase, and quartz) generated by direct crystallization of a relatively dry magma under the following assumptions: (i) growing sites are randomly distributed in the melt, (ii) crystals do not move just as they grow because of a large viscosity of the melt, (iii) since crystals growing in a viscous medium is surrounding by a zone of depletion of certain components necessary for crystal growth (because of their extraction from the melt), grains of the same generation are apart from each other and do not combine intergrowths.

Although several ways exist for measuring Markov properties, the spatial forms, which characterize a rock texture, have received little attention in the offered model. As Whitten and Dacey (1975) have demonstrated, transition probabilities of a rock, assuming given modal composition, vary with the grain size differences. Instead, for an ideal granite "a sequence of independent trials such that each individual grain conforms to a point located on the line closely to adjacent points" is considered (Vistelius and Faas, 1972, p. 1386), and equigranularity of mineral aggregates which serve to determine Markov properties is taken as a supplementary (implicit) premise. In fact, this line of approximation gives no way of taking into account the peculiarities of nature (commonly inequigranular) granitoid texture. Meanwhile, they are as such that prove to be useful in classifying acid igneous rock formation; the orogenic and postorogenic, K–Na-rich, 'small and moderate deep'-type granites are a good case in point (Beskin, Larin, and Marin, 1996). This nearly anchieutectic rocks (25–35 vol.% quartz and 65–70 vol.% feldspar), as shown for Paleozoic granitic plutons of Central Kazakhstan and other regions, may be categorized according to certain arrangement of quartz grains against the background of feldspathic matrix into three essentially different types:

(A) inequigranular, often porphyritic granite of standard type with 'tables' of feldspar and apparently disordering distribution of irregular morphology quartz grains in rock volume;

(B) equigranular granite of alaskite type with chain-like distribution of isometric quartz grains tended to form clusters of uniform ones;
(C) inequigranular granite of sub-alkali, that is amazonite type with porphyritic quartz grains of euhedral morphology distributed in fine-grained quartz–feldspathic matrix-like homogeneously (Fig. 1).

Each type is subdivided into four to five varieties, which are morphologically similar, but modify in grain size. These varieties, associated with the different types, appear as intrusive phases and make up the separated intrusive complexes within genetically related series. The earliest rocks of these series that follow granodiorite or granosyenite are represented by the 'A' granite of standard type, which forms batholitic plutons of appreciable lateral and vertical extent. The younger 'B' granite of alaskite-type shows itself mainly as central kind plutons of ring or arc shape and tens of kilometers in diameter. Their inner structure is characterized by sets of alternating layers composed of 'A' and 'B' granitic rocks, in places disconnected by the screens of older granites or host rocks. The youngest 'C' granite of sub-alkaline type emerges in linear belts of dykes and stocks, up to several kilometers in diameter, located in fault zones dissecting older granites.

Using the proposed systematics, the structure of plutons can be extrapolated to the depth even in the case, when only intrusion fragments are exposed. Similar prediction is of particular importance by virtue of the fact that only 'B' and 'C' granites are able to generate rare metal deposits, whereas 'A' granitic rocks are barren for the most part. In this context, the task of discrimination between these rocks is very topical and refers to understanding common processes that took place while intrusive mass formation. The aim of this work is to quantify the parameters that govern the textural development in granites and discuss reasons why the corresponding textural evolution occurred. To do this, a statistical approach combining fractal analysis of texture of rocks and computer simulation of nucleation and growth is used.

METHOD OF ANALYSIS

As noted above, when studying the texture of igneous rocks, two different aspects of rock fabric may be given much attention: the morphologies of mineral grains and the properties of their spatial distribution. First group of textural variables, as applyed to granitoids, is available to describe by means of techniques intended for description of complex shapes including Fourier transform (Brodskaya, 1990; Byerly, Mrakovich, and Malcuit, 1975; Ehrlich and others, 1972) and fractal analysis (Petford and others, 1993). Another one has been the subject of investigations designed for examining homogeneity and randomness of rock-forming aggregates including contact-area method (Mahan and Rogers, 1968;

