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Experimental Assessment of Gradual Deformation Method[1]

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Uncertainty in future reservoir performance is usually evaluated from the simulated performance of a small number of reservoir realizations. Unfortunately, most of the practical methods for generating realizations conditional to production data are only approximately correct. It is not known whether or not the recently developed method of Gradual Deformation is an approximate method or if it actually generates realizations that are distributed correctly. In this paper, we evaluate the ability of the Gradual Deformation method to correctly assess the uncertainty in reservoir predictions by comparing the distribution of conditional realizations for a small test problem with the standard distribution from a Markov Chain Monte Carlo (MCMC) method, which is known to be correct, and with distributions from several approximate methods. Although the Gradual Deformation algorithm samples inefficiently for this test problem and is clearly not an exact method, it gives similar uncertainty estimates to those obtained by MCMC method based on a relatively small number of realizations.

KEY WORDS: Markov chain, Monte Carlo, randomized maximum likelihood, local perturbation.

INTRODUCTION

The only practical methods for quantifying uncertainty in reservoir performance require the generation of multiple random reservoir models conditional to available data. By simulating the future production from each realization, an empirical distribution of production characteristics is obtained. The validity of the Monte Carlo method for quantifying uncertainty depends on the quality of the distribution of reservoir models generated. Methods for sampling from the a posteriori probability density function (pdf) of reservoir flow models conditioned to production data have been discussed by Hegstad and Omre (1997), Oliver, Cunha, and Reynolds (1997), Bonet-Cunha and others (1998), Roggero and Hu (1998), Hegstad and Omre (2001), Hu (2000), Holden and others (2001), and Omre (2001).

Gradual Deformation is a method for gradually deforming continuous geostatistical models to generate reservoir models that honor historic production data.

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This algorithm has been used by Hu and others (1999), Le Ravalec, Hu, and Nœtinger (1999), and Hu, Ravalec, and Blanc (2001) to incorporate historic production data to reduce uncertainty in production forecasts. Because the gradual deformation algorithm preserves the geostatistical parameters while deforming the model to honor the data, it seems intuitive that it might generate realizations from the pdf for model variables conditioned to data. The purpose of this study is to evaluate the ability of Gradual Deformation method to correctly assess uncertainty of reservoir model realizations. The assumptions and models used in this study are identical to those used previously by Liu and Oliver (2003) to assess uncertainty quantification of other common algorithms for conditional simulation.

PROBLEM DESCRIPTION

Evaluation of the ability of Gradual Deformation method to correctly sample reservoir models conditional to production data requires an appropriate test problem. The test problem should be small enough that a large number of realizations can be generated in a reasonable amount of time. It should also be highly nonlinear as some of the approximate methods are known to sample correctly for problems with linear relationships between the conditioning data and the model parameters. By choosing a single-phase transient flow problem with highly accurate pressure measurements, fairly large uncertainty in the values of porosity and permeability field, and a short correlation length, we were able to obtain a problem with multiple local maxima in the likelihood function, yet for which a flow simulation required only 0.02 seconds.

The test problem is a one-dimensional heterogeneous reservoir whose permeability and porosity fields are shown in Figure 1. The reservoir is discretized into 20 gridblocks, each of which is 50 ft in length. Both the log-permeability ($\ln k$) and porosity fields were assumed to be multivariate Gaussian with exponential covariance and a range of 175 ft. The prior means for porosity and log-permeability are 0.25 and 4.5, respectively. The standard deviation of the porosity field is 0.05 and the standard deviation of the log-permeability field is 1.0. The correlation coefficient between porosity and log-permeability is 0.5. The flow is single phase with an oil viscosity of 2 cp and a total compressibility of 4×10^{-6} psi^{-1}. The initial reservoir pressure is 3500 psi.

