GEOPHYSICS, VOL. 69, NO. 6 (NOVEMBER-DECEMBER 2004); P. 1530–1534, 3 FIGS.
10.1190/1.1836826

Short Note

Scattering attenuation in randomly layered structures with finite lateral extent: A hybrid *Q* model

Tobias M. Müller[1] and Serge A. Shapiro[2]

INTRODUCTION

Since the pioneering work of O'Doherty and Anstey (1971), much research has been devoted to understand the effect of stratigraphic filtering of seismic waves, i.e., the problem of multiple scattering in 1D random structures (e.g., Burridge et al., 1993; Shapiro and Hubral, 1999). For many subsurface structures, such as sedimentary basins, the assumption of layering is reasonable as a first approximation. However, real geostructures do not show perfect layering but exhibit a finite lateral extent in their elastic properties. This becomes particularly important when studying overburden effects in reflection seismology, where amplitude information is used in subsequent data analysis. For example, Malme et al. (2003) showed that the amplitude variation with offset (AVO) response for a vertical seismic profile (VSP) experiment over a North Sea field is significantly distorted by nonlayered overburden inhomogeneities. They demonstrated by seismic forward modeling that pointlike diffractors and large-scale, gas-filled sand bodies can be responsible for strong amplitude fluctuations.

In this short note, we study the transmission behavior of seismic primaries when the finite lateral extent of the inhomogeneities is accounted for. Physically speaking, we intend to quantify the combined effects of scattering attenuation due to thin layering and random diffractions and refractions. It is important to understand that our approach describes *scattering attenuation of seismic primaries* and not attenuation of the mean field (ensemble averaged wavefield) as presented in earlier works (e.g., Lerche, 1986). We use the model of an anisotropic 3D random medium. Anisotropy in this context means that the randomly distributed inhomogeneities have three different characteristic length scales and are characterized by a spatially anisotropic correlation function. We also assume that the medium is lossless, so that no intrinsic wave attenuation occurs. A sketch of such a structure is shown in Figure 1, resembling a realization of a 2D anisotropic random medium. Obviously, the applicability of this model implies that the statistical properties can be inferred from measurements such as well-log and seismic data, as shown for example by Imhof and Toksöz (2000) and Imhof and Kempner (2003). Also, statistical analysis of reflection traveltimes provides information about the horizontal scale of the inhomogeneities in the overburden of a large-scale reflector (Iooss et al., 2003; Kravtsov et al., 2003). In laboratory experiments, X-ray computed tomography is used to estimate sizes and shapes of heterogeneities and their spatial correlations (Hackert and Parra, 2001).

Scattering attenuation of seismic primaries results from the redistribution of wavefield energy from the vicinity of the first arrival into later arriving signals that is from the ballistic part of the pulse signal into the coda. Whereas in 1D inhomogeneous media only *multiple backscattering* causes scattering attenuation, in 3D inhomogeneous media *random diffraction* and *refraction* cause additional attenuation. The latter mechanisms become manifest in a random focusing of wavefield energy and can be interpreted as the occurrence of intersecting rays in the geometrical optics framework (Rytov et al., 1989). In general, for arbitrary inhomogeneous media, all three mechanisms occur. Our strategy is to compute the amount of scattering attenuation caused by these three mechanisms within the precision of the Rytov approximation. The latter is a perturbation approximation and allows the quantification of the coefficient of scattering attenuation, α, associated with the primary seismic wavefield. In particular, we use the generalized the O'Doherty-Anstey (ODA) approach of Shapiro and Hubral (1999), which corresponds to the Rytov approximation for primary waves in 1D random media and includes multiple backscattering. We also use the Rytov approximation to compute α in 3D anisotropic random media. This 3D Rytov approximation takes into account random diffraction and refraction but neglects backscattering (Müller and Shapiro, 2003). By

Manuscript received by the Editor August 14, 2003; revised manuscript received April 27, 2004.
[1]Curtin University of Technology, Department of Exploration Geophysics, GPO Box U1987, Perth, Western Australia 6845, Australia. E-mail: tobias.muller@geophy.curtin.edu.au.
[2]Freie Universität Berlin, Fachrichtung Geophysik, Bldg. D, Malteserstr. 74–100, 12249 Berlin, Germany. E-mail: shapiro@geophysik.fu-berlin.de.

combining heuristically the ODA and 3D Rytov approaches, we find a Q model that is applicable in 3D anisotropic random media, where all of the aforementioned attenuation mechanisms apply. Numerical experiments confirm the validity of the combined scattering Q model.