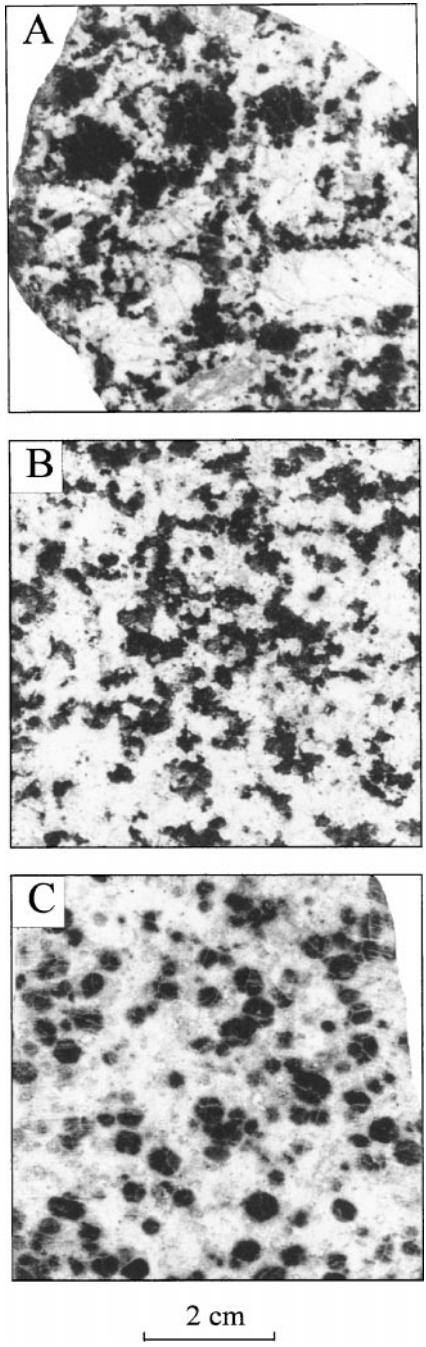

Figure 1. Morphological types of granites: (A) standard type, Severny pluton, North Chuckotka, sample 4046-3; (B) alaskite type, Aqshatau pluton, Central Kazakhstan, sample 1-3; (C) amazonite type, Aetyka pluton, Trans-Baikal region, sample Ae-1.

Rogers and Bogy, 1958; Sanderson, 1974) and cited analysis of grain transition probabilities (Vistelius and others, 1983; Vistelius and Harbaugh, 1980).

To emphasize principal significance, which is placed on the quartz grain arrangement in granites for their discriminating, the method used in this study was chosen from a compromise between two different views on rock texture essentials. Concentrating on morphometry of quartz aggregates, the method under review takes a power-law dependence of square of aggregate cuts (M) on their linear size (R)

$$M(L) \propto R^D$$

as a main algorithm for quantifying texture through its fractal dimension (D), and therefore may be defined as a certain kind of fractal approximation (Mandelbrot, 1982). Known as the concept of a fractal cluster, related approach, for instance, is typical for testing whether the objects of microworld (e.g. metal colloids, soot, or coagulated aerosols) exhibit a scale invariance (Feder, 1988, for a review).

For studying the fractal behavior of quartz aggregate distribution, 19 granitic samples from several intrusive masses of Russia, Kazakhstan, and Egypt were selected and divided into three groups according to their morphological type and magmatic formation assignment (Table 1). These groups were presented by series of medium-coarse grained granites without obvious signs of preferred grain orientation or tectonic pattern. For each sample, first cut in an arbitrary way into polish sections as large as 20×20 cm or 10×10 cm, the quartz aggregate images were digitized using a scanner at a resolution 300 pixels/in., and the parameter of a greatest length (R), along with the square value (M) are measured for each images. To determine fractal dimension, for every digitized set one calculated a slope coefficient of linear regression

$$\log M = \log a + D \log R$$

by a least-squares technique. To summarize the accuracy of fractal dimension estimation, a 95% confidence interval was computed for each D value obtained. In addition, the amount of quartz in the rock samples was evaluated.

RESULTS

Figure 2 shows typical log–log plot of M versus R for alaskite granite well described by straight-line relationship, as expected for a fractal. Similar results were also obtained for the other types of the granites; a linear least-square fit is valid for testing self-similarity of the data sets throughout the entire observed range of a grain size (about two orders of magnitude).