There are three wells in this linear reservoir: a constant rate producer in gridblock 13, and two observation wells in gridblocks 7 and 18. A reservoir flow simulator was used to compute pressures at each of the wells at 10 different times, then Gaussian random noise with a standard deviation of 0.5 psi was added to the data generated from the true reservoir model. The observed pressure data for all three wells are shown in Figure 2. Because of the random noise, the early

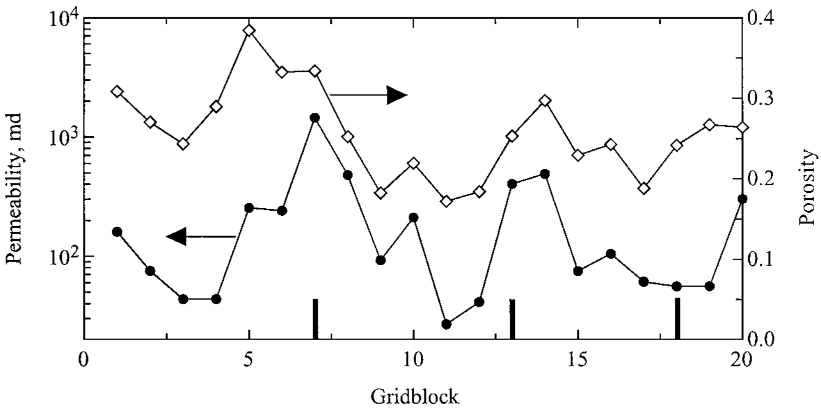

Figure 1. The true synthetic permeability and porosity fields, used to generate pressure data to test the sampling algorithms. Well locations are shown by solid bars along the base of the figure.

time pressure drop measurements at the observation wells are erratic. Porosity measurements at well locations were not included in this study as their introduction would have made the posteriori (conditional) pdf for model variables more nearly Gaussian.

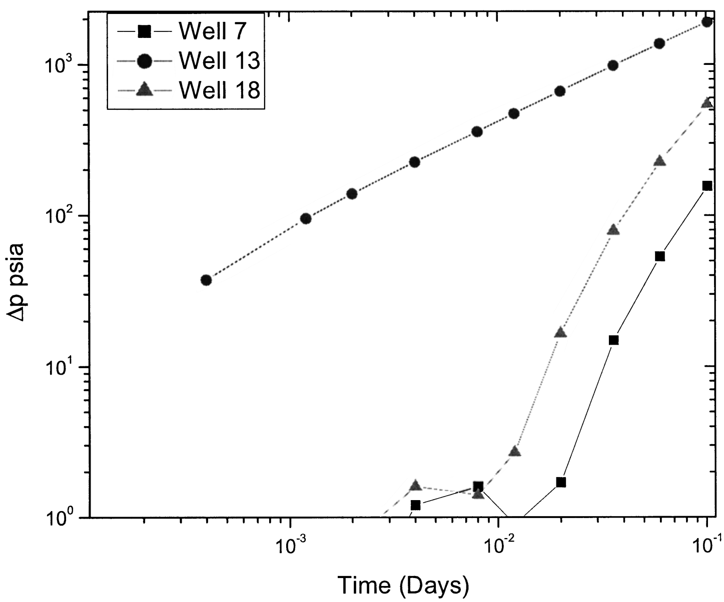

Figure 2. The observed pressure drop at all wells. Random noise added to the true pressure drop causes the nonphysical appearance at low values of Δp.

The conditional probability density for the model variables, given the pressure data, is provided by Bayes rule,

$$\begin{aligned} f_{\mathrm{M|D}}(m|d_{\mathrm{obs}}) &= f_{\mathrm{D|M}}(d_{\mathrm{obs}}|m)\, f_{\mathrm{M}}(m) \Big/ \int f_{\mathrm{D|M}}(d_{\mathrm{obs}}|m)\, f_{\mathrm{M}}(m)\, dm \\ &\propto \exp\left(-\frac{1}{2}(g(m)-d_{\mathrm{obs}})^T C_{\mathrm{D}}^{-1}(g(m)-d_{\mathrm{obs}}) \right) \\ &\times \exp\left(-\frac{1}{2}(m-\mu)^T C_{\mathrm{M}}^{-1}(m-\mu) \right) \end{aligned} \tag{1}$$

where $g(m)$ is the vector of theoretical pressure data obtained by running the simulator with log-permeability and porosity values given by the vector m. C_{D} and C_{M} are the data error covariance and the prior model parameter covariance respectively. Both of these matrices are assumed to be known for this study. If the algorithm used to generate reservoir realizations is an exact method, the samples should reflect this distribution, i.e., reservoir models with higher conditional probability density should be more likely to be generated.