The outline of this short note is as follows. In the next two sections, we briefly review the results of the ODA approach for scalar wave propagation and approximations for the scattering attenuation coefficient in 3D anisotropic random media based on the Rytov approximation. Then, a combined Q model by the joint ODA approach and the Rytov approximation is presented. Numerical results illustrate the properties of this Q model. Finally, we discuss some open problems and present the conclusions.

SCATTERING ATTENUATION IN THINLY LAYERED MEDIA — THE ODA APPROACH

For 1D random media, Shapiro and Hubral (1999) approximated the transmitted, primary wavefield with a second-order Rytov approximation. An essential feature of their wavefield description (the so-called generalized ODA approach) is its ability to describe the wavefield in a single realization of the random medium. This becomes possible due to the use of self-averaged wavefield attributes. In 1D random media, scattering attenuation is caused by interference of multiply backscattered waves.

For a plane acoustic wave vertically impinging on a stack of randomly distributed layers (for simplicity, we assume the density to be constant), the time-harmonic transmissivity $T(t,\omega)$ can be written as (equation 4.19 in Shapiro and Hubral, 1999)

$$T(t,\omega) = e^{-\alpha^{1D}L + i\varphi^{1D}L} e^{-i\omega t}, \quad (1)$$

with

$$\alpha^{1D} = k^2 \int_0^\infty dr\, B_v(r)\cos(2kr) \quad (2)$$

$$\varphi^{1D} = k + \frac{3}{2}kB_v(0) - k^2\int_0^\infty dr\, B_v(r)\sin(2kr), \quad (3)$$

where k denotes the wave number corresponding to wave propagation in the unperturbed background medium ($k = \omega/c_0$, where c_0 is the sound velocity) and L is the propagation distance in the z-direction. $B_v(r)$ is the correlation function of the relative, zero-average velocity fluctuations $\zeta_v : B_v(r) = \langle\zeta_v(z)\zeta_v(z+r)\rangle$, where r is the correlation lag in meters, and the angle brackets denote ensemble averaging. In statistically homogeneous media, $B_v(r)$ is characterized by two parameters: the variance of the fluctuations, $\sigma_v^2 = B_v(0)$, and the characteristic size of the inhomogeneities, i.e., the correlation length $a_\parallel$. For example, for exponentially correlated fluctuations, we have $B_v(r) = \sigma_v^2 \exp[-|r|/a_\parallel]$. The quantities α^{1D} and φ^{1D} are called the attenuation coefficient and the phase increment, respectively. We note that, for elastic random media, one obtains different expressions for α^{1D} and φ^{1D}, which are, however, of the same structure as expressions 2 and 3. That is to say, α^{1D} and φ^{1D} are then given by a sum of Fourier sine and cosine transforms of the correlation functions of the respective elastic properties. From equation 2, it follows that the reciprocal quality factor $Q^{-1} = 2\alpha/k$ is maximum if $ka_\parallel = 1$. Applicability of equation 1 is valid under the assumption

$$L < \frac{\max(2\pi/k, a_\parallel)}{\sigma_v^2}, \quad (4)$$

or equivalently,

$$\begin{cases} \sigma_v^2 \dfrac{L}{a_\parallel} ka_\parallel < 2\pi & \text{if} \quad ka_\parallel < 2\pi, \\ \sigma_v^2 \dfrac{L}{a_\parallel} < 1 & \text{if} \quad ka_\parallel > 2\pi, \end{cases} \quad (5)$$

which is deduced from equation 5.46 in Shapiro and Hubral (1999) and defines the regime of weak scattering.