Table 1. Data of Granitic Texture Analyses

Sample	Type of granite[a]	Mode of quartz, Q	Fractal dimension, D
Severny pluton, North Chuckotka			
4121	A-2	0.32	1.75 ± 0.05
4046-1	A-2	0.38	1.79 ± 0.08
4046-2	A-2	0.30	1.77 ± 0.07
4046-3	A-2	0.35	1.74 ± 0.12
4022-1	A-3	0.33	1.79 ± 0.10
4022-2	A-3	0.31	1.78 ± 0.12
S-1	A-3	0.27	1.77 ± 0.10
S-2	A-3	0.31	1.74 ± 0.09
Urmi pluton, Badzhal, Chabarovsky region			
82221-1	A-4	0.31	1.89 ± 0.10
82221-2	A-4	0.25	1.75 ± 0.09
82180	A-2	0.43	1.78 ± 0.08
Aqshatau pluton, Central Kazakhstan			
4\75-1	B-2	0.27	1.62 ± 0.10
4\75-2	B-2	0.28	1.70 ± 0.12
4\75-3	B-2	0.34	1.78 ± 0.05
1-1	B-2	0.34	1.69 ± 0.13
1-2	B-2	0.31	1.62 ± 0.09
1-3	B-2	0.38	1.59 ± 0.12
1-4	B-2	0.37	1.74 ± 0.08
1-5	B-2	0.35	1.66 ± 0.12
1046-1	B-2	0.35	1.62 ± 0.10
1046-2	B-2	0.34	1.65 ± 0.12
Kent pluton, Central Kazakhstan			
K-1	B-2	0.41	1.72 ± 0.10
K-2	B-2	0.37	1.72 ± 0.10
Karkaralinsky pluton, Central Kazakhstan			
Kr1-1	B-3	0.36	1.68 ± 0.07
Kr1-2	B-3	0.37	1.64 ± 0.09
Sibinsky pluton, Central Kazakhstan			
17\75	B-2	0.22	1.57 ± 0.12
Aetyka pluton, Trans-Baikal region			
Ae-1	C-2	0.26	1.55 ± 0.09
19\77	C-2	0.45	1.61 ± 0.07
Abu Dabbab pluton, Egypt			
AD1	C-3	0.35	1.46 ± 0.10
AD2	C-3	0.29	1.53 ± 0.11
AD3	C-3	0.12[b]	1.58 ± 0.11

[a] Coarse-medium (2), medium (3), and fine-grained (4) granites of standard (A), alaskite (B), and amazonite (C) type.
[b] Amount of porphyritic quartz grains only.

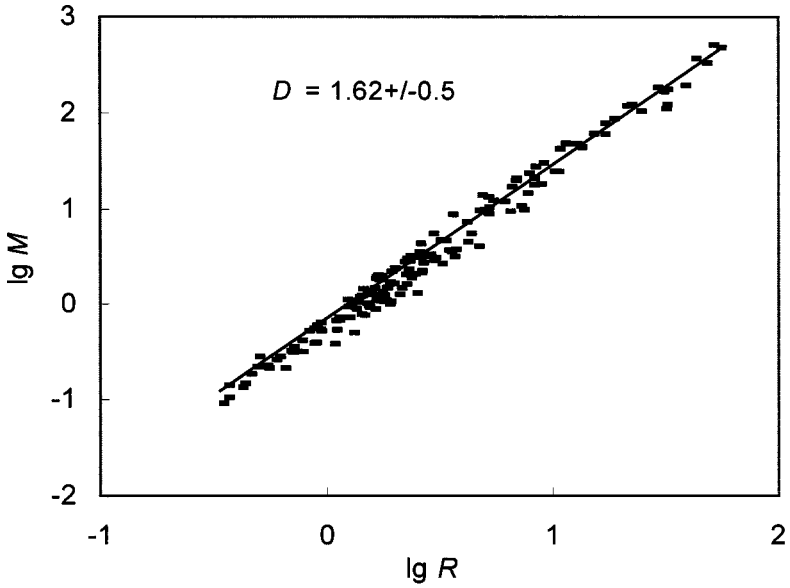

Figure 2. Plot of square of aggregate cuts (M) against their size (R) in log–log coordinates. The fractal exponent shown is consistent with the slope in the domain of large R. Alaskite granite, Aqshatau, Central Kazakhstan, sample 1046-1.

More detailed analysis of the data sets, however, made it possible to detect that slight nonlinearity of the fractal plots occurs, and two linear elements are required to satisfy experimental curves completely. In the literature (Orford and Whalley, 1983; Petford and others, 1993), these two linear unites are referred to in the context of the notion of pseudofractal, assuming textural and structural fractal dimensions convey self-similarity on different scale levels. It is thought that the former reflects the local peculiarities of objects, while the later corresponds to the global ones. With this in mind and having regard to relation between nonaggregative and cluster grain portions, the best linear fit in the domain of large R, i.e. the major domain of aggregate reality, for every set was used to evaluate its D value. As a criterion of best-fitting regression, the error of the slope factor served which was no more than ± 0.10–0.12.