GRADUAL DEFORMATION METHOD

The principal idea of the Gradual Deformation method is that new realizations of a random field Z can be written as the linear combination of a set of independent random Gaussian fields with expected mean μ and covariance C_Z, i.e.,

$$Z(K) = \sum_{i=1}^{n} k_i (Z_i - \mu) + \mu \tag{2}$$

The coefficients k_i are required to satisfy:

$$\sum_{i=1}^{n} k_i^2 = 1. \tag{3}$$

In this study, each of the Z_i is a vector of identical independent distributed deviates with expectation 0 and variance 1. By deforming the model in this way, the intent is to preserve the covariance and mean. Because the k_i depends on the Z_i through the minimization, the mean and covariance are not necessarily preserved, however, when long sequences of deformations are applied (Skjervheim, 2002). Le Ravalec, Hu, and Nœtinger (1999) have shown that the chain of realizations created in this way sometimes results in biased sampling.

We tested the most basic form of the Gradual Deformation algorithm in which pairs of vectors are combined:

$$Z(\rho) = Z_1 \cos(\pi\rho) + Z_2 \sin(\pi\rho) \tag{4}$$

where ρ is the deformation parameter with the range from 0 to 2. The procedure for generating a realization conditional to d_{obs} is as follows:

1. Generate an initial vector Z_1 of independent normal deviates.
2. Generate a second vector of independent normal deviates Z_2.
3. Search for the optimum ρ value which gives a reservoir realization minimizing the objective function S_{d} of Equation (7).
   Reservoir model realizations are generated using the LU decomposition method:

$$m(\rho) = m_{\text{prior}} + L\,Z(\rho) \tag{5}$$

   where

$$L L^{\text{T}} = C_{\text{M}}. \tag{6}$$

   Note that the objective function to be minimized contains the squared data mismatch only:

$$S_d(\rho) = \frac{1}{2}[g(m(\rho)) - d_{\text{obs}}]^T C_{\text{D}}^{-1}[g(m(\rho)) - d_{\text{obs}}]. \tag{7}$$

4. If the minimum value of the objective function is sufficiently small, then stop the procedure. Otherwise replace Z_1 with the optimal $Z(\rho)$ and return to step 2.

Because the Gradual Deformation algorithm involves minimization, the convergence or stopping criterion is expected to be important to the sampling. Unfortunately, none of the papers referenced here provided quantitative convergence criterion for this algorithm. In our tests, we used $S_d = 1.6\, n_{\text{d}}$ as a stopping criterion, as reduction to a lower level is difficult in a reasonable number of iterations (10,000).

Local Perturbation

When the historic production data are scattered spatially in the reservoir, adding an independent vector Z is likely to improve the fit in some gridblocks

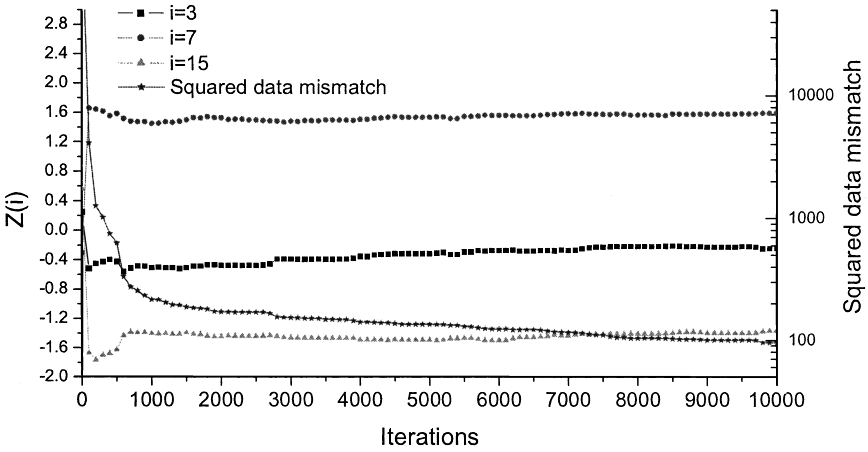

Figure 3. The 3rd, 7th, and 15th elements change with iterations using global perturbation.