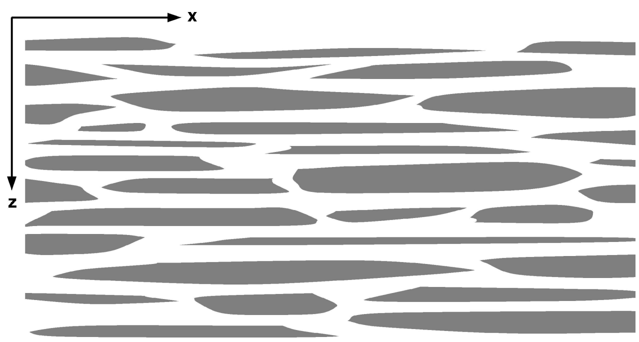

Figure 1. Schematic plot of a layered structure structure with finite lateral extent. The correlation scale in the x-direction, a_x, is much larger than that in the z-direction, a_z. Thus, the ratio of spatial anisotropy $\gamma = a_z/a_x$ is a small parameter. In this paper, we consider the case of normally incident waves that mainly propagate in the z-direction.

SCATTERING ATTENUATION CAUSED BY RANDOM DIFFRACTION AND REFRACTION

Diffraction and refraction of waves by randomly distributed inhomogeneities result in a random focusing and defocussing of wave energy and, consequently, result in an increase of the amplitude fluctuations measured at varying transversal positions with increasing propagation distances (Rytov et al., 1989). Shapiro and Kneib (1993) showed that the variance of the log-amplitude fluctuations, σ_χ^2, is directly related to the coefficient of scattering attenuation of a plane wave, α, via the relation

$$\alpha = \frac{\sigma_\chi^2}{L}. \quad (6)$$

This means that the key to the description of attenuation due to random diffraction and refraction is the computation of the log-amplitude variance σ_χ^2. For 3D anisotropic random media (a propagation medium with a constant background squared slowness $p^2 = 1/c_0^2$, superimposed by a field of statistically anisotropic fluctuations), Müller and Shapiro (2003) obtained

$$\sigma_\chi^2 = k^2 L \iint_{-\infty}^{+\infty} d^2\mathbf{r}_\perp \int_0^\infty dz\, F_n(z,\mathbf{r}_\perp)\left[\cos\left(\frac{z}{2k}r_\perp^2\right) - \frac{\sin\left(r_\perp^2 L/k\right)}{r_\perp^2 L/k}\right]. \quad (7)$$

$F_n(z,\mathbf{r}_\perp)$ is the Fourier transform of the correlation function $B_n(z,\mathbf{r}_\perp)$ in the transversal coordinates $\mathbf{r}_\perp = (x, y)^T$. In order to obtain explicit results from equation 7, we have to specify the correlation function B_n. For example, in the case of a Gaussian correlation function $B_n(\mathbf{r}) = \sigma_n^2 \exp(-\frac{x^2}{a_x^2} - \frac{y^2}{a_y^2} - \frac{z^2}{a_z^2})$ with $a_x = a_y = a_\perp, a_z = a_\parallel$, and the variance of the squared slowness fluctuations, σ_n^2, we obtain

$$\sigma_\chi^2 \approx \sigma_n^2 \frac{\sqrt{\pi}}{4} \frac{a_\parallel}{a_\perp} k^3 a_\perp^3 D \left[1 - \frac{\arctan(2D)}{2D} \right] \quad (8)$$

in three dimensions and

$$\sigma_\chi^2 \approx \sigma_n^2 \frac{\sqrt{\pi}}{4} \frac{a_\parallel}{a_\perp} k^3 a_\perp^3 D \left[1 - \frac{1}{\sqrt{2}D} \sqrt{\sqrt{1+4D^2} - 1} \right] \quad (9)$$

in two dimensions, where we used the dimensionless wave parameter $D = L/(ka_\perp^2)$. Note that in the case $a_\parallel = a_\perp$ these results could be derived from the expression of σ_χ^2 in the isotropic case (Müller et al., 2002). Therefore, the ratio $\gamma = a_\parallel / a_\perp$ additionally controls the magnitude of the log-amplitude variance in anisotropic random media. It is important to note that equation 7 is restricted by

$$\sigma_n^2 \gamma \frac{L}{a_\perp} (ka_\perp)^2 < 1 \quad (10)$$

and

$$ka_\parallel \gtrsim 1, \quad (11)$$

which are the conditions of weak wavefield fluctuations and small-angle scattering, respectively. We also note that the Rytov approximation for σ_χ^2 has an increased range of validity as compared to the isotropic result if $\gamma < 1$.