For different types of granites, the results of the calculations are given in Table 1. Generally, the regressed data define the structural fractal dimension that ranges from 1.46 to 1.89. It is particularly remarkable that within these limits the D values of sets may be divided into three virtually nonintersectional groups according to the morphological designations of rocks. Such conclusion is readily apparent from the statistical variations of fractal dimension digested in Table 2. These variations are such that the majority of D values changes in 1.75–1.81 for

Table 2. Variations of Fractal Dimension for Different Types of Granites

	Mean	Standard	Minimum	Maximum	Confident intervals ($p = 0.05$) (−95%)	(+95%)
A	1.78	0.04	1.74	1.89	1.75	1.81
B	1.67	0.06	1.57	1.78	1.63	1.70
C	1.55	0.06	1.46	1.61	1.48	1.62

standard granites, in 1.63–1.70 for alaskite granites, and in 1.48–1.62 for amazonite granites. Within the limits of each individual group the variations of fractal dimension are random, among them variations associated with different sections of the same specimen of rock. Significance of quantitative differences between the type of granites is verified through t test for comparison of group means, and analysis of their within-group and between-group variances. This solution provides the basis for the inference that the fractal model in question may be used for identification of granites from their texture.

DISCUSSION

Theory

To account for the data of morphological studies, the model of nucleation and growth occurring when the first-order phase transformation happens was chosen. This model covers the kinetics of melt crystallization or solid state recrystallization on the basis of statistical laws of the process without regard for physical aspects of phenomenon. As a result, the time quantities of the evolution of the system as well as the morphologies of aggregate texture originated (e.g. interface area, edge length, grain, or cluster size distribution) are discussed (Mahin, Hanson, and Morris, 1980; Meijering, 1953; Saetre and others, 1986).

Kolmogorov (1937) was the first who considered the matter of interest in a general case, followed by Johnson and Mehl (1939), who applied it to study the common processes of metal freezing. These authors proposed several postulates for describing the mechanism of phase transition, namely (i) limitless reaction space, (ii) Poisson (random) distribution of nuclei in transformed medium, (iii) a convex shape, and (iiii) a linear velocity of growth that are the same for all crystals at each instant of crystallization. Under listed conditions, a rate of nucleation (α) and a linear velocity of nuclei growth (ν), being arbitrary functions of time t, appear as parameters of the reaction. In the simplest case of the stationary process

such that

$$\alpha = \text{const} \quad \text{and} \quad \nu = \text{const},$$

the volume fraction of transformed phase for spherical nuclei is given by the equation

$$q(t) = \exp\left(-\frac{1}{3}\pi\alpha\nu^3 t^4\right),$$

which will plot as a curve of logistic type.

During the transformation, growing nuclei impinged to one another produce the clusters. At a certain value of phase transition degree (q_c), these ones unite into single aggregate extended through the whole system, causing drastic changes of its physical properties. In a much the same way behaved are many other systems with second-order phase transition-like conversion from an insulator to a conductor in the compound of insulated and conducted particles, breakdown of rocks under formation of enough amount of cracks, and so on. All of them may be adequately described in terms of the percolation theory that deals with the properties of random clusters forming on a regular lattice near percolation threshold. Kindred clusters are fractal with a power-law mass distribution and able to capture the cardinal features of many actual systems (Stauffer and Aharony, 1992).

As selected part of percolation theory, the continuum models concerned irregular lattice geometry are also considered. Of special interest here is the percolation model for overlapping spheres randomly distributed with random radii that, being treated as a time sequence, corresponds closely with the assumptions of nucleation and growth model (Orgzall and Lorenz, 1994). An important point is that last model conforms with the concept of ideal granite in every way except the type of the predicted linear grain sequences: in the case discussed similar succession may be defined as a sequence of Bernoulli type rather than zth-order ($z \geq 1$) Markov chain type for lack of limitation on the same type grain accretion.