while deteriorating the fit in other locations. This led Hu and others (1999) to develop a procedure for modifying the values of Z only within a limited region for which the data mismatch was large. Because it is not clear how to choose a limited region for interference tests, we examined the extreme case in which the region of change was limited to a single gridblock. The location of the gridblock to be modified was randomly chosen in each iteration. In this case, the Z vector in Equation (5) is calculated as

$$z_i(\rho) = \begin{cases} z_{1,k} & \text{for} \quad i \neq k, \\ \cos(\pi\rho) z_{1,i} + \sin(\pi\rho) z_{2,i} & \text{for} \quad i = k. \end{cases} \tag{8}$$

where k is a randomly selected perturbation location. $z_{2,i}$ is a realization of a random variable sampled from the Gaussian distribution with mean 0 and variance 1. $z_{1,i}$ is the ith element of the vector Z_1. Figures 3 and 4 show the permeability value in the 3rd, 7th, and 15th gridblock after every 100th perturbation for global and local perturbation, respectively. From these two figures, the squared data mismatch is seen to decrease even slower by local perturbation than by global perturbation. Part of the reason is that some of the local perturbations are applied in regions for which a change in property values has no effect on the data mismatch.

RESULTS

In this study, we ran the Gradual Deformation algorithm until the objective function was reduced to 50 or less. If the objective function was not reduced to

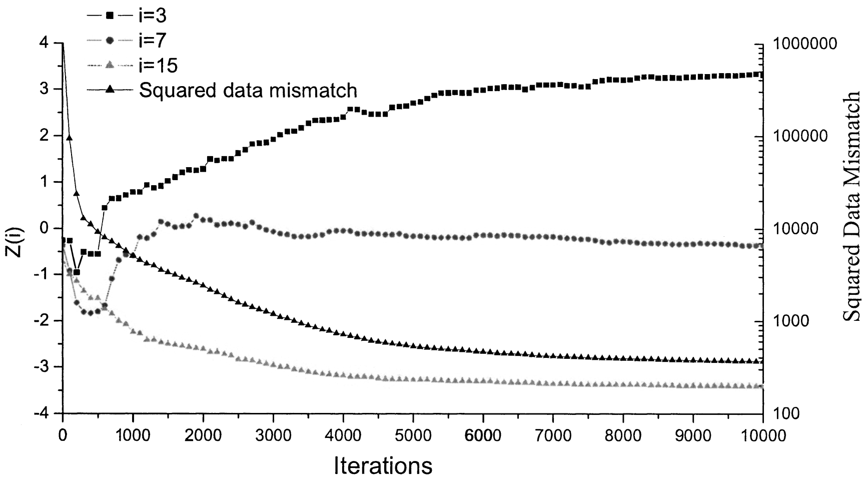

Figure 4. The 3rd, 7th, and 15th elements change with iterations using local perturbation.

50 by the 10,000th iteration, it was discarded. For comparison, over 99% of the MCMC (Markov Chain Monte Carlo) realizations have squared data mismatch values less than 50, so 50 is a relatively loose tolerance on the data mismatch. In each iteration, a line search is required to find the optimum ρ value, so 10,000 is a sensible maximum number of iterations. For the global perturbation method, 86 realizations were generated out of 1000 Gradual Deformation sequences. The local perturbation method performed slightly better with 11% of the sequences reaching the convergence criterion before the 10,000th iteration.

Several summary property values of each realization were calculated to compare the distribution of realizations from Gradual Deformation with distributions from other methods. We report the distribution of realizations of average reservoir porosity and effective permeability because these are important in predicting oil-in-place and recovery. Also, because some minimization methods generate extreme values in the property fields, we report the maximum permeability of each reservoir realization. We also report the distribution of the squared model mismatch to check the probability that the realization could be a sample from the prior distribution. Finally, we report the distribution of realizations of data mismatch because it is important that realizations honor the historical data. The distributions of realizations are summarized in the form of box plots. A key for interpreting the box plots is shown in Figure 5.