A HYBRID SCATTERING Q MODEL

Typically, the wavelength of seismic waves greatly exceeds the correlation length $a_\parallel$ that is associated with the thin layering. Thus, the application of the scattering attenuation approximation based on the Rytov approximation in 3D anisotropic random media becomes impossible because the constraint $ka_\parallel \gtrsim 1$ is violated. Physically, it means that the mechanism of backscattering becomes more important. On the other hand, the exclusive application of the 1D Q^{-1} estimate results in an underestimation of scattering attenuation while the horizontal correlation length $a_\perp$ is finite. To overcome these restrictions, one should look for a combination of both attenuation estimates as follows.

The simplest way to combine the attenuation estimates of the ODA theory and the diffraction analysis (see previous two sections) is the linear combination of the attenuation coefficients or, equivalently, the Q^{-1}-estimates. Hence, a hybrid Q-factor estimate can be constructed in the form

$$Q^{-1} = Q_{1D}^{-1} + Q_{\text{diff}}^{-1}, \quad (12)$$

where Q_{1D}^{-1} denotes the estimate for 1D random media (based on equation 2) and Q_{diff}^{-1} can be computed via $Q_{\text{diff}}^{-1} = 2\alpha/k = 2\sigma_\chi^2/kL$ using equation 7. Assuming a statistically homogeneous random medium, the evaluation of scattering attenuation according to equation 12 is then based on the functional form of the correlation function involving the correlation scales ($a_\parallel, a_\perp$) and the strength of the inhomogeneities (σ_v). Q^{-1} is also dependent on propagation distance and the frequency. We note that we implicitly used the approximation $\sigma_v^2 \approx \sigma_n^2$, which introduces only a negligibly small error if $\sigma_n^2, \sigma_v^2 \ll 1$. Comparing the ranges of applicability of the ODA and 3D Rytov approximations (equations 5 and 10), we find that equation 12 is valid for any $ka_\parallel$ and only limited by the weak wavefield fluctuation assumption (equation 10). The frequency dependence of Q^{-1} is shown in Figure 2 for the case of a Gaussian correlated random medium. It is interesting to note the increasing magnitude of Q^{-1} and the appearance of an additional maximum at higher frequencies for decreasing horizontal correlation lengths. This means that the more the propagation medium deviates from a 1D random medium, the more important the effect of random diffraction and refraction. Maximal attenuation no longer occurs at frequency $\omega = c_0/a_\parallel$ (as predicted by ODA) but at $\omega \approx c_0/a_\parallel \sqrt{L/a_\perp}$. We note that the magnitude and frequency dependence of Q^{-1} strongly depend on the used correlation function. From Figure 2, we can obtain an error estimate of Q^{-1} if the effect of random diffraction and refraction is neglected: in this example maximal