Simulation

For comparison between nature and model textures, the numerical experiments were conducted to evaluate the random configuration outlined above with particular reference to the fractal properties of "impinged grain" cluster cuts in the plane section. The simulation of the nucleation and growth processes was performed by formal algorithm of Johnson and Mehl (1939), according to which spherical nuclei are assumed to form at random throughout the entire reaction volume and grow as interpenetrating spheres. In line with these assumptions, the possibility of appearance (p) and the increment of radius (dr) of single nucleus

Table 3. Fractal Dimension of Random Clusters Cuts Referring to Nucleation and Growth Model

Phase transition degree, q	Fractal dimension, D
0.25	1.32 ± 0.10
0.28	1.42 ± 0.10
0.29	1.38 ± 0.08
0.29	1.46 ± 0.13
0.33	1.36 ± 0.09
0.33	1.39 ± 0.10
0.37	1.55 ± 0.05
0.38	1.37 ± 0.09
0.43	1.51 ± 0.10
0.44	1.53 ± 0.07
0.54	1.60 ± 0.10
0.65	1.73 ± 0.13
0.72	1.81 ± 0.11

at each cycle of phase transformation were taken as the parameters of the model. Additionally, the phase transformation degree was consider which varied between 0.25 and 0.72. The resolution of the model patterns thus constructed was equivalent to that of a $800 \times 600 \times 500$ grid. In operations these patterns were dissected by random planes with fitting of the large cluster cut sets was applied to determine their fractal dimension.

The results of the simulation are given in Table 3. As may be seen, D values presented here increase with the volume fraction of transformed phase right up to 1.81 ± 0.11, even though some transgressions of the regular trend take place which underline the statistical character of the fractal model. The cited figure agrees closely with the expected value of 1.89 ± 0.01 which is theoretical fractal dimension of the two-dimensional continuum percolation cluster with $q_c = 0.68$. In the range of $q = 0.25 - -0.40$ typical of granites, the largest values of D are no more than 1.53–1.55 and consistent with the value of 2.51 ± 0.01 which is fractal dimension of the three-dimensional continuum percolation cluster with $q_c = 0.30$ reasonably well (Orgzall and Lorenz, 1994; Rosso, 1989).

The fractal parameters deduced from the simulation may be compared with those in the granites. This clearly demonstrates an insufficiency of much difference between the fractal dimension of the quartz aggregate distribution in amazonite granites and that of the homogeneous Poisson model (with allowance made for given modal composition). It is notable that the nature and modal patterns are equally similar in appearance (Fig. 3). On the contrary, such a difference is found for other types of examined granitic rocks whose fractal dimension noticeably exceeds the theoretical value. Indeed, their cluster distributions appear as less

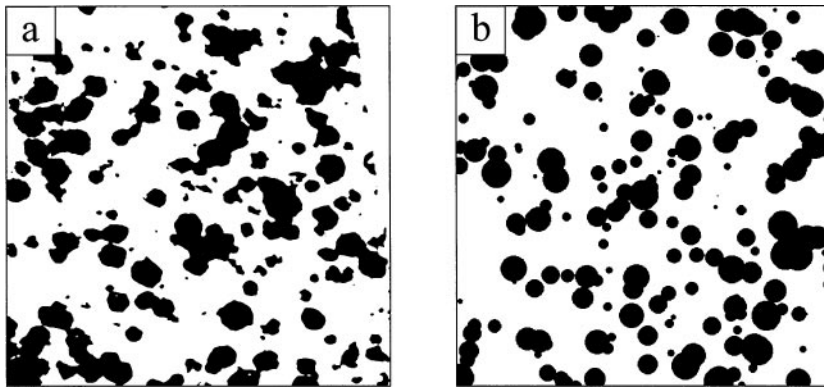

Figure 3. (a) Quartz aggregate distribution and (b) model pattern for amazonite granite. Mode of "quartz" nuclei $Q = 0.25$, fractal dimension $D = 1.32$.

homogeneous and clusters alone do as more compact as against the Poisson model (Figs. 4(a) and 5(a)).

Order of Crystallization

To explain such a textural variation, the order of crystallization of the granites should be taken into account. The fact is that only the amazonite type of them exhibits an early growth of quartz porphyritic phenocrysts from melt which

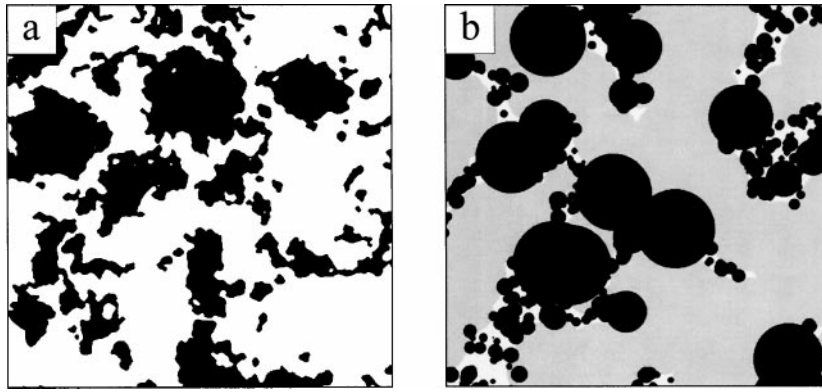

Figure 4. (a) Quartz aggregate distribution for standard granite and (b) model pattern originated from successive forming three generations of spherical nuclei: early "feldspar" (grey circles), intermediate (large black circles), and late "quartz" (small black circles) ones. $Q = 0.34$, $D = 1.80$.