Figure 6 shows comparisons of the property distributions from different sampling methods for the test problem described in this paper. The acronym MCMC indicates the Markov Chain Monte Carlo method based on 320 million realizations. The distributions for Randomized Maximum Likelihood (RML), Linearization

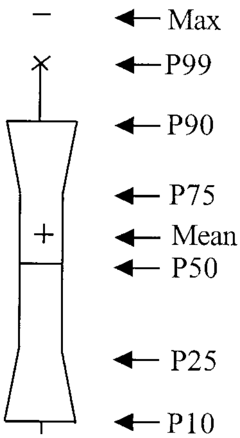

Figure 5. Key for interpretation of box plots. (*P* Values are percentiles.)

about the Maximum a Posterior Model (LMAP), Pilot Point (PP), and Unconditional (UC) are all based on 5000 realizations. The algorithm of each sampling method has been described in a previous paper (Liu and Oliver, 2003). As explained in that paper, we are quite confident that the results from MCMC are correct because we obtained consistent results from several different variants of the MCMC method.

DISCUSSION

The distributions of average porosity, effective permeability, and maximum permeability from Gradual Deformation were very similar to distributions from MCMC and RML. The small differences are easily attributed to the small number of samples from Gradual Deformation. The Gradual Deformation algorithm also did an excellent job of sampling the squared model mismatch distribution (shown in Figure 7), even though the standard implementation of this algorithm does not have squared model mismatch term in the objective function. The only distribution from Gradual Deformation that is poor is the squared data mismatch. Instead of being distributed approximately as χ^2, it is nearly a delta function at $S_\mathrm{d} = 50$.

The good performance of the Gradual Deformation algorithm in matching the prior model is quite discrepant with the results from Le Ravalec, Hu, and Nœtinger (2000) and Skjervheim (2002). Le Ravalec, Hu, and Nœtinger (2000) revealed that the distribution of samples from the Gradual Deformation method do not reflect the correct conditional distribution even for linear problems. As a result, they proposed an Enhanced Gradual Deformation method, which contains both the likelihood and the prior constraint terms explicitly in the objective function. For the classical Gradual Deformation method, the only constraint in the

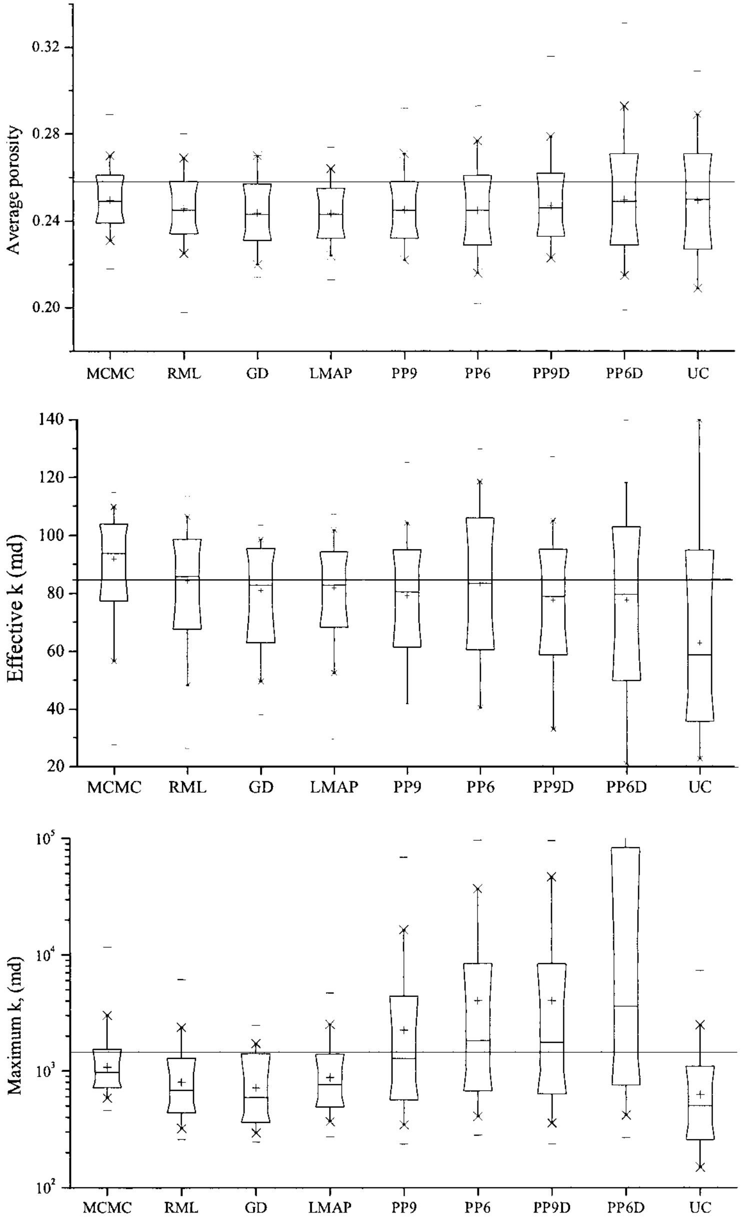

Figure 6. Distribution of conditional realizations of average porosity, effective permeability and maximum permeability from the approximate sampling algorithms and from the very long MCMC. The unconditional distribution and the true values are shown for comparison.