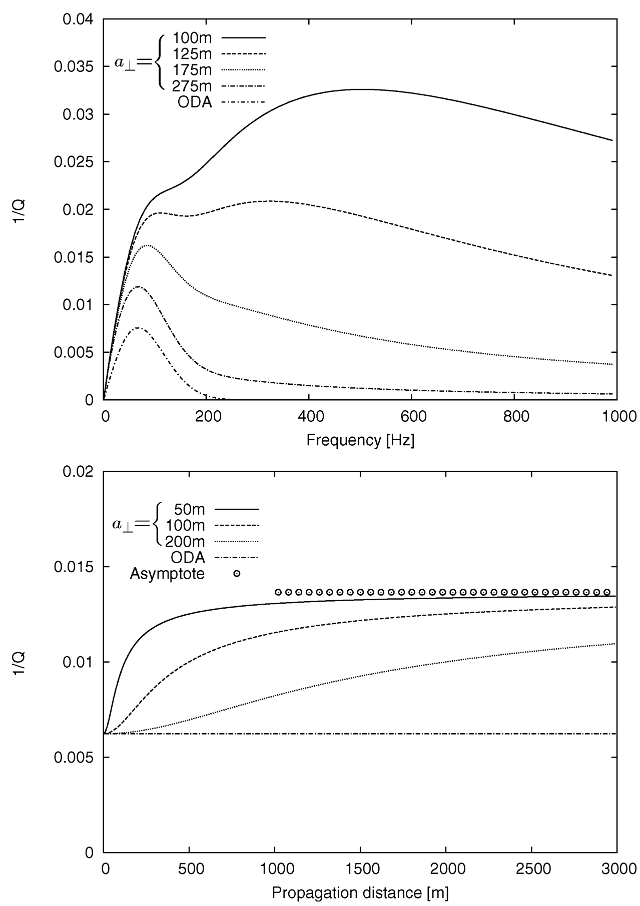

Figure 2. (Top) Frequency dependence of the reciprocal quality factor Q^{-1} according to equation 12 in an anisotropic random medium with $c_0 = 3000$ m/s, $\sigma_n = 0.1$, and $L = 4$ km. The inhomogeneities are Gaussian correlated with a vertical correlation length $a_z = 5$ m and varying horizontal correlation length $a_\perp$ indicated in the legend. The lowermost curve represents Q^{-1} according to the ODA approach or, equivalently, to equation 12 with $a_\perp = \infty$. (Bottom) Travel-distance dependence of Q^{-1} for the same medium at frequency 40 Hz and for varying $a_\perp$. For the case $a_\perp = 50$ m, the large travel-distance asymptote (equation 13) is also shown.

attenuation in the 1D random medium ($\gamma = 0$) can be observed at frequency $f \approx 75Hz$. But even for $\gamma = 0.018$ ($a_\perp = 275$ m is 55 times larger than $a_\parallel$), the overall attenuation is increased by 60%. This demonstrates that neglecting the contribution of diffraction results in a severe underestimation of the total amount of scattering attenuation.

The dependence of Q^{-1} on the propagation distance is also shown in Figure 2 (bottom). We note that the dependence of scattering attenuation on the propagation distance is a signature of 3D inhomogeneous media and is due to the nonlinear accumulation of the wavefield fluctuations with increasing propagation distance (see equations 8 and 9). It can be observed that the larger γ the stronger the L-dependence of Q^{-1} if L is small. On the other hand, for large propagation distances, Q^{-1} becomes independent of L for any value of γ.

Explicit analytical formulas for Q^{-1} based on equation 12 can be rather involved for certain correlation functions of interest. However, in some limiting cases, simpler results can be obtained. To do that, we consider the case of large propagation distances (where the value of L is still within the limitations of the weak-scattering approximation). Comparing formula 8 with the result for isotropic random media, ${}^{\mathrm{iso}}\sigma_\chi^2$, we obtain the simple formula $\sigma_\chi^2 = \gamma \cdot {}^{\mathrm{iso}}\sigma_\chi^2$ (see also Müller and Shapiro, 2003). That is, the log-amplitude variance in anisotropic random media can be obtained by multiplying the log-amplitude variance in isotropic random media with the parameter γ. It is well-known (Rytov et al., 1989) that, for large L, the log-amplitude variance is approximately given by ${}^{\mathrm{iso}}\sigma_\chi^2 \approx 2\pi k^2 L \int_0^\infty d\kappa \kappa \Phi^{3D}(\kappa)$, where Φ^{3D} denotes the fluctuation spectrum that is the 3D Fourier transform of the correlation function. It is also known that, in isotropic random media, the fluctuation spectra in one dimension and three dimensions are related through (Ishimaru, 1978) $\Phi^{3D}(\kappa) = -\frac{1}{2\pi\kappa}\frac{d}{d\kappa}\Phi^{1D}(\kappa)$, where Φ^{1D} denotes the 1D fluctuation spectrum. Using the latter relation and the fact the attenuation coefficient of the ODA theory can be expressed as $\alpha^{1D} = 0.5\Phi^{1D}(2k)$, we finally obtain the simple formula for large travel distances:

$$Q^{-1} \simeq k\left[2\gamma \Phi_{a_\perp}^{1D}(0) + \Phi_{a_\parallel}^{1D}(2k)\right], \tag{13}$$

where the subscript of Φ^{1D} denotes the correlation length that has to be used in conjunction with the respective fluctuation spectrum. In Figure 2, we plot equation 13 as a function of propagation distance for Gaussian correlated random media ($\Phi_{a_\parallel}^{1D}(\kappa) = \sqrt{\pi}\sigma_n^2 a_\parallel \exp[-\kappa^2 a_\parallel^2/4]$). The asymptotic coincidence with Q^{-1} based on equation 12 can be observed. Equation 13 can be easily interpreted: Scattering attenuation due to random diffraction and refraction is described by the first term, and only the spatial wavenumber $\kappa = 0$ yields a contribution. The second term in equation 13 refers to scattering attenuation in the ODA approximation, where only the spatial wavenumber $\kappa = 2k$ contributes.