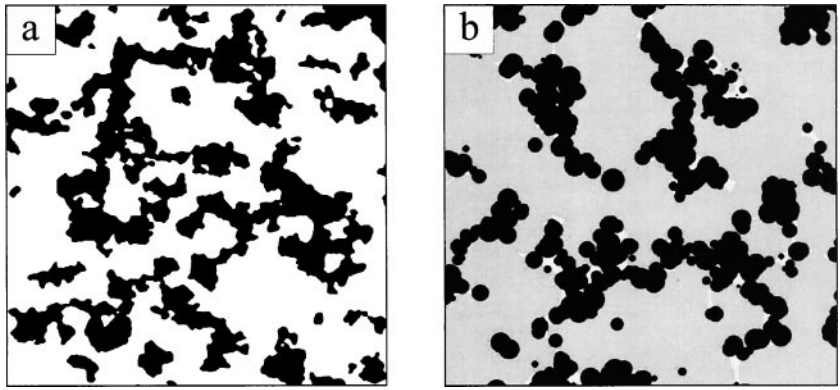

Figure 5. (a) Quartz aggregate distribution for alaskite granite and (b) model pattern originated from successive forming two generations of spherical nuclei: early "feldspar" (grey) and late "quartz" (black) ones. $Q = 0.29$, $D = 1.62$.

resulted in the random distribution of their growing sites regardless of the other mineral phase emergence. For porphyritic granites of the standard type, the actual features of the freezing process are two different-sized generations of quartz crystals forming after feldspar phenocryst framework appears. This order of crystallization may be simulated by the heterogeneous Poisson model, which is set through superposition of several, sequentially appearing random distributions. So each of them mimics the growth of one mineral phase or generation, the locations of nuclei during crystallization are thus dictated by a presence of earlier growing sites. By fitting parameters of cited distributions, the model patterns can be obtained that coincide with those of nature (Fig. 4(b)). As would be expected, their fractal dimension, being measured, takes distinctly high value.

For alaskite granites, the similar scheme of genesis is unsuitable to interpret the texture because of equigranularity of the rocks. Instead, two competitive hypothesis may be touched which connected with different mechanisms of their origin. The first one admits the influence of microheterogeneity of acid silicate melt upon the texture of rock (Beskin, Larin, and Marin, 1996) and are beyond the present discussion. The second one suggests that the autometamorphic recrystallization plays key part in the granitic texture development and merits more detailed consideration here. It is useful to note in this connection the data of microscopic study, by which the specified processes were favorable to solid-state transformation of the perthitic alkali-feldspar phenocrysts of granites into two dissimilar phases, microcline and albite, yielding a secondary aggregate that is equigranular in texture instead (Buzkova, 1986). From this line of observation, the primary magmatic origin of alaskite texture may be assumed with equidimensionality of individual quartz as a necessary condition for initiation of quartz "chains"

arrangement (Fig. 5(b)). The eventual appearance of the rock was acquired in the course of the secondary processes of deuteric changes induced by recrystallization of feldspathic matrix.

CONCLUSIONS

Morphometry of quartz aggregate cuts in terms of fractal approach can be applied to quantify texture of igneous rocks with reference to the processes of their origin. Used in this way, the fractal dimension of mineral cluster sets can be taken as a multiscale descriptor for discriminating between the different types of granites. Its application to petrogenesis implies that the spatial distribution of quartz grains in rock matrix will be simulated through nucleation and growth and will satisfy the homogeneous or heterogeneous Poisson models, with both order of crystallization and autometamorphism important in explaining textural variability. This enables the granitic texture to be discussed with respect to the configuration of the three-dimensional percolation cluster.

Such a view is far from a generally taken concept of rock grain framework as distorted regular lattice and relevant for describing rock texture evolution in terms of self-organization of magmatic system near the critical points in which the second-order transition occurs. The position of these points as well as the correlation length of percolation cluster can be determined numerically, with the latter quantity being a characteristic size of rock-forming mineral aggregate. Notice that it increases indefinitely in the vicinity of a percolation threshold to form essentially nonperiodic texture of rock.

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