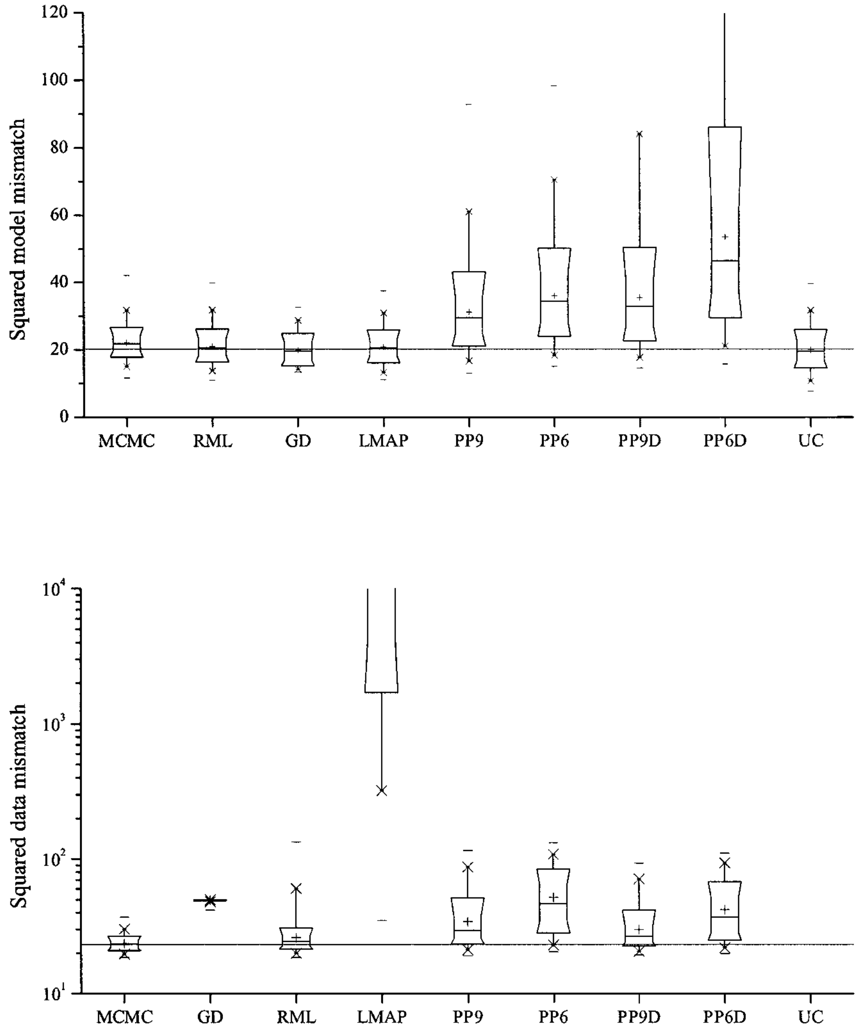

Figure 7. Distribution of conditional realizations of squared model mismatch and squared data mismatch from the approximate sampling algorithms and from the very long MCMC. The unconditional distribution and the true values are shown for comparison.

objective function is the likelihood term, and the prior information is only honored approximately by the method of combining realizations. After many combinations, the influence of the prior was small in their example, implying that the stopping criteria may be important (see also Holden and others, 2001). Skjervheim (2002) tested the Gradual Deformation on small two-parameter problems, linear and nonlinear, and drew a similar conclusion. Both Le Ravalec, Hu and Nœtinger (2000) and Skjervheim (2002) concluded that, for linear problems, the standard Gradual Deformation method does not sample from the posteriori distribution, but the Enhanced Gradual Deformation does sample correctly; for nonlinear problems, neither algorithm samples well, but the distributions of variables for the Enhanced Gradual Deformation are better.