In order to numerically validate the proposed Q^{-1} model, we performed finite-difference simulations of seismic wave propagation in 2D anisotropic random media. An initially plane wave (a Ricker wavelet with dominant frequency of 45 Hz) propagating in the homogeneous background medium impinges on a realization of the random medium. The wavefield is recorded at geophone lines that are placed perpendicular to the direction of wave propagation. These numerical experiments are described in detail in Müller and Shapiro (2003). The evaluation of Q estimates is always associated with numerical instabilities. A quantity that is more robustly estimated is the log-amplitude variance, which is connected to the attenuation coefficient via equation 6 and can be understood as the cumulative attenuation. Figure 3 displays the numerically determined log-amplitude variance as a function of propagation distance (squares) for an experiment with $a_\parallel = 11.25$ m, $a_\perp = 135$ m ($\gamma = 1/12$). The σ_χ^2 values according to the ODA theory (the straight dashed line) clearly underestimates the numerically determined values for σ_χ^2. Also the log-amplitude variance calculated with the 2D Rytov approximation (equation 9) does not agree with the numerical result of σ_χ^2. However, the sum of both approximations fits the numerical values quite well (solid curve), indicating that the hybrid Q^{-1} model is applicable in this case.

DISCUSSION AND CONCLUSIONS

It has been shown that the importance of diffraction effects scales with the parameter $\gamma = a_\parallel / a_\perp$. In situations where the correlation length associated with thin layering becomes relatively small (say $a_\parallel < 1m$) so that γ becomes small too, the contribution of random diffraction to Q^{-1} can be practically neglected. However, analyzing the correlation properties of well-log data shows that there is often an additional vertical correlation scale involved which exceeds that of the thin layering and corresponds to larger but still subwavelength inhomogeneities (Goff and Holliger, 1999). That means in real geostructures an effective value of γ can be sufficiently large, and the effect of random diffraction can significantly increase the amount of scattering attenuation.

The hybrid Q^{-1}-model is able to describe scattering attenuation of seismic primaries in thinly layered structures with finite lateral extent and is an extension to the generalized ODA theory of Shapiro and Hubral (1999). In such quasi-1D structures, seismic scattering attenuation is not only caused by backscattering from thin layers but also due to diffractions and

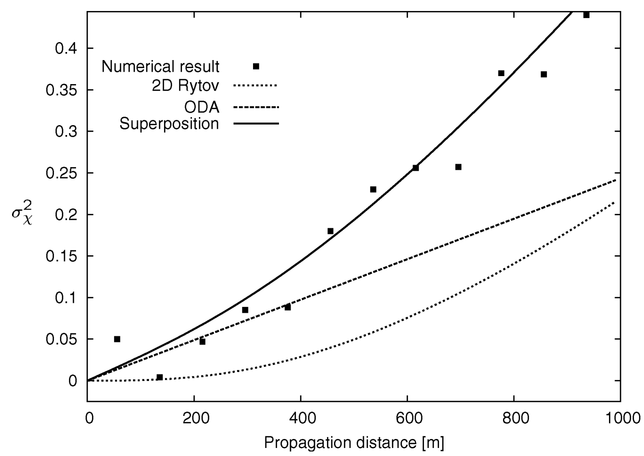

Figure 3. The log-amplitude variance, which is a measure of the plane-wave cumulative attenuation, as a function of travel distance. For the numerical experiment, we used $c_0 = 3000$ m/s, $\sigma_n = 0.1$, $a_\perp = 135$ m, $a_\parallel = 11.25$ m, and a dominant frequency of 45 Hz. Neither the ODA approach nor the 2D Rytov approach approximates the numerically obtained log-amplitude variances (squares). However, the superposition of both approximations (solid line) yields a reasonable agreement with the numerical experiment.