In our nonlinear test problem, we found that the standard algorithm gave good results. It is possible that the difference in results could be due to the difference in the dimension of the test problems (2 variables vs. 40 variables). This same property of sampling well for large models but not for small has been observed with Randomized Maximum Likelihood method (Oliver, He, and Reynolds (1996). Liu and Oliver (2003)). It also seems that the drift of the covariance and mean observed in the small examples of Le Ravalec, Hu, and Nœtinger (2000) and Skjervheim (2002) are less likely to occur in larger models if the number of deformations is relatively modest. A chain of 10,000 local deformations was not sufficient to deform the structure of the initial model for our example.

The fact that the mean of the squared data mismatch distribution is larger than expected is simply a result of our choice of stopping criterion. If we had chosen a smaller criterion, we would have had better agreement in the mean with the correct distribution. Estimates of uncertainty in near future pressure predictions would still be in error, but it would be straight forward to modify the objective function to add errors to the measured data as illustrated in Le Ravalec, Hu, and Nœtinger (2000). In that case the sampling of data mismatch would probably be similar to RML.

Although our focus was not on efficiency, it was clear that the Gradual Deformation method is inefficient in the neighborhood of the minimum compared, for example, to the method of randomized maximum likelihood. One reason for the inefficiency is that the new vector in each iteration of the Gradual Deformation method provides a random direction for minimization, instead of a downhill direction. At early iterations, a reduction in the mismatch can be achieved in almost any direction, but at later iterations, the likelihood that a reduction in the mismatch can be obtained in a randomly chosen direction is small. Figure 8 shows the data objective function as a function of ρ for the first and the 1000th iterations. In the first iteration, a large reduction is obtained by choosing $\rho = 0.88$. At the 1000th iteration, on the other hand, no reduction was possible. From the squared data mismatch curve in Figures 3 and 4, it is clear that the rate of improvement in the data match is slow at late iterations.

It does seem very likely that the rate of convergence would improve dramatically if multiple independent realizations were used at each step in the deformation. Le Ravalec and Nœtinger (2002) have compared the schemes of combining different number of realizations to form a chain and found that the convergence rate has a exponential relation with the number of unconditional realizations combined

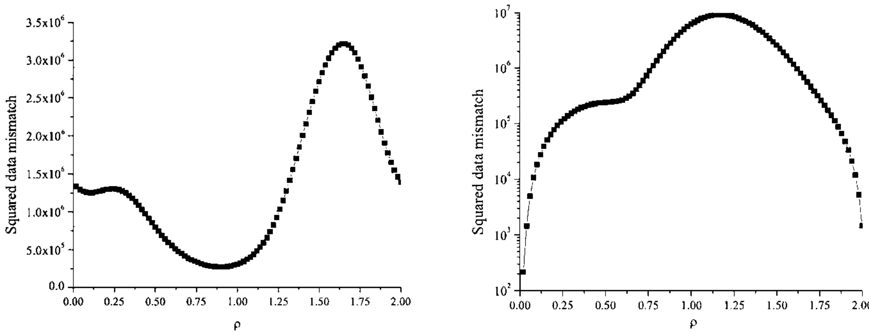

Figure 8. The shape of squared data mismatch function at the 1st (left) and 1000th (right) iteration.

into the chain in each iteration. Instead of a line search for an optimal ρ, it would be necessary to perform a multidimensional search for the coefficients of the expansion. It appears however, that the objective function to be minimized for Gradual Deformation can be multimodal, even when the data constraints are linear.

CONCLUSIONS

The method of Gradual Deformation produced acceptable distributions of reservoir properties from the 40-parameter nonlinear model in a reasonable number of iterations. These distributions were similar to the distributions from MCMC, which is known to reflect the true distribution. This indicates that, although the method is only approximate, because the constraint of the prior model is approximately reflected in the new realizations to be combined and the number of iterations is relatively small in practice, the realizations will approximately honor the prior model. So based on these experiments, the Gradual Deformation method may be a reasonable method for uncertainty evaluation in reservoir studies. Although efficiency was not the primary focus of this investigation, it was clear that the method of Gradual Deformation was less efficient for generating realizations than other approximate methods.

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