refractions from randomly distributed inhomogeneities. Since the generalized ODA approach handles conversion scattering, the presented approximations can be also used to model the attenuation of P- and S-waves in elastic random media. We restricted this analysis to wave propagation perpendicular to the large characteristic length scale of the inhomogeneities, which corresponds to the zero angle-of-incidence case in the layered media limit. For arbitrary angles of incidence, we expect (1) that the contribution of backscattering to the attenuation decreases with increasing angle of incidence (in agreement with the ODA theory) and (2) that the amplitude fluctuations increase and therefore attenuation due to diffraction increases (amplitude fluctuations are maximum for waves propagating along the long characteristic length scale of the inhomogeneities). A full analysis of this problem is a subject of future research.

ACKNOWLEDGMENTS

This work was kindly supported by the Deutsche Forschungsgemeinschaft (contract MU 1725/1-1) and the sponsors of the Wave Inversion Technology Consortium (WIT). The numerical modeling was performed on a parallel computer (Cray T3E) at ZIB Berlin. Comments and suggestions of the two reviewers significantly improved the quality of the manuscript.

REFERENCES

Burridge, R., M. V. de Hoop, K. Hsu, L. Le, and A. Norris, 1993, Waves in stratified viscoelastic media with microstructure: Journal of the Acoustic Society of America, **94**, 2884–2894.

Goff, J. A., and K. Holliger, 1999, Nature and origin of upper crustal seismic velocity fluctuations and associated scaling properties: Combined stochastic analyses of KTB velocity logs and lithology logs: Journal of Geophysical Research, **104**, 13169–13182.

Hackert, C. L., and J. O. Parra, 2003, Estimating scattering attenuation from vugs or karsts: Geophysics, **68**, 1182–1188.

Imhof, M. G., and W. C. Kempner, 2003, Seismic heterogeneity cubes and their equiprobable simulations: Journal of Seismic Exploration, **12**, 1–16.

Imhof, M. G., and M. N. Toksöz, 2000, Estimation of stratigraphic heterogeneity using seismic reflection data: Journal of Seismic Exploration, **9**, 41–60.

Iooss, B., D. Gaerets, T. Mukerji, Y. Samuelides, M. Touati, and A. Galli, 2003, Inferring the statistical distribution of velocity heterogeneities by statistical traveltime tomography: Geophysics, **68**, 1714–1730.

Ishimaru, A., 1978, Wave propagation and scattering in random media: Academic Press.

Kravtsov, Y. A., T. M. Müller, S. A. Shapiro, and S. Buske, 2003, Statistical properties of reflection traveltimes in 3-D randomly inhomogeneous and anisomeric media: Geophysical Journal International, **154**, 841–851.

Lerche, I., 1986, Stratigraphic filter theory: Combined effects of parallel bedding and random inhomogeneities: Geophysics, **51**, 1531–1537.

Malme, T. N., M. Landrø, and R. Mittet, 2003, Effects of overburden distortions on 3D seismic data—A sensitivity study: 65th Annual Meeting, European Association of Exploration Geophysicists, Expanded Abstracts, C42.

Müller, T. M., and S. A. Shapiro, 2003, Amplitude fluctuations due to diffraction and refraction in anisotropic random media: Implications for seismic scattering attenuation estimates: Geophical Journal International, **155**, 139–148.

Müller, T. M., S. A. Shapiro, and C. M. A. Sick, 2002, Most probable ballistic waves in random media: A weak-fluctuation approximation and numerical results: Waves Random Media, **12**, 223–246.

O'Doherty, R. F., and N. A. Anstey, 1971, Reflections on amplitudes: Geophysical Prospecting, **19**, 430–458.

Rytov, S. M., Y. A. Kravtsov, and V. I. Tatarskii, 1989, Wave propagation through random media: Springer Verlag.

Shapiro, S. A., and P. Hubral, 1999, Elastic waves in random media: Springer.

Shapiro, S. A., and G. Kneib, 1993, Seismic attenuation by scattering: Theory and numerical results: Geophysical Journal International, **114**, 373–391.